



WEIGHT OPTIMIZATION OF TRUSS STRUCTURES USING WATER CYCLE ALGORITHM

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ABSTRACT

Water cycle algorithm (WCA) is a new metaheuristic algorithm which the fundamental concepts of WCA are derived from nature and are based on the observation of water cycle process and how rivers and streams flow to sea in the real world. In this paper, the task of sizing optimization of truss structures including discrete and continuous variables carried out using WCA, and the optimization results were compared with other well-known optimizers. The obtained statistical results show that the WCA is able to provide faster convergence rate and also manages to achieve better optimal solutions compared to other efficient optimizers.

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KEY WORDS: water cycle algorithm; truss structures, sizing optimization; metaheuristics; constraint optimization

1. INTRODUCTION

Over the last decades, various algorithms have been used for truss optimization problems which are very popular in the field of structural optimization. In general, there are three main categories in structural optimization applications: a) sizing optimization (cross-sectional areas of the members are considered as design variables (discrete and continuous) [1,2]), b) shape optimization (nodal coordinates are considered as design variables [2]) and c) topology optimization (the location of links in which connect nodes, are considered as

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design variables [3]). Recently, metaheuristic methods such as genetic algorithms (GAs) [4], particle swarm optimization (PSO) [5] and other stochastic searching methods are used to optimize the trusses.

GAs are based on the genetic process of biological organisms [6]. Over many generations, natural populations evolve according to the principles of natural selection, (i.e., survival of the fittest). Goldberg and Samtani [7], and Rajeev and Krishnamoorthy [8] have applied sizing optimization on truss structures. Krishnamoorthy et al. [9] used GAs to optimize the space truss structure within an object-oriented framework. Sivakumar et al. [10] presented an optimization technique using GA for steel lattice towers. Gero et al. [11] used GAs for the design optimization of 3D steel structures.

PSO is an evolutionary computation technique for solving global optimization problems developed by Kennedy and Eberhart [12]. Li et al. [13] developed a heuristic particle swarm optimization (HPSO) for truss structures, which was proven computationally efficient and reliable, was applied on several truss problems and the obtained results have been compared with hybrid PSO with passive congregation [14] (PSOPC) and standard particle swarm optimization (PSO).

Recently, Sadollah et al. [15] developed an optimization method named as mine blast algorithm (MBA) which the concepts are from explosion of mine bomb. The proposed MBA was examined using truss structures with discrete variables [15].

In this paper, application of a novel metaheuristic algorithm for optimizing discrete and continuous problems is conducted. The proposed method is called water cycle algorithm (WCA), and is based on the observation of water cycle process in nature [16].

Recently, the WCA was implemented for constrained and engineering benchmark problems [16]. The obtained statistical results showed that the superiority of the WCA over other optimizers in terms of convergence rate and accuracy for benchmark constrained problems.

The remaining of this paper is organized as follow: formulation of the discrete valued optimization problems is presented in Section 2. In Section 3, the concepts of WCA are introduced, briefly. Section 4 marks for application of WCA for sizing optimization of truss structures with discrete and continuous design variables. In this section, two well-known truss structures have been optimized using WCA and the obtained results have been compared with numerous algorithms. Finally, conclusions are presented in Section 5.

2. DISCRETE STRUCTURAL OPTIMIZATION PROBLEMS

Structural optimization problem with discrete variables can be formulated as a non-linear programming problem (NLP). For sizing optimization of truss structures, the cross-section areas of the members are considered as the design variables.

Usually, each design variables is chosen from a list of discrete cross-sections based on production standards. Typically, the objective function is the structure weight, while the design must also satisfy certain (stress, displacement, etc) constraints. Any structural optimization with discrete variables can be presented as follow [13]:

$$\min \quad f(x_1, x_2, \dots, x_i) \quad i = 1, 2, \dots, N \quad (1)$$

subject to:

$$g_j(x_1, x_2, \dots, x_N) \leq 0 \quad j = 1, 2, \dots, m \quad (2)$$

$$x^d \in S_d = \{X_1, X_2, \dots, X_p\} \quad (3)$$

where $f(X)$ is the objective function which describe the weight of the truss. N and m are the number of design variables and inequality constraints ($g_j(X) \leq 0$), respectively. S_d consists of all permissive discrete variables (X_1, X_2, \dots, X_p), in which P denotes the number of available variables [13].

3. WATER CYCLE ALGORITHM

The idea of the WCA is inspired from nature and based on the observation of water cycle and how rivers and streams flow downhill towards the sea in the real world. Similar to other metaheuristic algorithms, the WCA begins with an initial population so called the raindrops.

First, we assume that we have rain or precipitation. The best individual (best raindrop) is chosen as a sea. Then, a number of good raindrops are chosen as a river and the rest of the raindrops are considered as streams which flow to the rivers and sea.

Depending on their magnitude of flow, each river absorbs water from the streams. In fact, the amount of water in a stream entering a rivers and/or sea varies from other streams. In addition, rivers flow to the sea which is the most downhill location [16].

As in nature, the streams are created from the raindrops and join each other to form new rivers. Some of the streams may also flow directly to the sea. All rivers and streams end up in sea (best optimal point). Figure 1 shows the schematic view of stream's flow towards a specific river. As shown in Figure 1, star and circle represent river and stream, respectively.

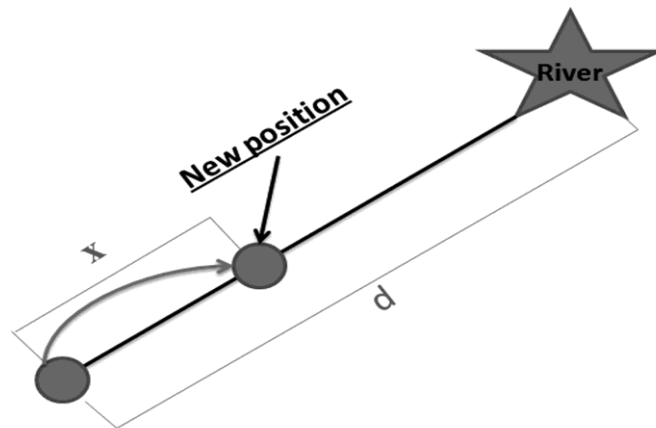


Figure 1. Schematic view of stream's flow to a specific river.

As illustrated in Figure 1, a stream flows to the river along the connecting line between them using a randomly chosen distance given as follow:

$$X \in (0, C \times d), \quad C > 1 \quad (4)$$

where C is a value between 1 and 2 (near to 2). The best value for C may be chosen as 2. The current distance between stream and river is represented as d . The value of X in Eq. (4) corresponds to a distributed random number (uniformly or may be any appropriate distribution) between 0 and $(C \times d)$.

The value of C being greater than one enables streams to flow in different directions towards the rivers. This concept may also be used in flowing rivers to the sea. Therefore, the new position for streams and rivers may be given as [16]:

$$X_{Stream}^{i+1} = X_{Stream}^i + rand \times C \times (X_{River}^i - X_{Stream}^i) \quad (5)$$

$$X_{River}^{i+1} = X_{River}^i + rand \times C \times (X_{Sea}^i - X_{River}^i) \quad (6)$$

where $rand$ is a uniformly distributed random number between 0 and 1. If the solution given by a stream is better than its connecting river, the positions of river and stream are exchanged (i.e. stream becomes river and river becomes stream). Such exchange can similarly happen for rivers and sea. Figure 2 depicts the exchange of a stream which is the best solution among other streams and the river where star represents river and black color circle shows the best stream among other streams.

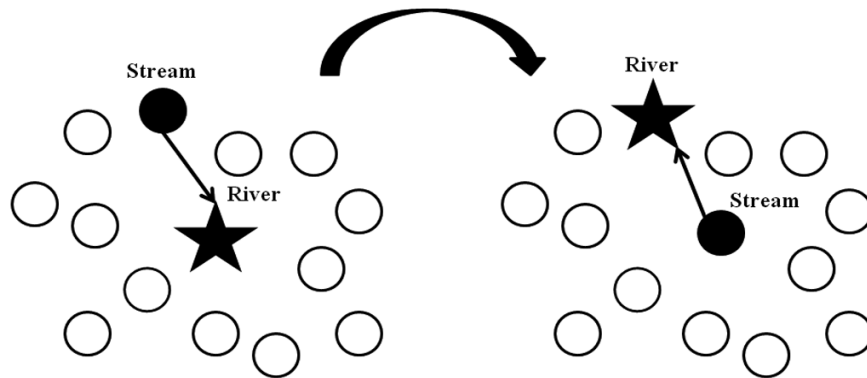


Figure 2. Exchanging the positions of the stream and the river.

Introducing another operator, evaporation process is one of the most important factors that can prevent the algorithm from rapid convergence (immature convergence). In the WCA, the evaporation process causes the sea water to evaporate as rivers/streams flow to the sea. This assumption is proposed in order to avoid getting trapped in local optima. The following Pseudocode shows how to determine whether or not river flows to the sea [16].

$$\text{if } |X_{Sea}^i - X_{River}^i| < d_{max} \quad i = 1, 2, 3, \dots, N_{sr} - 1$$

Evaporation and raining process

end

where d_{max} is a small number (close to zero). After satisfying the evaporation process, the raining process is applied. In the raining process, the new raindrops form streams in the different locations (acting similar to mutation operator in the GAs).

The schematic view of the WCA is illustrated in Figure 3 where circles, stars, and the diamond correspond to streams, rivers, and sea, respectively. From Figure 3, the white (empty) shapes refer to the new positions found by streams and rivers. Figure 3 is an extension of Figure 1.

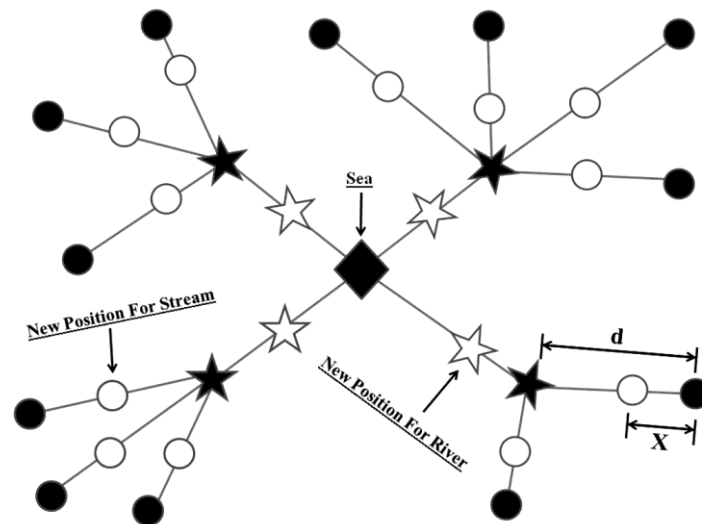


Figure 3. Schematic view of WCA processes

4. NUMERICAL EXAMPLES

In this section, the WCA is applied for discrete and continuous optimization benchmark problems including two well-known truss structures. The proposed WCA was implemented in MATLAB programming software and runs were performed on Pentium IV 2.53 GHz CPU with 4 GB RAM. For considered truss structures, N_{total} , N_{sr} and d_{max} (maximum distance between sea and river) were chosen 25, 8 and $1e-5$, respectively, as user parameters.

The analysis of all trusses has been performed via the finite element method (FEM). The number of design variables for 10 and 15-bar is 10 and 15, respectively. The number of constraints for 10 and 15-bar is 32 (10 tension constraints, 10 compression constraints, and 12 displacement constraints) and 46 (15 tension constraints, 15 compression constraints, and 16 displacement constraints), respectively. In order to have acceptable statistical results, the task of optimization was carried out using 50 independent runs for each truss structure.

4.1 10-bar truss structure

The 10-bar truss, shown in Figure 4, has been extensively analyzed by many researchers, such as Rajeev and Krishnamoorthy [8], Li et al. [13], Sadollah et al. [15], Ringertz [17], Kaveh and Rahami [18], Shih and Yang [19], and, Kaveh and Hassani [20].

The material density and the modulus of elasticity are 0.1 lb/in^3 (0.0272 N/cm^3) and $E=10^4 \text{ ksi}$ (68947.57 MPa), respectively. The stress limitation for each member of this structure is equal to 25 ksi ($\pm 172.37 \text{ MPa}$) for compression and tension stresses. The allowable displacement for each node in both directions is $\pm 2 \text{ in}$ ($\pm 0.0508 \text{ m}$). The weight optimization of 10-bar truss was carried out using 2 types including discrete and continuous design variables.

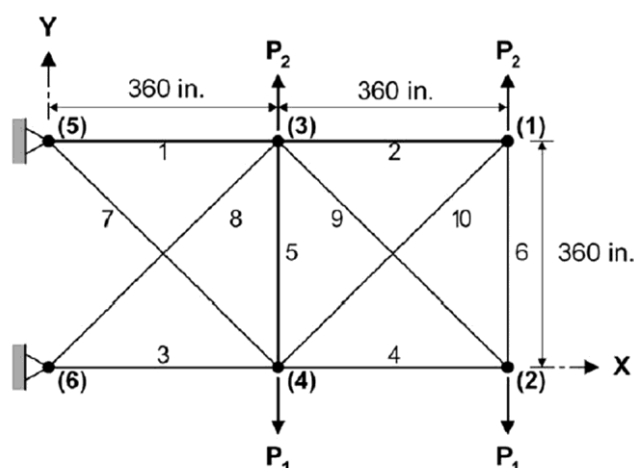


Figure 4. 10-bar planar truss

4.1.1 Discrete

The vertical load in nodes number 2 and 4 is equal to $P_1=10^5 \text{ lbs}$ and in nodes number 1 and 3 is equal to $P_2=0$. In this problem, two cases for discrete design variables were studied. In the first case, discrete variables were selected from the set $D=[1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50]$ (in^2), and in the second case, they were selected from the set $D=[0.1, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5, 16.0, 16.5, 17.0, 17.5, 18.0, 18.5, 19.0, 19.5, 20.0, 20.5, 21.0, 21.5, 22.0, 22.5, 23.0, 23.5, 24.0, 24.5, 25.0, 25.5, 26.0, 26.5, 27.0, 27.5, 28.0, 28.5, 29.0, 29.5, 30.0, 30.5, 31.0, 31.5]$ (in^2).

A maximum number of 1000 iterations was used to compare WCA with other algorithms. For the first case (discrete), the WCA is compared with GA [8], standard particle swarm optimization (PSO), particle swarm optimizer with passive congregation (PSOPC), and heuristic particle swarm optimizer (HPSO) [13], Kaveh and Rahami [18], Shih and Yang [19], Kaveh and Hassani [20], and mine blast algorithm (MBA) [15].

The standard deviation of WCA optimizer for Case 1 is zero, while the standard deviation of PSO, PSOPC, HPSO are 664.07891, 12.84174 and 3.8402, respectively. The best and mean number of function evaluations (NFEs), for the 10-bar truss for Case 1, are 2200 and 6450, respectively. Table 1 represents the comparison of optimal design results for Case 1.

Table 1: Comparison of optimal design for the 10-bar truss for Case 1

| Variables (in^2) | GA [8] | PSO [13] | PSOC [13] | HPSO [13] | Shih & Yang [19] | Kaveh & Rahami [18] | Kaveh & Hassani [20] | MBA [15] | WCA |
|----------------------|--------|----------|-----------|-----------|------------------|---------------------|----------------------|----------|---------|
| A1 | 33.5 | 30 | 30 | 30 | 33.5 | 33.5 | 33.5 | 30 | 33.5 |
| A2 | 1.62 | 1.62 | 1.8 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 |
| A3 | 22 | 30 | 26.5 | 22.9 | 22.9 | 22.9 | 22.90 | 22.90 | 22.9 |
| A4 | 15.5 | 13.5 | 15.5 | 13.5 | 15.5 | 14.2 | 14.2 | 16.9 | 14.2 |
| A5 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 |
| A6 | 1.62 | 1.8 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 |
| A7 | 14.2 | 11.5 | 11.5 | 7.97 | 7.97 | 7.97 | 11.5 | 7.97 | 7.97 |
| A8 | 19.9 | 18.8 | 18.8 | 26.5 | 22 | 22.9 | 22 | 22.9 | 22.9 |
| A9 | 19.9 | 22 | 22 | 22 | 22 | 22 | 19.9 | 22.9 | 22 |
| A10 | 2.62 | 1.8 | 3.09 | 1.8 | 1.62 | 1.62 | 1.62 | 1.62 | 1.62 |
| Weight (lb) | 5613.8 | 5581.76 | 5593.4 | 5531.9 | 5491.7 | 5490.738 | 5517.72 | 5507.7 | 5490.73 |

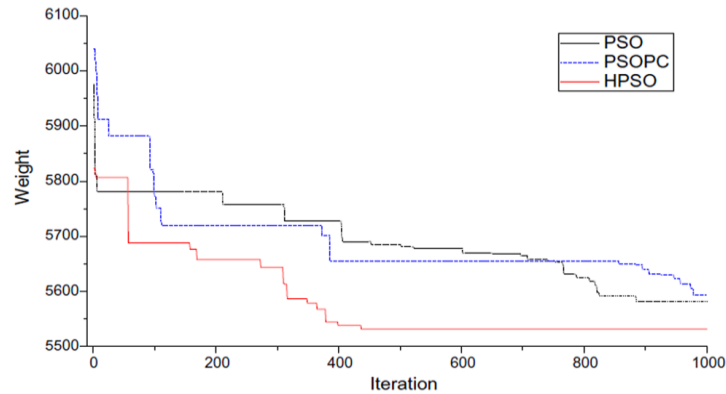
For the second case, the WCA is compared with Ringertz [17], PSO, PSOPC, HPSO, and MBA. The statistical results of WCA optimizer for the 10-bar truss for Case 2 include worst, mean, best solution and standard deviation which are 5074.787, 5068.296, 5067.331 and 1.744, respectively. The best and mean NFEs for the 10-bar truss for Case 2 are 1800 and 13500, respectively.

Table 2 presents the comparison of optimal design results obtained from various algorithms for the 10-bar truss for the second case. As shown in Table 2, WCA, similar to MBA, reached the second best optimal design and outperformed other algorithms except Ringertz's method.

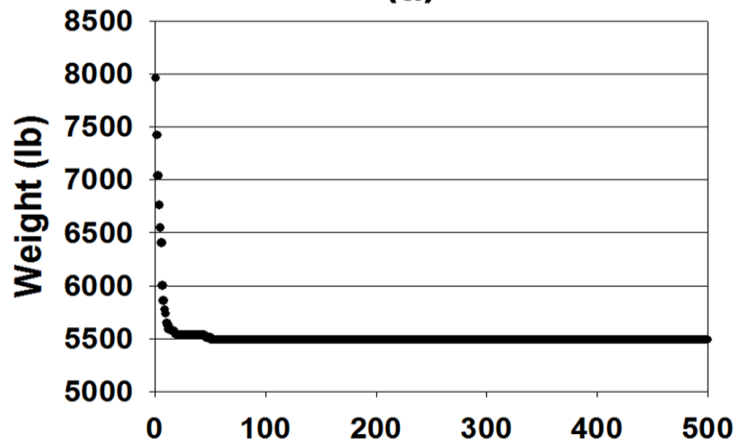
Table 2: Comparison of optimal design for the 10-bar truss for Case 2

| Variables (in^2) | Ringertz [17] | PSO [13] | PSOPC [13] | HPSO [13] | MBA [15] | WCA |
|----------------------|---------------|----------|------------|-----------|----------|---------|
| A ₁ | 30.5 | 24.5 | 25.5 | 31.5 | 29.5 | 29.5 |
| A ₂ | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| A ₃ | 23 | 22.5 | 23.5 | 24.5 | 24 | 24 |
| A ₄ | 15.5 | 15.5 | 18.5 | 15.5 | 15 | 15 |
| A ₅ | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| A ₆ | 0.5 | 1.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| A ₇ | 7.5 | 8.5 | 7.5 | 7.5 | 7.5 | 7.5 |
| A ₈ | 21 | 21.5 | 21.5 | 20.5 | 21.5 | 21 |
| A ₉ | 21.5 | 27.5 | 23.5 | 20.5 | 21.5 | 22 |
| A ₁₀ | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| Weight (lb) | 5059.9 | 5243.71 | 5133.16 | 5073.51 | 5067.33 | 5067.33 |

For comparison in terms of convergence rate, Figures 5 and 6 are presented. For the Case 1, WCA obtained the best solution at 44 iterations (2200 function evaluations as shown in Figure 5b), while HPSO found the best solution at more than 400 iterations (more than 20,000 function evaluations). In contrast, PSO, PSOPC did not detect the best solution after 1000 iterations as shown in Figure 5a. Compared to the reported algorithms, WCA has the fastest convergence rate.



(a)



(b)

Figure 5. Comparison of convergence rates for the 10-bar truss for Case 1 using: (a) HPSO [13], (b) WCA

For the second case, as shown in Figures 6a and 6b, WCA reached the minimum weight at 36 iterations (1800 function evaluations), while HPSO reached the minimum more than 500 iterations (more than 25000 function evaluations). In contrast, PSOPC and PSO did not give the best solution after 1000 iterations as shown in Figure 6a. The convergence rate of the WCA outperforms the other considered algorithms. For more clarification for the convergence rate, Figures 5b and 6b are given the weight values only for 500 iterations.

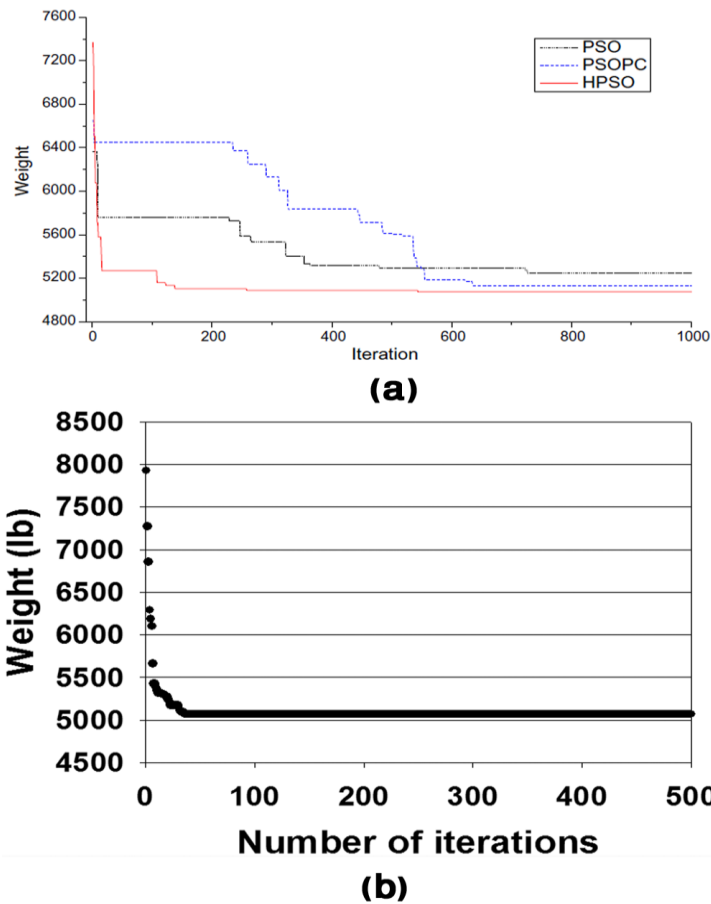


Figure 6: Comparison of convergence rates for the 10-bar truss for Case 2 using: (a) HPSO [13], (b) WCA

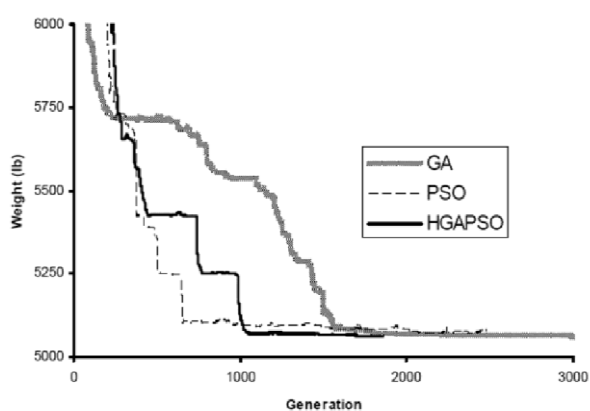
4.1.2 Continuous

For continuous 10-bar truss, decision variables vary from 0.1 to 35.0 in² (from 0.6452 cm² to 225.806 cm²). The 10-bar truss is subjected to loading condition as $P_1 = 100$ kips (444.8 kN) and $P_2 = 0$. For the continuous 10-bar truss, the WCA were applied and the obtained results were compared with other researchers and methods including GA [8], Kaveh and Rahami [18], Kaveh and Hassani [20], Kaveh and Kalatjari [21], Rizzi [22], Khan et al. [23], Dobbs and Nelson [24], Schmit and Farshi [25], Schmit and Miura [26], Gellatly and Berke [27], Venkayya [28], HGAPSO [29], Haug and Arora [30], and Ghasemi et al. [31]. Table 3 represents the comparison of optimal solution by numerous optimizers.

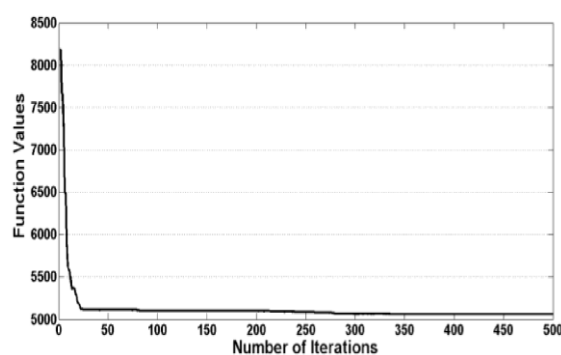
The PSO reached its best solution after 3000 iterations (150,000 function evaluations) [13], while WCA detected its optimal configuration after 500 iterations (12,500 function evaluations). Figure 7 depicts the function values (weight) versus the number of iterations for 10-bar truss (continuous) using WCA and HGAPSO [29]. By observing Figure 7, in terms of convergence rate, WCA has found its optimal configuration faster and more accurate than the HGAPSO at 500 iterations (12,500 function evaluations).

Table 3: Comparison of optimal design for the 10-bar truss for continuous design variables using several optimizers

| Methods | A ₁ | A ₂ | A ₃ | A ₄ | A ₅ | A ₆ | A ₇ | A ₈ | A ₉ | A ₁₀ | Weight (lb) |
|------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|-------------|
| PSO [13] | 33.46 | 0.11 | 23.17 | 15.47 | 3.64 | 0.11 | 8.32 | 23.34 | 23.01 | 0.19 | 5529.50 |
| GA [8] | 28.92 | 0.10 | 24.07 | 13.96 | 0.10 | 0.56 | 7.69 | 21.95 | 22.09 | 0.10 | 5067.31 |
| Kaveh & Rahami [18] | 30.67 | 0.10 | 22.87 | 15.34 | 0.10 | 0.46 | 7.48 | 20.96 | 21.70 | 0.10 | 5061.90 |
| Kaveh & Hassani [20] | 30.86 | 0.10 | 23.55 | 15.01 | 0.10 | 0.22 | 7.63 | 21.65 | 21.32 | 0.10 | 5095.46 |
| Kaveh & Kalatjari [21] | 29.50 | 0.10 | 23.50 | 15.50 | 0.10 | 0.50 | 7.50 | 21.50 | 21.50 | 0.10 | 5067.3 |
| Rizzi [23] | 30.73 | 0.10 | 23.93 | 14.73 | 0.10 | 0.10 | 8.54 | 20.95 | 21.84 | 0.10 | 5076.66 |
| Khan et al. [23] | 30.98 | 0.10 | 24.17 | 14.81 | 0.10 | 0.41 | 7.54 | 21.05 | 20.94 | 0.10 | 5066.98 |
| Dobbs & Nelson [24] | 30.50 | 0.10 | 23.29 | 15.43 | 0.10 | 0.21 | 7.65 | 20.98 | 21.82 | 0.10 | 5080 |
| Venkayya [28] | 30.42 | 0.13 | 23.41 | 14.91 | 0.10 | 0.10 | 8.7 | 21.08 | 21.08 | 0.19 | 5084.9 |
| Schmit & Miura [26] | 30.67 | 0.10 | 23.76 | 14.59 | 0.10 | 0.10 | 8.59 | 21.07 | 20.96 | 0.10 | 5076.85 |
| Schmit & Farshi [25] | 33.43 | 0.10 | 24.26 | 14.26 | 0.10 | 0.10 | 8.39 | 20.74 | 19.69 | 0.10 | 5089 |
| Gellatly & Berke [27] | 31.35 | 0.10 | 20.03 | 15.60 | 0.14 | 0.24 | 8.35 | 22.21 | 22.06 | 0.10 | 5112 |
| HGAPSO [29] | 30.63 | 0.10 | 23.06 | 15.01 | 0.10 | 0.59 | 7.49 | 21.10 | 21.56 | 0.10 | 5061.4 |
| Haug & Arora [30] | 30.03 | 0.10 | 23.27 | 15.28 | 0.10 | 0.55 | 7.46 | 21.19 | 21.61 | 0.10 | 5060.92 |
| Ghasemi et al. [31] | 25.73 | 0.10 | 24.85 | 16.35 | 0.10 | 0.10 | 8.70 | 21.41 | 22.30 | 0.12 | 5095.65 |
| WCA | 30.53 | 0.10 | 23.05 | 15.03 | 0.10 | 0.56 | 7.48 | 21.12 | 21.63 | 0.10 | 5061.02 |



(a)



(b)

Figure 7. Weight reduction history for 10-bar truss using: (a) HGAPSO [30], (b) WCA

4.2. 15-bar truss structure

The 15-bar truss, shown in Figure 8, has been studied by Zhang et al. [32] and Li et al. [13]. The material density and the modulus of elasticity are 7800 kg/m^3 and $E=200 \text{ MPa}$ respectively. The stress limitation for each member of this structure is equal to $\pm 120 \text{ MPa}$. The allowable displacement for each node in both directions is $\pm 10 \text{ mm}$.

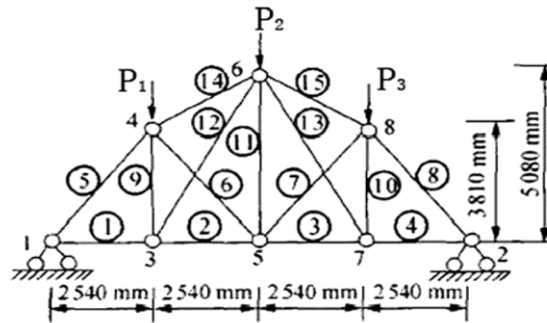


Figure 8. 15-bar planar truss

In this problem, design variables were selected from the set $D = [113.2, 143.2, 145.9, 174.9, 185.9, 235.9, 265.9, 297.1, 308.6, 334.3, 338.2, 497.8, 507.6, 736.7, 791.2, 1063.7]$ (mm^2). Three load cases were considered: Case 1: $P_1=35 \text{ kN}$, $P_2=35 \text{ kN}$, $P_3=35 \text{ kN}$; Case 2: $P_1=35 \text{ kN}$, $P_2=0 \text{ kN}$, $P_3=35 \text{ kN}$; Case 3: $P_1=35 \text{ kN}$, $P_2=35 \text{ kN}$, $P_3=0 \text{ kN}$.

For comparison with other algorithms in identical situation, a maximum number of 500 iterations was imposed. The standard deviation of the WCA and MBA are zero for all considered cases. Table 4 shows the optimal design of WCA which is compared with improved hybrid genetic algorithm (HGA) [32], PSO, PSOPC, HPSO, and MBA for Case 1. It is evident from Table 4 that WCA reached the optimal design value equal or better than other algorithms. For the first case, the best and mean NFEs are 750 and 1850, respectively.

Table 4: Comparison of optimal design for the 15-bar truss for Case 1.

| Variables (in^2) | HGA [32] | PSO [13] | PSOPC [13] | HPSO [13] | MBA [15] | WCA |
|-----------------------------|----------|----------|------------|-----------|----------|---------|
| A1 | 308.6 | 185.9 | 113.2 | 113.2 | 113.2 | 113.2 |
| A2 | 174.9 | 113.2 | 113.2 | 113.2 | 113.2 | 113.2 |
| A3 | 338.2 | 143.2 | 113.2 | 113.2 | 113.2 | 113.2 |
| A4 | 143.2 | 113.2 | 113.2 | 113.2 | 113.2 | 113.2 |
| A5 | 736.7 | 736.7 | 736.7 | 736.7 | 736.7 | 736.7 |
| A6 | 185.9 | 143.2 | 113.2 | 113.2 | 113.2 | 113.2 |
| A7 | 265.9 | 113.2 | 113.2 | 113.2 | 113.2 | 113.2 |
| A8 | 507.6 | 736.7 | 736.7 | 736.7 | 736.7 | 736.7 |
| A9 | 143.2 | 113.2 | 113.2 | 113.2 | 113.2 | 113.2 |
| A10 | 507.6 | 113.2 | 113.2 | 113.2 | 113.2 | 113.2 |
| A11 | 279.1 | 113.2 | 113.2 | 113.2 | 113.2 | 113.2 |
| A12 | 174.9 | 113.2 | 113.2 | 113.2 | 113.2 | 113.2 |
| A13 | 297.1 | 113.2 | 185.9 | 113.2 | 113.2 | 113.2 |
| A14 | 235.9 | 334.3 | 334.3 | 334.3 | 334.3 | 334.3 |
| A15 | 265.9 | 334.3 | 334.3 | 334.3 | 334.3 | 334.3 |
| Weight (lb) | 142.117 | 108.84 | 108.96 | 105.735 | 105.735 | 105.735 |

In addition, WCA as for MBA was examined for Cases 2 and 3 as presented in Table 5. For Case 2, the best and mean NFEs are 1200 and 1650, respectively. Similarly, for Case 3, the best and averaged NFEs are 1350 and 1700, respectively. The WCA and MBA detected the same results for Cases 2 and 3.

The results of the comparison of convergence rate are shown in Figure 9 for Case 1. As shown in Figures 9a and 9b, WCA found the best solutions at 15 iterations (750 function evaluations), while HPSO found the best one at almost 150 iterations (almost 7500 function evaluations), respectively.

However, PSOPC and PSO did not reach the best solution after 1000 iterations as shown in Figure 9a. The WCA converged to the optimal design solution faster than the other considered algorithms. For Cases 2 and 3, WCA reached the minimum weight at 24 and 27 iterations (1200 and 1350 function evaluations), as shown in Figures 10a and 10b, respectively, while the MBA reached the minimum weight at 66 and 62 iterations (3300 and 3100 function evaluations), respectively [15].

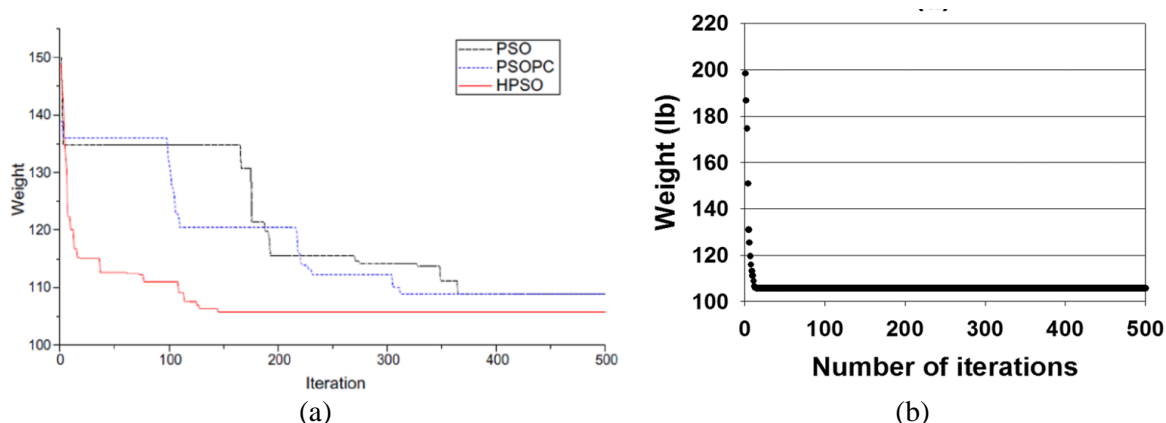


Figure 9. Comparison of convergence rates for the 15-bar truss for Case 1 using: (a) HPSO [13], (b) WCA.

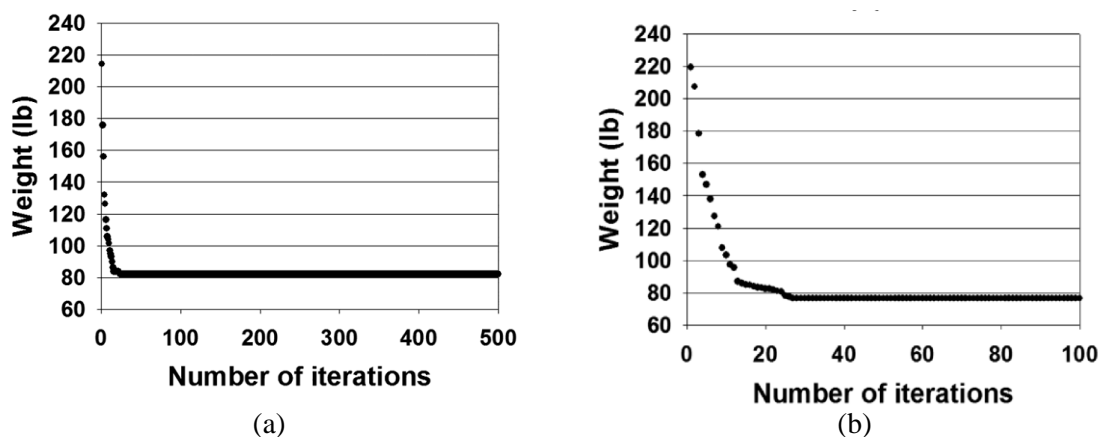


Figure 10. Weight (lbs) evolution history for the 15-bar truss using WCA: (a) Case 2, (b) Case 3.

Table 5: Optimal design solution for the 15-bar truss using WCA and MBA for Cases 2 and 3

| Variables (in ²) | WCA | | MBA | |
|---------------------------------|--------|--------|--------|--------|
| | Case 2 | Case 3 | Case 2 | Case 3 |
| A1 | 113.2 | 113.2 | 113.2 | 113.2 |
| A2 | 113.2 | 113.2 | 113.2 | 113.2 |
| A3 | 113.2 | 113.2 | 113.2 | 113.2 |
| A4 | 113.2 | 113.2 | 113.2 | 113.2 |
| A5 | 497.8 | 497.8 | 497.8 | 497.8 |
| A6 | 113.2 | 113.2 | 113.2 | 113.2 |
| A7 | 113.2 | 113.2 | 113.2 | 113.2 |
| A8 | 497.8 | 265.9 | 497.8 | 265.9 |
| A9 | 113.2 | 113.2 | 113.2 | 113.2 |
| A10 | 113.2 | 113.2 | 113.2 | 113.2 |
| A11 | 113.2 | 113.2 | 113.2 | 113.2 |
| A12 | 113.2 | 113.2 | 113.2 | 113.2 |
| A13 | 113.2 | 113.2 | 113.2 | 113.2 |
| A14 | 185.9 | 265.9 | 185.9 | 265.9 |
| A15 | 185.9 | 235.9 | 185.9 | 235.9 |
| Weight (lb) | 82.095 | 76.692 | 82.095 | 76.692 |

5. CONCLUSIONS

Water cycle algorithm (WCA) which the ideas behind of this method are inspired from water cycle process was applied for sizing optimization of truss structures with discrete and continues decision variables. Based on the computational results obtained from two truss optimization benchmark problems, WCA has found the optimum structural configuration equal or better than other optimizers. In addition, fast convergence rate to reach the best solution, better solution, and also low computational effort (in terms of number of function evaluations) are considered as other advantages of using WCA for solving structural optimization problems.

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