



## **PREDICTION OF SLOPE STABILITY STATE FOR CIRCULAR FAILURE: A HYBRID SUPPORT VECTOR MACHINE WITH HARMONY SEARCH ALGORITHM**

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### **ABSTRACT**

The slope stability analysis is routinely performed by engineers to estimate the stability of river training works, road embankments, embankment dams, excavations and retaining walls. This paper presents a new approach to build a model for the prediction of slope stability state. The support vector machine (SVM) is a new machine learning method based on statistical learning theory, which can solve the classification problem with small sampling, non-linearity and high dimension. However, the practicability of the SVM is influenced by the difficulty of selecting appropriate SVM parameters. In this study, the proposed hybrid harmony search (HS) with SVM was applied for the prediction of slope stability state, in which HS was used to determine the optimized free parameters of the SVM. A dataset that includes 55 data points was applied in current study, while 45 data points (80%) were used for constructing the model and the remainder data points (10 data points) were used for assessment of degree of accuracy and robustness. The results obtained indicate that the SVM-HS model can be used successfully for the prediction of slope stability state for circular failure.

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**KEY WORDS:** slope stability state; harmony search; support vector machine.

### **1. INTRODUCTION**

Slope stability analysis is an important area in geotechnical engineering. Most textbooks on soil mechanics include several methods of slope stability analysis. A detailed review of

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equilibrium methods of slope stability analysis is presented by Duncan [1]. These methods include the ordinary method of slices, force equilibrium methods, Morgenstern and Price's method; Janbu's generalized procedure of slices, Spencer's method and Bishop's modified method. These methods, in general, require the soil mass to be divided into slices. The directions of the forces acting on each slice in the slope are assumed. This assumption is a key role in distinguishing one limit equilibrium method (LEM) from another. The LEMs require a continuous surface passes the soil mass. This surface is essential in calculating the minimum safety factor (SF) against sliding or shear failure [2-4]. Generally, LEMs were developed before the advent of computers; computationally complex methods followed later. These methods require data about the geometrical parameters and the strength parameters of the soil. In LEMs, slope stability is analyzed by first computing the SF. The SF is defined as the ratio of reaction over action, expressed in terms of moments or forces, and eventually in terms of stresses, depending on the geometry of the assumed slip surface. In circular mechanisms of failure, SF is defined in terms of moments about the center of the failure arc, as the ratio of the moment of shear strength along the failure arc over the moment of weight of the failure mass. With the development of cheaper personal computer, numerical methods have been increasingly used in slope stability analysis [5, 6]. Also, during the last years, intelligence system approaches such as support vector machine (SVM) have been used successfully, often in different separation and technological applications, mainly due to its powerfulness modeling and classification. SVM is a popular pattern classification method with many diverse applications is an emerging data classification technique proposed by Vapnik [7], and has been widely adopted in various fields of classification problems in recent years [8, 9].

Kernel parameter setting in the SVM training procedure, along with the feature selection, significantly influences the classification accuracy. Several kernel functions help the SVM in obtaining the optimal solution. The most frequently used such kernel functions are the polynomial, sigmoid and radial basis kernel function (RBF) [10]. The RBF is generally applied most frequently, because it can classify multi-dimensional data, unlike a linear kernel function. Additionally, the RBF has fewer parameters to set than a polynomial kernel. RBF and other kernel functions have similar overall performance. Consequently, RBF is an effective option for kernel function. Therefore, this study applies an RBF kernel function in the SVM to obtain optimal solution. Two major RBF parameters applied in SVM,  $C$  and  $\sigma$ , must be set appropriately. Parameter  $C$  represents the cost of the penalty. The choice of value for  $C$  influences on the classification outcome. If  $C$  is too large, then the classification accuracy rate is very high in the training phase, but very low in the testing phase. If  $C$  is too small, then the classification accuracy rate unsatisfactory, making the model useless. Parameter  $\sigma$  has a much greater influence on classification outcomes than  $C$ , because its value affects the partitioning outcome in the feature space. An excessively large value for parameter  $\sigma$  results in over-fitting, while a disproportionately small value leads to under-fitting [11].

The value of parameters  $C$  and  $\sigma$  that lead to the highest classification accuracy rate in this interval can be found by setting appropriate values for the upper and lower bounds (the search interval) and the jumping interval in the search. Meta-heuristic approaches are commonly employed to help in looking for the best feature subset, such as genetic algorithm (GA) [12, 13], particle swarm optimization (PSO) [14, 15], Grid search [16, 17], and ant colony optimization (ACO) [18, 19].

In the present paper, for the achievement of the above-mentioned purpose, a fast, robust and easy to used method so-called HS is applied as the searching strategy for finding the optimal value of user-defined parameters.

In this paper, the HS is capable of improving the performance of SVM through determining their free parameters. Integration of SVM model and HS algorithm produced a model, which can to predict of slope stability state for circular failure with good precision.

## 2. METHODOLOGY

### 2.1 Support vector regression

The purpose of the SVM classification is to find optimal separating hyperplane by maximizing the margin between the separating hyperplane and the data. Assume that, a set of data  $T = \{x_i, y_i\}_{i=1}^m$  is given, where,  $x_i$  denotes the input vectors,  $y_i \in \{+1, -1\}$  stands for two classes, and  $m$  is the sample number. It is possible to determine the hyperplane  $f(x)=0$  that separates the given data when two classes are linearly separable:

$$f(x) = w \cdot x + b = 0 \tag{1}$$

where,  $w$  denotes the weight vector, and  $b$  denotes the bias term.  $w$  and  $b$  are used to define the position of separating hyperplane. The separating hyperplane should be satisfying the constraints:

$$y_i f(x_i) = y_i (w \cdot x_i + b) \geq 1, \quad i = 1, 2, \dots, m \tag{2}$$

Positive slack variables  $\xi_i$  are introduced to measure the distance between the margin and the vectors  $x_i$  that lying on the wrong side of the margin. Then, the optimal hyperplane separating the data can be obtained by the following optimization problem:

$$\begin{aligned} & \underset{w, b, \xi}{\text{Minimize}} \quad \frac{1}{2} \|w\|^2 + C \sum_{i=1}^m \xi_i, \quad i = 1, \dots, m \\ & \text{Subject to:} \quad \begin{cases} y_i (w \cdot x_i + b) + \xi_i - 1 \geq 0 \\ \xi_i \geq 0. \end{cases} \end{aligned} \tag{3}$$

where,  $C$  is the error penalty. By the lagrangian multipliers  $\alpha_i$  introduced, the above mentioned optimization problem is transformed into the dual quadratic optimization problem, that is:

$$\begin{aligned} & \text{Maximize} \quad L(\alpha) = \sum_{i=1}^m \alpha_i - \frac{1}{2} \sum_{i,j=1}^m \alpha_i \alpha_j y_i y_j (x_i \cdot x_j) \\ & \text{Subject to} \quad \sum_{i=1}^m \alpha_i y_i = 0, \quad \alpha_i \geq 0, \quad i = 1, \dots, m \end{aligned} \tag{4}$$

Thus, the linear decision function is created by solving the dual optimization problem, which is defined as:

$$f(x) = \text{sign} \left( \sum_{i,j=1}^m \alpha_i y_i (x_i x_j) + b \right) \quad (5)$$

The SVM can also be used in non-linear classification by using kernel function. By using the non-linear mapping function  $\varphi(\bullet)$  the original data  $x$  is mapped into a high-dimensional feature space, where the linear classification is possible. The non-linear decision function is:

$$f(x) = \text{sign} \left( \sum_{i,j=1}^m \alpha_i y_i K(x_i x_j) \right) + b \quad (6)$$

Where,  $K(x_i x_j)$  is called the kernel function,  $K(x_i x_j) = \varphi(x_i) \varphi(x_j)$ . The SVM constructed by radial basis function ( $K(x_i x_j) = \exp(-\|x_i - x_j\|/2\sigma^2)$ ) has excellent non-linear classification ability.

## 2.2 Harmony search

In order to explain the harmony Search (HS) in more detail, first idealize the improvisation process by a skilled musician. When a musician is improvising, he or she has three possible choices: (1) play any famous piece of music (a series of pitches in harmony) exactly from his or her memory; (2) play something similar to a known piece (thus adjusting the pitch slightly); or (3) compose new or random notes [20]. Geem et al. [21, 22] formalized these three options into quantitative optimization process, and the three corresponding components become: usage of harmony memory, pitch adjusting, and randomization [21, 22]. The usage of harmony memory is important, as it is similar to the choice of the best-fit individuals in the GA. This will ensure that the best harmonies will be carried over to the new harmony memory. In order to use this memory more effectively, it is typically assigned as a parameter  $r_{\text{accept}} \in [0, 1]$ , called harmony memory accepting or considering rate. If this rate is too low, only few best harmonies are selected and it may converge too slowly. If this rate is extremely high (near 1), almost all the harmonies are used in the harmony memory, then other harmonies are not explored well, leading to potentially wrong solutions.

The second component is the pitch adjustment determined by a pitch bandwidth  $b_{\text{range}}$  and a pitch adjusting rate  $r_{\text{pa}}$ . Though in music, pitch adjustment means to change the frequencies, it corresponds to generate a slightly different solution in the HS algorithm [21, 22]. In theory, the pitch can be adjusted linearly or nonlinearly, but in practice, linear adjustment is used.

$$x_{\text{new}} = x_{\text{old}} + b_{\text{range}} \times \mathcal{E} \quad (7)$$

Where  $x_{\text{old}}$  is the existing pitch or solution from the harmony memory, and  $x_{\text{new}}$  is the new

pitch after the pitch adjusting action. This essentially produces a new solution around the existing quality solution by varying the pitch slightly by a small random amount [21-23]. Here  $\varepsilon$  is a random number generator in the range of [-1,1]. Pitch adjustment is similar to the mutation operator in genetic algorithms. We can assign a pitch-adjusting rate ( $r_{pa}$ ) to control the degree of the adjustment. A low pitch adjusting rate with a narrow bandwidth can slow down the convergence of HS because the limitation in the exploration of only a small subspace of the whole search space. On the other hand, a very high pitch-adjusting rate with a wide bandwidth may cause the solution to scatter around some potential optima as in a random search. Thus, we usually use  $r_{pa}=0.1\sim 0.5$  in most applications. The third component is the randomization, which is to increase the diversity of the solutions. Although adjusting pitch has a similar role, but it is limited to certain local pitch adjustment and thus corresponds to a local search. The use of randomization can drive the system further to explore various diverse solutions so as to find the global optimality [20]. The three components in harmony search can be summarized as the pseudo code shown in Fig. 1. In this pseudo code, we can see that the probability of randomization is

$$P_{random} = 1 - r_{accept} \tag{8}$$

and the actual probability of adjusting pitches is

$$P_{pitch} = r_{accept} \times r_{pa} \tag{9}$$

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Harmony Search

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begin
  Objective function  $f(\mathbf{x})$ ,  $\mathbf{x}=(x_1, x_2, \dots, x_d)^T$ 
  Generate initial harmonics (real number arrays)
  Define pitch adjusting rate ( $r_{pa}$ ), pitch limits and bandwidth
  Define harmony memory accepting rate ( $r_{accept}$ )
  while (  $t < \text{Max number of iterations}$  )
    Generate new harmonics by accepting best harmonics
    Adjust pitch to get new harmonics (solutions)
    if (  $\text{rand} > r_{accept}$  ), choose an existing harmonic randomly
    else if (  $\text{rand} > r_{pa}$  ), adjust the pitch randomly within limits
    else generate new harmonics via randomization
    end if
    Accept the new harmonics (solutions) if better
  end while
  Find the current best solutions
end

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Figure 1. Pseudo code of the Harmony Search algorithm [20]

The HS was used in different applications by various researchers worldwide [24-27].

### 2.3 Parameters optimization of the support vector machine based on harmony search

The selection of the parameters  $C$  and  $\sigma$  has a great influence on the performance of the SVM. In this study, the HS was used to determine the optimized  $C$  and  $\sigma$ . Fig. 2 presents the

process of optimizing the SVM parameters with the HS.

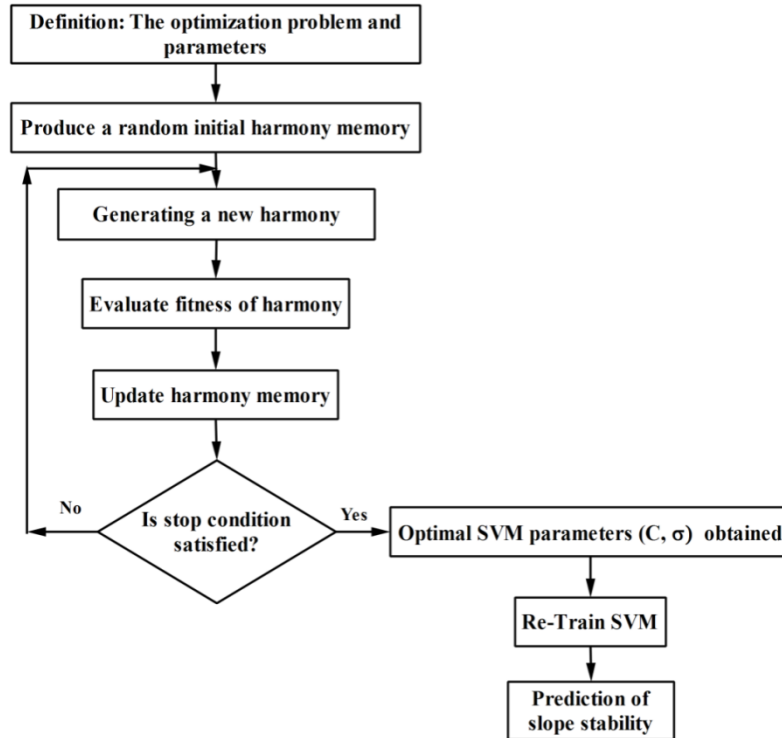


Figure 2. The process of optimizing the SVM parameters with the HS

As shown in Fig. 2, in step 4, the SVM model is trained with the parameters  $C$  and  $\sigma$  included in current harmony. The fivefold cross validation, which offers the best compromise between computational cost and reliable parameter estimates, is used to evaluate fitness. In fivefold cross validation, the training data set is randomly divided into 5 mutually exclusive subsets of approximately equal size, in which 4 subsets are used as the training set and the last subset is used as validation. The above-mentioned procedure is repeated 5 times, so that each subset is used once for validation. The fitness function is defined as the  $1-CA_{validation}$  of the fivefold cross validation method on the training data set, which is shown in Eq. (10). The solution with a bigger  $CA_{validation}$  has a smaller fitness value.

$$Fitness = 1 - CA_{validation} = 1 - \frac{1}{5} \sum_{i=1}^5 \left| \frac{y_c}{y_c + y_f} \right| \times 100\% \quad (10)$$

where,  $y_c$  and  $y_f$  represent the number of true and false classifications, respectively.

### 3. INPUTS AND OUTPUT DATA

The main scope of this work is to implement the above methodology in the problem of the

slope stability state prediction for circular failure.

The input data for the SVM-HS model has been taken from information published by Sah et al. [28] and Xu and Xie [29]. The input layer data consists of six input parameters in the case of circular failure. The parameters that have been selected are related to the geometry and the geotechnical properties of each slope. More specifically, the parameters used for circular failure were height (H), cohesion (c), slope angle ( $\beta$ ), unit weight ( $\gamma$ ), pore water pressure ( $r_u$ ), and angle of internal friction ( $\phi$ ).

A total of 55 circular slopes were considered. Of the 46 cases from Sah et al. [28], 30 failed and 16 were stable; of the 9 cases from Xu and Xie [29], 2 failed and 7 were stable.

The original data covering the 55 case studies are presented in Table 1 that 45 cases were used for training and 10 cases were used for testing.

Table 1: Circular slope failures reported by Sah et al. [28]\*, and Xu and Xie [29]\*\*

Case No.	Input parameters						Output parameter	Location
	$\gamma$ (kN/m <sup>3</sup> )	C (kPa)	$\phi$ (°)	$\beta$ (°)	H (m)	$r_u$	Failed/Stable***	
1	18.68	26.34	15	35	8.23	0	Failed=0	Congress street, open cut slope, Chicago, USA
2	16.5	11.49	0	30	3.66	0	Failed=0	Brightlingsea slide UK
3	18.84	14.36	25	20	30.5	0	Stable=1	Unknown
4	18.84	57.46	20	20	30.5	0	Stable=1	Unknown
5	28.44	29.42	35	35	100	0	Stable=1	Case 1: open pit iron ore mine, India
6	28.44	39.23	38	35	100	0	Stable=1	Case 2: open pit iron ore mine, India
7	20.6	16.28	26.5	30	40	0	Failed=0	Open pit chromite mine, Orissa, India
8	14.8	0	17	20	50	0	Failed=0	Sarukuygi landslide, Japan
9	14	11.97	26	30	88	0	Failed=0	Case 1: open pit iron ore mine, Goa, India
10	25	120	45	53	120	0	Stable=1	Mercoirol open pit coal mine, France
11	26	150.05	45	50	200	0	Stable=1	Marquesade open pit iron ore mine, Spain
12	18.5	25	0	30	6	0	Failed=0	Unknown
13	18.5	12	0	30	6	0	Failed=0	Unknown
14	22.4	10	35	30	10	0	Stable=1	Case 1: Highvale coal mine, Alberta, Canada
15	21.4	10	30.34	30	20	0	Stable=1	Case 2: Highvale coal mine, Alberta, Canada
16	22	20	36	45	50	0	Failed=0	Case 1: open pit coal mine, Newcastle coalfield, Australia
17	22	0	36	45	50	0	Failed=0	Case 2: open pit coal mine,

18	12	0	30	35	4	0	Stable=1	Newcastle coalfield, Australia
19	12	0	30	45	8	0	Failed=0	Unknown
20	12	0	30	45	4	0	Stable=1	Unknown
21	12	0	30	45	8	0	Failed=0	Unknown
22	23.47	0	32	37	214	0	Failed=0	Pima open pit mine, Arizona, USA
23	16	70	20	40	115	0	Failed=0	Case 1: Wyoming, USA
24	20.41	33.52	11	16	10.67	0.35	Stable=1	Seven Sisters Landslide, UK
25	19.63	11.97	20	22	12.19	0.405	Failed=0	Case 1: The Northolt slide, UK
26	21.82	8.62	32	28	12.8	0.49	Failed=0	Selset Landslide, Yorkshire, UK
27	20.41	33.52	11	16	45.72	0.2	Failed=0	Saskatchewan dam, Canada
28	18.84	15.32	30	25	10.67	0.38	Stable=1	Case 2: The Northolt slide, UK
29	18.84	0	20	20	7.62	0.45	Failed=0	Sudbury slide, UK
30	21.43	0	20	20	61	0.5	Failed=0	Folkstone Warren slide, Kent, UK
31	19.06	11.71	28	35	21	0.11	Failed=0	River bank side, Alberta, Canada
32	18.84	14.36	25	20	30.5	0.45	Failed=0	Unknown
33	21.51	6.94	30	31	76.81	0.38	Failed=0	Unknown
34	14	11.97	26	30	88	0.45	Failed=0	Case 2: open pit iron ore mine, Goa, India
35	18	24	30.15	45	20	0.12	Failed=0	Athens slope, Greece
36	23	0	20	20	100	0.3	Failed=0	Open pit coal mine Allori coalfield, Italy
37	22.4	100	45	45	15	0.25	Stable=1	Case 1: open pit coal mine, Alberta, Canada
38	22.4	10	35	45	10	0.4	Failed=0	Case 2: open pit coal mine, Alberta, Canada
39	20	20	36	45	50	0.25	Failed=0	Case 3: open pit coal mine, Newcastle coalfield, Australia
40	20	20	36	45	50	0.5	Failed=0	Case 4: open pit coal mine, Newcastle coalfield, Australia
41	20	0	36	45	50	0.25	Failed=0	Case 5: open pit coal mine, Newcastle coalfield, Australia
42	20	0	36	45	50	0.5	Failed=0	Case 6: open pit coal mine, Newcastle coalfield, Australia



43	22	0	40	33	8	0.35	Stable=1	Case 1: Harbour slope, Newcastle, Australia
44	24	0	40	33	8	0.3	Stable=1	Case 2: Harbour slope, Newcastle, Australia
45	20	0	24.5	20	8	0.35	Stable=1	Case 3: Harbour slope, Newcastle, Australia
46	18	5	30	20	8	0.3	Stable=1	Case 4: Harbour slope, Newcastle, Australia
47	26.49	150	33	45	73	0.15	Stable=1	China
48	26.70	150	33	50	130	0.25	Stable=1	China
49	26.89	150	33	52	120	0.25	Stable=1	China
50	26.57	300	38.7	45.3	80	0.15	Failed=0	China
51	26.78	300	38.7	54	155	0.25	Failed=0	China
52	26.81	200	35	58	138	0.25	Stable=1	China
53	26.43	50	26.6	40	92.2	0.15	Stable=1	China
54	26.7	50	26.6	50	170	0.25	Stable=1	China
55	26.8	60	28.8	59	108	0.25	Stable=1	China

\*Case 1–46. \*\*Case 47–55.

\*\*\*Mapping the state of failure has been considered as two possibilities: failed and stable, which are respectively represented by two state values: 0 and 1.

#### 4. PREDICTION OF SLOPE STABILITY STATE FOR CIRCULAR FAILURE USING HYBRID SVM-HS MODEL

Popular methods in the SVM multi-class classification are including: ‘One against-all’, ‘one-against-one’ and ‘binary tree’. The ‘binary tree’ SVM classification algorithm needs only  $k - 1$  two-class SVM classifiers for a case of  $k$  classes, while the ‘one-against-all’ SVM classification algorithm needs  $k$  two-class SVM classifiers where each one is trained with all of samples and the ‘one-against-one’ SVM classification algorithm needs  $k(k - 1)/2$  two-class SVM classifiers where each one is trained on data from two classes [14]. Obviously less two-class classifiers expedite the rate of training and recognition. In this paper, the ‘one-against-all’ SVM classification algorithm is adopted for prediction and classification of the slope stability state (Failed/Stable), using the MATLAB environment (Fig. 3).

As it can be seen in Fig. 3, height ( $H$ ), cohesion ( $c$ ), slope angle ( $\beta$ ), unit weight ( $\gamma$ ), pore water pressure ( $r_u$ ), and angle of internal friction ( $\phi$ ) were defined as input parameters into the SVM-HS model and the slope stability state (Failed/Stable) as output.

The total data (55 data sets) are divided into two data sets: the training data (39 data sets) and the testing data (16 data sets), in which the training data sets are used to calculate the fitness function and train the SVM, and the testing data sets are used to examine the classification and prediction accuracy.

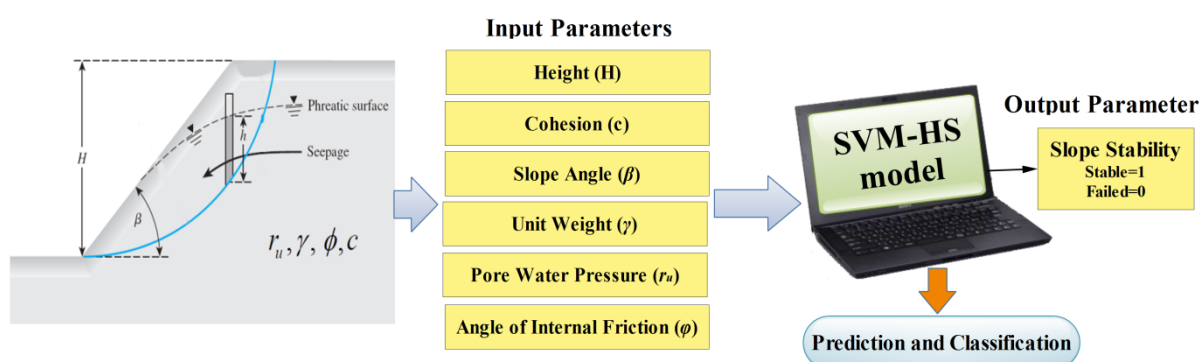


Figure 3. Diagnostic model of prediction of the slope stability state based on the SVM-HS model

In this study applies an RBF kernel function in the SVM to obtain optimal solution. In HS-SVM model, the parameters  $\sigma$ ,  $C$  of SVM model are optimized by HS. The related parameters  $C$  and  $\sigma$  for this kernel were varied in the arbitrarily fixed ranges [1, 4000] and [0, 6] so that to cover high and small regularization of the prediction model, and fat as well as thin kernels, respectively. In the harmony search, there are several coefficients, which their values can be adjusted to produce a better rate of convergence. Table 2 shows the coefficient values in the HS.

Table 2: Coefficient values in the HS.

Maximum number of iterations	100
Harmony memory size (HMS)	40
Number of new harmonies	10
Harmony memory consideration rate (HMCR)	0.95
Pitch adjustment rate (PAR)	0.3
Band width (bw)	bw, max=59.98 bw, min=0.1

The adjusted parameters ( $\sigma$ ,  $C$ ) with maximal classification and prediction accuracy are selected as the most appropriate parameters. Then, the optimal parameters are utilized to train the SVM model. The classification and prediction accuracies and optimal parameters of the SVM model estimated by the HS are given in Table 3.

Table 3: The classification and prediction accuracies and optimal parameters

	Optimal values of $\sigma$ parameters	Optimal values of $C$ parameters	Optimal classification and prediction accuracies of SVM (%)
HS-SVM	2.184	1031.7	97.45

The performance indices obtained in Table 5 indicate the high performance of the

SVM-HS model that can be used successfully for the prediction of slope stability state (Failed/Stable).

## 5. DISCUSSION AND CONCLUSION

The prediction of a circular slope failure using SVM-HS suggests that this might prove to be a useful alternative, with distinct advantages over the LEMs. The advantage of the SVM-HS model in the analysis of slope stability problems over traditional LEMs is that no assumption needs to be made in advance about the shape or location of the failure surface, slice side forces and their directions. The SVM-HS model could be successful for predicting the state of slope stability depends upon the input data available. For predicting the state of slope stability considered in this paper, 55 cases were available; these showed good agreement with the observed state of stability (Failed/Stable). The SVM-HS model considered as input factors: unit weight, cohesion, internal friction angle, slope angle, slope height and pore pressure ratio. It is believed that a number of other factors could also be influential, for example the history of slope movement, engineering disturbance, climate and vegetation. However, lack of measurements prevents their direct incorporation. Subsequently, caution needs to be exercised in the practical implementation of a trained SVM-HS model, recognizing the limitations of the available input data. It would appear that SVM-HS model already provides a viable tool for situations where significant quantities of data are available.

In this paper, a new approach namely SVM optimized by HS is proposed for the prediction of slope stability state (Failed/Stable). In our methodology, HS is employed as an optimization tool for determining the optimal value of user-defined parameters existing in the formulation of SVM. The optimization implementation increases the performance of SVM model. Moreover, this method requires less time for setting optimal value in comparison with other algorithms, which are usually used for finding these values. This study shows that the SVM combined with HS can be employed as a powerful tool for modeling of some problems involved in civil engineering.

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