

STRUCTURAL RELIABILITY ASSESSMENT UTILIZING FOUR METAHEURISTIC ALGORITHMS

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ABSTRACT

The failure probability of the structures is one of the challenging problems in structural engineering. To obtain the reliability index introduced by Hasofer and Lind, one needs to solve a nonlinear equality constrained optimization problem. In this study, four of the most recent metaheuristic algorithms are utilized for finding the design point and the failure probability of problems with continuous random variables. These algorithms consist of Improved Ray Optimization, Democratic Particle Swarm Optimization, Colliding Bodies Optimization, and Enhanced Colliding Bodies Optimization. The performance of these algorithms is tested on nineteen engineering optimization problems.

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KEY WORDS: structural reliability; reliability index; improved ray optimization (IRO); democratic particle swarm optimization (DPSO); colliding bodies optimization (CBO); enhanced colliding bodies optimization (ECBO).

1. INTRODUCTION

The evaluation of the safety of structures has been one of the subjects of interest for engineers. The safety of a structure depends on the resistance of the structure and the actions on the structure that are functions of random variables. Due to the presence of inherent uncertainties in mechanical and geometrical properties of structural systems and external loads achievement of absolute safety is impossible [1-3].

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The methods of probability analysis for the sake of structural reliability can generally be classified into three categories: Moment methods [4-6], Simulation methods [7-11] and Heuristic methods [12-17]. In this paper, structural reliability assessment is studied based on metaheuristic algorithms. In this category, the shortest distance of the limit state function from the origin of the standard normal coordinate system is considered as a fitness function and is obtained by heuristic methods.

This paper proposes a reliability model based on metaheuristic algorithms, which are population-based methods to find the optimum result. These algorithms consist of Improved Ray Optimization (IRO) [18], Democratic Particle Swarm Optimization (DPSO) [19], Colliding Bodies Optimization (CBO) [20] and Enhanced Colliding Bodies Optimization (ECBO) [21]. The IRO is the enhanced version of the ray optimization [22] which was conceptualized using the relationship between the angles of incidence and fraction based on Snell's law. Each agent is modeled as a ray of light that moves in the search space to find the global or near-global optimum solution. Particle Swarm Optimization (PSO) has become one of the most popular optimization techniques that simulates the social interaction behavior of birds flocking and fish schooling [23]. The main emphasis in DPSO is placed upon addressing the problem of premature convergence which is believed to be one of the PSO's main drawbacks. In CBO, each agent is modeled as a body with a specified mass and velocity. A collision occurs between pairs of objects and the new positions of the colliding bodies are updated based on the collision laws. ECBO is proposed to improve the exploration capability of the CBO and thus to remove the problem of premature convergence in some problems.

The rest of this paper is organized as follows. In Section 2, brief presentation of the concepts of structural reliability is provided. In Section 3, IRO, DPSO, CBO and ECBO methods are briefly presented. Nineteen benchmark functions of the literature are studied in Section 4, and conclusions are derived in Section 5.

2. FAILURE PROBABILITY ASSESSMENT

The structure failure probability, P_f , is given by the following formula:

$$p_f = p(G(X) \leq 0) = \int_{G(X) \leq 0} f(X) dX \quad (1)$$

where $X = [x_1, x_2, \dots, x_n]^T$ represents a vector of stochastic variables of the reliability problem, the superscript T denotes the transpose, and $f(X)$ represents a joint probability density function in X -space. The operation of each structure can be expressed by a function of basic random variables of that structure called limit state function ($G(X)=0$) so that $G(X)>0$ indicates a safe state and $G(X)<0$ indicates a failure state. Therefore the integral represents the volume of the joint probability density function in the failure domain. The evaluation of this multi-fold probability integral is a fundamental problem in structural reliability theory because direct calculation of this integral is very difficult, especially for real structures.

Hasofer and Lind [24] proposed the concept of design point which is the most probable failure point. Based on this definition, the reliability index (β) is the shortest distance between the limit state function and the center of the standard normal space (the distance between the origin and the design point).

The independent normal distribution stochastic variables x can be standardized by

$$u_i = \frac{x_i - \mu_i}{\sigma_i} \quad (2)$$

where x_i is the i th component of x , μ_i and σ_i are the mean value and the standard variance of x_i , respectively.

Finding the Hasofer-Lind's reliability index can be treated as an optimization problem. In other words, the objective function is supposed to determine the coordinates of a specific point on failure surface which is closest to the origin in the U-space (independent standard normal distribution). The mathematical formulation of these problems can be expressed as follows:

$$\begin{aligned} &\text{Find} && \{U\} = [u_1, u_2, \dots, u_n] \\ &\text{to minimize} && \beta(U) = \sqrt{u^T u} \\ &\text{subjected to:} && G(U) = 0 \end{aligned} \quad (3)$$

In order to handle the constraints, the penalty approach is employed. Thus, the objective function is redefined as:

$$\beta(U) = (1 + \varepsilon_1 \nu)^{\varepsilon_2} \times \beta(U), \quad \nu = \sum_{j=1}^{nc} |G(u)| \quad (4)$$

where ν denotes the sum of the violations of the constraints and nc is the number of the constraints. The constants ε_1 and ε_2 are selected considering the exploration and the exploitation rates of the search space. In this study, ν is considered closed to 0 if its absolute value is lower than 10^{-4} [13].

3. OPTIMIZATION ALGORITHMS

3.1 Improved Ray Optimization (IRO)

Ray Optimization (RO), which is based on the transition of ray from one medium to another from physics, was introduced by Kaveh and Khayatazad [22]. As the light passes through a surface of two certain transparent materials, its path is changed slightly. This phenomenon was formulated by the Snell's law. In RO by utilizing Snell's refraction law and a number of random terms, each agent moves in the search space to find the global or near-global optimum solution. Kaveh et al. [18] developed Improved Ray Optimization (IRO) and it is proved to be

competent in structural optimization problems. The formulation of generating solution vectors and returning violated agents to feasible search space is changed in IRO. This algorithm consists of the following steps:

Step 1: Initialization

The initial positions of the agents are determined randomly in the search space and the objective function is evaluated for each agent. A memory which saves all the best positions as local best memory (**LBM**) is considered, and the position of the best agent is saved as the global best (**GB**).

Step 2: Initial velocity

The initial movement vectors of agents are stated as:

$$V_{ij} = -1 + 2.rand \quad j=1,2,\dots,n \quad (5)$$

where V_{ij} is the initial movement value of the j th variable for the i th agent. New positions are obtained by adding the position of each agent with its movement vector.

Step 3: Regenerating

If any component of each agent violates a boundary, it must be regenerated by the following formula:

$$X_{ij}^{k+1} = X_{ij}^k + 0.9(Int_{ij} - X_{ij}^k) \quad (6)$$

where X_{ij}^{k+1} and X_{ij}^k are the refined component and component of the j th variable for the i th agent in $(k+1)$ th and k th iteration, respectively. Int_{ij} is the intersection point (in case an agent violates a boundary, it intersects the boundary at a specified point, because of having definite movement vector). After regenerating all violated components, the goal function is evaluated for each agent and the **LBM** and **GB** are updated.

Step 4: Evaluating the Origin

Consider the origin as a point which each agent wants to move toward it and specified by

$$O_i^k = \frac{(ite+k).GB + (ite-k).LB}{2.ite} \quad (7)$$

where O_i^k is the origin of the i th agent for the k th iteration, and ite is the total number of iterations for the optimization process. For each agent, **LB** is a solution vector which is selected randomly from local best memory (**LBM**). Target vector is defined as:

$$Tv_i = O_i - X_i \quad (8)$$

where Tv_i and X_i are target vector and current position of the i th agent, respectively.

Step 5: Evaluating the direction of movement vector

Direction of the new movement vector is defined by

$$V_i^{k+1} = \alpha.Tv_i^k + \beta.V_i^k \quad (9)$$

$$\alpha = 1 + \left(\frac{k}{ite}\right) \quad (10)$$

$$\beta = 1 - 0.5\left(\frac{k}{ite}\right) \quad (11)$$

where \mathbf{V}_i^{k+1} and \mathbf{V}_i^k are movement vectors for the i th agent in $(k+1)$ th iteration and k th iteration, respectively. Finally all the \mathbf{V}_i^{k+1} vectors should be normalized.

Step 6: Evaluating the magnitude of movement vector

Possibility like *stoch* is considered and the magnitude of movement vectors are defined as:

a. with probability like *stoch*,

$$\mathbf{V}_{ij}^{k+1} = -1 + 2.rand \quad (12)$$

$$\mathbf{V}_i^{k+1} = \frac{\mathbf{V}_i^{k+1}}{norm(\mathbf{V}_i^{k+1})} \cdot \frac{a}{d} .rand \quad (13)$$

$$a = \sqrt{\sum_{i=1}^n (\mathbf{X}_{j,max} - \mathbf{X}_{j,min})^2} \quad (14)$$

where d is a number that divides a into smaller length for effective search. $\mathbf{X}_{j,max}$ and $\mathbf{X}_{j,min}$ are the upper and lower bounds for the j th design variable, respectively. In this study d is set to 20.

b. with probability like $(1-stoch)$,

If $norm(\mathbf{O}_i^k - \mathbf{X}_i^k) = 0$, then

$$\mathbf{V}_i^{k+1} = \frac{\mathbf{V}_i^k}{norm(\mathbf{V}_i^k)} .rand.0.001 \quad (15)$$

Otherwise,

$$\mathbf{V}_i^{k+1} = \mathbf{V}_i^{k+1} .norm(\mathbf{X}_i^k - \mathbf{O}_i^k) \quad (16)$$

Step 7: Terminating Criterion Control

The optimization process is terminated after a fixed number of iterations. If it is not fulfilled, each movement vector is added with its current position vector and the process of optimization is continued from Step 3.

3.2 Democratic Particle Swarm Optimization (DPSO)

Particle Swarm Optimization (PSO) initially developed by Kennedy and Eberhart [25] is a metaheuristic algorithm which mimics the social behavior of certain species of animals like birds flocking and fishes schooling. PSO is one of the most widely used population-based technique having some advantages such as few parameters to adjust, easiness of implementation, global search capability in some problems and, in general, fast convergence.

However, despite having the above-mentioned benefits, the standard PSO is infamous of premature convergence. Democratic Particle Swarm Optimization (DPSO) is proposed by Kaveh and Zolghadr [19] to improve the exploration capabilities of the PSO and thus to address the problem of premature convergence. Each particle only leads by its historically best position and the historically best position of the entire swarm in standard PSO but in the DPSO all eligible particles have the right to be involved in decision making in order to overcome the problem of premature convergence. The steps of the DPSO are outlined in the following.

Step 1: Initialization

The positions of all particles are randomly set within predefined ranges and their associated velocities are set to 0. The objective function is evaluated for each particle and $xlbest_i$ (historically best position of i th particle) and $xgbest$ (historically best position of the entire swarm) are stored.

Step 2: Velocity and position updating

The velocity and position update rules are given by:

$$v_{i,j}^{k+1} = \chi(\omega v_{i,j}^k + c_1 \cdot r_1 \cdot (xlbest_{i,j}^k - x_{i,j}^k) + c_2 \cdot r_2 \cdot (xgbest_j^k - x_{i,j}^k) + c_3 \cdot r_3 \cdot d_{i,j}^k) \quad (17)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (18)$$

where $v_{i,j}^k$ is the velocity or the amount of change of the design variable j of particle i in k th iteration, $x_{i,j}^k$ is the current value of the j th design variable of the i th particle. r_1 , r_2 and r_3 are three random numbers uniformly distributed in the range (1, 0), c_1 , c_2 and c_3 are three constant parameters. χ parameter is used to avoid divergence behavior. In this study, c_1 , c_2 , c_3 and χ are set to 2, 2, 4 and 0.5, respectively.

ω is the inertia weight for the previous iteration's velocity and it can be set in order to control the exploration of the algorithm. This parameter is defined as [26]:

$$\omega = 0.4[1 + \min(cov, 0.6)] \quad (19)$$

where cov is the coefficient of variation of the swarm's objective function.

In Eq. (17), $d_{i,j}^k$ is the j th variable of the democratic vector i , D_i , in the k th iteration. The vector D represents the democratic effect of the other particles of the swarm on the movement of the i th particle is evaluated by:

$$D_i = \sum_{k=1}^n Q_{ij} (x_k - x_i) \quad (20)$$

$$Q_{ik} = \frac{E_{ik} \frac{obj_{best}}{obj(k)}}{\sum_{j=1}^n E_{ij} \frac{obj_{best}}{obj(j)}} \quad (21)$$

$$E_{ik} = \begin{cases} 1 & \frac{obj(k) - obj(i)}{obj_{worst} - obj_{best}} > rand \vee obj(k) < obj(i) \\ 0 & else \end{cases} \quad (22)$$

Where n is the number of particles and the \vee symbol stands for union. obj_{worst} and obj_{best} are the values of the objective function for the worst and the best particles in the current iteration, respectively.

Step 3: Updating $xlbest$ and $xgbest$

The objective function is evaluated for each particle and $xlbest$ and $xgbest$ are updated.

Step 4: Terminating criterion controlling

Repeat the optimization process until a fixed number of iterations is completed. Otherwise, go to Step 2 for a new round of iteration.

3.3 Colliding Bodies Optimization (CBO) and its Enhanced version (ECBO)

3.3.1 CBO

Colliding Bodies Optimization (CBO) is a population-based meta-heuristic optimization algorithm introduced by Kaveh and Mahdavi [20], which is based on the governing laws of one dimensional collision between two bodies from the physics. One object collides with other object and they move toward minimum energy level. The CBO is simple in concept, shows fast-converging behavior and depends on no internal parameters. The following steps are developed to introduce the details of this method.

Step 1: Initialization

The initial positions of all Colliding Bodies (CBs) are determined randomly in search space.

Step 2: Defining mass

Each CB has a specified mass defined as:

$$m_k = \frac{\frac{1}{fit(k)}}{\sum_{i=1}^n \frac{1}{fit(i)}}, \quad k = 1, 2, \dots, n \quad (23)$$

Where $fit(i)$ represents the objective function value of the i th CB and n is the number of colliding bodies. CBs are sorted according to their objective function values in an increasing order. To select the pairs of CBs for collision, they are divided into two equal groups: (i) stationary group, (ii) moving group.

Step 3: Criteria before the collision

The velocity of stationary bodies before collision is zero and moving objects move toward stationary objects:

$$v_i = 0, \quad i = 1, 2, \dots, \frac{n}{2} \quad (24)$$

$$v_i = x_{i-\frac{n}{2}} - x_i, \quad i = \frac{n}{2} + 1, \frac{n}{2} + 2, \dots, n \quad (25)$$

Step 4: Criteria after the collision

The velocities of stationary and moving bodies after the collision are evaluated by:

$$v_i' = \frac{(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}})v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}} \quad i = 1, 2, \dots, \frac{n}{2} \quad (26)$$

$$v_i' = \frac{(m_i - \varepsilon m_{i-\frac{n}{2}})v_i}{m_i + m_{i-\frac{n}{2}}} \quad i = \frac{n}{2} + 1, \frac{n}{2} + 2, \dots, n \quad (27)$$

ε is the coefficient of restitution (COR) that decreases linearly from unit to zero. Thus, it is stated as

$$\varepsilon = 1 - \frac{iter}{iter_{max}} \quad (28)$$

where $iter$ is the current iteration number and $iter_{max}$ is the total number of iteration for optimization process.

Step 5: Updating CBs

The new position of each stationary CB is:

$$x_i^{new} = x_i + rand \circ v_i', \quad i = 1, 2, \dots, \frac{n}{2} \quad (29)$$

where x_i^{new} , x_i and v_i' are the new position, previous position and the velocity after the collision of the i th CB, respectively. $rand$ is a random vector uniformly distributed in the range of $[-1, 1]$ and the sign " \circ " denotes an element-by-element multiplication. The new position of each moving CB is calculated by:

$$x_i^{new} = x_{i-\frac{n}{2}} + rand \circ v_i', \quad i = \frac{n}{2} + 1, \frac{n}{2} + 2, \dots, n \quad (30)$$

Step 6: Terminal condition check

The optimization process is terminated after a fixed number of iterations. If this criterion is not satisfied go to Step 2 for a new round of iteration.

3.3.2 ECBO

Enhanced Colliding Bodies Optimization (ECBO) was introduced by Kaveh and Ilchi

Ghazaan [21] in order to improve the performance of CBO yet preserving some above mentioned strength points of the standard formulation. Colliding Memory (CM) is considered to save some historically best CB vectors and their related mass and objective function values to improve the performance of the CBO and reduce the computational cost. In each iteration, the solution vectors saved in CM are added to the population, and the same numbers of current worst CBs are deleted. To prevent premature convergence, a parameter like **Pro** within (0, 1) is introduced and it is specified whether a component of each CB must be changed or not. For each colliding body **Pro** is compared with m_i ($i=1,2,\dots,n$) which is a random number uniformly distributed within (0, 1). If $m_i < \mathbf{Pro}$, one dimension of the i th CB is selected randomly and its value is regenerated as follows:

$$x_{ij} = x_{j,\min} + \mathit{random}(x_{j,\max} - x_{j,\min}) \quad (31)$$

where x_{ij} is the j th variable of the i th CB. $x_{j,\min}$ and $x_{j,\max}$ respectively, are the lower and upper bounds of the j th variable. In order to protect the structures of CBs, only one dimension is changed. In this paper, the value of **Pro** set to 0.35 and the size of the CM is taken as $n/10$ (n is the number of colliding bodies).

4. NUMERICAL EXAMPLES

In this section the IRO, DPSO, CBO and ECBO algorithms are applied to nineteen widely used test problems in literature, representing a broad range of possible limit states that can occur in practice. The final results are compared to the solutions of some other methods to demonstrate the validity and effectiveness of these algorithms.

For all examples a population of 20 agents is utilized. In order to investigate the effect of the initial solution on the final result and because of the stochastic nature of the meta-heuristic algorithms, each example is independently solved for 20 times with random initial designs. Afterwards the best results are reported for performance evaluation of proposed algorithms. In all the algorithms, the predefined maximum iteration is considered as 200. The total number of limit state function evaluations is the same for all of the algorithms (population size \times maximum number of iterations).

The constants ε_1 of penalty function is set to unity for all examples except G_5 from Table 1, G_3 from Table 5 and G_4 from Table 9. For these functions ε_1 is set to 2, 3 and 3, respectively. In all examples, in the first steps of the search process, ε_2 is set to 1.5 and ultimately increased linearly to 3.

Table 1: Limit state function (the components are independent standard normal distribution)

$G_1(x) = 5 - 0.5(x_1 - 0.1)^2 - x_2$
$G_2(x) = \exp(0.4(x_1 + 2) + 6.2) - \exp(0.3x_2 + 5) - 200$
$G_3(x) = \exp(0.2x_1 + 1.4) - x_2$
$G_4(x) = 2 + \sum_{i=1}^9 x_i^2 - x_{10}$

$G_5(x) = 3 - x_2 + (4x_1)^4$
$G_6(x) = 2 - x_2 - 0.1x_1^2 + 0.06x_1^3$
$G_7(x) = 0.1(x_1 - x_2)^2 - \frac{(x_1 + x_2)}{\sqrt{2}} + 2.5$
$G_8(x) = -0.5(x_1 - x_2)^2 - \frac{(x_1 + x_2)}{\sqrt{2}} + 3$

Eight benchmark limit state functions with independent standard normal random variables are summarized in Table 1. The optimal reliability index and corresponding probability of failure obtained by crude Monte-Carlo [27], PSO [13], CSS [17] and proposed algorithms are shown in Table 2. As can be observed, the results achieved by different methods are approximately identical. The optimal design points obtained by proposed methods are given in Table 3. Statistical information is presented in Table 4. Convergence history diagrams of average optimal reliability index for the first limit state function from Table 1 are depicted in Fig. 1.

Table 2: Comparison of failure probability and reliability index obtained by different methods

		Grooteman [27]	Elegbede [13]	Kaveh et al. [17]	IRO	DPSO	CBO	ECBO
G_1	P_f	-	0.00183	0.00183	0.001831	0.001832	0.001832	0.001832
	β	-	2.9056	2.906	2.9058	2.9056	2.9057	2.9056
G_2	P_f	-	0.00337	0.003365	0.003334	0.003361	0.003354	0.003314
	β	-	2.7099	2.71	2.7129	2.7103	2.7109	2.7149
G_3	P_f	-	0.000405	0.000404	0.000404	0.000404	0.000404	0.000404
	β	-	3.4971	3.35	3.3496	3.3496	3.3496	3.3496
G_4	P_f	0.00534	-	0.0227498	0.022464	0.022746	0.022752	0.022747
	B	2.103	-	2	2.005333	2.0000	1.9999	2.0000
G_5	P_f	0.00018	-	0.0013498	0.001350	0.001350	0.001350	0.001350
	β	2.925	-	3	2.9999	2.9999	2.9999	2.9999
G_6	P_f	0.0347	-	0.022750	0.022756	0.022756	0.022756	0.022755
	β	1.996	-	2	1.9999	1.9999	1.9999	1.9999
G_7	P_f	0.00416	-	-	0.006211	0.006210	0.00621	0.006211
	β	2.481	-	-	2.4999	2.4999	2.4999	2.4999
G_8	P_f	0.105	-	-	0.04863	0.048629	0.048629	0.048628
	β	1.625	-	-	1.6582	1.6582	1.6582	1.6583

Table 3: Design points obtained by algorithms

	Design point	IRO	DPSO	CBO	ECBO
G_1	x_1	-2.75145	-2.74074	-2.74499	-2.74014
	x_2	0.93450	0.96498	0.95296	0.96677
G_2	x_1	-2.5872	-2.52197	-2.56748	-2.47706
	x_2	0.81629	0.99268	0.87017	1.11142
G_3	x_1	-1.67975	-1.68462	-1.66811	-1.68013
	x_2	2.89798	2.89521	2.9047	2.89777
G_4	x_1	0.04490	-0.01055	0.00574	-0.00449

	x ₂	-0.03938	0.00725	0.00347	-0.00461
	x ₃	0.04674	-0.0095	0.00452	-0.00867
	x ₄	0.06378	0.00037	-0.00460	-0.01044
	x ₅	-0.0192	0.01194	-0.0019	-0.01201
	x ₆	0.01470	-0.0067	-0.01274	0.00206
	x ₇	-0.07475	-0.00161	-0.00243	0.00320
	x ₈	-0.06726	0.00509	0.00127	0.00357
	x ₉	-0.00213	-0.01465	0.00012	0.01327
	x ₁₀	2.00020	1.99991	1.99990	1.99991
G ₅	x ₁	-0.00325	-0.00042	-2.91e-008	-0.00625
	x ₂	2.99990	2.9999	2.9999	2.99991
G ₆	x ₁	-0.00174	0.00102	-2.35e-008	0.00132
	x ₂	1.99989	1.9998	1.9999	1.99990
G ₇	x ₁	1.76760	1.76209	1.75802	1.76847
	x ₂	1.76778	1.7733	1.77742	1.7669
G ₈	x ₁	1.4699	1.46880	1.47398	-0.76116
	x ₂	-0.76747	-0.7697	-0.75980	1.47329

Table 4: Statistical optimization results obtained by algorithms

		IRO	DPSO	CBO	ECBO
	Best β	2.9058	2.9056	2.9057	2.9056
	Worst β	3.1025	3.1430	3.5352	3.0951
	Avg β	2.9946	2.9881	3.0307	2.9274
G ₁	Std β	0.0957	0.0990	0.1768	0.0573
	Iteration	100	52	38	190
	Avg Iteration	105.8	121.25	39.75	118.85
	Std Iteration	27.2968	36.5180	14.0295	50.7805
	Best β	2.7129	2.7103	2.7109	2.7149
	Worst β	6.3233	5.4257	3.9674	5.6122
	Avg β	3.5772	3.7728	3.0350	3.2572
G ₂	Std β	1.1430	0.8326	0.3724	0.6505
	Iteration	162	88	47	118
	Avg Iteration	132.35	127.7	51.55	118.3
	Std Iteration	24.2036	40.9712	12.2193	37.9502
	Best β	3.3496	3.3496	3.3496	3.3496
	Worst β	3.3519	3.3653	3.5560	3.3620
	Avg β	3.3500	3.3522	3.3670	3.3513
G ₃	Std β	0.0005	0.0041	0.0479	0.0029
	Iteration	90	109	30	54
	Avg Iteration	82.5	108.75	27.4	99.25
	Std Iteration	19.1874	53.0698	10.049	52.3529
	Best β	2.0053	2.0000	1.9999	2.0000
G ₄	Worst β	2.1244	2.0045	6.9571	6.2084
	Avg β	2.0293	2.0009	3.0399	2.2107

	Std β	0.0271	0.0010	1.5502	0.9409
	Iteration	185	111	128	121
	Avg Iteration	174.15	137.55	157.95	132
	Std Iteration	20.3812	16.8693	30.8075	25.0620
	Best β	2.9999	2.9999	2.9999	2.9999
	Worst β	3.0005	2.9999	3.0053	3.0237
	Avg β	2.9999	2.9999	3.0001	3.0020
G ₅	Std β	0.0001	5.17e-007	0.0012	0.0057
	Iteration	66	36	20	149
	Avg Iteration	71.3	40.6	27.15	127.15
	Std Iteration	13.4363	7.8028	7.6796	47.8476
	Best β	1.9999	1.9999	1.9999	1.9999
	Worst β	2.0059	2.0000	2.0004	2.1130
	Avg β	2.0006	1.9999	1.9999	2.0056
G ₆	Std β	0.0016	3.12e-005	0.000126	0.0252
	Iteration	47	63	26	43
	Avg Iteration	93.2	60.8	27.7	68.65
	Std Iteration	34.1984	22.4232	8.4859	37.4858
	Best β	2.4999	2.4999	2.4999	2.499
	Worst β	2.5013	2.5012	2.5256	2.5149
	Avg β	2.5000	2.5001	2.5031	2.5009
G ₇	Std β	0.0003	0.0002	0.0057	0.0033
	Iteration	67	32	39	19
	Avg Iteration	78.05	64.25	27.05	55.1
	Std Iteration	28.9945	31.6525	7.5077	61.3170
	Best β	1.6582	1.6582	1.6582	1.6583
	Worst β	1.6679	1.6617	1.7081	1.6609
	Avg β	1.6595	1.6589	1.6663	1.6588
G ₈	Std β	0.0022	0.0010	0.0122	0.0005
	Iteration	98	42	20	73
	Avg Iteration	96.25	84.8	36.05	78.35
	Std Iteration	16.9049	41.6092	9.5832	47.2966

Avg β = average optimized β ; Std β = standard deviation on optimized β ; Iteration = required number of iteration for the best run; Avg Iteration = average required number of iteration; Std Iteration = standard deviation on required number of iteration.

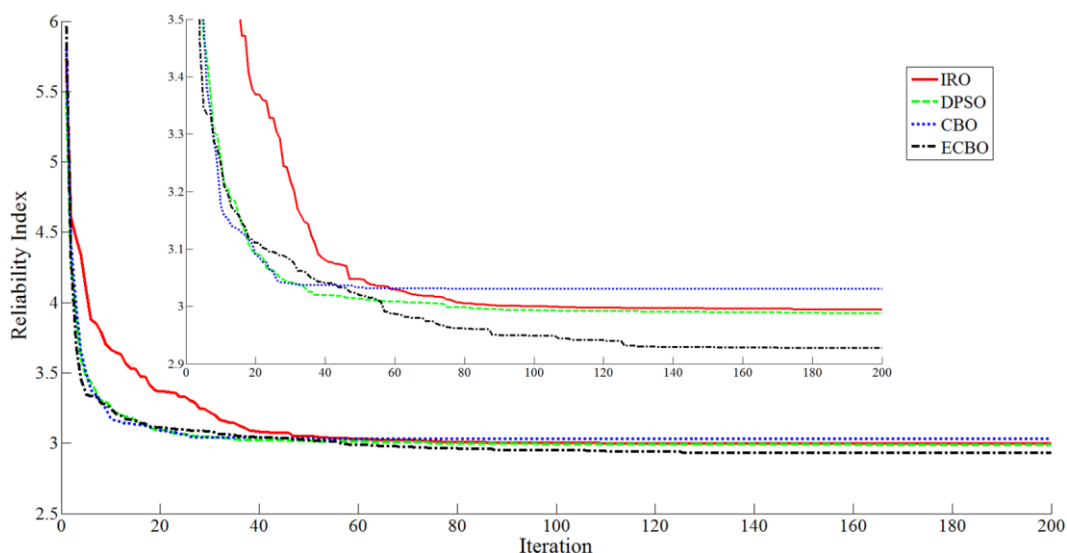


Figure 1. Comparison of average values of reliability index recorded in the optimization history for the different algorithms (G_1 from Table 1)

Table 5 provides five limit state functions with independent normal random variables. Column 2 gives the mean and standard deviation values of the variables. Table 6 lists the optimal reliability index and corresponding probability of failure obtained by different methods. It can be seen that the optimal β values achieved by various techniques are approximately the same. The optimal design points in normal and standard normal spaces are shown in Table 7. In Table 8, the statistical results of independent runs are summarized. The average convergence curves of proposed algorithms for the first limit state function from Table 5 are compared in Fig. 2.

Table 5: Limit state function (the components are independent normal distribution)

Limit state function	Stochastic variables
$G_1(x) = 18.46154 - 7.476923 \times 10^{10} \frac{x_1}{x_2}$	$x_1: N(0.001, 0.0002)$ $x_2: N(250, 37.5)$
$G_2(x) = x_1 - \frac{x_2}{x_3}$	$x_1: N(600, 3)$ $x_2: N(1000, 33)$ $x_3: N(2, 0.1)$
$G_3(x) = 1 - \frac{\sqrt{3(1-\mu^2)}}{\pi x_1 x_2^2 \cos^2 x_3} \left(\frac{x_6}{2\gamma} + \frac{x_5}{\eta x_4} \right)$	$x_1: N(70000, 3500)$ $x_2: N(0.0025, 0.000125)$ $x_3: N(0.524, 0.01048)$ $x_4: N(0.9, 0.0225)$ $x_5: N(80000, 6400)$ $x_6: N(70000, 5600)$
$G_4(x) = x_1 x_2 - 146.14$	$x_1: N(78064.4, 11709.7)$ $x_2: N(0.0104, 0.00156)$
$G_5(x) = 2.5 - 0.2357(x_1 - x_2) + 0.00463(x_1 + x_2 - 20)^4$	$x_1: N(10, 3)$ $x_2: N(10, 3)$

Table 6: Comparison of failure probability and reliability index obtained by different methods

		Grooteman [27]	Elegbede [13]	Kaveh et al. [17]	IRO	DPSO	CBO	ECBO
G_1	P_f	-	0.009879	0.009878	0.009879	0.009878	0.009878	0.009875
	β	-	2.3309	2.33092	2.3309	2.3309	2.3309	2.3310
G_2	P_f	-	0.01161	0.01161	0.011394	0.009608	0.0116	0.008621
	β	-	2.2697	2.26969	2.2769	2.3413	2.2701	2.3814
G_3	P_f	-	5.232e-7	8.07e-7	8.07e-007	8.05e-007	7.95e-007	7.97e-007
	β	-	4.883	4.79654	4.796378	4.79704	4.7995	4.798937
G_4	P_f	1.46e-07	-	4.82e-8	4.72e-008	4.35e-008	4.53e-008	4.67e-008
	β	5.443	-	5.33329	5.33702	5.3516	5.3442	5.3387
G_5	P_f	0.00286	-	-	0.006211	0.006210	0.006211	0.006210
	β	2.431	-	-	2.4999	2.4999	2.4999	2.4999

Table 7: Design points obtained by algorithms

	Design point	IRO	DPSO	CBO	ECBO
G_1	x_1	0.00111	0.00111	0.00111	0.00111
	x_2	165.48	165.48	165.53	165.24
	u_1	0.59394	0.5944	0.59895	0.57012
	u_2	-2.25397	-2.2538	-2.2526	-2.2602
G_2	x_1	559.71	565.22	555.71	540.56
	x_2	1032.3	1044.8	1030.3	1012.9
	x_3	1.8444	1.8484	1.854	1.8738
	u_1	-1.34300	-1.15916	-1.47619	-1.9812
	u_2	0.98002	1.35638	0.91719	0.39036
	u_3	-1.55578	-1.5160	-1.46048	-1.26244
G_3	x_1	6.38e+010	6.39e+010	6.42e+010	6.39e+010
	x_2	0.00199	0.00199	0.00199	0.00199
	x_3	0.52789	0.52788	0.52867	0.52888
	x_4	0.88833	0.8858	0.8862	0.88624
	x_5	90364	90136	90752	90883
	x_6	74181	74424	74333	73990
	u_1	-1.75342	-1.73945	-1.65693	-1.71645
	u_2	-4.04282	-4.0395	-4.03658	-4.01175
	u_3	0.37142	0.37056	0.44558	0.46518
	u_4	-0.51851	-0.63105	-0.61343	-0.61147
	u_5	1.61930	1.58382	1.67996	1.70040
	u_6	0.74657	0.79006	0.77378	0.71258
G_4	x_1	19516	51622	20470	55456
	x_2	0.00748	0.00283	0.00713	0.00263
	u_1	-4.9999	-2.25816	-4.91856	-1.93070
	u_2	-1.86650	-4.8519	-2.0901	-4.97742
G_5	x_1	15.304	15.314	15.303	15.319
	x_2	4.6974	4.708	4.6965	4.7125

u_1	1.76788	1.77142	1.76760	1.77295
u_2	-1.76753	-1.7640	-1.76781	-1.7625

Table 8: Statistical optimization results obtained by algorithms

	IRO	DPSO	CBO	ECBO	
G_1	Best β	2.3309	2.3309	2.3309	2.3310
	Worst β	2.4651	2.4613	2.6532	2.4026
	Avg β	2.3546	2.3497	2.3614	2.3447
	Std β	0.0320	0.0346	0.0813	0.0193
	Iteration	85	67	33	76
	Avg Iteration	126.15	139.35	38.3	117.2
	Std Iteration	30.9060	49.9634	13.5805	38.7292
G_2	Best β	2.2769	2.3413	2.2701	2.3814
	Worst β	5.1388	5.9199	4.6905	5.0723
	Avg β	3.2534	3.6394	2.5852	3.2628
	Std β	0.9307	1.0863	0.5750	0.8835
	Iteration	170	31	43	88
	Avg Iteration	163.05	125.75	60.6	121.8
	Std Iteration	18.7714	46.2133	11.4542	40.2852
G_3	Best β	4.7963	4.7970	4.7995	4.7989
	Worst β	4.8038	4.8100	5.1449	4.8434
	Avg β	4.7986	4.8001	4.8548	4.8139
	Std β	0.0021	0.0033	0.0805	0.0119
	Iteration	85	152	77	141
	Avg Iteration	133.1	123.35	91.1	161.45
	Std Iteration	38.7147	48.3030	23.5011	23.7807
G_4	Best β	5.3370	5.3516	5.3442	5.3387
	Worst β	5.4181	5.4278	5.4277	5.4280
	Avg β	5.3568	5.3946	5.4008	5.3878
	Std β	0.0261	0.0256	0.0247	0.0317
	Iteration	57	117	46	53
	Avg Iteration	90.8	110	55.6	104.9
	Std Iteration	42.3712	46.6713	14.4637	42.4932
G_5	Best β	2.4999	2.4999	2.4999	2.4999
	Worst β	2.5005	2.5009	2.5149	2.5038
	Avg β	2.5000	2.5001	2.5015	2.5009
	Std β	0.0001	0.0003	0.0035	0.0011
	Iteration	60	38	23	30
	Avg Iteration	67.65	54.95	25.3	63.7
	Std Iteration	16.6647	24.1997	6.5862	55.2821

Avg β = average optimized β ; Std β = standard deviation on optimized β ; Iteration = required number of iteration for the best run; Avg Iteration = average required number of iteration; Std Iteration = standard deviation on required number of iteration.

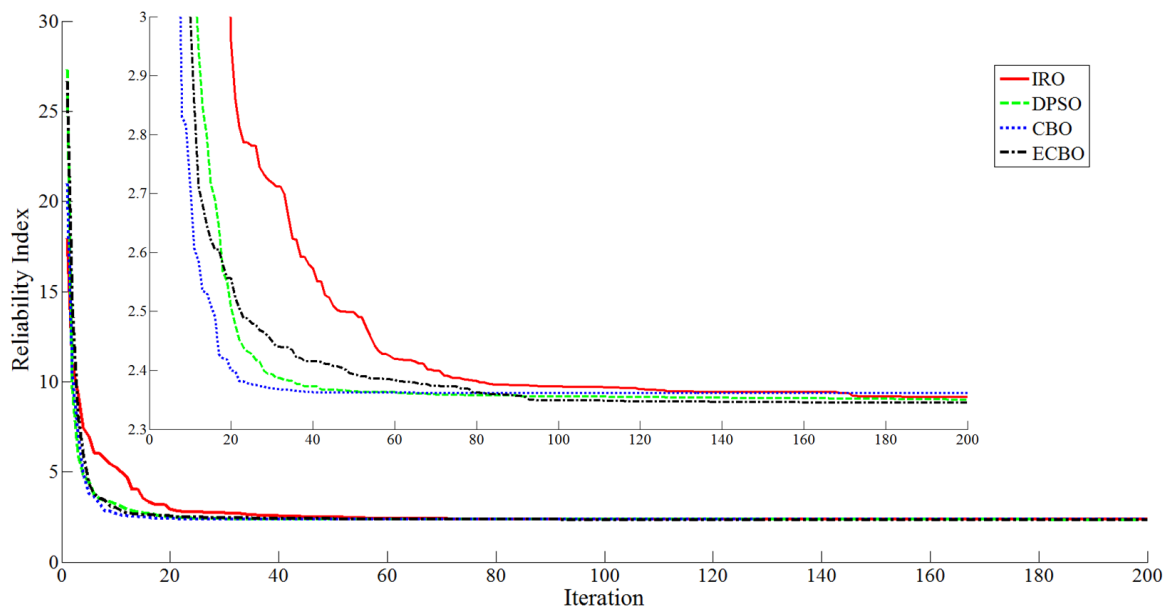


Figure 2. Comparison of average values of reliability index recorded in the optimization history for the different algorithms (G_1 from Table 5)

Six limit state functions are listed in Table 9. All of the random variables are independent standard normal. Table 10 summarizes the optimal reliability index and the corresponding probability of failure obtained by crude Monte-Carlo and proposed algorithms. Again the results achieved by different methods are approximately identical. The optimal design points are shown in Table 11 and statistical results are provided in Table 12. Fig. 3 depicts the average convergence history for the optimal reliability index of the first limit state function from Table 9.

Table 9: Limit state function (the components are independent standard normal distribution)

Limit state function	Description
$g_1 = 2.677 - x_1 - x_2$ $g_2 = 2.500 - x_2 - x_3$ $g_3 = 2.323 - x_3 - x_4$ $g_4 = 2.250 - x_4 - x_5$	Parallel system
$G_1(x) = \max(g_1, g_2, g_3, g_4)$ $g_1 = -x_1 - x_2 - x_3 + 3\sqrt{3}$ $g_2 = -x_3 + 3$	Series system
$G_2(x) = \min(g_1, g_2)$ $g_1 = -x_1 - x_2 - x_3 + 3\sqrt{3}$ $g_2 = -x_3 + 3$	Parallel system
$G_3(x) = \max(g_1, g_2)$	

$g_1 = 2 - x_2 + \exp(-0.1x_1^2) + (0.2x_1)^4$	
$g_2 = 4.5 - x_1x_2$	Series system
$G_4(x) = \min(g_1, g_2)$	
$g_1 = 2 - x_2 + \exp(-0.1x_1^2) + (0.2x_1)^4$	
$g_2 = 4.5 - x_1x_2$	Parallel system
$G_5(x) = \max(g_1, g_2)$	
$g_1 = 0.1(x_1 - x_2)^2 - \frac{(x_1 + x_2)}{\sqrt{2}} + 3$	
$g_2 = 0.1(x_1 - x_2)^2 + \frac{(x_1 + x_2)}{\sqrt{2}} + 3$	
$g_3 = x_1 - x_1x_2 + 3.5\sqrt{2}$	Series system
$g_4 = -x_1 + x_1x_2 + 3.5\sqrt{2}$	
$G_6(x) = \max(g_1, g_2, g_3, g_4)$	

Table 10: Comparison of failure probability and reliability index obtained by different methods

		Grooteman [27]	IRO	DPSO	CBO	ECBO
G_1	P_f	2.11e-04	0.003529	0.003442	0.003439	0.003363
	β	2.738	2.6940	2.7023	2.7026	2.7100
G_2	P_f	2.57e-03	0.00135	0.001350	0.001350	0.001350
	β	2.953	2.9999	2.9999	2.9999	2.9999
G_3	P_f	1.23e-04	0.000365	0.000364	0.000365	0.000364
	β	3.434	3.3780	3.3781	3.3780	3.3782
G_4	P_f	3.54e-03	0.001350	0.001350	0.00135	0.001350
	β	2.925	2.9999	2.9999	2.9999	2.9999
G_5	P_f	2.50e-04	0.000647	0.000647	0.000647	0.000647
	β	3.219	3.2171	3.2172	3.2171	3.2172
G_6	P_f	2.18e-03	0.001350	0.001350	0.001350	0.001350
	β	2.925	2.9999	2.9999	2.9999	2.9999

Table 11: Design points obtained by algorithms

	Design point	IRO	DPSO	CBO	ECBO
G_1	x_1	1.08776	1.03139	0.99898	1.18811
	x_2	1.58933	1.64823	1.67792	1.49069
	x_3	0.93877	0.86184	0.82198	1.0100
	x_4	1.38594	1.4611	1.5020	1.40707
	x_5	0.8640	0.80269	0.74781	0.84296
G_2	x_1	-0.00010	0.00337	-2.03e-008	0.00751
	x_2	-0.02114	0.00657	1.56e-008	-0.00311
	x_3	2.99990	2.9999	2.9999	2.9999
G_3	x_1	1.10801	1.08304	1.10151	1.11241

	x_2	1.08813	1.11315	1.09463	1.08349
	x_3	2.99990	2.99995	2.9999	3.00017
G_4	x_1	0.00839	0.00211	-5.56e-008	-5.43e-005
	x_2	2.99989	2.99989	2.9999	2.99990
G_5	x_1	1.61840	1.61854	1.61840	1.61856
	x_2	2.78044	2.78047	2.78044	2.78045
G_6	x_1	-2.12052	-2.1243	-2.12428	-2.12016
	x_2	-2.12197	-2.1182	-2.11821	-2.1223

Table 12: Statistical optimization results obtained by algorithms

		IRO	DPSO	CBO	ECBO
G_1	Best β	2.6940	2.7023	2.7026	2.7100
	Worst β	2.8248	2.8696	4.7878	2.8324
	Avg β	2.7277	2.7413	2.9720	2.7592
	Std β	0.0420	0.0446	0.4713	0.0350
	Iteration	177	128	104	157
	Avg Iteration	160.55	146.4	110.4	138.05
	Std Iteration	21.3380	42.7285	22.2365	49.5160
G_2	Best β	2.9999	2.9999	2.9999	2.9999
	Worst β	3.0089	3.0007	2.9999	3.0412
	Avg β	3.0022	3.0000	2.9999	3.0070
	Std β	0.0026	0.0001	1.45e-012	0.0104
	Iteration	76	61	29	52
	Avg Iteration	116.05	84.15	31.35	113.35
	Std Iteration	22.4768	34.6581	5.1837	58.6481
G_3	Best β	3.3780	3.3781	3.3780	3.3782
	Worst β	3.4034	3.3869	3.3955	3.3967
	Avg β	3.3840	3.3811	3.3808	3.3823
	Std β	0.0070	0.0023	0.0040	0.0047
	Iteration	111	172	56	98
	Avg Iteration	127.3	127.2	50.05	116.35
	Std Iteration	25.1732	44.2880	14.5872	39.3864
G_4	Best β	2.9999	2.9999	2.9999	2.9999
	Worst β	3.0508	3.0077	3.0388	3.0083
	Avg β	3.0081	3.0003	3.0035	3.0003
	Std β	0.0131	0.0017	0.0093	0.0018
	Iteration	106	34	28	90
	Avg Iteration	112.65	64.85	28.25	81.35
	Std Iteration	47.0668	38.7777	5.9105	41.3995
G_5	Best β	3.2171	3.2172	3.2171	3.2172
	Worst β	3.2221	3.2209	3.3381	3.2194
	Avg β	3.2178	3.2182	3.2304	3.2180
	Std β	0.0013	0.0009	0.0297	0.0006
	Iteration	84	72	54	36

	Avg Iteration	119.05	100.75	62.5	108.75
	Std Iteration	24.9071	42.2970	24.3688	45.5653
	Best β	2.9999	2.9999	2.9999	2.9999
	Worst β	3.0014	3.0027	3.0070	3.0038
	Avg β	3.0001	3.0005	3.0008	3.0006
G_6	Std β	0.0004	0.0007	0.0016	0.0011
	Iteration	74	108	21	95
	Avg Iteration	81.05	76.7	27.95	83.95
	Std Iteration	12.9917	32.5383	8.5129	56.0549

Avg β = average optimized β ; Std β = standard deviation on optimized β ; Iteration = required number of iteration for the best run; Avg Iteration = average required number of iteration; Std Iteration = standard deviation on required number of iteration.

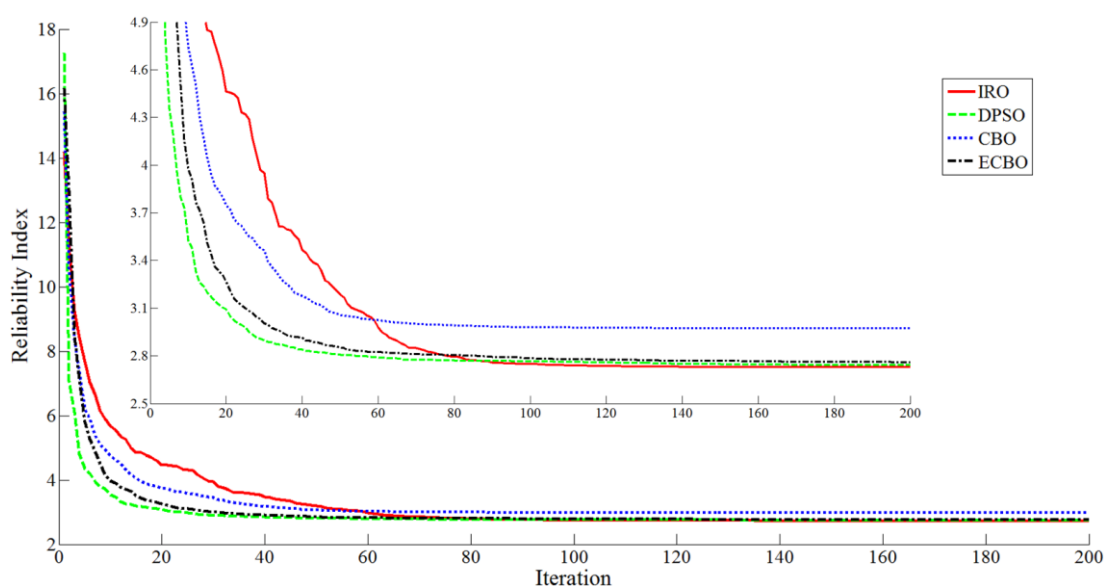


Figure 3. Comparison of average values of reliability index recorded in the optimization history for the different algorithms (G_1 from Table 9)

5. CONCLUSION

Structural reliability analysis utilizing the recently developed metaheuristic algorithms consisting of Improved Ray Optimization, Democratic Particle Swarm Optimization, Colliding Bodies Optimization, and Enhanced Colliding Bodies Optimization is studied in this paper. The various problems serve to demonstrate the efficiency and robustness of these optimization methods. The optimal reliability indexes yielded by the proposed algorithms are approximately identical with respect to those results reported in literature such as crude Monte-Carlo that can be obtained a near exact solution by evaluating many times of the limit state function. Therefore the proposed methods always converge to the optimal values of reliability index and their accuracies are proved. Also, a little standard deviation from the mean value of the twenty

independent runs and the low computing time of these zero order algorithms show the reliability of search and the efficiency of proposed methods, respectively. Generally, comparison of the results proves the suitability and efficiency of the proposed algorithms.

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