

SIZE AND GEOMETRY OPTIMIZATION OF TRUSSES USING TEACHING-LEARNING-BASED OPTIMIZATION

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ABSTRACT

A novel optimization algorithm named teaching-learning-based optimization (TLBO) algorithm and its implementation procedure were presented in this paper. TLBO is a meta-heuristic method, which simulates the phenomenon in classes. TLBO has two phases: teacher phase and learner phase. Students learn from teachers in teacher phases and obtain knowledge by mutual learning in learner phase. The suitability of TLBO for size and geometry optimization of structures in structural optimal design was tested by three truss examples. Meanwhile, these examples were used as benchmark structures to explore the effectiveness and robustness of TLBO. The results were compared with those of other algorithms. It is found that TLBO has advantages over other optimal algorithms in convergence rate and accuracy when the number of variables is the same. It is much desired for TLBO to be applied to the tasks of optimal design of engineering structures.

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KEY WORDS: teaching-learning-based optimization (TLBO); size and geometry optimization; truss structure

1. INTRODUCTION

Structural optimal design has always been a concern for engineers in practice. The focus is not only in construction cost, but also in geometry of structures. It is responsible for engineers to design structures with high reliability and low cost. For these purposes, Many optimal

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algorithms were investigated to accomplish the tasks including the classical methods and the innovative algorithms.

In the early 1990s, the genetic algorithm was presented by Goldberg [1], then, it was made significant achievements in structural optimization fields [2-5]. After that, more attentions were paid by researchers to meta-heuristic optimization algorithms. Tremendous amount of meta-heuristic optimization algorithms were created and used in structural optimization tasks soon afterwards. Teaching-learning-based optimizer (TLBO) is a recently proposed meta-heuristic algorithm [6-8]. The easy and effectiveness of TLBO were supported by research works of many other researchers published [9-12]. In the problem of size and geometry optimization of truss structures, the cross-sectional area and the geometry of primary structures both increase the dimension of the design space. It has been proved that TLBO algorithm performs well in problems with large dimensions [7-13]. The main work introduced in this paper is about verifying the optimization capability of the TLBO in truss structures.

2. TEACHING-LEARNING-BASED OPTIMIZATION

TLBO is a population based algorithm similar to the ant colony optimizer (ACO) proposed by Dorigo et al [14], harmony search (HS) developed by Geem et al [15] and particle swarm optimizer (PSO) created by Kennedy et al [16]. It simulates the teaching-learning process proceeded in classroom. Students in class constitute the population in TLBO. The different subjects offered to students are considered as different constrains and the students' marks are analogous to the 'fitness'. Teacher, who obtains the highest marks among students, will do his/her best to increase the average marks of students according to his or her capability. The process of TLBO is divided into two parts. The first part is 'Teacher Phase' and the second part is 'Learner Phase'. The 'Teacher Phase' means learning from the teacher and the 'Learner Phase' means learning through the interaction between learners. The following section briefly describe about the implementation of TLBO in trusses. The notations used for describing the TLBO are as following:

n : number of learners in class (i.e., class size);

m : dimension of a learner;

ps : population which has a matrix size of n rows and m columns. Each row represents a feasible solution (i.e., a learner)

(1) First, the population ps was randomly generated in constrained spaces, then the results $W(x)$ ($x = 1, 2, 3, \dots, n$) were calculated according to the objective. The results were sorted in ascending order corresponding to ps (ascending order is convenient for finding minimum value, maximum value can be obtained by multiply by -1 before the objective).

$$ps = \begin{pmatrix} A_1^1 & A_2^1 & \cdots & \cdots & A_{m-1}^1 & A_m^1 \\ A_1^2 & A_2^2 & \cdots & \cdots & A_{m-1}^2 & A_m^2 \\ \vdots & \vdots & \cdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \vdots & \vdots \\ A_1^{n-1} & A_2^{n-1} & \cdots & \cdots & A_{m-1}^{n-1} & A_m^{n-1} \\ A_1^n & A_2^n & \cdots & \cdots & A_{m-1}^n & A_m^n \end{pmatrix} \begin{matrix} \rightarrow W(1) \\ \rightarrow W(2) \\ \vdots \\ \vdots \\ \rightarrow W(n-1) \\ \rightarrow W(n) \end{matrix} \quad (1)$$

where $W(1) < W(2) \dots < W(n-1) < W(n)$. Assume $A^1 = (A_1^1 \ A_2^1 \ \dots \ A_m^1)$. A^1 is considered as a teacher.

(2) In teacher phase. The mean parameter is given as:

$$A^{mean} = \{mean(\sum_{i=1}^n A_1^i) \ mean(\sum_{i=1}^n A_2^i) \ \dots \ mean(\sum_{i=1}^n A_m^i)\} \tag{2}$$

The teacher improves the average score of the whole class

$$A^{new,i} = A^i + r_i(A^1 - T_F A^{mean}) \quad (i = 1, 2, 3, \dots, n) \tag{3}$$

where r_i is the random number in the range [0,1]. T_F is decided randomly with equal probability as:

$$T_F = round[1 + rand(1)] \tag{4}$$

If $W(A^{new,i}) < W(A^i)$, then, update the solution set $A^i = A^{new,i}$

(3) In learner phase. A learner interacts randomly with other learners for enhancing his or her knowledge. Randomly select two learners A^i and A^j ($i \neq j$).

$$\begin{aligned} A^{new,i} &= A^i + r_i(A^i - A^j) && \text{if } W(A^i) < W(A^j) \\ A^{new,i} &= A^i + r_i(A^j - A^i) && \text{if } W(A^j) < W(A^i) \end{aligned} \tag{5}$$

If $W(A^{new,i}) < W(A^i)$, then, update the solution set $A^i = A^{new,i}$

(4) The duplicate solutions is modified in order to avoid trapping in the local optima. Duplicate solutions are modified by mutation on randomly selected dimensions of the duplicate solutions before executing the next iteration.

(5) Sort the results in ascending order corresponding to ps . Repeat process (2) to (4) until the termination condition is fulfilled.

More details of TLBO can be refereed in literatures [6] to [8].

3. MATHEMATICAL MODEL FOR SIZING AND GEOMETRY OPTIMIZATION OF TRUSS

Usually, there are two types of variables in the mathematical model for the size and geometry optimization of the truss structures i.e. the cross-sectional area variables and the node coordinate variables, which determine the geometry of the structures. Compared with the truss size optimization problems which have been extensively studied, the size and geometry

optimization introduces node coordinate variables. This not only makes the design space is of higher dimension, but also greatly enhances the degree of nonlinearity, moreover, the optimization may lead to a local optima. The mathematical model of size and geometry optimization problems can be expressed as follows:

$$\begin{aligned}
 \min .Weight(A_i, C_j) &= \sum_{i=1}^N \rho_i A_i L_i \\
 L_i &= L_i(C_j) \\
 s.t. \quad g_i^\sigma &= [\sigma_i] - \sigma_i \geq 0 (i = 1, 2, \dots, k) \\
 g_{jl}^u &= [u_{jl}] - u_{jl} \geq 0 (j = 1, 2, \dots, M; l = 1, 2, \dots, N) \\
 A_i &\in S (i = 1, 2, \dots, k)
 \end{aligned} \tag{6}$$

where k is the total number of truss elements; M is the number of nodes; N is the number of nodal freedoms; A_i , L_i and ρ_i represents the cross-sectional area, the length and the density of the i th bar respectively; C_j represents the coordination of j th node; g_i^σ and g_{jl}^u are the constraint violations for member stress (include buckling stress) and joint displacements of the structure. σ_i is the stress of the i th bar due to loading condition, $[\sigma_i]$ is its allowable stress. u_{jl} is the nodal displacement of the l th translational degree of the j th node, $[u_{jl}]$ is its allowable joint displacements. S is a set of discrete cross-section of bars.

4. NUMERICAL EXAMPLES

In this section, three pin-connected structures used in literatures were selected as benchmark structures to test the TLBO. Thirty independent runs were carried out for each design examples. The best result, the worst result, the number of structural analyses, the average result and the standard deviation (std. dev) of 30 independent runs are presented. The same terminal conditions were used in TLBO in this article for the convenience of comparing with literatures. That is, TLBO is terminated when times of structural analyses reaches 50000. However, the exact number of structural analyses is difficult to know, as the duplicate solutions are randomly modified in the duplicate elimination step of the TLBO. The total number of structural analyses in the TLBO algorithm is referred as $((2 \times \text{population} \times \text{number of iterations}) + (\text{structural analyses required for duplicate elimination}))$ [17].

The TLBO is coded in Matlab 2008b and implemented on a desktop computer having Intel Core 3.20GHz processor with 3.47GB RAM.

All examples are analyzed by the finite element method (FEM). The constraints are handled by using 'fly-back mechanism' created by He et al. [18], the method can briefly describe if infeasible designs exist, then it will be forced to fly back to the previous position to guarantee a feasible solution. The efficiency of the method was previously verified for optimization of truss structures [19].

The groups search optimizer (GSO) [20] and the heuristic particle swarm optimizer

(HPSO) [21] are applied to compare with the new algorithm presented in this paper. For these two algorithms, the number of structural analyses is limited to 50000 and the population size is set to at 50. For the GSO algorithm, 20% of the population were selected as rangers; the initial head angle φ_0 of each individual is set to be $\pi/4$. The constant a is given by $\text{round}(\sqrt{n+1})$. The maximum pursuit angle θ_{max} is π/a^2 . The maximum turning angle α is set to be $\pi/2\alpha^2$. For the HPSO algorithm, the inertia weight ω decrease linearly from 0.9 to 0.4, and the value of acceleration constants c_1 and c_2 are set to be the same and equal to 0.8. The passive congregation coefficient c_3 is given as 0.6, the maximum velocity is set as the difference between the upper bound and the lower bound of variables. More details about the GSO can be found in [22].

4.1. A 40-bar planar truss structure

The 40-bar planar truss is shown in Figure 1. The material density is 7800 kg/m^3 and the modulus of elasticity is 196.13GPa . The stress limits of the members are subjected to $\pm 156.91\text{MPa}$. Node 4 and 5 are subjected to the displacement limits of $\pm 0.035\text{m}$ ($1/600$ span) in y directions. There are 40 members, which fall into 19 groups, as follows: (A_1) 1, 7; (A_2) 2, 6; (A_3) 3, 5; (A_4) 4; (A_5) 8, 14; (A_6) 9, 13; (A_7) 10, 12; (A_8) 8; (A_9) 15, 12; (A_{10}) 16, 21; (A_{11}) 17, 20; (A_{12}) 18, 19; (A_{13}) 23, 36; (A_{14}) 24, 35; (A_{15}) 25, 34; (A_{16}) 26, 33; (A_{17}) 30, 29; (A_{18}) 31, 28; (A_{19}) 32, 27.

Discrete values considered for this example are taken from the set $D=[0.001, 0.05] \text{ (m}^2\text{)}$ and the interval is 0.001 m^2 . With the symmetry, the geometry variables group and side constraints are given as, $1 \leq y_9 = y_{16} \leq 5$, $1 \leq y_{10} = y_{15} \leq 5$, $1 \leq y_{11} = y_{14} \leq 5$, $1 \leq y_{12} = y_{13} \leq 5$ (m). Node 2, 3, 4, 5, 6 and 7 are acted by P , 10t in y direction. The optimal weight of 40-bar truss under 30 independent runs for different population is shown in Table 1. The best results of TLBO for population size 20 were selected to compare with those of other algorithms and were shown in Table 2.

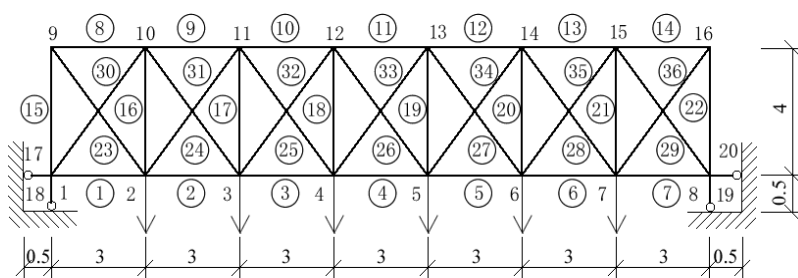


Figure 1. A 40-bar planar truss

Table 1: Results of sensitivity analysis of 40-bar truss for 30 independent runs

| PS | Number of structural analyses averaged | Number of best result for structural analyses | Best (kg) | Mean (kg) | Worst (kg) | std. dev |
|----|----------------------------------------|-----------------------------------------------|-----------|-----------|------------|----------|
| 20 | 50018 | 50008 | 2058.805 | 2200.081 | 2301.954 | 53.314 |
| 30 | 50030 | 50032 | 2067.641 | 2272.684 | 2363.284 | 93.464 |
| 40 | 50044 | 50076 | 2058.805 | 2219.201 | 2288.156 | 75.725 |
| 50 | 50054 | 50092 | 2088.306 | 2228.173 | 2371.685 | 88.011 |

Table 2: Optimal results of 40-bar truss with ps size 20

| variables | HPSO [22] | GSO [22] | TLBO | Variables | HPSO [22] | GSO [22] | TLBO |
|-----------------|-----------|----------|--------|-----------------|-----------|-----------|-----------|
| A ₁ | 0.0055 | 0.0015 | 0.001 | A ₁₃ | 0.001 | 0.001 | 0.001 |
| A ₂ | 0.001 | 0.001 | 0.001 | A ₁₄ | 0.001 | 0.001 | 0.001 |
| A ₃ | 0.0105 | 0.001 | 0.001 | A ₁₅ | 0.0015 | 0.001 | 0.001 |
| A ₄ | 0.001 | 0.001 | 0.001 | A ₁₆ | 0.005 | 0.001 | 0.001 |
| A ₅ | 0.001 | 0.001 | 0.001 | A ₁₇ | 0.004 | 0.0025 | 0.0025 |
| A ₆ | 0.0025 | 0.003 | 0.003 | A ₁₈ | 0.001 | 0.001 | 0.001 |
| A ₇ | 0.003 | 0.0035 | 0.0035 | A ₁₉ | 0.001 | 0.001 | 0.001 |
| A ₈ | 0.0245 | 0.0035 | 0.0035 | y ₉ | 1.006 | 1.069 | 1.004 |
| A ₉ | 0.0025 | 0.001 | 0.001 | Y ₁₀ | 2.791 | 2.307 | 2.412 |
| A ₁₀ | 0.001 | 0.001 | 0.001 | Y ₁₁ | 3.541 | 2.851 | 2.737 |
| A ₁₁ | 0.001 | 0.001 | 0.001 | Y ₁₂ | 3.396 | 3.287 | 3.314 |
| A ₁₂ | 0.001 | 0.001 | 0.001 | Weight (kg) | 3653.0103 | 2080.6733 | 2058.8055 |

It is observed from Table 1 that strategy with population size of 20 and 40 produced the best result than other strategies did, the corresponding number of iterations are 1246 and 623 respectively. However, it is observed that the standard deviation (std. dev) is relatively large, in other words, the result is easy to fall into the local optimum. The increasing of population makes no much difference for the results. In addition, It can be seen from the mean results that TLBO possesses a good global search ability although it has a weak local search capacity.

It is obvious from Table 2 that the TLBO found the better designs than those of the HPSO and GSO under the same number of structural analyses. The optimum design obtained by the GSO is slightly heavier than the TLBO. The result of the HPSO is the worst. Figure 2 and Figure 3 is the convergence curves of TLBO and the optimized structure respectively.

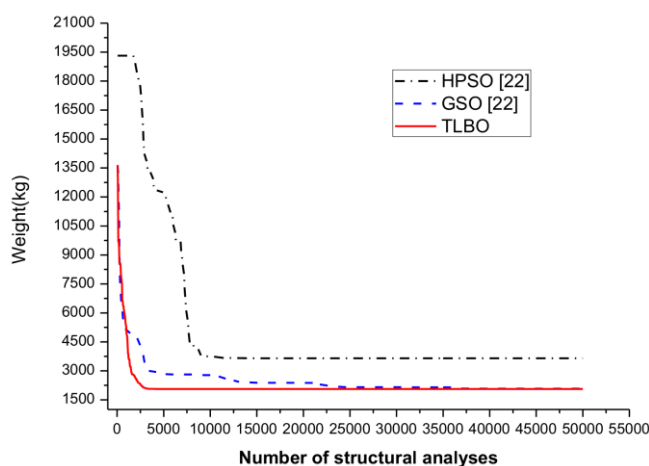


Figure 2. Convergence of TLBO with ps size 20

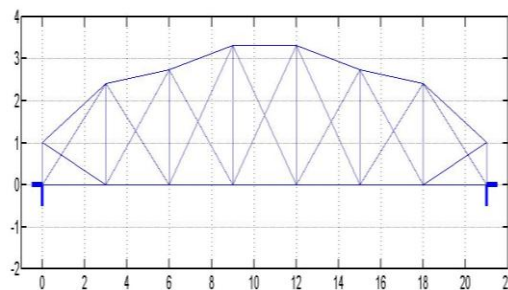


Figure 3. The optimized 40-bar truss structure

It is clear from Figure 2 that the TLBO shows better convergence capability than the HPSO

and GSO did at the early stage of optimization process.

4.2. A 18-bar planar truss structure

The 18-bar planar truss is shown in Figure 4. The material density is 0.1 lb/in³, and the modulus of elasticity is 10000 ksi. The stress limits of the members are subjected to ± 20 ksi. Euler buckling stress constraints are $\sigma_i = \alpha EA_i / L_i^2$, where buckling coefficient $\alpha=4$. Node 1, 2, 4, 6 and 8 have -20 kips in y direction. Size variables are $A_1 = A_4 = A_8 = A_{12} = A_{16}$, $A_2 = A_6 = A_{10} = A_{14} = A_{18}$, $A_3 = A_7 = A_{11} = A_{15}$, $A_5 = A_9 = A_{13} = A_{17}$. The cross-sectional area variables are set [2.00, 21.75] (in²) and the interval is 0.25 in². Side constraints for geometry variables are $-225 \leq y_3, y_5, y_7, y_9 \leq 245$, $775 \leq x_3 \leq 1225$, $525 \leq x_5 \leq 975$, $275 \leq x_7 \leq 725$, $25 \leq x_9 \leq 475$ (in).

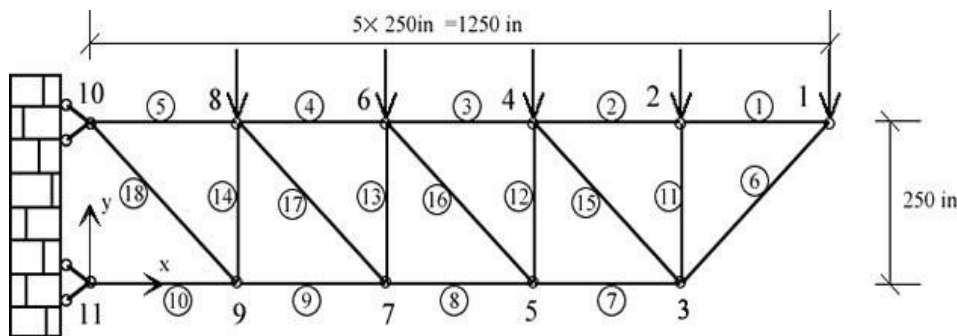


Figure 4. The geometry of the 18-bar planar truss

The optimal weight of 18-bar truss for different population under 30 independent runs is shown in Table 3.

Table 3: Results of sensitivity analysis of the 18-bar truss for 30 independent runs

| PS | Number of structural analyses averaged | Number of best results for structural analyses | Best (lb) | Mean (lb) | Worst (lb) | std. dev |
|----|----------------------------------------|------------------------------------------------|-----------|-----------|------------|----------|
| 20 | 50021 | 50034 | 4543.834 | 4672.787 | 5135.951 | 116.962 |
| 30 | 50030 | 50059 | 4532.538 | 4622.168 | 4815.306 | 64.001 |
| 40 | 50024 | 50029 | 4535.251 | 4590.072 | 4750.639 | 53.406 |
| 50 | 50019 | 50003 | 4526.708 | 4597.752 | 4727.466 | 54.070 |

It is observed from Table 3 that strategy with population size of 50 and number of iterations of 500 produced the best result than other strategies. The standard deviation (std. dev) is relatively large as well, this indicates that computation is easy to trap in local optimum. Similarly, the increase of the population has little impact on the results when the number of structural analyses is about the same. Good global search ability and weak local search ability are also expressed. The best results of TLBO for population size 50 were selected to contrast with those obtained from other algorithms and were shown in Table 4. Figure 5 and Figure 6 is the convergence curves of TLBO and the optimized 18-bar structure respectively.

Table 4: Optimal results of TLBO with ps size 50 for 18-bar truss

| Variables | Rajeev [23] | Hasanqehi [24] | Kaveh [25] | GSO [22] | TLBO |
|-------------|-------------|----------------|------------|----------|----------|
| A_1 | 12.5 | 12.5 | 13 | 12.25 | 12.5 |
| A_2 | 16.25 | 18.25 | 18.25 | 18.25 | 18 |
| A_3 | 8 | 5.5 | 5.5 | 4.75 | 5.25 |
| A_4 | 4 | 3.75 | 3 | 4.25 | 3.75 |
| X_3 | 891.9 | 933 | 913 | 916.9 | 914.524 |
| Y_3 | 145.3 | 188 | 182 | 191.971 | 188.793 |
| X_5 | 610.6 | 658 | 648 | 654.224 | 647.351 |
| Y_5 | 118.2 | 148 | 152 | 156.1 | 149.683 |
| X_7 | 385.4 | 422 | 417 | 423.5 | 416.831 |
| Y_7 | 72.5 | 100 | 103 | 102.571 | 101.332 |
| X_9 | 184.4 | 205 | 204 | 207.519 | 204.165 |
| Y_9 | 23.4 | 32 | 39 | 28.579 | 31.662 |
| Weight (lb) | 4616.800 | 4574.280 | 4566.210 | 4538.768 | 4526.708 |

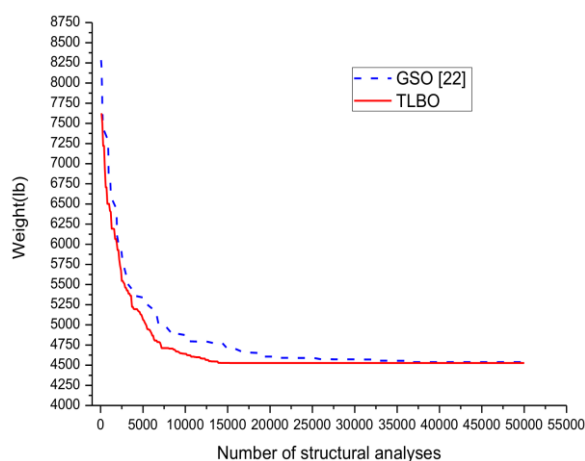


Figure 5. Convergence of TLBO with ps size 50

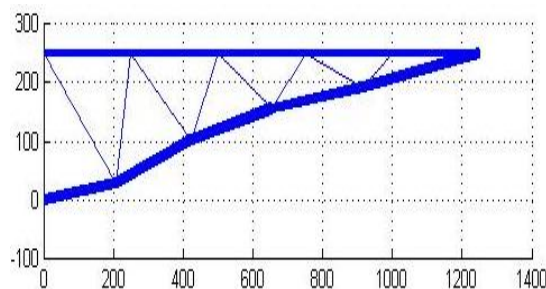


Figure 6. The optimized 18-bar truss

TLBO has almost the same number of structural analyses with GSO. It is obvious from Table 4 that the result of TLBO is the best. It is obvious that the TLBO requires less computation effort to reach convergence and its convergence rate is faster than that of GSO.

4.3. A 25-bar space truss structure (Model I)

The 25-bar space truss is shown in Figure 7. The material density is 70.1 lb/in^3 and the modulus of elasticity is 10000 ksi. The stress of members are limited to ± 40 ksi. Node 1, 2, 3, 4, 5 and 6 are subjected to the displacement limits of ± 0.35 in. The cross-sectional area variables set is $D = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.2, 3.4]$ (in^2). Side constraints for geometry variables are $20 \leq x_4 = x_5 = -x_3 = -x_6 \leq 60$, $40 \leq y_3 = y_4 = -y_5 = -y_6 \leq 80$, $90 \leq z_3 = z_4 = z_5 = z_6 \leq 130$, $40 \leq x_8 = x_9 = -x_7 = -x_{10} \leq 80$, $100 \leq y_7 = y_8 = -y_9 = -y_{10} \leq 140$ (in). Tables 5 and 6 tabulate

the element grouping and loading of the 25-bar space truss, respectively.

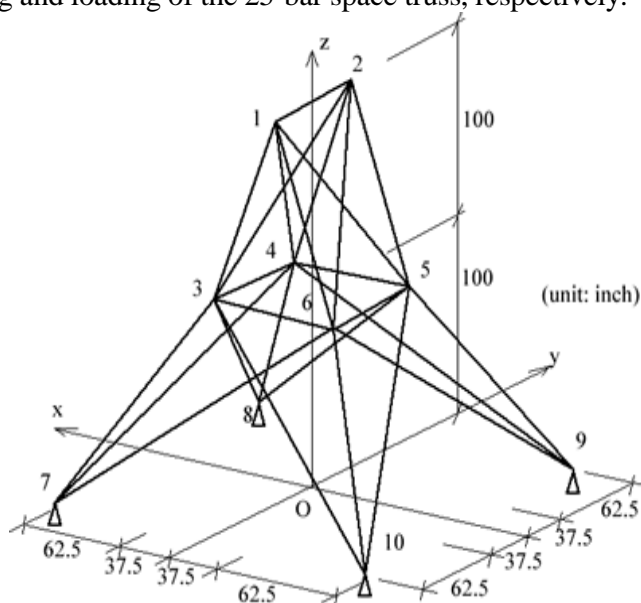


Figure 7. The geometry of the 25-bar truss

Table 5: Details of 25-bar truss

| Variables | Members | End nodes |
|----------------|----------------|-----------------------------|
| A ₁ | 1 | (1,2) |
| A ₂ | 2, 3, 4, | (1,4), (2,3), (1,5), (2,6) |
| A ₃ | 6, 7, 8, 9 | (2,5), (2,4), (1,3), (1,6) |
| A ₄ | 10, 11 | (3,6), (4,5) |
| A ₅ | 12, 13 | (3,4), (5,6) |
| A ₆ | 14, 15, 16, 17 | (3,10), (6,7), (4,9), (5,8) |
| A ₇ | 18, 19, 20, 21 | (3,8), (4,7), (6,9), (5,10) |
| A ₈ | 22, 23, 24, 25 | (3,7), (4,8), (5,9), (6,10) |

Table 6: Load case of 25-bar spatial truss

| Node | F _x | F _y | F _z |
|------|----------------|----------------|----------------|
| | Kips | | |
| 1 | 1.0 | -10.0 | -10.0 |
| 2 | 0.0 | -10.0 | -10.0 |
| 3 | 0.5 | 0.0 | 0.0 |
| 6 | 0.6 | 0.0 | 0.0 |

The optimal weight of 25-bar truss for different population under 30 independent runs is shown in Table 7.

Table 7: Results of sensitivity analysis of 25-bar truss (Model I) for 30 independent runs

| PS | Number of structural analyses averaged | Number of best result for structural analyses | Best (lb) | Mean (lb) | Worst (lb) | std. dev |
|----|----------------------------------------|-----------------------------------------------|-----------|-----------|------------|----------|
| 20 | 50016 | 50012 | 118.852 | 125.079 | 152.404 | 7.199 |
| 30 | 50030 | 50044 | 117.320 | 121.148 | 129.652 | 2.679 |
| 40 | 50011 | 50000 | 117.271 | 120.043 | 126.222 | 1.798 |
| 50 | 50007 | 50004 | 117.258 | 119.846 | 122.697 | 1.594 |

It is observed from Table 7 that strategy with population size of 50 and number of iterations

of 500 produced the best result than other strategies. The standard deviation is reduced and the result is getting better with increasing populations. The best results of TLBO for population size 50 were selected to contrast with those of other algorithms and were shown in Table 8.

Under the same number of structural analyses, the TLBO has the best performance of these algorithms, which is clearly expressed in Table 8. Figure 8 and Figure 9 is the convergence curves of TLBO and the optimal structures respectively.

Table 8: Results of TLBO with ps size 50 for 25-bar truss (Model I)

| Variables | Wu [26] | Kaveh [25] | HPSO [22] | GSO [22] | TLBO |
|-------------|----------|------------|-----------|----------|---------|
| A_1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| A_2 | 0.2 | 0.1 | 0.2 | 0.1 | 0.1 |
| A_3 | 1.1 | 1.1 | 1 | 1 | 1 |
| A_4 | 0.2 | 0.1 | 0.1 | 0.1 | 0.1 |
| A_5 | 0.3 | 0.1 | 0.1 | 0.1 | 0.1 |
| A_6 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| A_7 | 0.2 | 0.1 | 0.1 | 0.2 | 0.1 |
| A_8 | 0.9 | 1 | 1 | 0.9 | 0.9 |
| z_1 | 41.070 | 36.230 | 34.084 | 32.149 | 37.657 |
| x_2 | 53.470 | 58.560 | 50.650 | 52.742 | 54.496 |
| z_2 | 124.600 | 115.590 | 129.978 | 128.230 | 130.000 |
| x_6 | 50.800 | 46.460 | 47.838 | 42.401 | 51.887 |
| z_6 | 131.480 | 127.950 | 129.584 | 132.603 | 139.521 |
| Weight (lb) | 136.1977 | 124.0015 | 124.6025 | 121.3684 | 117.258 |

It is clear from Figure 8 that the TLBO performs better not only in the convergence rate but also in the convergence accuracy.

4.4. A 25-bar space truss structure (Model II)

Model II is extended on the basis of the model I. The Euler stress constraints were added in this case and the buckling coefficient is $\alpha = 12.5$. The optimal weight of 25-bar truss for different population under 30 independent runs is shown in Table 9.

Table 9: Sensitivity analysis of 25-bar truss (Model II) with independent 30 runs

| PS | Number of structural analyses averaged | Number of best result for structural analyses | Best(lb) | Mean(lb) | worst(lb) | std. dev |
|----|----------------------------------------|-----------------------------------------------|----------|----------|-----------|----------|
| 20 | 50017 | 50020 | 226.0832 | 232.943 | 236.7720 | 5.122 |
| 30 | 50026 | 50059 | 226.0832 | 234.543 | 239.8022 | 5.814 |
| 40 | 50035 | 50054 | 226.0833 | 228.185 | 234.2589 | 3.437 |
| 50 | 50052 | 50013 | 226.0832 | 229.903 | 232.4256 | 3.042 |

Table 9 shows that TLBO get the same best solution even if the population is different, and the standard deviation is reduced with the increase of populations.

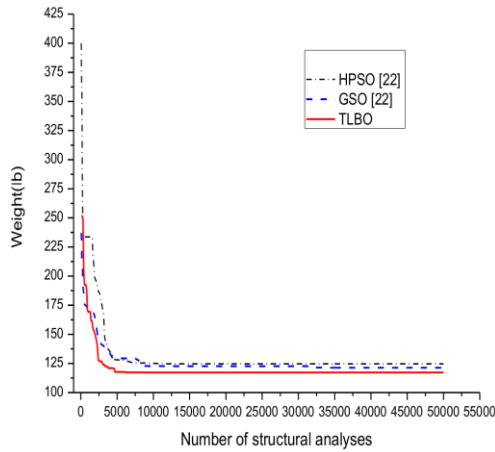


Figure 8. Convergence of TLBO for 25-bar truss (Model I)

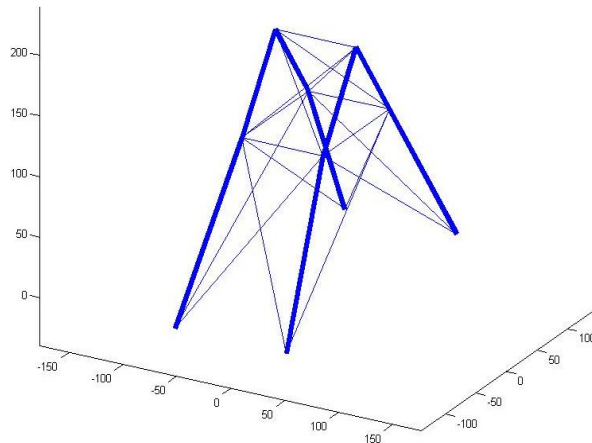


Figure 9. The optimized 25-bar truss (Model I)

The best results of TLBO for population size 50 at iteration times 499 were selected to contrast with those of other algorithms and were shown in Table 10.

Table 10. Optimal results of TLBO for 25-bar truss (Model II) with *ps* size 50

| Variables | Wu [26] | Chuang [27] | HPSO [22] | GSO [22] | TLBO |
|-------------|----------|-------------|-----------|----------|----------|
| A_1 | 0.9 | 0.1 | 0.5 | 0.1 | 0.1 |
| A_2 | 0.8 | 0.9 | 0.9 | 0.9 | 0.9 |
| A_3 | 1.3 | 1.2 | 1.1 | 1.2 | 1.2 |
| A_4 | 0.5 | 0.1 | 0.1 | 0.1 | 0.1 |
| A_5 | 0.3 | 0.2 | 0.2 | 0.2 | 0.2 |
| A_6 | 0.6 | 0.3 | 0.4 | 0.3 | 0.3 |
| A_7 | 1.2 | 0.9 | 1 | 0.9 | 0.9 |
| A_8 | 1.6 | 1.2 | 1.4 | 1.2 | 1.2 |
| z_1 | 22.22 | 20.143 | 20 | 20.758 | 20.143 |
| x_2 | 49.01 | 52.235 | 47.526 | 51.878 | 52.235 |
| z_2 | 106.98 | 97.152 | 105.186 | 96.777 | 97.152 |
| x_6 | 44.6 | 40 | 40 | 40.005 | 40 |
| z_6 | 102.44 | 100 | 100 | 100.051 | 100 |
| Weight (lb) | 301.5968 | 226.0832 | 246.7083 | 226.1846 | 226.0832 |

It is observed from Table 10 that the result of the TLBO is as good as that of Chuang [27] (PSO-SA), and is better than those of other algorithms. Figure 10 and Figure 11 is the convergence curves of TLBO and the optimized 25-bar truss structure (Model II) respectively. Figure 10 demonstrates that the convergence capability of the TLBO is better than the HPSO or the GSO.

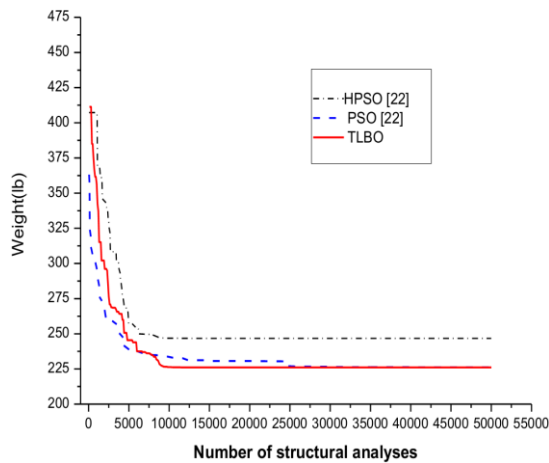


Figure 10. Convergence of TLBO for 25-bar truss (Model II)

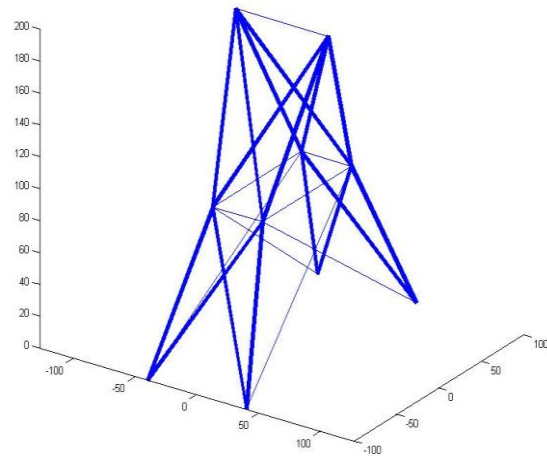


Figure 11. Optimized 25-bar truss (Model II)

From the examples studied in this paper, it is clear that the effects of populations on optimal results is not so pronounced for bigger population size than for small one. It is worth mentioning that a number of numerical results show the design space of size and geometry optimization problem is complex and full of trapping local optima. If topology variables are introduced, the nonlinearity of the design space will increase. Therefore, the future research issues are most related with using energy and force method to improve the efficiency of the algorithm [28].

6. CONCLUSIONS

In this paper, a new algorithm named teaching-learning-based optimization (TLBO) is introduced to solve the size and geometry optimization problem of truss structures. Compared with other intelligent optimization algorithms, the main characteristic of the TLBO is that it is an algorithm-specific parameter-less algorithm. It does not require any algorithm-specific parameters, only the common control parameters are needed. Within the examples considered, the results of TLBO obtained are as good as or better than that of other algorithms in terms of both convergence rate and convergence accuracy.

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