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A COMPRATIVE STUDY OF THREE METAHEURISTICS FOR OPTIMUM DESIGN OF TRUSSES

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ABSTRACT

In the present study, the computational performance of the particle swarm optimization (PSO) harmony search (HS) and firefly algorithm (FA), as popular metaheuristics, is investigated for size and shape optimization of truss structures. The PSO was inspired by the social behavior of organisms such as bird flocking. The HS imitates the musical performance process which takes place when a musician searches for a better state of harmony, while the FA was based on the idealized behavior of the flashing characteristics of natural fireflies. These algorithms were inspired from different natural sources and their convergence behavior is focused in this paper. Several benchmark size and shape optimization problems of truss structures are solved using PSO, HS and FA and the results are compared. The numerical results demonstrate the superiority of FA to HS and PSO.

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KEY WORDS: optimum design; truss structure; metaheuristic; particle swarm optimization; harmont search; firefly algorithm

1. INTRODUCTION

As the material cost is one of the major factors in the construction of a structure, it is preferable to reduce it by minimizing the weight of the structural system. All of the methods used for minimizing the weight intend to achieve an optimum design having a set of design variables under certain design criteria. Optimum design of structures is usually achieved by selecting the design variables such that an objective function is minimized while all of the design constraints are satisfied [1]. Since truss structures are widely used for structural

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applications, optimum design of this type of structures has a great importance. Generally, in design optimization of truss structures, the objective is to find the best feasible structure with a minimum weight [2]. The great development of structural optimization took place in the early 1960s and from then on, various general approaches have been developed and adopted to structural optimization.

The main idea behind designing the metaheuristic algorithms is to tackle complex optimization problems where other optimization methods have failed to be effective. These methods are now recognized as one of the most practical approaches for solving many real-world problems. The practical advantage of metaheuristics lies in both their effectiveness and general applicability. In fact, metaheuristics are the most general kinds of stochastic optimization algorithms, and are applied to a very wide range of problems. In recent years, metaheuristic algorithms are emerged as the global search approaches which are responsible to tackle the complex optimization problems. By taking a glance at literature it can be observed that the most popular metaheuristics are genetic algorithm (GA) [3], ant colony optimization (ACO) [4], particle swarm optimization (PSO) [5], harmony search (HS) [6] and firefly algorithm (FA) [7]. Lamberti and Pappalettere [8] achieved a comprehensive review of the metaheuristics and their applications in the field of structural optimization.

The FA is an optimization technique, developed recently by Yang [7] at Cambridge University. It is inspired by social behavior of fireflies and the phenomenon of bioluminescent communication. The superiority of FA to PSO and GA was demonstrated using various test functions [7, 9]. Gandomi *et al.* [10] utilized the FA to solve benchmark mixed-variable and non-convex optimization problems. In [2] FA was employed to achieve shape optimization of structures. In the present study, PSO, HS and FA algorithms are employed to achieve size and shape optimization of truss structures and the results are compared.

2. PROBLEM FORMULATION

In structural optimization problems the aim is usually to minimize an objective function, under some behavioural constraints. This problem may be expressed as follows:

$$\begin{array}{c} \text{Minimize} \quad f(X) \\ \text{Subject to} \quad g_i(X) \le 0 \ , \quad i = 1, \dots, m \\ X_j^l \le X_j \le X_j^u \ , \quad j = 1, \dots, n \end{array} \tag{1}$$

where, X is the vector of design variables; $f(X)$ is the objective function to be minimized; $g_i(X)$ is the ith behavioral constraints; X_j^l and X_j^u are the lower and the upper bounds on a typical design variable X_j.

In this study, to transform the constrained structural optimization problem into an unconstrained one the exterior penalty function method (EPFM) is employed. The above mentioned constrained optimization problem can be converted into an unconstrained problem by constructing a function of the following form:

$$\Phi(X, r_p) = f(X) + r_p \sum_{i=1}^{m} [\max\{0, g_i(X)\}]^2 \tag{2}$$

where, Φ and r_p are the pseudo objective function, and positive penalty parameter, respectively.

3. METAHEURISTICS

Stochastic optimization is the general class of techniques which employ some degree of randomness to find optimal solutions to hard problems. In order to comprehensively explore a larger fraction of the design space, stochastic search techniques reveal their promising abilities in comparison with gradient-based optimization methods. Metaheuristics are the most general of these kinds of algorithms, and are applied to a very wide range of problems. Metaheuristics solve instances of problems that are believed to be hard in general, by exploring the usually large solution search space of these instances. These algorithms achieve this by reducing the effective size of the space and by exploring that space efficiently. Metaheuristics serve three main purposes: solving problems faster, solving large problems, and obtaining robust algorithms. Moreover, they are simple to design and implement, and are very flexible. The past 20 years have witnessed the development of numerous metaheuristics in various communities that sit at the intersection of several fields, including artificial intelligence, computational intelligence and soft computing. Most of the metaheuristics mimic natural metaphors to solve complex optimization problems such as evolution of species, annealing process, ant colony, particle swarm, immune system, bee colony, and wasp swarm. Metaheuristics are more and more popular in different research areas. In the present paper, PSO, HS and FA metaheuristic algorithms are considered and their essential concepts are briefly described below.

3.1. Particle swarm optimization

The PSO is based on the social behaviour of animals such as fish schooling, insect swarming and bird flocking. The PSO has been proposed to simulate the graceful motion of bird swarms as a part of a socio-cognitive study.

The PSO involves a number of particles, which are randomly initialized in the search space. These particles are referred to as swarm. Each particle of the swarm represents a potential solution of the optimization problem. The particles fly through the search space and their positions are updated based on the best positions of individual particles and the best of the swarm in each iteration. The objective function is evaluated for each particle at each grid point and the fitness values of particles are obtained to determine the best position in the search space [11]. In iteration t, the swarm is updated using the following equations:

$$V_i^{t+1} = \omega^t V_i^t + c_1 r_1 (P_i^t - X_i^t) + c_2 r_2 (P_g^t - X_i^t) \tag{3}$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \tag{4}$$

where X_i and V_i represent the current position and the velocity of the ith particle, respectively; P_i is the best previous position of the ith particle (*pbest*) and P_g is the best global position among all the particles in the swarm (*gbest*); r_1 and r_2 are two uniform random sequences generated from interval [0, 1]; c_1 and c_2 are the cognitive and social scaling parameters, respectively. The inertia weight used to discount the previous velocity of particle preserved is expressed by ω.

Due to the importance of ω in achieving efficient search behavior the updating criterion can be taken as follows:

$$\omega = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{t_{max}} . t \tag{5}$$

where ω_{max} and $\omega_{\min}$ are the maximum and minimum values of ω, respectively. Also, t_{max}, and t are the numbers of maximum iterations and present iteration, respectively.

The main steps in the standard PSO may be stated as follows:

(a) Initialize a swarm of particles by randomly selecting particles from design space.
(b) Update particles' velocity.
(c) Update particles' position.
(d) Analyze the swarm to evaluate the fitness value of each particle.
(e) Update the *pbest* and the *gbest*.
(f) Repeat (b) to (f) until a termination condition is satisfied.

In the field of structural engineering many successful application of PSO have been reported in literature. A number of such applications can be found in [12-14].

3.2. Harmony search

The HS is based on the musical performance process that achieves when a musician searches for a better state of harmony. Jazz improvisation seeks musically pleasing harmony similar to the optimum design process which seeks optimum solutions. The pitch of each musical instrument determines the aesthetic quality, just as the objective function value is determined by the set of values assigned to each design variable. In the process of musical production a musician selects and brings together number of different notes from the whole notes and then plays these with a musical instrument to find out whether it gives a pleasing harmony. The musician then tunes some of these notes to achieve a better harmony. Similarly it is then checked whether this candidate solution improves the objective function or not. This candidate solution is then checked to find out whether it satisfies the objective function or not, similar to the process of finding out whether euphonic music is obtained or not. The HS consists of five basic steps which can be summarized as follows:

A possible range for each design variable is specified. The number of solution vectors in harmony memory (HM) or size of HM (HMS), the harmony considering rate (HMCR), the pitch adjusting rate (PAR) and the maximum number of searches are also specified.

An initial harmony memory matrix (HM) is produce. The HM is a matrix in which each row contains the values of design variables which are randomly selected from the design space. If the optimization problem includes n design variables the HM has the following form:

$$\text{HM} = \begin{bmatrix} X^1 \\ X^2 \\ \vdots \\ X^j \\ \vdots \\ X^{HMS-1} \\ X^{HMS} \end{bmatrix} = \begin{bmatrix} x_1^1 & x_2^1 & \dots\ x_i^1\ \dots & x_{n-1}^1 & x_n^1 \\ x_1^2 & x_2^2 & \dots\ x_i^2\ \dots & x_{n-1}^2 & x_n^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^j & x_2^j & \dots\ x_i^j\ \dots & x_{n-1}^j & x_n^j \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{HMS-1} & x_2^{HMS-1} & \dots\ x_i^{HMS-1}\ \dots & x_{n-1}^{HMS-1} & x_n^{HMS-1} \\ x_1^{HMS} & x_2^{HMS} & \dots\ x_i^{HMS}\ \dots & x_{n-1}^{HMS} & x_n^{HMS} \end{bmatrix} \tag{6}$$

where x_i^j is the value of the ith design variable in the jth solution vector.

To improvise new HM, a new harmony vector is generated. Thus the new value of the ith design variable can be chosen from the possible range of ith column of the HM with the probability of HMCR or from the entire possible range of values with the probability of 1-HMCR as follows:

$$x_i^{\text{new}} = \begin{cases} x_i^j \in \{x_i^1, x_i^2, \dots, x_i^{HMS}\}^{\text{T}} & \text{with the probability of HMCR} \\ x_i \in \varDelta_i & \text{with the probability of } (1 - \text{HMCR}) \end{cases} \tag{7}$$

where $\varDelta_i$ is the set of the potential range of values for ith design variable. The HMCR is the probability of choosing one value from the significant values stored in the HM, and (1-HMCR) is the probability of randomly choosing one practical value not limited to those stored in the HM.

As the third operation components of the new harmony vector, is examined to determine whether it should be pitch-adjusted. Pitch adjusting is performed only after a value has been chosen from the HM as follows:

$$\text{pitch adjusment of } x_i^{\text{new}}\ ?\ \begin{cases} \text{Yes} & \text{with the probability of PAR} \\ \text{No} & \text{with the probability of } (1 - \text{PAR}) \end{cases} \tag{8}$$

If the pitch-adjustment decision for x_i^{new} is "Yes", then a neighboring value with the probability of PAR%×HMCR is taken for it as follows:

$$x_i^{new} \leftarrow \begin{cases} x_i^{\text{new}} \pm u(-1,+1) \times bw & \text{with the probability of PAR} \times \text{HMCR} \\ x_i^{\text{new}} & \text{with the probability of PAR} \times (1 - \text{HMCR}) \end{cases} \tag{9}$$

where u(-1,+1) is a uniform distribution between -1 and +1; also bw is an arbitrary distance bandwidth for the continuous design variables.

This operation increases the chance of reaching the global optimum.

After selecting the new values for each design variables the objective function value is calculated for the new harmony vector. In this case x_i^{new} is analyzed using FEM and its objective function value is determined. If x_i^{new} is better than the worst vector in the HM, the new harmony is substituted by the existing worst harmony. The HM is then sorted in descending order by the objective function value.

The optimization process of HS is repeated by continuing improvising new harmonies until a termination criterion is satisfied.

However HS is a relatively new metaheuristic algorithm, its efficiency for solving complex structural optimization problems has been demonstrated in many researches. For example, one can see [15-17].

3.3. Firefly algorithm

The FA is a new metaheuristic optimization algorithm inspired by the flashing behavior of fireflies. FA is a population-based algorithm, which may share many similarities with PSO. Fireflies communicate, search for pray and find mates using bioluminescence with varied flashing patterns [10]. In order to develop the firefly algorithm, natural flashing characteristics of fireflies have been idealized using the following three rules [7]:

a. All of the fireflies are unisex; therefore, one firefly will be attracted to other fireflies regardless of their sex.
b. Attractiveness of each firefly is proportional to its brightness, thus for any two flashing fireflies, the less bright firefly will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly.
c. The brightness of a firefly is determined according to the nature of the objective function.

The attractiveness of a firefly is determined by its brightness or light intensity which is obtained from the objective function of the optimization problem. However, the attractiveness β, which is related to the judgment of the beholder, varies with the distance between two fireflies. The attractiveness β can be defined by [18]:

$$\beta = \beta_0 e^{-\gamma . r^2} \tag{10}$$

where r is the distance of two fireflies, β_0 is the attractiveness at $r = 0$, and γ is the light absorption coefficient.

The distance between two fireflies i and j at X_i and X_j respectively, is determined using the following equation:

$$r_{ij} = \left\| X_i - X_j \right\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \tag{11}$$

where $x_{i,k}$ is the k-th parameter of the spatial coordinate x_i of the i-th firefly.

In the firefly algorithm, the movement of a firefly i towards a more attractive (brighter) firefly j is determined by the following equation [18]:

$$X_i = X_i + \beta_0 e^{-\gamma . r_{ij}^2} (X_j - X_i) + \alpha(\text{rand} - 0.5) \tag{12}$$

where the second term is related to the attraction, while the third term is randomization with α being the randomization parameter. Also rand is a random number generator uniformly distributed in [0, 1].

In this paper, a slight modification is accomplished on the third term of Eq. (12). In this case, α is changed dynamically according to the following equation:

$$\alpha = \alpha_{max} - \frac{\alpha_{max} - \alpha_{min}}{t_{max}} . t \tag{13}$$

where α_{max}=1 and α_{min}=0.2. Also, t_{max} and t are the numbers of maximum iterations and present iteration, respectively. It should be noted that, various values are examined for α_{max} and α_{min} and the best results are obtained in the case of reported values.

We observed that by using Eq. (13) the convergence behavior of FA was improved. Therefore, in this paper the following updating equation is used:

$$X_i = X_i + \beta_0 e^{-\gamma . r_{ij}^2} (X_j - X_i) + (\alpha_{max} - \frac{\alpha_{max} - \alpha_{min}}{t_{max}} . t)(\text{rand} - 0.5) \tag{14}$$

As FA is a very new metaheuristic algorithm, there are a few papers on application of FA to structural optimization [2, 10, 19].

4. NUMERICAL RESULTS

In the present paper, two size optimization examples, including a 10-bar planar truss and a 72-bar space truss, and two shape optimization examples including a 15-bar planar truss and a 25-bar space truss structures are optimized by PSO, HS and FA to investigate the computational performance of the algorithms. The selected test examples have been optimized by other researchers and the results obtained in this paper are compared with their results. For all examples, the size of population is considered to be 20 and the maximum number of iterations is 500. All of the required computer programs are coded in MATLAB platform. Also, to achieve optimization processes a personal Pentium IV 3000MHz is employed.

4.1. Example 1: size optimization of 10-bar planar truss

The 10-bar truss structure is shown in Figure 1. The loads applied to the truss are $P_1 = 10^5$ lbs, $P_2 = 0$. The material density is 0.1 lb/in^3 and the modulus of elasticity is 10,000 ksi. The members are subjected to stress limitations of ±25 ksi. All nodes in both directions are subjected to displacement limitations of ±2.0.

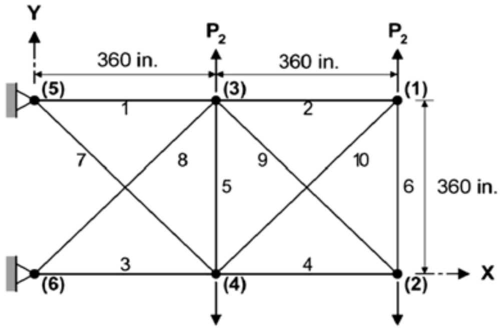

Figure 1. 10-bar truss structure

There are 10 design variables and two design cases in this example are considered. In case 1: the discrete variables are selected from the set D = {1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50} (in^2);

In case 2: the discrete variables are selected from the set D = {0.1, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5, 16.0, 16.5, 17.0, 17.5, 18.0, 18.5, 19.0, 19.5, 20.0, 20.5, 21.0, 21.5, 22.0, 22.5, 23.0, 23.5, 24.0, 24.5, 25.0, 25.5, 26.0, 26.5, 27.0, 27.5, 28.0, 28.5, 29.0, 29.5, 30.0, 30.5, 31.0, 31.5} (in^2).

The comparison of optimal designs for the 10-bar planar truss structure under two load cases are given in Tables 1 and 2, respectively. In these tables the maximum nodal deflection and element stress are expressed by $|d_{max}|$ and $|\sigma_{max}|$, respectively. To investigate the computational performance of the PSO, HS and FA metaheuristics in this example, 20 independent runs are implemented and statistical results of these runs for both cases are given in Table 3.

Table 1. Comparison of optimal designs for the 10-bar truss structure (Case 1)

Design variables No.	**Li et al. [20]**		**Gholizadeh [1]**	**Present study**		
	PSOPC	**HPSO**	**IPSO**	**PSO**	**HS**	**FA**
1	30.00	30.00	33.50	33.50	30.00	33.50
2	1.80	1.62	1.62	1.62	1.8	1.62
3	26.50	22.90	22.90	22.90	22.90	22.90
4	15.50	13.50	14.20	14.20	13.50	14.20
5	1.62	1.62	1.62	1.62	1.62	1.62
6	1.62	1.62	1.62	1.62	1.62	1.62
7	11.50	7.97	7.97	7.97	11.50	7.97
8	18.80	26.50	22.90	22.90	22.00	22.90
9	22.00	22.00	22.00	22.00	22.90	22.00
10	3.09	1.80	1.62	1.62	1.99	1.62
Weight (lb)	5593.44	5531.98	5490.74	5490.74	5544.6	5490.74
Number of analyses	50000	50000	4512	10000	10000	10000
$\|d_{max}\|$	1.995	1.999	1.999	1.999	1.999	1.999
$\|\sigma_{max}\|$	11.482	14871	14197	14.197	10.9833	14.197

Table 2. Comparison of optimal designs for the 10-bar truss structure (Case 2)

Design variables No.	Li et al. [20]		Gholizadeh [1]	Present study		
	PSOPC	**HPSO**	**IPSO**	**PSO**	**HS**	**FA**
1	25.5	31.5	29.5	29.5	28.5	31.0
2	0.1	0.1	0.1	0.1	0.1	0.1
3	23.5	24.5	24.0	24.0	21.5	23.0
4	18.5	15.5	15.0	15.0	14.0	15.0
5	0.1	0.1	0.1	0.1	0.1	0.1
6	0.5	0.5	0.5	0.5	0.5	0.5
7	7.5	7.5	7.5	7.5	8.0	7.5
8	21.5	20.5	21.5	21.5	23.0	21.0
9	23.5	20.5	21.5	21.5	23.5	21.5
10	0.1	0.1	0.1	0.1	0.1	0.1
Weight (lb)	5133.16	5073.51	5067.33	5067.33	5104.9	5060.6
Number of analyses	50000	50000	5600	10000	10000	10000
$\|d_{\max}\|$	2.000	1.999	1.999	1.999	1.996	2.000
$\|\sigma_{\max}\|$	24.714	24.394	24.604	24.604	23.941	24.947

Table 3. Results of 20 runs of PSO, HS and FA for the 10-bar truss

Evaluation metrics	Case 1			Case 2		
	PSO	**HS**	**FA**	**PSO**	**HS**	**FA**
Best weight	5490.73	5544.60	5490.73	5067.33	5104.90	5060.60
Worst weight	5676.81	5898.71	5597.73	5170.20	5471.60	5084.10
Average weight	5591.71	5778.72	5531.30	5107.40	5283.20	5064.90
Standard deviation	72.20	90.91	37.49	40.41	98.10	5.40

It can be observed from the tables that the computational performance of FA is better than that of the PSO and HS. Also the results demonstrate the superiority of PSO to HS. The results also reveal that, the IPSO algorithm proposed by Gholizadeh [1] is better than the slightly modified FA in terms of required structural analyses. This implies that the computational performance of FA may be modified by hybridizing it with other metaheuristics.

4.2. Example 2: size optimization of 72-bar space truss

The 72-bar spatial truss structure is shown in Figure 2. The material density is 0.1 lb/in^3 and the modulus of elasticity is 10,000 ksi. The members are subjected to stress limitations of ±25 ksi. The uppermost nodes are subjected to displacement limitations of ±0.25 in both the x and y directions. Two load cases are listed in Table 4. There are 72 members, which are divided into 16 groups, as follows:

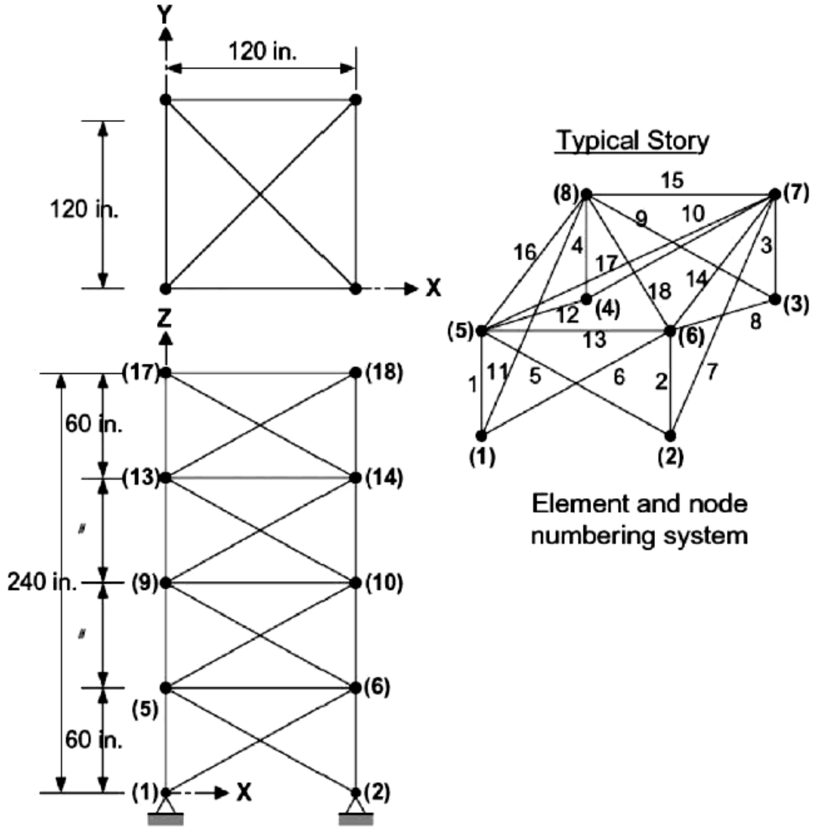

Figure 2. 72-bar space truss structure

(1) A1–A4, (2) A5–A12, (3) A13–A16, (4) A17–A18, (5) A19–A22, (6) A23–A30 (7) A31–A34, (8) A35–A36, (9) A37–A40, (10) A41–A48, (11) A49–A52, (12) A53–A54, (13) A55–A58, (14) A59–A66 (15) A67–A70, (16) A71–A72.

Two optimization cases are considered as follows:

Case 1: The discrete variables are selected from the set D = {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8,0.9,1.0, 1.1,1.2, 1.3,1.4, 1.5,1.6, 1.7, 1.8, 1.9, 2.0, 2.1,2.2, 2.3,2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1,3.2} (in^2).
Case 2: The discrete variables are selected from Table 5.

Optimal design results for the 72-bar space truss structure, for two mentioned cases, are compared in Tables 6 and 7, respectively. In these tables the maximum nodal deflection and element stress are presented by $|d_{max}|$ and $|\sigma_{max}|$, respectively.

Table 4. The load cases for the 72-bar space truss structure

Nodes	Load Case 1			Load Case 2		
	P_x (kips)	P_y (kips)	P_z (kips)	P_x (kips)	P_y (kips)	P_z (kips)
17	5.0	5.0	–5.0	0.0	0.0	–5.0
18	0.0	0.0	0.0	0.0	0.0	–5.0
19	0.0	0.0	0.0	0.0	0.0	–5.0
20	0.0	0.0	0.0	0.0	0.0	–5.0

Table 5. The available cross-sectional areas for second example

No.	Cross-sectional area (in^2)	No.	Cross-sectional area (in^2)
1	0.111	33	3.840
2	0.141	34	3.870
3	0.196	35	3.880
4	0.250	36	4.180
5	0.307	37	4.220
6	0.391	38	4.490
7	0.442	39	4.590
8	0.563	40	4.800
9	0.602	41	4.970
10	0.766	42	5.120
11	0.785	43	5.740
12	0.994	44	7.220
13	1.000	45	7.970
14	1.228	46	8.530
15	1.266	47	9.300
16	1.457	48	10.850
17	1.563	49	11.500
18	1.620	50	13.500
19	1.800	51	13.900
20	1.990	52	14.200
21	2.130	53	15.500
22	2.380	54	16.000
23	2.620	55	16.900
24	2.630	56	18.800
25	2.880	57	19.900
26	2.930	58	22.000
27	3.090	59	22.900
28	1.130	60	24.500
29	3.380	61	26.500
30	3.470	62	28.000
31	3.550	63	30.000
32	3.630	64	33.500

Table 6. Comparison of optimal designs for the 72-bar truss structure (case 1)

Design variables No.	Li et al. [7]		Gholizadeh [1]	Present study		
	PSOPC	HPSO	IPSO	PSO	HS	FA
1	3.0	2.1	2.0	2.0	3.2	1.9
2	1.4	0.6	0.5	0.5	0.5	0.5
3	0.2	0.1	0.1	0.1	0.1	0.1
4	0.1	0.1	0.1	0.1	0.1	0.1
5	2.7	1.4	1.2	1.2	1.4	1.3
6	1.9	0.5	0.5	0.5	0.4	0.5
7	0.7	0.1	0.1	0.1	0.1	0.1
8	0.8	0.1	0.1	0.1	0.1	0.1
9	1.4	0.5	0.6	0.6	0.7	0.6
10	1.2	0.5	0.5	0.5	0.5	0.5
11	0.8	0.1	0.1	0.1	0.1	0.1
12	0.1	0.1	0.1	0.1	0.1	0.1
13	0.4	0.2	0.2	0.2	0.2	0.2
14	1.9	0.5	0.6	0.6	0.5	0.6
15	0.9	0.3	0.4	0.4	0.4	0.4
16	1.3	0.7	0.6	0.6	0.7	0.6
Weight (lb)	1069.79	388.94	385.54	385.54	403.46	385.54
Number of analyses	50000	50000	4176	10000	10000	10000
$\|d_{max}\|$	0.1	0.25	0.2502	0.2502	0.2496	0.2500
$\|\sigma_{max}\|$	5.726	3.293	20.368	20.368	20.595	20.380

In this example, the computational performance of the PSO, HS and FA metaheuristics are investigated through 20 independent runs and the results of cases 1 and 2 are given in Tables 8.

The numerical results demonstrate the superiority of FA to both PSO and HS. Also it is observed that PSO is better than the HS. As well as the first example, it is also reveal that the IPSO algorithm [1] is better than the FA in terms of required structural analyses. This implies that, some computational strategies can be adopted to improve the computational performance of FA.

Table 7. Comparison of optimal designs for the 72-bar truss structure (case 2)

Design variables No.	**Li et al. [7]**		**Gholizadeh [1]**	**Present study**		
	PSOPC	**HPSO**	**IPSO**	**PSO**	**HS**	**FA**
1	4.490	4.970	1.800	1.800	1.228	1.800
2	1.457	1.228	0.563	0.563	0.994	0.563
3	0.111	0.111	0.111	0.111	0.111	0.111
4	0.111	0.111	0.111	0.111	0.111	0.111
5	2.620	2.880	1.228	1.266	1.228	1.266
6	1.130	1.457	0.442	0.442	0.563	0.442
7	0.196	0.141	0.111	0.111	0.111	0.111
8	0.111	0.111	0.111	0.111	0.111	0.111
9	1.266	1.563	0.563	0.563	0.563	0.563
10	1.457	1.228	0.563	0.563	0.563	0.563
11	0.111	0.111	0.111	0.111	0.111	0.111
12	0.111	0.196	0.111	0.111	0.111	0.111
13	0.442	0.391	0.196	0.196	0.196	0.196
14	1.457	1.457	0.563	0.563	0.442	0.563
15	1.228	0.766	0.442	0.442	0.563	0.442
16	1.457	1.563	0.602	0.602	0.543	0.602
Weight (lb)	941.82	933.09	388.56	389.45	424.88	389.45
Number of analyses	50000	50000	5968	10000	10000	10000
$\lvert d_{max} \rvert$	0.1	0.1	0.2503	0.2496	0.2497	0.2496
$\lvert \sigma_{max} \rvert$	9.491	10.272	20.709	20.713	21.187	20.713

Table 8. Results of 20 runs of PSO, HS and FA for the 72-bar truss

Evaluation metrics	**Case 1**			**Case 2**		
	PSO	**HS**	**FA**	**PSO**	**HS**	**FA**
Best weight	385.54	403.46	385.54	388.56	424.88	388.56
Worst weight	411.90	438.94	389.35	411.85	446.47	402.84
Average weight	391.31	417.22	387.20	395.06	427.62	391.12
Standard deviation	8.92	14.13	1.00	8.36	8.90	4.11

4.3. Example 3: shape optimization of 15-bar planar truss

A fifteen-bar truss is shown in Figure 3. The material density is 0.1 lb/in^3 and the modulus of elasticity is 10^4 ksi. In this example there are 23 design variables including two categories:
Sizing variables: A_i , i=1,2,…,15
Geometry variables: $x_2 = x_6$; $x_3 = x_7$; y_2; y_3; y_4; y_6; y_7; y_8.
The size variables are selected from the following set:

D = { 0.111, 0.141, 0.174, 0.220, 0.270, 0.287, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180} (in.^{2}).

Also side constraints for geometry variables are as follows:
100 in. $\le x_2 \le$ 140 in.; 220 in. $\le x_3 \le$ 260 in.; 100 in. $\le y_2 \le$ 140 in.; 100 in. $\le y_3 \le$ 140 in.;
50 in. $\le y_4 \le$ 90 in.; −20 in. $\le y_6 \le$ 20 in.; −20 in. $\le y_7 \le$ 20 in.; 20 in. $\le y_8 \le$ 60 in.;

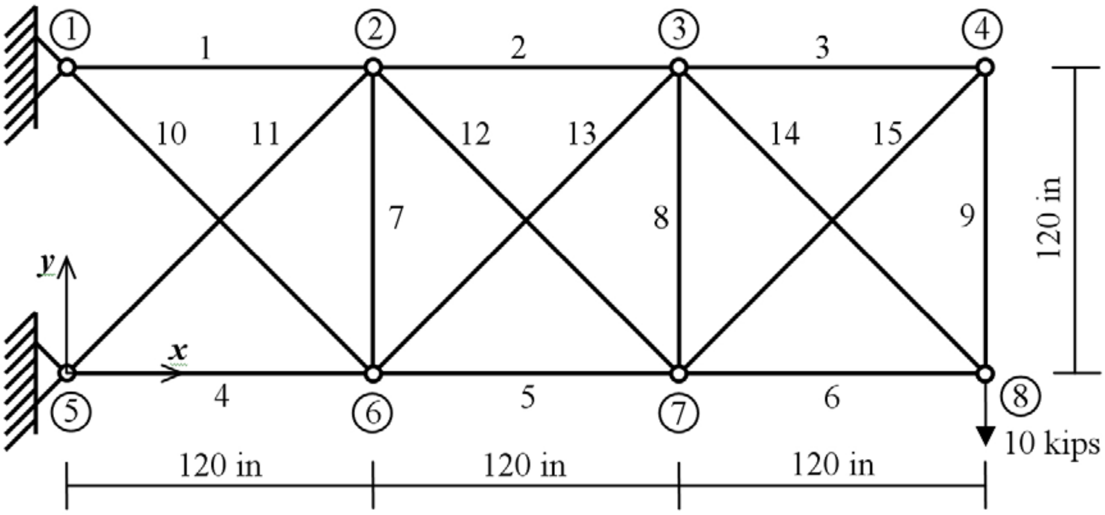

Figure 3. A fifteen-bar truss

Stress limitation for all elements is 25 ksi. In this example, optimal design results obtained are compared with those of Tang et al. [21] and Rahami et al. [22] in Table 9.

The computational performance of the PSO, HS and FA metaheuristics in this example are investigated through 20 independent runs and the results are summarized in Table 10.

The numerical results indicate that the FA converges to a solution which is better than that of those reported by the other researchers in [21, 22], however, FA requires more computational cost (10000 analyses versus 8000 ones). The results also demonstrate the superiority of FA to both PSO and HS metaheuristics. In this example also, PSO is better than the HS.

Table 9. Comparison of optimal designs for the 15-bar planar truss

Design variables	Tang et al. [21]	Rahami et al. [22]	Present study		
			PSO	HS	FFA
A_1	1.081	1.081	0.954	1.081	0.954
A_2	0.539	0.539	0.539	0.954	0.539
A_3	0.287	0.287	0.111	0.27	0.27
A_4	0.954	0.954	0.954	0.954	1.081
A_5	0.954	0.539	0.539	0.539	0.539
A_6	0.220	0.141	0.270	0.270	0.174
A_7	0.111	0.111	0.111	0.111	0.111
A_8	0.111	0.111	0.111	0.141	0.111
A_9	0.287	0.539	0.111	0.220	0.440
A_{10}	0.220	0.440	0.440	0.220	0.440
A_{11}	0.440	0.539	0.539	0.440	0.347
A_{12}	0.440	0.270	0.220	0.111	0.220
A_{13}	0.111	0.220	0.287	0.440	0.220
A_{14}	0.220	0.141	0.347	0.287	0.174
A_{15}	0.347	0.287	0.111	0.220	0.270
x_2	133.612	101.577	106.290	137.260	113.65
x_3	234.752	227.910	248.980	220.000	254.47
y_2	100.449	134.790	140.000	138.520	128.97
y_3	104.738	128.220	140.000	127.410	115.73
y_4	73.762	54.860	50.000	50.000	59.364
y_6	-10.067	-16.440	-7.8109	19.180	-12.733
y_7	-1.339	-13.300	8.1291	2.800	3.5467
y_8	50.402	54.850	52.701	38.330	59.290
Weight (lb)	79.82	76.68	77.04	80.36	73.93
Number of analyses	8000	8000	10000	10000	10000
$\|\sigma_{max}\|$	23.876	24.9992	24.998	24.994	24.998

Table 10. Results of 20 runs of PSO, HS and FA for the 15-bar truss

Evaluation metrics	PSO	HS	FA
Best weight	77.04	80.36	73.93
Worst weight	90.38	92.90	82.48
Average weight	84.60	85.24	79.85
Standard deviation	3.82	4.45	2.29

4.4. Example 4: SHAPE optimization of 25-bar space truss

A 25-bar truss is considered as shown in Figure 4. Loading data is provided in Table 11.

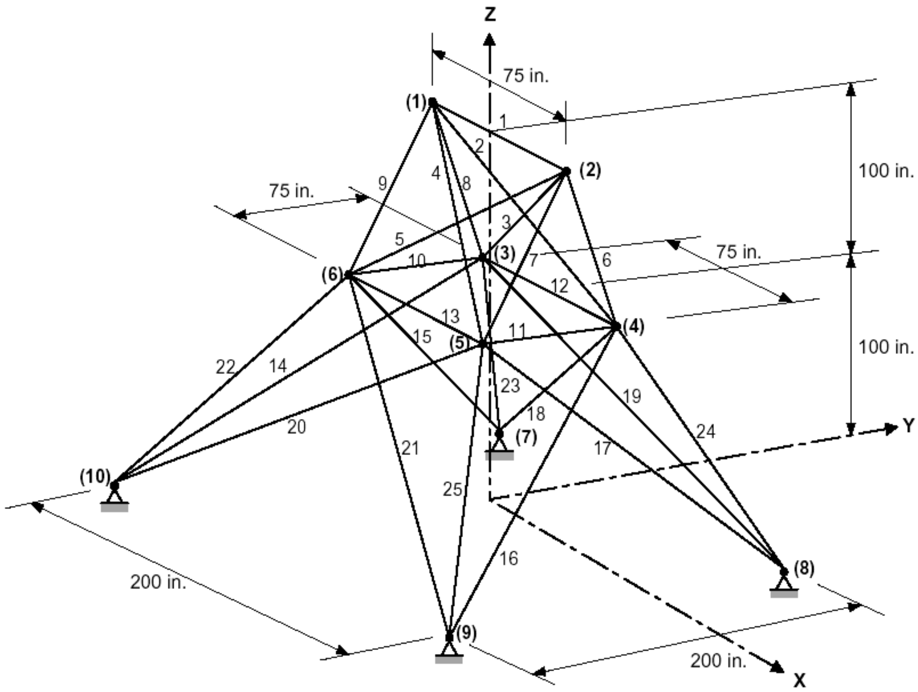

Figure 4. 25-bar space truss

Table 11. Loading data for the 25-bar truss

Node	F_x (kips)	F_y (kips)	F_z (kips)
1	1.0	-10.0	-10.0
2	0.0	-10.0	-10.0
3	0.5	0.0	0.0
6	0.6	0.0	0.0

The material density is 0.1 lb/in^3 and the modulus of elasticity is 10^4 ksi. Stress limitation fro all elements is $\pm 40\,\text{ksi}$ also displacement constraint in all directions is 0.35 in. There are 13 design variables including two categories:
Size variables: A_1; $A_2 = A_3 = A_4 = A_5$; $A_6 = A_7 = A_8 = A_9$; $A_{10} = A_{11}$; $A_{12} = A_{13}$; $A_{14} = A_{15} = A_{16} = A_{17}$; $A_{18} = A_{19} = A_{20} = A_{21}$; $A_{22} = A_{23} = A_{24} = A_{25}$
Geometry variables: $x_4 = x_5 = -x_3 = -x_6$; $x_8 = x_9 = -x_7 = -x_{10}$; $y_3 = y_4 = -y_5 = -y_6$; $y_7 = y_8 = -y_9 = -y_{10}$; $z_3 = z_4 = z_5 = z_6$
The size variables are selected from the following set:

D = {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.2, 3.4} (in.^{2}).

Also side constraints for geometry variables are as follows:
$20 \le x_4 \le 60$ in.; $40 \le x_8 \le 80$ in.; $40 \le y_4 \le 80$ in.; $100 \le y_8 \le 140$ in.; $90 \le z_4 \le 130$ in.;
In this example, optimal design results are given in Table 12.

Table 12. Comparison of optimal designs for the 25-bar space truss

Design variables	Tang et al. [21]	Rahami et al. [22]	Present study		
			PSO	**HS**	**FA**
A_1	0.1	0.1	0.1	0.1	0.1
A_2	0.1	0.1	0.1	0.1	0.1
A_3	1.0	1.1	0.9	1.0	1.0
A_4	0.1	0.1	0.1	0.1	0.1
A_5	0.1	0.1	0.1	0.1	0.1
A_6	0.2	0.1	0.1	0.1	0.1
A_7	0.2	0.2	0.1	0.1	0.1
A_8	0.7	0.8	1.0	1.0	0.9
x_4	35.47	33.048	36.749	32.950	37.401
y_4	60.37	53.5667	63.478	68.185	55.379
z_4	129.07	129.90	115.950	107.370	129.290
x_8	45.06	43.782	48.033	47.360	51.807
y_8	137.04	136.83	137.980	136.020	139.560
Weight (lb)	124.94	120.115	118.93	122.62	117.35
Number of analyses	6000	10000	10000	10000	10000
$\lvert d_{max}\rvert$	0.350	0.3500	0.3499	0.345	0.3498
$\lvert \sigma_{max}\rvert$	18.350	16.828	17.699	17.757	19.194

Similar to the previous examples, in this example the computational performance of the PSO, HS and FA metaheuristics are investigated through 20 independent runs and the results are summarized in Table 13.

Table 13. Results of 20 runs of PSO, HS and FA for the 25-bar truss

Evaluation metrics	PSO	HS	FA
Best weight	118.93	122.62	117.35
Worst weight	135.04	135.6	134.28
Average weight	127.53	128.19	125.33
Standard deviation	5.07	3.95	2.59

Compared to the results reported in [21, 22], the FA converges to a better solution. The results also demonstrate the superiority of FA to PSO and HS. It is observed in this example that the PSO performs better than the HS.

5. CONCLUSIONS

The main contribution of the present study is to investigate computational performance of three popular metaheuristics, PSO, HS and FA, for size and shape optimization of planar and space truss structures. In order to achieve this purpose, four benchmark size and shape optimization problems are tackled using the mentioned metaheuristics and the obtained results are compared to those of the other papers. Also in the case of all the examples, the results of 20 independent runs of each employed are reported. The numerical results demonstrate that the FA converges to a better solution in comparison with the PSO, HS and some other algorithms reported in the literature. Finally, it is expected that by hybridizing the FA with other metaheuristics, the computational performance of the hybrid algorithm will be improved.

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