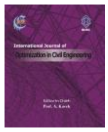
**INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING**
***Int. J. Optim. Civil Eng., 2013; 3(3):359-370***



# TOPOLOGY OPTIMIZATION OF SPACE STRUCTURES USING ANT COLONY METHOD

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## ABSTRACT

In this article, the ant colony method is utilized for topology optimization of space structures. Strain energy of the structure is minimized while the material volume is limited to a certain amount. In other words, the stiffest possible structure is sought when certain given materials are used. In addition, a noise cleaning technique is addressed to prevent undesirable members in optimum topology. The performance of the method for topology optimization of space structures are demonstrated by three numerical examples.

Received: 10 February 2013; Accepted: 18 July 2013

KEY WORDS: space structure, topology optimization, Ant Colony Algorithm

## 1. INTRODUCTION

In general, three optimization steps are considered to design optimal structures [1]. First step is called topology optimization and the aim is to find general layout of the structure. In other words, location and number of holes are sought during the optimization process. Figure 1(a) shows topology optimization of a discrete system. Topology optimization has received enormous attention since the introduction of the ‘homogenization approach to topology optimization’ by Bendsøe and Kikuchi in 1988 [2] but its origin goes back to the minimum weight structures of Michell in 1904 [3].

The second step is devoted to optimize the boundary of the structure and called shape

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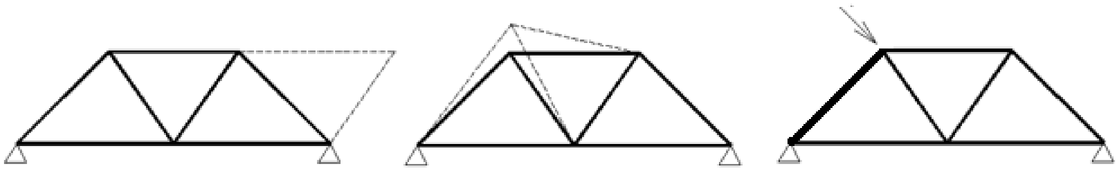
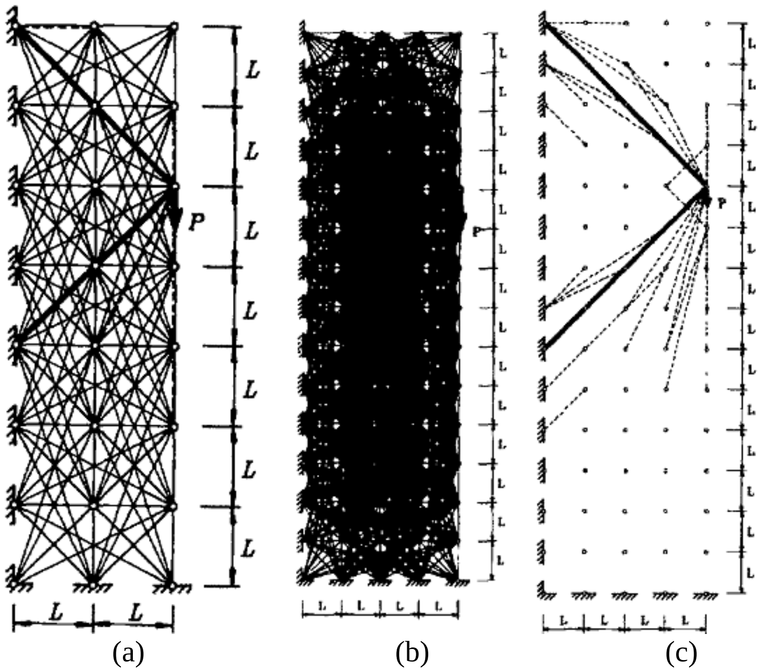
optimization. In this part, the topology is constant during optimization process (Figure 1(b)). In the last step, size of members of the structure such as thickness or dimension of members' section is minimized which is called size optimization. Shape and topology of the structure is assumed to be invariable in this stage (Figure 1(c)).



(a) (b) (c)

Figure 1. Three steps of design of a truss

Topology optimization of a truss is often started from a ground structure including all possible members and the optimum layout is obtained by removing unnecessary members during optimization process. For instance, topology optimization of a deep beam-like truss has been shown in Figure 2. It can be observed that providing a ground structure for large scale space structures is complicated especially when a fine mesh is chosen.



(a) (b) (c)

Figure 2. Topology optimization of a clamped deep beam like truss by the ground structure method (a) a coarse ground structure with solution (b) a fine ground structure (c) solution [22].

In order to solve the topology optimization problem any non-linear mathematical programming methods such as CONLIN by Fleury [4] and Methods of Moving Asymptotes (MMA) by Svanberg [5] can be used. Also, optimality criteria [6-8], the so-called evolutionary structural optimization [9-10] and natural process based methods such as

Genetic Algorithm (GA) [11-13] and Ant Colony (ACO) [14-15] have demonstrated to be well dealt with topology optimization problems during the last decade.

One of the latest and more promising meta-heuristic and evolutionary algorithms is called ant colony optimization (ACO). This method has been inspired by the behavior of colonies of ants when they try to get food. Ant Algorithm is a developed random bionics algorithm that was proposed by Dorigo et al [16]. As a new evolutionary optimization, this method has successfully been applied to several optimization problems, such as TSP (Traveling Salesman Problem), QAP (Quadratic Assignment Problem) and so on [17-21]. The positive feeding-back, coordination and implicit parallelism of ant algorithm have made it an attractive tool for optimization.

In this article, the optimum topology of space structures is sought by using ant colony method. A standard space structure including periodic simple space trusses (truss cell) is considered and the topology algorithm tries to maintain certain amount of members. In other words, the stiffest possible structure is sought to carry the applied loads to the supports by considering the coarsest mesh. In order to achieve this, strain energy of the structure is considered as the objective function and there is a material volume constraint that is practically assumed to be fixed number of members.

## 2. TOPOLOGY OPTIMIZATION PROBLEM

In structural topology optimization, the problem is how to distribute the material in order to minimize the objective function. In other words, the goal can be thought of as determination of the optimal spatial material distribution. It is important to note that the problem type is, from a computational point of view, inherently large scale with the number of design variables proportional to the number of the finite elements in the discretized domain. The problem at hand is defined as finding the stiffest possible structure when a certain amount of material is given. A structure with maximum global stiffness provides a minimum for the strain energy [1]. Therefore, the topology optimization problem can be constructed as below

$$\begin{array}{ll} Min & U(\mathbf{u}) \\ subject\ \ to & equilibrium, \\ & V_s \le \overline{V}_s\,, \end{array} \tag{1}$$

where $\mathbf{u}$ is the displacement field, $U$ is the strain energy, $\overline{V}_s$ is the amount of material available and $V_s$ represent the volume of solid material in each design. The strain energy function, $U(\mathbf{u})$, after discretization of the domain can be written as

$$U(\mathbf{u}) = \frac{1}{2}\sum_{e=1}^{N}\int_{V^e} \boldsymbol{\varepsilon}^T(\mathbf{u}) D^e \boldsymbol{\varepsilon}(\mathbf{u})\, dV \tag{2}$$

where $\boldsymbol{\varepsilon}$ denotes strains, $V^e$ is the entire volume of the element e and $D^e$ is the constitutive matrix of the element and $N$ is the number of finite elements in the discretized domain.

In this research, the density of each member of a space structure is considered as design variable and assumed to be one or zero for solid and empty members, respectively. Therefore, the structure can be described by a discrete function $\chi$, defined at each member as

$$\chi^m = \begin{cases} 1 \ \mathit{if} \quad m \ \mathit{be} \ \text{ a solid element} \\ 0 \ \mathit{if} \quad m \ \mathit{be} \ \text{ an empty element} \end{cases} \tag{3}$$

The elasticity matrix for a typical member $m$ of space structure can be written as

$$\mathbf{D}^m = \chi^m \mathbf{D}^0 \tag{4}$$

where $\mathbf{D}^0$ is the elasticity matrix of the solid material.

## 3. ANT COLONY ALGORITHM FOR TOPOLOGY OPTIMIZATION

One of the recently developed meta-heuristic approaches is the Ant Colony Optimization (ACO). The basic idea in the ACO algorithm is simulation of the natural metaphor of real ant colonies behavior. Real ants are capable of finding the shortest path from a food source to their nest without using visual cues but exploiting a chemical substance called pheromone. While walking, ants deposit pheromone trail on the ground which is added to the previously deposited by other ants.

The ACO has successfully been employed to solve the TSP which is a well known combinatorial optimization problem [18]. Also, this method has shown reasonable results in size optimization of skeletal structures [14] and structural topology optimization in continua [15].

Overall objective of the problem, which is along to the pheromone trail of a segment of a route, is here denoted by $\tau_i(t)$. The parameter $t$ represents the time of development of ants which is equivalent to the cycles of iteration within the algorithm. Inspired by the procedure employed in TSP [18], and ignoring the effect of the local heuristic values, the ant decision index $\mathrm{a_i(t)}$ can be written as

$$\mathrm{a_i(t)} = \frac{[\tau_\mathrm{i}(\mathrm{t})]^\alpha}{\sum_{j=1}^{\mathrm{N}} [\tau_\mathrm{j}(\mathrm{t})]^\alpha} \tag{5}$$

where α is a parameter that controls the relative weight of the pheromone trail, N is the number of finite elements or space structure members and t is an indication of the present cycle which is analogous to the *t*-th time of deploying our ants. Note that here the probability of an element being chosen by a typical ant is the same as the decision index as defined in Equation (5). Two ants have devised two paths with different topology in Figure 3.

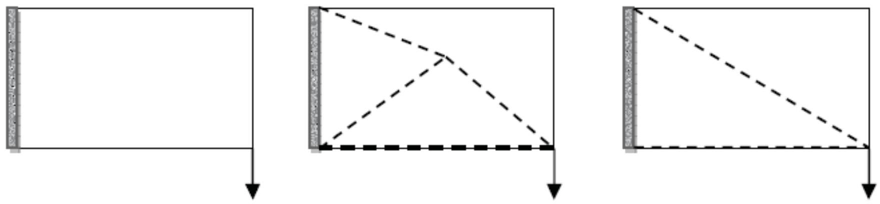


Figure 3. Design domain and paths of two imaginary ants (two designs) to transfer the food (point load) to the nest (supports).

After completion of a cycle of designs by all ants, each ant $k$ deposits a quantity of pheromone $\Delta\tau_i^k$ on each element based on its relative strain energy, as shown below, which is an index of the performance of the element, i.e. for a better design a larger amount of pheromone is deposited.

$$\Delta\tau_i^k = \frac{(U_i^k)^\lambda}{\sum_{j=1}^{N}(U_j^k)^\lambda} \tag{6}$$

where $U_i^k$ is the strain energy in each element of design and the exponent $\lambda$ is a tuning parameter for improvement of performance of the algorithm and its convergence. Note that as $\lambda$ is increased, the search space becomes limited and if enough attention is not paid to its selection, it is likely that the algorithm converges towards layouts which are not globally optimal.

The amount of pheromone in each element is due to addition of new pheromone as well as evaporation which is implemented within the algorithm via the following rule:

$$\tau_i(t+1) = \rho\tau_i(t) + \Delta\tau_i \tag{7}$$

where $\Delta\tau_i = \sum_{k=1}^{m}\Delta\tau_i^k$ and $m$ is the number of ants used in each cycle. The rate of evaporation coefficient $\rho \in (0, 1]$ is applied for taking into account the pheromone decay to avoid quick convergence of the algorithm towards a suboptimal solution. An initial amount of pheromone $\tau_i(0)$ is introduced and a small positive constant value $\tau(0)$ is considered for all elements in the first cycle.

Following Stützle and Hoos in [19] and Bullnheimer *et al* [21], another modification is employed in order to increase the chance of selection of elements with a higher level of accumulated pheromone in the later cycles. This idea is here implemented by sorting elements based on their contribution to the trailing function. Hence, the amount of pheromone at a percentage, σ, of elements with the highest rank is further increased. This percentage is, in general, problem-dependent and is decided according to the problem definition which is usually taken as 10 to 15 percent.

## 4. NOISE CLEANING TECHNIQUES

As a further improvement, the so called noise cleaning techniques can be employed to prevent creation of undesirable members in the resulted optimum layout. This technique was proposed by Sigmond [23-25] for structural topology optimization in continua. Here, the method is developed for discontinuous space structures. Inspired by image processing techniques the strain energy of the member *i* is substituted by weighted average of strain energies of the member *i* and its neighbors *e*. Similar to the conventional impulse response matrix in the image processing texts, a filter $H_e$ is defined as

$$H_e = \begin{cases} 1 & if \quad e = i \\ w_e & if \quad e \in \{1,2,...,n^i\} \end{cases} \tag{8}$$

where $w_e$ is weight of the connected member *e* which is considered between zero and 1 ($0 \le w_e \le 1$). $n^i$ is the number of neighbors of the member *i*. In this case, the strain energy of an element will be modified as

$$\Delta\tau_i^k = \frac{(U_i^k)^\lambda}{\sum_{j=1}^{N}(U_j^k)^\lambda} \tag{9}$$

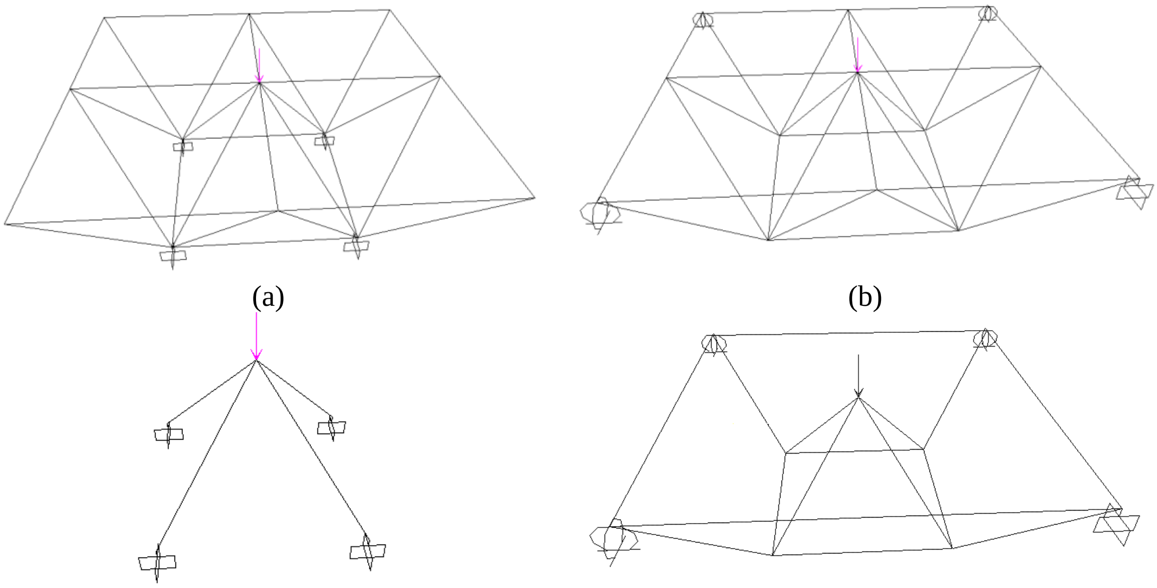
For implementation of this technique into the ACO algorithm, it is enough to replace $U_i$ with the modified strain energy $\hat{U}_i$ in equation (6). As will be illustrated by the following examples, using this technique results in better layouts. Furthermore, the ACO algorithm becomes less sensitive to its tuning parameters.

## 5. NUMERICAL EXAMPLES

### *5.1 Example 1*

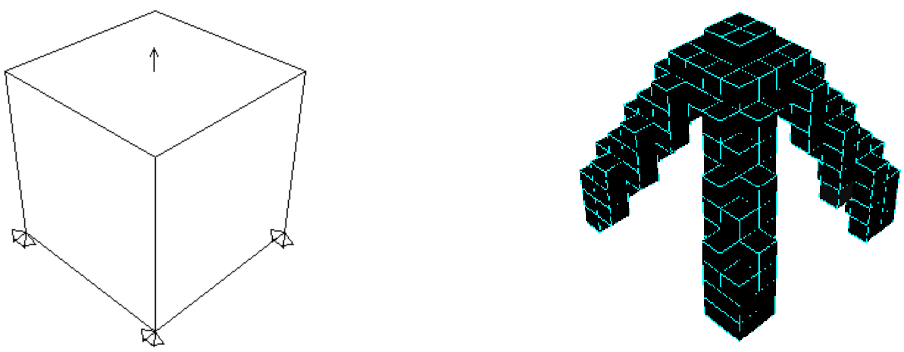
In this example, the stiffest space structure cell is sought by different type of support arrangements in a double-layer space truss. For this purpose a double layer truss with 32 members is considered as shown in Figures 4(a) and 4(b). The volume fraction is assumed to be 15 percent for 4(a) and 50 percent for 4(b). Two ants are considered to travel around the feasible design domain, i.e. number of design iterations, in each cycle of the optimization problem. The tuning parameters $\lambda$ and $\rho$ are considered as 3 and 0.6, respectively.

The results are shown in Figures 4(c) and (d). For the sake of comparison, the results are compared with a similar three dimensional example in continua. In this example, the stiffest possible structure is also searched and the optimality criteria method is used to optimize the objective function. For this purpose the code written by Tavakkoli and Hassani [26] is used. The obtained layouts are depicted in Figures 5 and 6. Noticed that from topological point of view the results in Figures 4 (c) and 5 (b) and Figures 4 (d) and 6 (b-d) are the same. Also, the results might be a good reason for considering these simple trusses as periodic cells to construct large double-layer space structures.



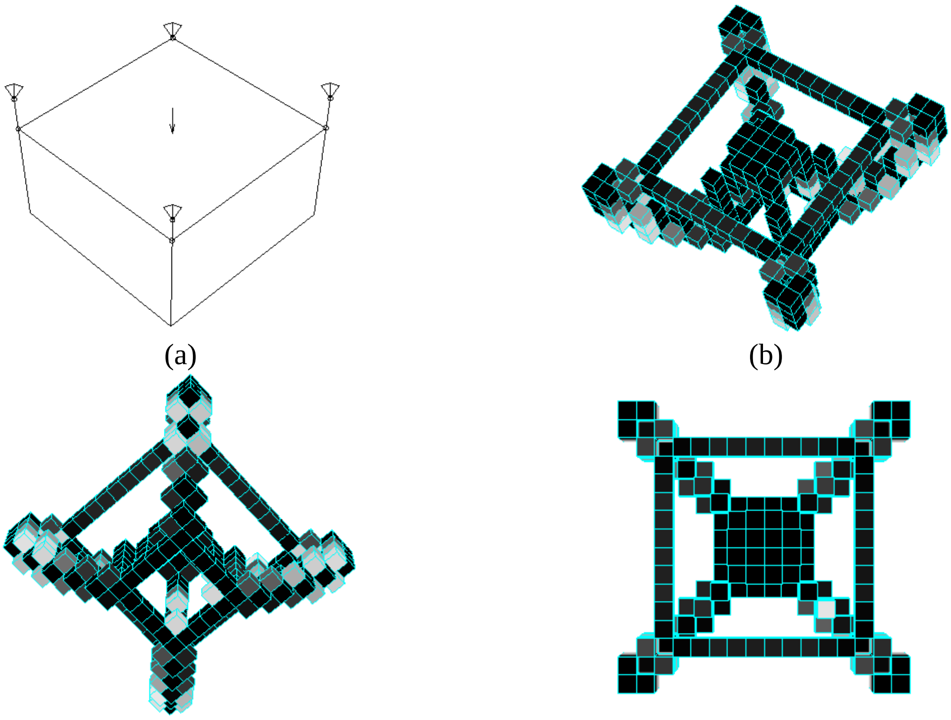
(a) (b) (c) (d)

Figure 4. (a)(b) Ground Structures with different support arrangement, (c)(d) obtained optimum topology pertaining to support arrangement (a) and (b), respectively.



(a) (b)

Figure 5. (a) Problem definition (b) Optimum Topology by using optimality criteria [26].



(a) (b)

(c) (d)

Figure 6. (a) Design domain, (b),(c),(d) different views of the optimum topology [26]

*5.2 Example 2*

In this example a double-layer space structure including 20 bays in each side is considered as shown in Figure 7. It contains 3200 elements and 841 nodes to carry a point load on the top center joint of the truss to the four bottom corner supports. The volume fraction is 30 percent and the tuning parameters are the same as previous example. The number of design iterations is assumed to be 5 in each cycle and the results are obtained after 30 cycles.

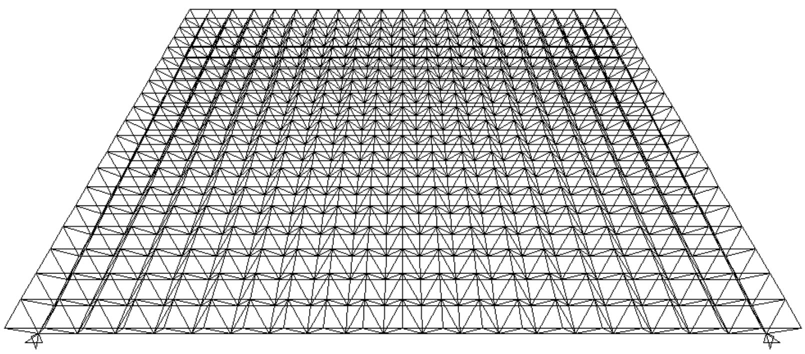
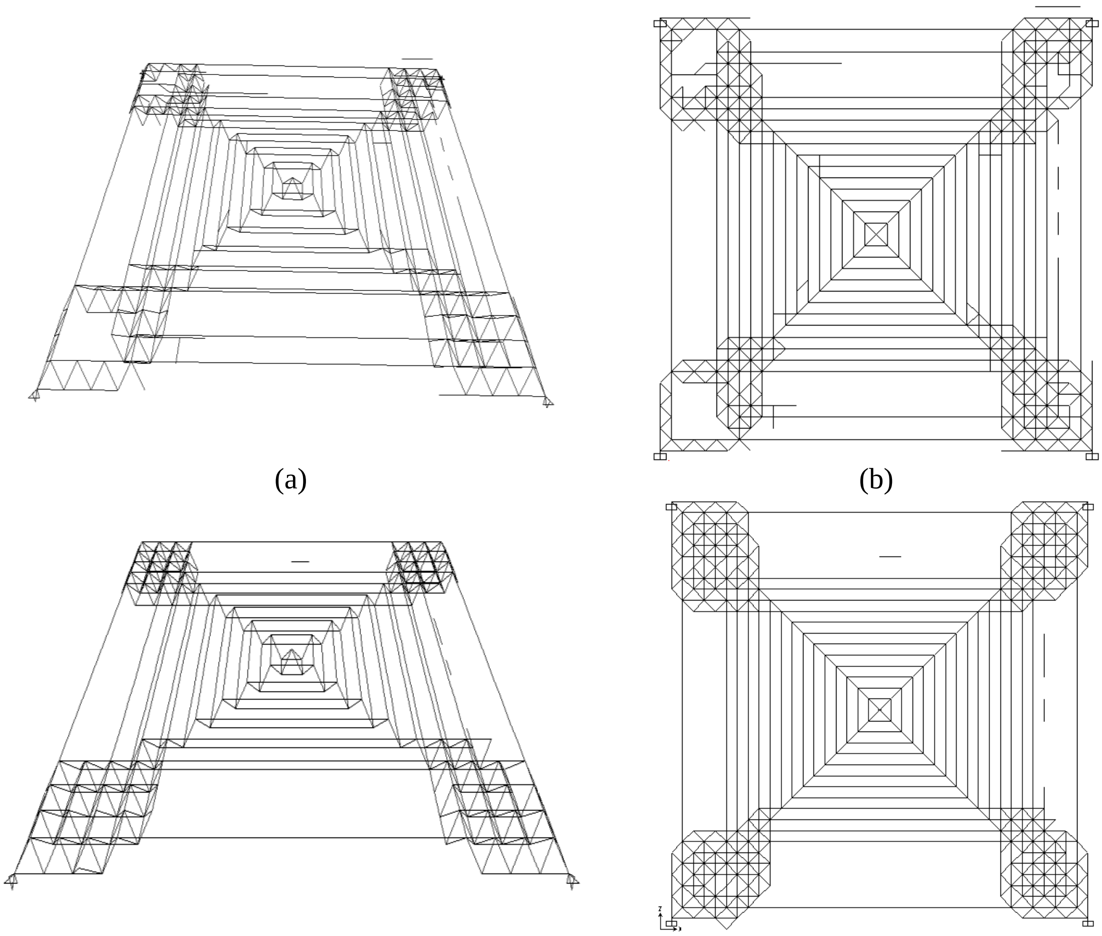


Figure 7. Ground structure for example 2

The effect of using the noise cleaning technique as described in section 4 is discussed in this example. The optimum structure without using the technique is illustrated in Figures

8(a) and 8(b) in different views. Also the result with noise cleaning is depicted in Figures 8(c) and 8(d). It is shown that by using this technique undesirable members are removed during the optimization process and more reasonable layout is obtained.

From practical point of view, the results can give us an idea of the most effective members of the ground structure (Figure 7) to control the deflection of the truss which is important in retrofitting problems. In other words, an existing structure can be imagined as the ground structure (Figure 7) and after optimization the obtained members should be retrofitted in order to increase the stiffness of the structure. It is also noticed that the center truss cell in optimum topology is exactly the same as results in example 1 shown in Figure 4(c-d).



(a) (b)

(c) (d)

Figure 8. Optimum topology (a),(b) without (c),(d) by using noise cleaning technique

*5.3 Example 3*

A three-layer space structure including 1268 elements and 302 nodes is considered as shown in Figure 9. The volume fraction is assumed to be 30 percent and number of design iterations is 5 in each cycle. The tuning parameters $\lambda$ and $\rho$ are assumed similar to previous examples. The optimum topology is depicted in Figure 10. This topology can help the designer to know the most effective layout to control the deflection of the ground space structure. The iteration history of strain energy is illustrated in Figure 11.

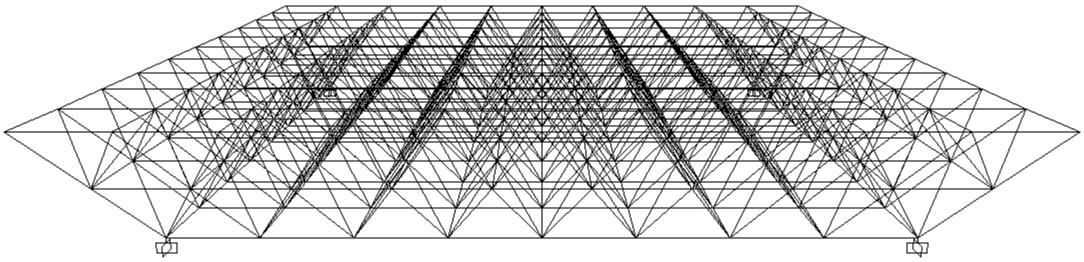


Figure 9. Ground space structure for example 3.

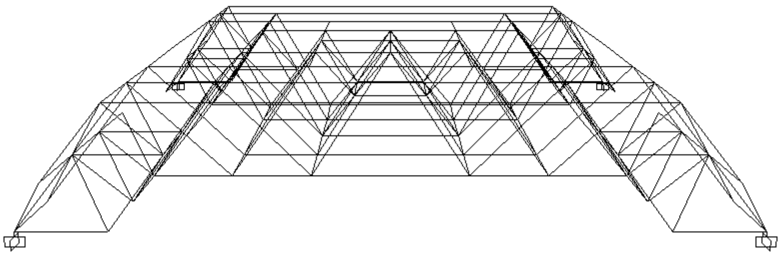


Figure 10. Obtained optimum topology

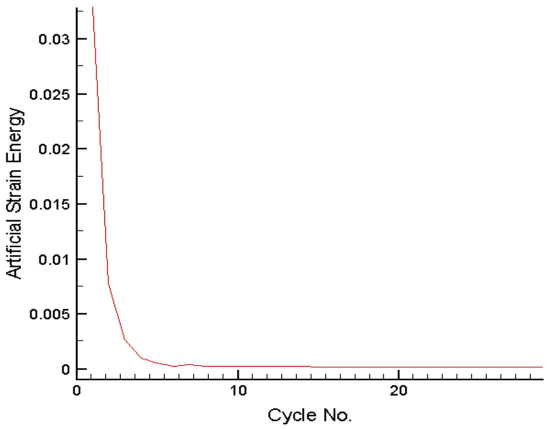



Figure 11. History of minimizing the strain energy in Example 3.

## 6. CONCLUSIONS

In this paper, the optimum topology of space structures is sought by using ant colony methodology. The objective is minimization of the strain energy and the constraints are equilibrium as well as using a certain amount of material. In practical point of view the optimum layouts of such problem definition provide the designer the most effective members to control the deflection of the ground structure which is helpful in case of retrofitting of the structure. In order to have practical optimum topology a noise cleaning technique is suggested in discrete topology optimization problems. The result of a discrete topology optimization problem is compared to optimum topology of similar problem in continua and observed that the results are comparable. Reasonable results are obtained when ant colony is applied for topology optimization of nearly large scale double and three-layer space structures.

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