

## IMPROVING THE SEISMIC BEHAVIOR OF NONLINEAR STEEL STRUCTURES USING OPTIMAL MTMDS

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### ABSTRACT

In this research, optimal design and assessment of multiple tuned mass dampers (MTMDs) capability in mitigating the damage of nonlinear steel structures subjected to earthquake excitation has been studied. Optimal parameters of TMDs on nonlinear multi-degree-of-freedom (MDOF) structures have been determined based on minimizing the maximum relative displacement (drift) of structure where for solving the optimization problem the genetic algorithm (GA) has been used successfully. For numerical analysis, three and nine storey 2-D moment resisting nonlinear steel frames subjected to far-field and near-field earthquakes and optimal MTMDs has been designed for different values of mass ratio and TMDs number. According to the results of numerical simulations, it can be said that MTMDs mechanism could reduce the damage of nonlinear steel structures where the effectiveness increases by increasing TMDs mass ratio. Also the performance of MTMDs depends on earthquake characteristics, mass ratio and TMDs configuration where in this research; the effective case has been locating TMDs on top floor in parallel configuration.

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KEY WORDS: Nonlinear steel structure, seismic behavior, multiple tuned mass damper (MTMD), genetic algorithm (GA)

### 1. INTRODUCTION

During past decades structural control systems have been used as alternative to support the structures against lateral loads such as wind and earthquake excitations which includes passive, active, semi-active and hybrid control mechanisms [1]. Tuned mass damper (TMD) as a kind of passive control system has been used extensively in theoretical and

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experimental studies and in some cases in practical applications [2]. Through the researches, it has been shown that while single TMD could be effective in reducing the response of structures, especially under wind induced excitations, but it has some limitations in practical application such as sensitivity to tuning frequency and damping ratio as well as need to a heavy mass and space for installation. To overcome these limitations and enhance the robustness of single TMD, using more than one TMD called multiple tuned mass dampers (MTMDs) has been proposed [3]. MTMDs consists of several tuned mass damper with uniformly or non uniformly distributed natural frequencies where TMDs can be used in different configurations such as parallel, series or distributing over the floors of a building structural system. About assessment the performance of MTMDs on multi-degree-of-freedom (MDOF) structures and its designing, extensive studies have been conducted which results show that the performance of MTMDs depends on the total number of dampers, damping ratio, frequency range selected for designing optimal MTMDs and the distribution of TMDs on the floors [4-6], moreover, MTMDs has been less sensitive to the uncertainty of the system parameters [7-8]. For designing MTMDs on MDOF structures with linear behavior different approaches have been utilized in previous researches such as designing MTMDs using identical distribution for mass and damping of TMDs [3] or different values for the parameters of each TMD [9], determining the optimum parameters of MTMDs for an undamped system based on minimizing the steady-state displacement of the main system [10], dividing MTMDs to several groups which each group consists of several TMDs and distributing each group on different floors [11], designing MTMDs based on tuning to several modes of structure vibration and determining the number of dampers based on the number of controlled modes [12], minimization of the displacement dynamic magnification factor(DDMF) and the acceleration dynamic magnification factor(ADMF) of a structure subjected to ground acceleration [13], designing multiple TMDs to minimize a quadratic performance index by using a gradient based nonlinear programming algorithm to find the optimal parameters of TMDs [14], distributing TMDs vertically and in plan [15-16] and designing optimal MTMDs for linear structures based on defining an optimization problem and minimizing different objective functions using genetic algorithm(GA) to determine the optimal parameters of TMDs [17-18]. Kenarangi and Rofooei [19] studied the effectiveness of TMDs and MTMDs in controlling the nonlinear behavior of 3-D structures by considering the soil-structure interaction effect where the results have shown the dependency of MTMDs performance to soil condition.

Most of researches on application of MTMDs on MDOF structures have been focused on linear structures while in reality under moderate and sever earthquakes, most of structures show nonlinear behavior and may undergo high nonlinearity. Therefore, in designing MTMDs under earthquake excitations, the nonlinear behavior of structures should be considered in design procedure. The research on MTMDs design and assessment its effectiveness on nonlinear structures has been very limited and on the other hand in nonlinear structures the stiffness of the structure changes during lateral deformation, hence the methods developed for linear structures by assuming constant dynamic properties such as stiffness and frequency, could not be used for designing optimal MTMDs for nonlinear structures. Therefore, in this paper, it has been decided to design and study the performance of MTMDs on nonlinear structures. To achieve exact information from this research, steel

moment resisting frames with real nonlinear behavior have been selected as case study also for optimal design of MTMDs on nonlinear steel structures; the method proposed by Mohebbi et al. [17] has been extended to be used. In this method an optimization problem has been defined which considers the parameters of TMDs as design variables and minimizing the maximum relative displacement (drift) of structure as objective function. By solving the optimization problem, the optimum values of TMDs parameters are determined.

## 2. EQUATION OF NONLINEAR STRUCTURE-MTMDs MOTION

For a  $n$ -degree of freedom nonlinear structure equipped with  $N$  TMDs, the equation of motion can be written as follows:

$$M\ddot{X}(t) + F_D(\dot{X}(t)) + F_S(X(t)) = Me\ddot{X}_g \quad (1)$$

where  $t$ =time,  $\ddot{X}_g$ =ground acceleration,  $X$ ,  $\dot{X}$  and  $\ddot{X}$ =displacement, velocity and acceleration vectors relative to ground respectively,  $M=(n+N)\times(n+N)$  mass matrix,  $F_D=(n+N)$ -dimensional vector of damping forces which is a function of velocity,  $F_S=(n+N)$ -dimensional vector of restoring forces which is a function of displacement,  $e=[-1,-1,\dots,-1]^T=(n+N)$  ground acceleration-mass transformation vector.

The equation of motion during the time interval  $((k-1)\Delta t, (k)\Delta t)$  can be considered as:

$$M\Delta\ddot{X}(t) + C^*\Delta\dot{X}(t) + K^*\Delta X(t) = \Delta F(t) \quad (2)$$

where:

$$\Delta\ddot{X}(t) = \ddot{X}_k - \ddot{X}_{k-1} \quad (3)$$

$$\Delta\dot{X}(t) = \dot{X}_k - \dot{X}_{k-1} \quad (4)$$

$$\Delta X(t) = X_k - X_{k-1} \quad (5)$$

$$\Delta F(t) = Me(\ddot{X}_{g_k} - \ddot{X}_{g_{k-1}}) \quad (6)$$

where  $k$ =integration time step,  $C^*$  and  $K^*$ =tangential damping and stiffness matrices at  $t=(k)\Delta t$  respectively. According to TMDs distribution on structure such as series or parallel configuration and structure dynamic properties, damping ( $C^*$ ) and stiffness ( $K^*$ ) matrices at each time step are developed, properly. The equation of motion could be solved using numerical integration methods where in this research the Newmark nonlinear numerical integration method has been used.

### 3. DESIGNING OPTIMAL MTMDS

In this research, following the method proposed by Mohebhi et al. [17] for linear structures, optimal parameters of TMDs has been determined by solving an optimization problem which considers the parameters of TMDs as design variables and minimization of a specified response of structure as objective function. Since in nonlinear structures, the maximum lateral relative displacement of stories (drift) can be considered as an index of structure damage, hence in this paper TMDs is designed to minimize the maximum drift,  $Y_{max}$ , of structure while some constraints on TMDs response and its parameters are applied. Therefore, the optimization problem is defined as follows:

$$Find: m_{d_1}, c_{d_1}, k_{d_1}, \dots, m_{d_{N_{tmd}}}, c_{d_{N_{tmd}}}, k_{d_{N_{tmd}}} \quad i=1, 2, \dots, N \quad (7)$$

$$Minimize: Y_{max} = \max ( |y_k(j)|, k=1, 2, \dots, k_{max} ) , \quad j=1, 2, \dots, n \quad (8)$$

$$Subject \text{ to: } X_{\max(\text{TMD})} \leq X_L \quad (9)$$

$$0 \leq m_{d_i} \leq m_{d_{max}} \quad i=1, 2, \dots, N_{tmd} \quad (10)$$

$$0 \leq k_{d_i} \leq k_{d_{max}} \quad i=1, 2, \dots, N_{tmd} \quad (11)$$

$$0 \leq c_{d_i} \leq c_{d_{max}} \quad i=1, 2, \dots, N_{tmd} \quad (12)$$

where  $y_k(j)$ = drift of structure at each time step( $k$ ),  $k_{max}$  = total number of time steps,  $N$ =number of TMDs ,  $X_L$  = the maximum stroke length of TMDs,  $m_{d_i}$ ,  $c_{d_i}$  and  $k_{d_i}$  are the mass, damping and stiffness of the  $i^{th}$  TMD. Also  $m_{d_{max}}$ ,  $c_{d_{max}}$  and  $k_{d_{max}}$  are the maximum values of TMDs parameters which could be selected by designer.

Solving the nonlinear optimization problem defined for determining TMDs parameters by using the traditional optimization methods, needs an extensive numerical computations. On the other hand Genetic Algorithm (GA) has been extensively used for solving complicated optimization problem in most fields of engineering [20-21] such as designing TMD or MTMDs for linear and nonlinear structures [17, 22-24]. Therefore; it has been decided to use Genetic Algorithm for designing optimal MTMDs for nonlinear steel frames.

### 4. GENETIC ALGORITHM (GA)

When the functions of objective function and the constraints of the optimization problem are not continuous, it is not possible to calculate the gradient of the functions. In this kind of problems, it is not possible to solve the optimization problem using the traditional gradient based methods. An alternative for solving the optimization problem is using genetic algorithms (GAs) developed by Holland [25]. In GAs, a design vector is considered

as a chromosome, its design components as the genes, and its value of the objective function as a measure of the fitness. Chromosomes could be represented by bit strings or real-valued coding. Whilst binary binary-coded GAs are suitable to complex problems, it has been found that using real-valued coding representations for real-valued numerical optimization problems has some advantages such as simple programming, less memory required and greater ability to use different genetic operators [26-27]. In this paper the real-valued coding has been used for representing the chromosomes.

Genetic algorithm includes selection, cross over and mutation operators. In every generation, a set of chromosomes is selected for mating based on their relative fitness. Different methods have been proposed for selection operator for real-valued coding representations of variables where in this paper the stochastic universal sampling method [28] has been used. In this method a number of chromosomes for mating, are selected based on their fitness values in the current population as:

$$P(\mathbf{x}_i) = \frac{F(\mathbf{x}_i)}{\sum_{i=1}^{N_{ind}} F(\mathbf{x}_i)} \quad i=1,2,\dots, N_{ind} \quad (13)$$

where  $F(\mathbf{x}_i)$ =fitness of chromosome  $\mathbf{x}_i$  and  $P(\mathbf{x}_i)$ =probability of selection of  $\mathbf{x}_i$  also  $N_{ind}$ =number of individuals.

Cross over produces new individuals that have some parts of both parents genetic material. For real-valued coding representation of variables different algorithms could be used for cross over. In this paper intermediate cross over method [29] has been used in which each pair of parents can produce two newborns and each newborn can get its genes from their parents with equal probability as follows:

$$O = P_1 + \alpha(P_2 - P_1) \quad (14)$$

where  $P_1$  and  $P_2$  are the parent chromosomes genes,  $O$  is the newborn gene, and  $\alpha$  is a scaling factor chosen randomly over [-0.25, 1.25] interval typically. This method uses a new  $\alpha$  for each pair of parents genes.

To escape from local minima in solving optimization problem, mutation is used in low rate in GAs. To keep the best chromosomes of the current generation from any changing and allow them to go to the next generation, in this paper the elitist strategy has been used.

## 5. NUMERICAL EXAMPLE

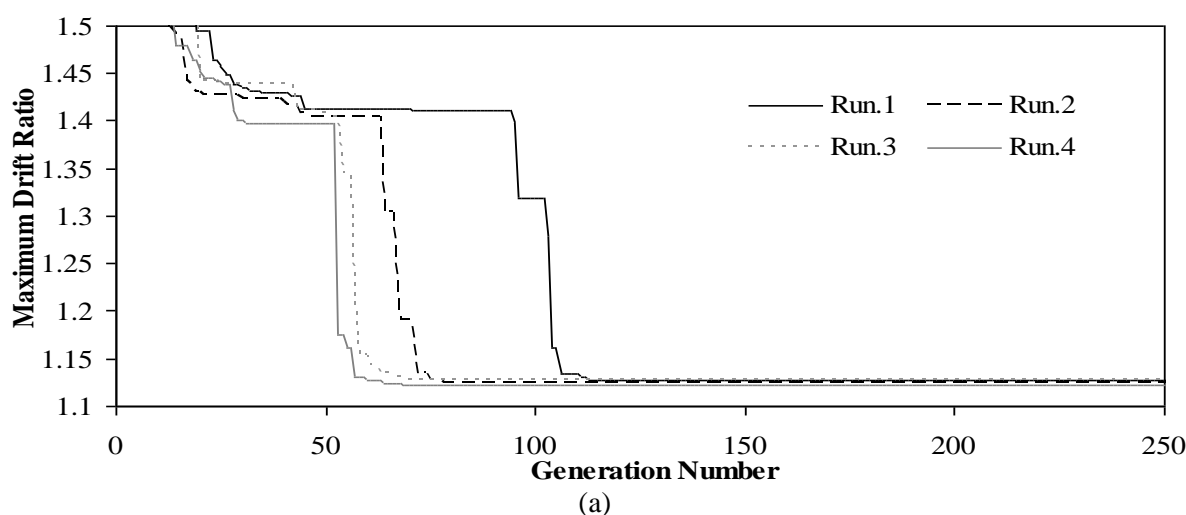
To assess the performance of MTMDs in improving the seismic behavior of nonlinear steel structures, in this paper 2-D moment resisting frames with three and nine storey denoted as SAC-3 and SAC-9 are selected. These frames have been designed for phase II of SAC project and meet the requirements of the 1994 UBC seismic design requirements for Los Angeles, California regions. More details about the structural properties as well as nonlinear behavior of selected frames could be found in [30]. Also, to assess the effect of

design record in performance of MTMDs, El-Centro (1940, PGA=0.34g), and Hachinohe (1968, PGA=0.23g) records as far-field earthquakes as well as Northridge (1994, 0.84g) and Kobe (1995, 0.83g) records as near-field earthquakes have been considered for optimal design of MTMDs.

### 5.1 Designing optimal MTMDs for SAC-3 when $N=3$ and $\mu=3\%$

To explain the procedure of designing optimal MTMDs on nonlinear steel frames, SAC-3 frame subjected to El-Centro excitation and for  $N=3$  and  $\mu=3\%$  optimal TMDs have been designed where TMDs located in parallel configuration at top floor of structure. In this paper to simplify the procedure of designing optimal MTMDs, uniform distribution for TMDs mass has been considered. By assuming a specified value for the total mass ratio,  $\mu$ , optimal values for TMDs stiffness and damping have been determined based on minimizing the maximum drift of structure. Hence, for this case there are 6 variables which should be determined by solving the optimization problem. Genetic Algorithm (GA) has been used for solving the optimization problem with the following parameters:

Number of individuals in each generation=50, Number of elites in each generation =5, Number of the newborns in each generation=50, Mutation rate=0.05. Also the maximum stroke length of TMDs has been assumed  $X_L=100$  cm. To determine the optimum point, different runs have been conducted where in Figure 1(a) the convergence of GA to optimum point for 4 runs has been reported. Also Figure 1(b) shows the value of normalized objective function (maximum drift ratio= $Y_{max}/h$  where  $h$  is the height of storey) of individuals at the final generation. According to the results, it is clear that all runs have the same final optimum answer but with different number of required generations also, at the final generation most of individuals have the same fitness. Therefore, the convergence behavior of design procedure has been shown.



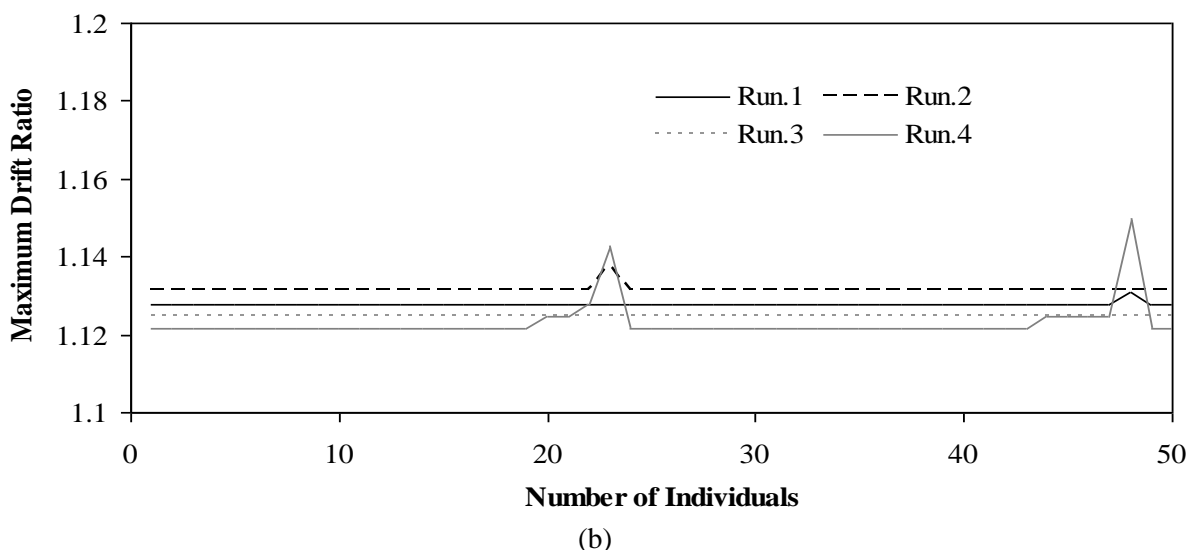


Figure 1. (a) Convergence of GA to optimum answer for  $N=3$  and  $\mu=3\%$ ; (b) Drift ratio for individuals at final generation for  $N=3$  and  $\mu=3\%$

5.2 Designing optimal MTMDs for different mass ratio and TMDs number

Following the same procedure explained for  $N=3$  and  $\mu=3\%$ , optimal MTMDs has been designed for SAC-3 assuming different values for mass ratio and TMDs number under different excitations. The maximum drifts of uncontrolled and controlled structures have been divided to storey height and have been reported as drift ratio in Figures 2-5 under different records. Also Figures 6-9 show the reduction in maximum drift of controlled SAC-3 frame for different mass ratio and TMDs number.

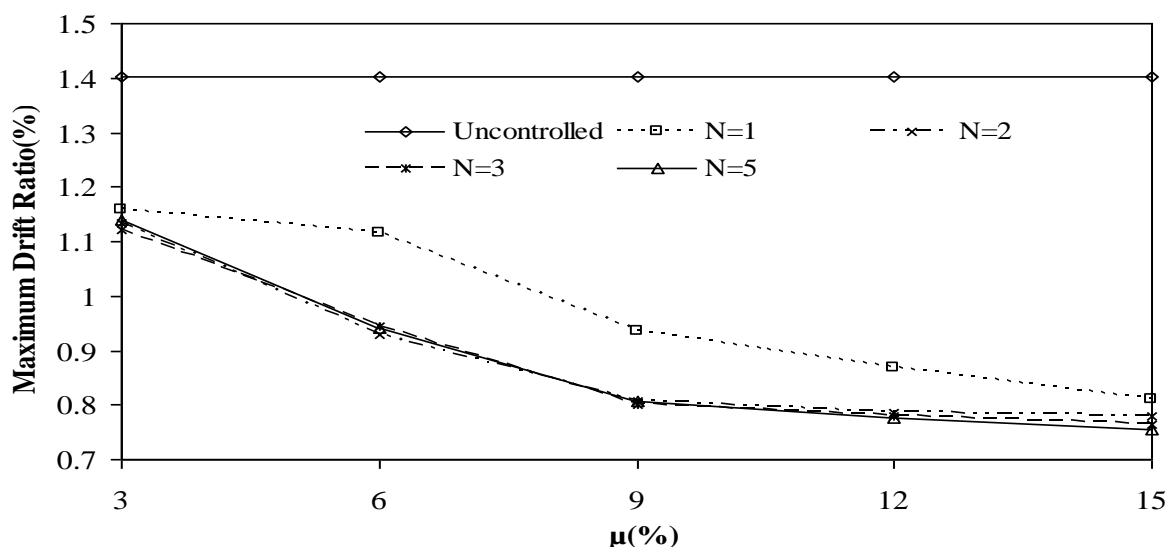


Figure 2. Uncontrolled and controlled structures maximum drift ratio versus different mass ratio and TMDs number under El-Centro excitation

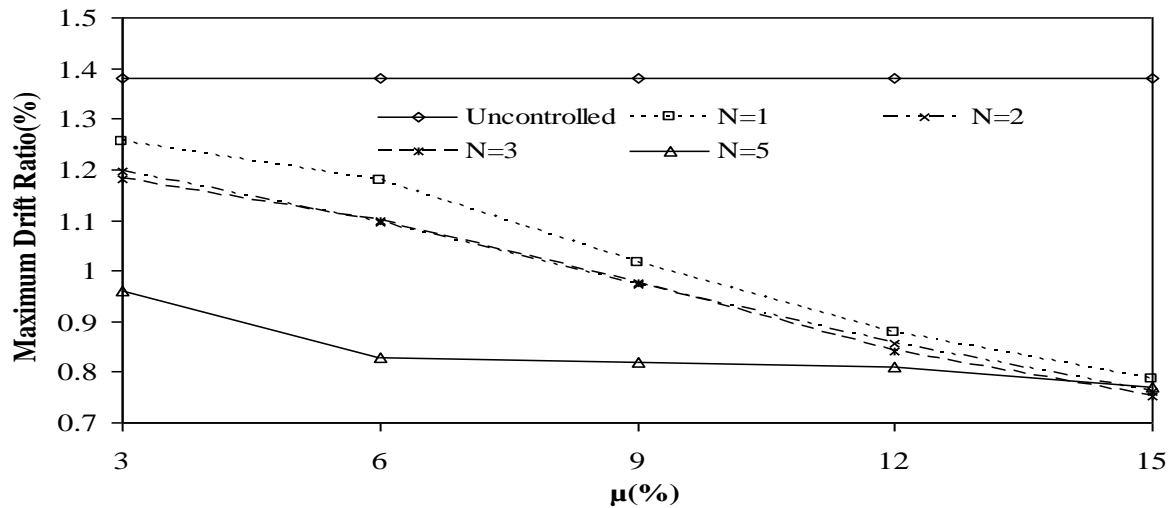


Figure 3. Uncontrolled and controlled structures maximum drift ratio versus different mass ratio and TMDs number under Hachinohe excitation

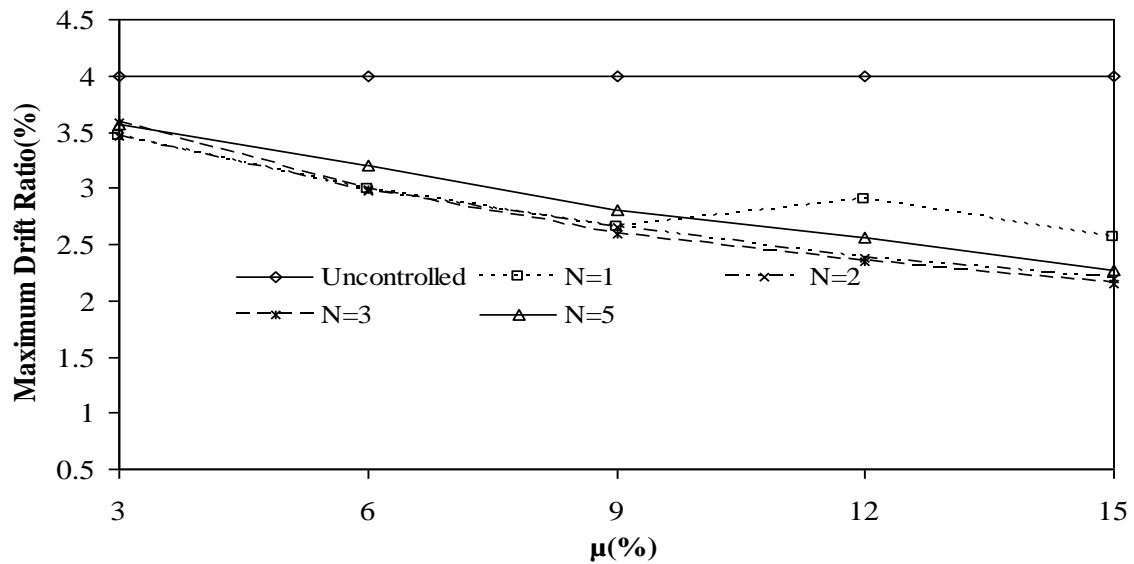


Figure 4. Uncontrolled and controlled structures maximum drift ratio versus different mass ratio and TMDs number under Kobe excitation

Results show that by using MTMDs on nonlinear SAC-3 steel structure the maximum drift could be mitigated where the reduction value depends on TMDs mass ratio and TMDs number as well as excitation record characteristics. Assessment the effect of TMDs mass ratio shows that in case study of this research, increasing the mass ratio has led to more reduction in maximum drift, consequently more reduction in damage of structure. Hence, in designing MTMDs to reduce the damage of the structure to a desired value under a specified excitation, a proper mass ratio should be selected. Also, in most cases MTMDs has worked better than single TMD.



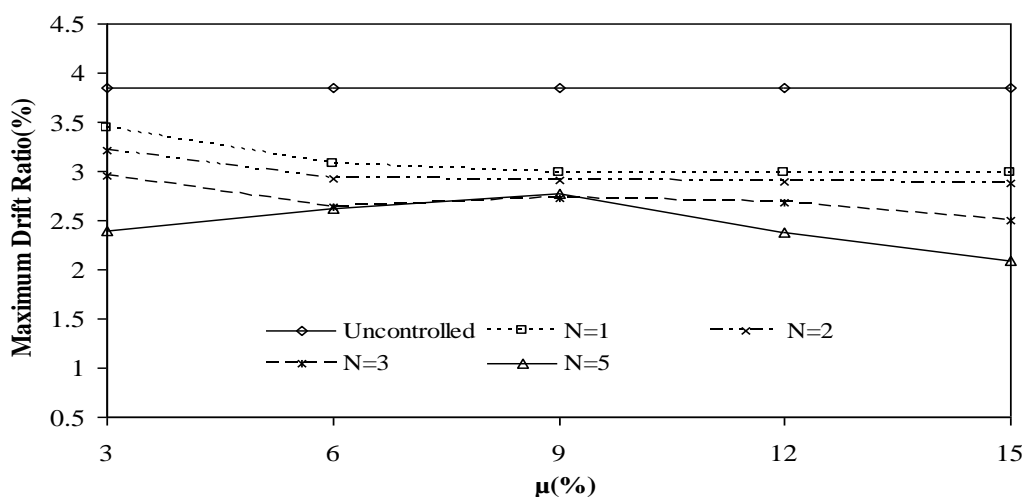


Figure 5. Uncontrolled and controlled structures maximum drift ratio versus different mass ratio and TMDs number under Northridge excitation

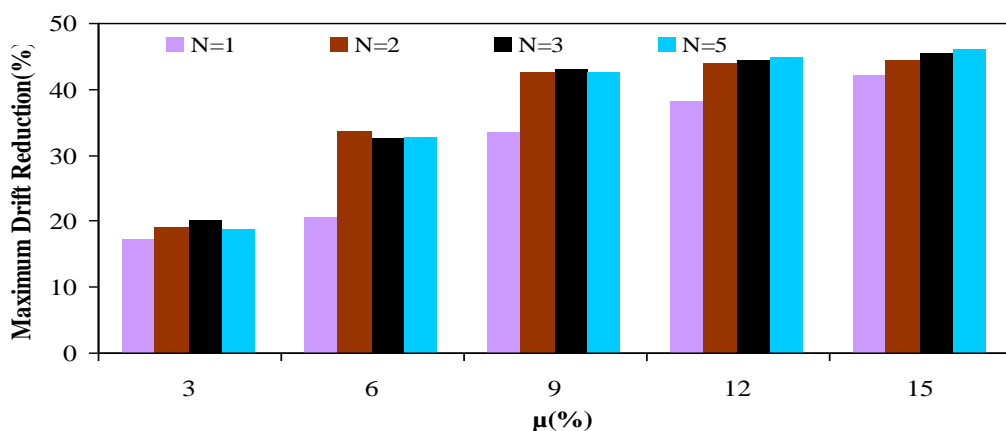


Figure 6. Reduction in maximum drift ratio versus different mass ratio and TMDs number under El-Centro excitation

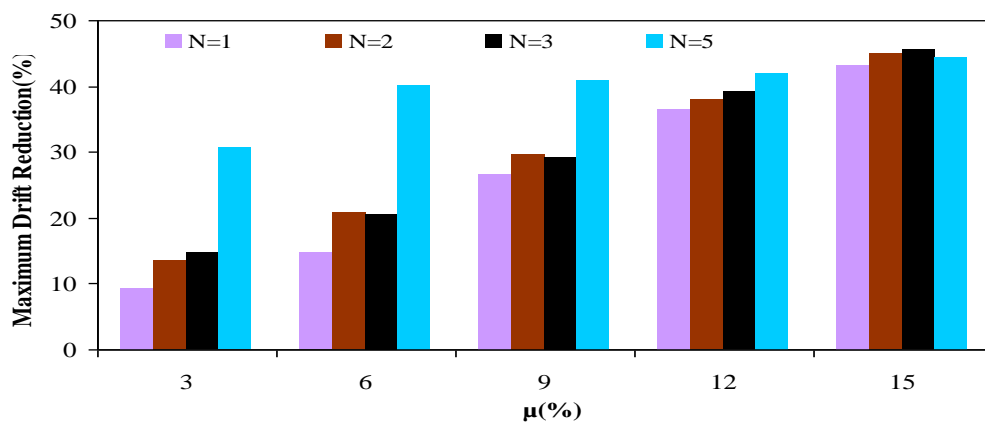


Figure 7. Reduction in maximum drift ratio versus different mass ratio and TMDs number under Hachinohe excitation

Since the performance of MTMDs depends on the input excitation characteristics, hence it can be recommended as design guideline to use the design record of each area when designing optimal MTMDs in a specific area.

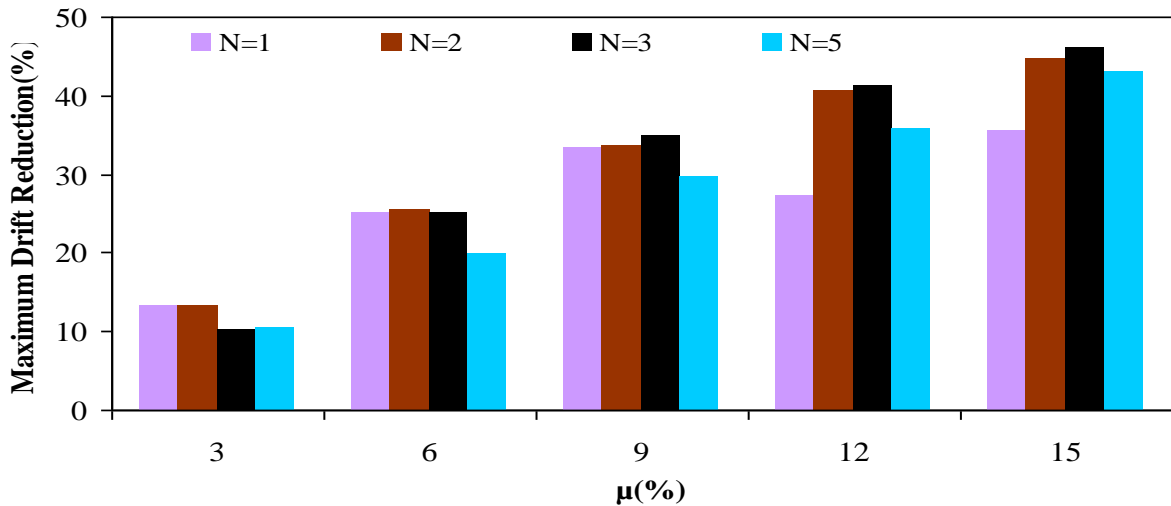


Figure 8. Reduction in maximum drift ratio versus different mass ratio and TMDs number under Kobe excitation

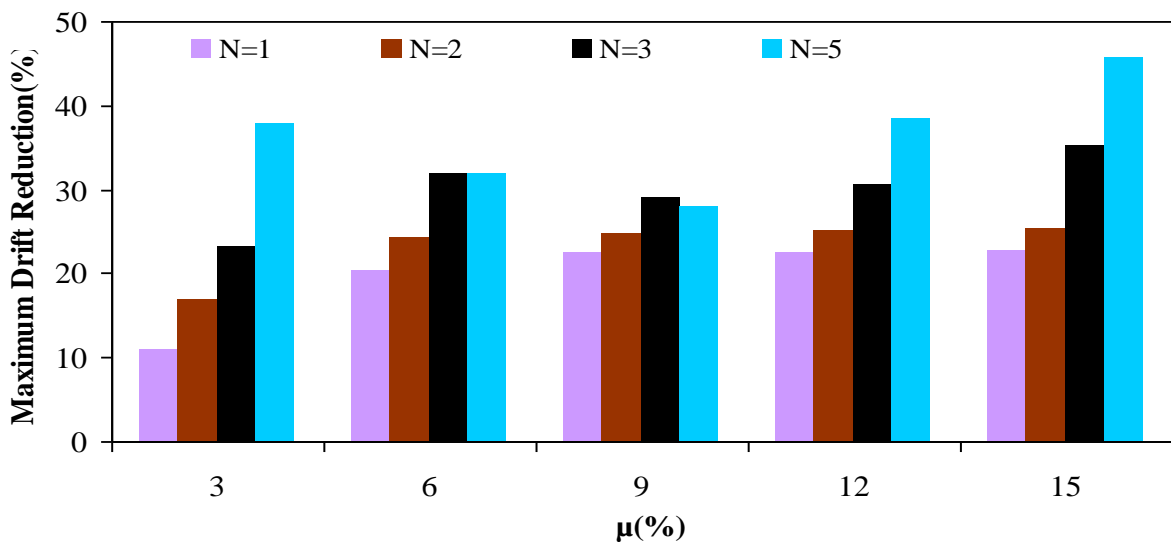


Figure 9. Reduction in maximum drift ratio versus different mass ratio and TMDs number under Northridge excitation

### 5.3. Effect of TMDs configuration

To assess the effect of TMDs configuration on the performance of multiple TMDs, for different values of MTMDs mass ratio,  $\mu$ , optimal MTMDs have been designed for SAC-3 frame for  $N=3$  when one TMD located on each floor. Figures 10-11 compare the reduction in maximum drift of structure for two cases of locating TMDs including parallel

configuration on top floor and distributing TMDs on each floor uniformly.

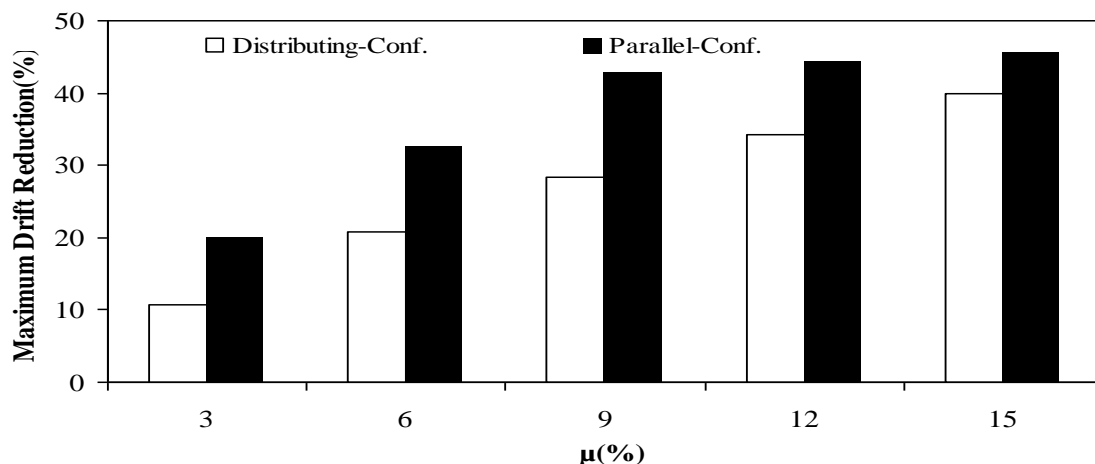


Figure 10. Reduction in maximum drift ratio versus different mass ratio and TMDs number under El-Centro excitation

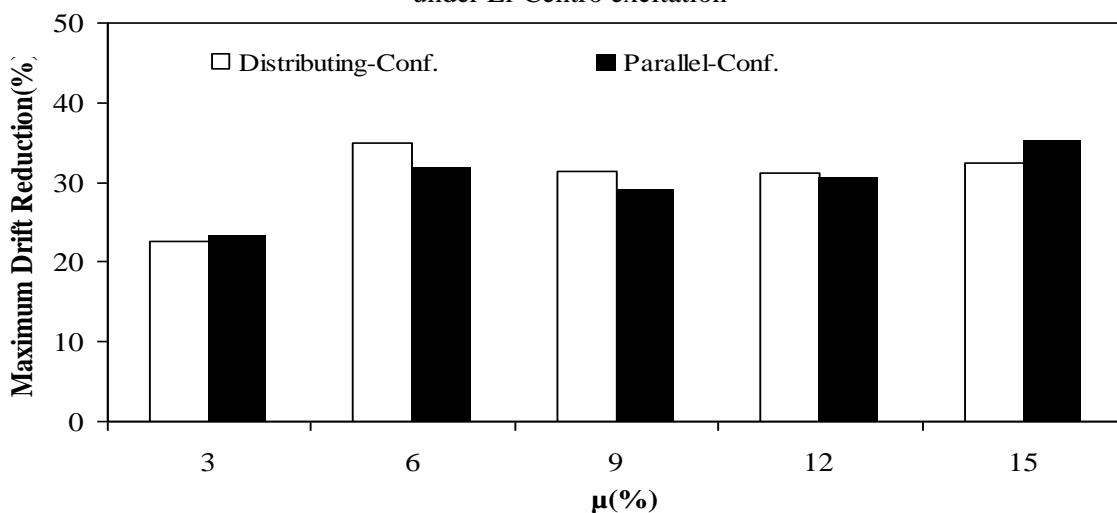


Figure 11. Reduction in maximum drift ratio versus different mass ratio and TMDs number under Northridge excitation

According to the results, it can be said that in this case study under El-Centro excitation, using parallel configuration has helped MTMDs to have more reduction in structure damage while under Northridge excitation there is no significant difference between both cases. Hence, in practical application of MTMDs on nonlinear structures, a proper configuration should be considered for TMDs which based on the results of this research and previous studies [17], using parallel configuration of TMDs on top floor could be recommended for most cases.

#### 5.4 Designing Optimal MTMDs for SAC-9

As the second example, to study the characteristics of MTMDs on different steel structures,

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SAC-9 frame subjected to El-Centro excitation and for different number of TMDs optimal MTMDs has been designed when  $\mu = 3\%$  and  $12\%$ . In Table 1 the maximum drift ratio of controlled structure and reduction in maximum drift have been reported while the maximum drift ratio of uncontrolled structure has been  $1.27\%$ . Results shows that similar to SAC-3, using MTMDs could be effective in reducing the damage of nonlinear steel structure which the performance of MTMDs has been affected by mass ratio. Also comparing the reduction obtained for SAC-9 with SAC-3 shows that for  $\mu = 3\%$ , MTMDs has worked better in SAC-9 while for  $\mu = 12\%$ , the results have been similar.

Table 1: Maximum drift ratio (%) and reduction in maximum drift (%) of SAC-9 under El-Centro excitation

$\mu(\%)$	$N=1$		$N=2$		$N=5$	
	Drift Ratio	Reduction	Drift Ratio	Reduction	Drift Ratio	Reduction
3%	0.95	25.24	0.90	28.81	0.91	28.16
12%	0.76	39.68	0.70	44.55	0.71	44.1

## 6. CONCLUSIONS

Optimal design of multiple tuned mass dampers (MTMDs) and assessment its effectiveness in reducing the maximum drift of nonlinear steel structures has been studied in this research. For designing optimal MTMDs for nonlinear frames, a method has been developed in which the parameters of TMDs are determined so that the maximum drift of structure is minimized. For solving the optimization problem genetic algorithms (GAs) has been used. To explain the design procedure of MTMDs and evaluate the performance of MTMDs on nonlinear steel structures, two three and nine storey 2-D nonlinear steel moment resisting frames have been considered. For different values of MTMDs mass ratio and TMDs number, optimal MTMDs has been designed when the structures subjected to real near-field and far-field excitations which have different characteristics. The results of numerical simulations show the simplicity of the design method and also the capability of GA in solving nonlinear optimization problem. Assessment the reduction obtained in maximum drift shows that MTMDs could be effective in mitigating the damage of nonlinear steel frames where the performance has been affected significantly by mass ratio and input excitation. According to designing different MTMDs for both frames using different number of TMDs, it has been concluded that MTMDs has worked better than single TMD in most cases. To evaluate the effect of TMDs configuration in performance of MTMDs, for both parallel and distributing configurations of TMDs, optimal MTMDs has been designed for different values of mass ratio which the results have shown the effectiveness of parallel configuration in this case study.

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