

DETERMINATION OF OPTIMUM LOCATION FOR FLEXIBLE OUTRIGGER SYSTEMS IN NON-UNIFORM TALL BUILDINGS USING ENERGY METHOD

R. Kamgar and R. Rahgozar*[†]

*Department of Civil Engineering, Faculty of Engineering, Shahid Bahonar University of
Kerman, Kerman, Iran*

ABSTRACT

In this paper, based on maximizing the outrigger-belt truss system's strain energy, a methodology for determining the optimum location of a flexible outrigger system is presented. Tall building structures with combined systems of framed tube, shear core, belt truss and outrigger system are modeled using continuum approach. In this approach, the framed tube system is modeled as a cantilevered beam with box cross section. The effect of outrigger and shear core systems on framed tube's response under lateral loading is modeled by a rotational spring placed at the location of belt truss and outrigger system. Optimum location of this spring is obtained when energy absorbed by the spring is maximized. For this purpose, first derivative of the energy equation with respect to spring location as measured from base of the structure, is set to zero. Optimum location for outrigger and belt truss system is calculated for three types of lateral loadings, i.e. uniformly and triangularly distributed loads along structure's height, and concentrated load at top of the structure. Accuracy of the proposed method is verified through numerical examples. The results show that the proposed method is reasonably accurate. In addition, for different stiffness of shear core and outrigger system, several figures are presented that can be used to determine the optimum location of belt truss and outrigger system.

Keywords: tall building; outrigger-belt truss system; optimum location; equivalent continuum model; strain energy.

Received: 20 March 2015; Accepted: 15 July 2015

*Corresponding author: Department of Civil Engineering, Faculty of Engineering, Shahid Bahonar University of Kerman, Kerman, Iran

[†]E-mail address: Rahgozar@mail.uk.ac.ir

1. INTRODUCTION

In recent decades, outrigger and belt truss systems have been widely utilized in tall buildings in order to decrease structure's deformation and increase its resistance to lateral loads. Outrigger systems connect exterior columns to the interior shear core. Consequently, the exterior columns and belt truss system resist the rotation of central shear core and hence decrease the lateral deformation as well as bending moment at base of the structure ([1-9]).

Hoenderkamp [10] presented a graphical method of analysis for preliminary design of outrigger truss-braced high-rise shear wall structures with non-fixed foundation conditions subject to horizontal loading. Hoenderkamp et al. [11], presented a simple method of analysis for preliminary design of outrigger braced high-rise shear walls subjected to horizontal loading. Raj Kiran Nanduri et al. [12], used a 30-storey three dimensional models of outrigger and belt truss system subjected to wind and earthquake loads. These structures were analyzed and compared to determine the lateral displacement reduction as related to the outrigger and belt truss system location. For a 30-storey model, 23% maximum displacement reduction was achieved by providing an outrigger at top of the structure. Assadi Zeidabadi et al. [13], presented a general analysis for a pair of coupled shear walls, stiffened by an outrigger and a heavy beam in an arbitrary position along the height. They presented a parametric model to investigate structure's behavior. Optimum location of the outrigger and parameters affecting its position were also investigated.

A correct knowledge of outrigger and belt truss system optimum location is crucial in decreasing deformation and moments of tall structures effectively. Many researchers have studied the optimum location of an outrigger-belt truss system in a tall structure subjected to lateral loading with the objective of reducing lateral displacement at top of the structure. Stanford Smith and Salim[14], developed formulae for estimating optimum levels of outriggers while minimizing the drift in outrigger braced buildings. These formulae are developed by applying multiple regression analysis to results from relatively complex compatibility analyses of structures with up to four outriggers. Rutenberg and Tal [15] presented the results of an investigation on drift reduction in uniform and non-uniform belted structures with rigid outriggers under several lateral load distributions which are likely to be encountered in practice. Zhang et al. [16], carried out a case study to analyze the horizontal top deflection and the mutation of restraining moments caused by variations in outrigger location. Bayati et al. [17], presented the results of an investigation on drift reduction in uniform belted structures with rigid outriggers, by analyzing a sample structure that was built at Tehran's Vanak Park. Gerasimidis et al. [18], calculated optimum outrigger location for a framed high rise structure reinforced by shear core wall and outrigger systems subjected to wind loading. Optimum location of the outrigger was calculated based on drift control criteria and balance of moments transferred by the outriggers to exterior columns. Rahgozar and Sharifi [19], presented a mathematical model for the combined system consisting of framed tube, shear core, belt truss and outrigger system with the objective of determining the optimum location of belt truss along building's height. The effect of belt truss and shear core on a framed tube was modeled as a concentrated moment applied at belt truss location. Optimum location of the belt truss was obtained by minimizing the deflection at top of the structure. Fawzia and Fatima [20], compared deflection variations while using up to three belt truss and outrigger systems for the same height. Similar studies were

conducted by Fawzia et al. [21], their investigations were based on deflection control and frequency optimization while using belt truss and outrigger systems for different building heights.

Control of deformation is an important criterion considered in building codes; another criterion is the control of stresses so not to exceed a certain value ([22-23]). Naturally, simultaneous consideration of both criteria should yield better results. Energy criterion is another reliable measure in engineering problems. Energy which is equivalent to the product of force and displacement is a more comprehensive criterion than displacement or stress by themselves, when computing the optimum location of belt truss and outrigger system in a tall building structure.

In this paper, optimum location of belt truss and outrigger system is obtained using energy criterion. A tall building structure with combined system of framed tube, shear core, belt truss and outrigger system is modeled using continuum approach. Here the framed tube system is modeled as a cantilevered beam with box cross section, [24]. Effect of outrigger-belt truss and framed tube systems on shear core under lateral loading is modeled by a rotational spring placed at outrigger-belt truss location ([1], [3]). Then, optimum location of the spring is obtained when energy absorbed by the spring becomes maximum. For this purpose, derivative of the energy functional with respect to spring location as measured from base of the structure is set to zero. Accuracy of the proposed method is verified through numerical examples.

2. PROGRAM FORMULATION AND SOLUTION

In this section, the following assumptions are made:

1. Floor slabs are not deformable in their planes and have no motion perpendicular to their plane.
2. Effect of the belt truss and outrigger system is considered as a rotational spring with constant rotational stiffness, which acts at the position of belt truss and outrigger system.
3. Spacing of columns and beams are constant throughout building's height.
4. Shear core and columns are fully fixed at the base.
5. Structure's material is linear elastic, homogeneous and obeys Hook's law.
6. The structure is symmetric in plan for all stories and therefore cannot twist.
7. The thickness of the shear core is variable along the height of the structure.
8. The dimension of members in belt truss-outrigger system and framed tube are constant and does not vary with height.
9. The outrigger system is flexible and it connects to exterior columns and shear core as pinned connection.
10. Belt truss is assumed to be rigid.

Based on the assumptions, a tall building structure with combined system of framed tube and shear core subjected to lateral loading can be modeled as a cantilevered beam with variable cross section (see Fig. 1) ([1], [3]). Outrigger-belt truss system under lateral loading acts as a rotating spring and causes changes in moment distribution [4]. Its role is to reduce the moment at structure's base and the displacement at structure's top. It is important to know the optimum location of a belt truss and outrigger system so to design the largest

possible reduction in base moment and displacement at structure's top. Most research studies determine the optimum location of outrigger-belt truss system in tall buildings based on displacement criterion at top of the structure, which have led to valuable findings. As mentioned earlier, the purpose of this research is to obtain optimum location of outrigger-belt truss systems by using energy criterion. The work done by external loads is stored as strain energy in structural members when the structure is subjected to external lateral loads. A portion of this energy is stored by the outrigger-belt truss (or equivalent spring). Hence, the optimum location of outrigger can be calculated when energy absorbed by the rotational spring model becomes maximum.

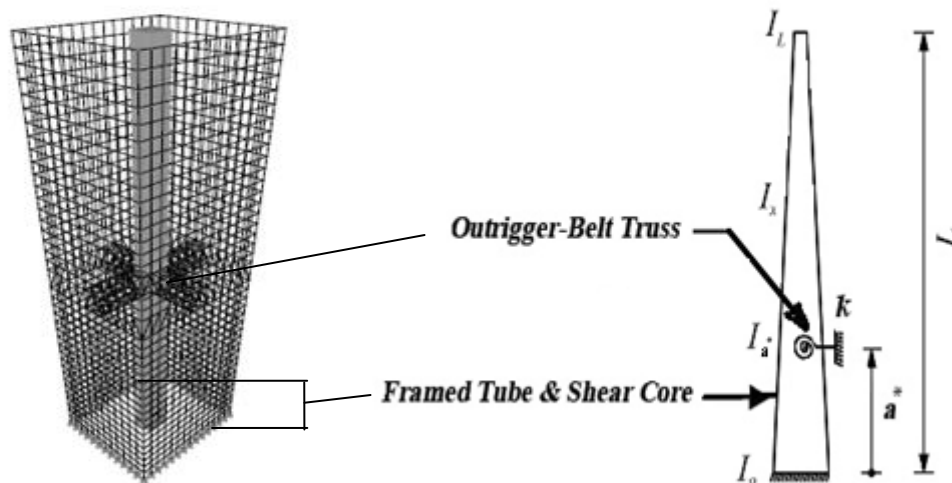


Figure 1. Combined system of framed tube, shear core and outrigger-belt truss system with variable cross section

Strain energy of the equivalent rotational spring (E) is:

$$E = \frac{1}{2} k \theta^2 \quad (1)$$

where θ and k are rotation and stiffness of the equivalent spring respectively. In finding optimum location of the equivalent spring, first derivative of the energy equation with respect to location of the spring as measured from base of the structure (a^*) should be zero (i.e. $dE/da^* = 0$). For this purpose θ and k should be presented as a function of parameter a^* . The θ parameter is related to loading type, three types of lateral loading are considered here, a) uniformly distributed load, b) triangularly distributed load and c) concentrated load at top of the structure. In the following section, stiffness of the equivalent spring and rotation of a tall building is computed.

3. STIFFNESS OF THE EQUIVALENT SPRING

As mentioned earlier, a structure with combined system of framed tube and shear core can be modeled as a cantilevered beam with variable cross section. Effects of outrigger and belt truss systems on the primary structure can be modeled via a rotational spring placed at the location of outrigger and belt truss system, somewhere along structure's height. In this sections, the equivalent stiffness of the flexible outrigger system which is modeled by a rotational spring is presented.

If the flexural stiffness of shear core and outrigger system can be shown as EI and EI_o , respectively, by considering the Fig. 2 and the slope-deflection relationship, torsional moment M_{ab} can be computed as follows:

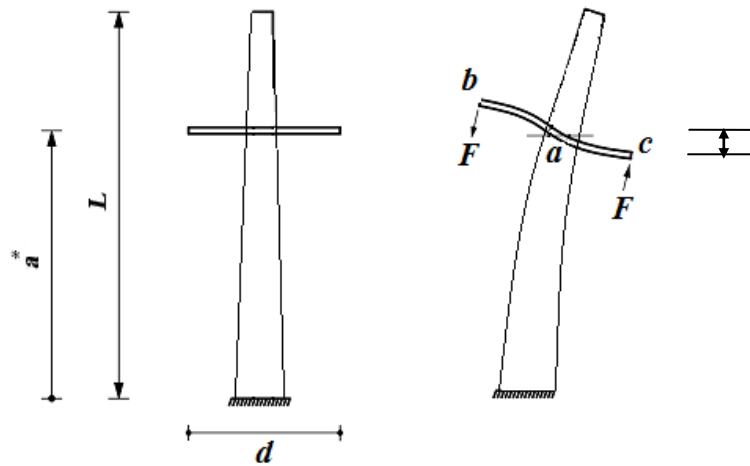


Figure 2. The effect due to torsional moment modeled as couple force while considering deformability of the outrigger system

$$M_{ab} = \frac{2EI}{d} a^* (3\theta_a - \frac{6\delta}{d}) \tag{2}$$

where, θ_a , δ , d and M_{ab} are respectively, the rotation of the structure at location a , axial deformation of exterior columns, the distance between exterior columns and the moment of beam and outrigger system at point a . Also, considering the Fig. 2, the value of M_{ab} can be determined as $M_{ab} = \frac{Fd}{2}$. Denoting the location of the belt truss and outrigger system as a^* for the case in which the cross section of the exterior columns (A) are not altered along the height of the structure, the axial force of these columns can be computed as $F = \frac{AE\delta}{a^*}$, in which E is structure's modulus of elasticity. By considering the above relations for F and M_{ab} , one can obtain:

$$\delta = \frac{2a^* M_{ab}}{AE d} \quad (3)$$

Substituting Eq. (3) in Eq. (2), one can obtain the equivalent stiffness of outrigger system as follows:

$$k = \left(\frac{(12EI_{a^*})(Ad^2)}{Ad^3 + 24a^*I_{a^*}} \right) \quad (4)$$

Note from Eq. (4) that stiffness of the equivalent spring, k , is inversely proportional to its location, a^* , as measured from structure's base.

The θ parameter is a function of spring location (a^*). This relationship is a function of lateral loading type applied to the structure. In the next section, three types of lateral loading, the uniformly distributed load, triangularly distributed load and concentrated load applied at top of the structure, are considered.

4. ROTATION OF EQUIVALENT SPRING FOR SHEAR CORE WITH VARIABLE STIFFNESS

At first, in this section, it is assumed that the moment of inertia for the combined system varies linearly with structure's height. Therefore, the structure can be modeled as shown in Fig. 1.

In this section, three types of lateral loading, the uniformly distributed load, triangularly distributed load and concentrated load placed at top of the structure are considered. Since, the stiffness of framed tube is assumed constant with respect to structure's height, the stiffness of equivalent spring is given by Eq. (4).

4.1 Calculation of the optimum location for outrigger-belt truss system subjected to uniformly distributed lateral loading

It is assumed that the structure is subjected to uniformly distributed lateral load W ; and rotation of the beam at location a^* is measured by angle θ .

Based on superposition principle, θ is

$$\theta = \theta_1 + \theta_2 \quad (5)$$

The value of moment in a section at distance x from the base of the structure is $M_x = wLx - \frac{wL^2}{2} - \frac{wx^2}{2}$. In addition, a linear relationship describing the moment of inertia for shear core is assumed as:

$$I_x = I_0 + \frac{\Delta I}{L} x \tag{6}$$

Hence, the rotation due to w at a^* is

$$\theta_1 = \frac{wL}{E\Delta I} \left[\left(L + \frac{I_0 L}{\Delta I} \right) a^* - \left(L^2 \frac{I_0}{\Delta I} + L^2 / 2 + \frac{I_0^2 L^2}{2\Delta I^2} \right) \ln \left(\frac{\Delta I}{I_0} \frac{a^*}{L} + 1 \right) - \frac{a^{*2}}{4} - \frac{1}{2} \frac{I_0}{\Delta I} \frac{a^*}{L} \right] \tag{7}$$

Since $M = k\theta$, then angular rotation θ_2 due to moment M at location a^* is computed as follows

$$\theta_2 = \frac{-k\theta}{EI_0} \times \ln \left(\frac{\Delta I}{I_0} \frac{a^*}{L} + 1 \right) \tag{8}$$

For sake of clarity and simplicity the following dimensionless parameters are introduced:

$$\alpha = \frac{a^*}{L}, \quad \lambda = \frac{\Delta I}{I_0}, \quad \gamma = \frac{Ad^2}{I_0} \tag{9}$$

Optimum location of the outrigger-belt truss system can be obtained by replacing k and θ from Eqs. (4-5) and (7-8) in $(\frac{dE}{da^*} = 0)$ and solving for a^* . Hence from Eqs. (1), (4-5), (7-9) and $(\frac{dE}{da^*} = 0)$, one can obtain:

$$\begin{aligned} & \left[\alpha\lambda + 0.5\gamma \ln(\alpha\lambda + 1) \right] \left\{ \lambda^2\alpha + 0.5\lambda\alpha - \frac{3\lambda^2\alpha^2}{4} + \right. \\ & \left. (\lambda + 0.5\lambda^2 + 0.5) \left[\ln(\alpha\lambda + 1) - \frac{2\alpha\lambda}{\alpha\lambda + 1} \right] \right\} + \\ & \gamma \left[\ln(\alpha\lambda + 1) - \frac{\alpha\lambda}{\alpha\lambda + 1} \right] \left[\lambda^2\alpha + 0.5\lambda\alpha - \frac{\lambda^2\alpha^2}{4} - \right. \\ & \left. (\lambda + 0.5\lambda^2 + 0.5) \ln(\alpha\lambda + 1) \right] = 0 \end{aligned} \tag{10}$$

Solving the Eq. (10) for different values of λ and γ , the normalized optimum location for belt truss and outrigger system (α) can be determined. These optimum locations are shown in Fig. 3 for uniformly distributed loading.

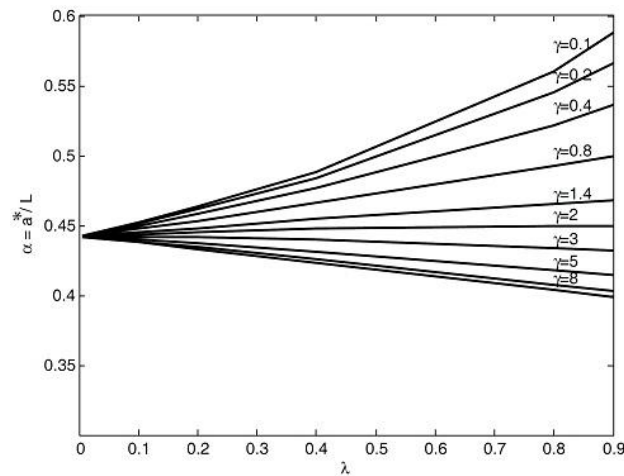


Figure 3. Optimum location of flexible outrigger system under uniformly distributed loading for combined system with variable moment of inertia

As shown in Fig. 3, for constant moment of inertia ($\lambda = 0$), the value of α is equal to 0.441 for all values of γ .

4.2 Calculation of the optimum location for outrigger-belt truss system subjected to triangularly distributed lateral loading

Since moment in a section at distance x from the base of the structure is

$$M_x = \frac{cL^2x}{2} - \frac{cL^3}{3} - \frac{cx^3}{6}, \text{ then } \theta_1 \text{ is obtained as}$$

$$\theta_1 = \frac{cwL}{EI} \left[\left(\frac{L^2}{2} - \frac{I_0^2 L^2}{6\Delta I^2} \right) a^3 + \frac{I_0 L a^2}{12\Delta I} - \frac{a^3}{18} - \left(\frac{I_0 L^3}{2\Delta I} + \frac{L^3}{3} - \frac{I_0^3 L^3}{6\Delta I^3} \right) \ln \left(\frac{\Delta I}{I_0} \frac{x}{L} + 1 \right) \right] \quad (11)$$

Angular rotation θ_2 due to moment M is the same the value of θ_2 for uniformly distributed loading.

Optimum location of the outrigger-belt truss system can be obtained by replacing k and θ from Eqs. (4-5), (8) and (11) in $(dE/da^* = 0)$ and solving for a^* . Hence from Eqs. (1), (4-5), (8-9), (11) and $(dE/da^* = 0)$, one can obtain:

$$\left[\alpha\lambda + \frac{\gamma}{2} \ln(\alpha\lambda + 1) \right] \left[\frac{\lambda^3 \alpha}{2} - \frac{\lambda \alpha}{6} + \frac{\lambda^2 \alpha^2}{4} - \frac{5\lambda^3 \alpha^3}{18} + \left(\frac{\lambda^2}{2} + \frac{\lambda^3}{3} - \frac{1}{6} \right) \left\{ \ln(\alpha\lambda + 1) - \frac{2\alpha\lambda}{\alpha\lambda + 1} \right\} \right] \\ + \gamma \left[\ln(\alpha\lambda + 1) - \frac{\alpha\lambda}{\alpha\lambda + 1} \right] \left[\frac{\lambda^3 \alpha}{2} - \frac{\lambda \alpha}{6} + \frac{\lambda^2 \alpha^2}{12} - \frac{\lambda^3 \alpha^3}{18} - \left(\frac{\lambda^2}{2} + \frac{\lambda^3}{3} - \frac{1}{6} \right) \ln(\alpha\lambda + 1) \right] = 0 \quad (12)$$

Solving the Eq. (12) for different values of λ and γ , the normalized optimum location of

belt truss and outrigger system (α) can be determined. These optimum locations are shown in Fig. 4 for triangularly distributed loading.

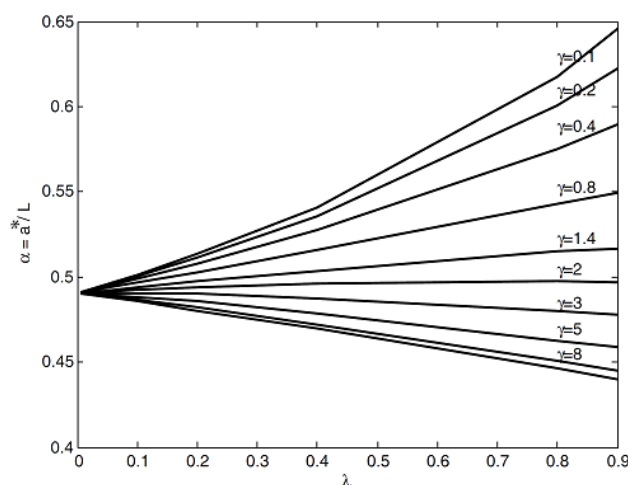


Figure 4. Optimum location of flexible outrigger system under triangularly distributed loading for combined system with variable moment of inertia

As shown in Fig. 4, for constant moment of inertia ($\lambda = 0$), the value of α is equal to 0.49 for all values of γ .

4.3 Calculation of the optimum location for outrigger-belt truss system subjected to concentrated load applied at top of the structure

Since moment in a section at distance x from the base of the structure is $M_x = Px - PL$, θ_1 is as follow

$$\theta_1 = \frac{PL}{E\Delta I} \left[a^* - \left(\frac{I_0 L}{\Delta I} + L \right) \ln \left(\frac{\Delta I}{I_0} \frac{a^*}{L} + 1 \right) \right] \tag{13}$$

Angular rotation θ_2 due to moment M is the same the value of θ_2 for uniformly distributed loading.

Optimum location of the outrigger-belt truss system can be obtained by replacing k and θ from Eqs. (4-5), (8) and (13) in $(dE/da^* = 0)$ and solving for a^* . Hence from Eqs. (1), (4-5), (8-9), (13) and $(dE/da^* = 0)$, one can obtain:

$$\left[\alpha\lambda + \frac{\gamma}{2} \ln(\alpha\lambda + 1) \right] \left\{ \alpha\lambda + (1 + \lambda) \left[\ln(\alpha\lambda + 1) - \frac{2\alpha\lambda}{\alpha\lambda + 1} \right] \right\} + \gamma \left[\ln(\alpha\lambda + 1) - \frac{\alpha\lambda}{\alpha\lambda + 1} \right] \left[\alpha\lambda - (1 + \lambda) \ln(\alpha\lambda + 1) \right] = 0 \tag{14}$$

Solving the Eq. (14) for different values of λ and γ , the optimum location of belt truss and outrigger system (α) can be determined. These optimum locations are shown in Fig. 5 for concentrated load applied at top of the structure.

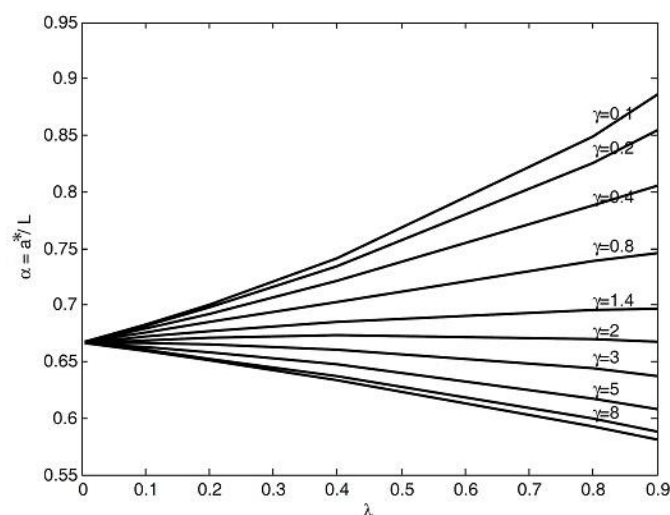


Figure 5. Optimum location of flexible outrigger system under concentrated load applied at top of the structure

As shown in Fig. 5, for constant moment of inertia ($\lambda = 0$), the value of α is equal to 0.667 for all values of γ .

5. ACCURACY OF THE RESULTS

In Figs. 3, 4 and 5, the optimum location of flexible outrigger and rigid belt truss system (α) has been drawn for different values of μ and η . To verify, the results of the proposed method, it is compared with the results of Ref. [7]. Table 1 shows the results of these two methods for a few points. As shown in this table, the proposed method shows that the accuracy of the proposed method is better in compared with Stafford Smith and Coull method's. The differences between the two methods is because of the effect of the flexibility of outrigger system which has been considered in the proposed method.

Table 1: Comparing between the results of the proposed method and Stafford Smith and Coull method

Number of Example	(Stafford Smith and Coull, [7])	Proposed Method	μ	η
First Example	0.62	0.5058	2.5	0.1
Second example	0.675	0.5215	10	1.4
Third Example	0.545	0.4382	15	5

6. CONCLUSION

As shown in Figs. 3, 4 and 5, the optimum location for outrigger-belt truss system is a function of loading type, axial stiffness of exterior columns, distance between the exterior columns and the moment of inertia due to shear core and outrigger system. In general, by increasing the axial stiffness of exterior columns (η or γ), the optimum location of this system is lowered. In addition, it can be seen from these figures that there is no uniform pattern when parameters μ or λ are increased. In fact, by increasing these parameters, different behaviors such as increasing, decreasing or constant for the optimum location of outrigger-belt truss system is predicted.

In the case of rigid outrigger system; for any value of η , normalized optimum location becomes $\alpha = 0.441$, $\alpha = 0.49$ and $\alpha = 0.667$ for uniformly distributed loading, triangularly distributed loading and concentrated load, respectively. Furthermore, for all loading types, it can be deduced that the η parameter has important effect on optimum location of the belt truss and outrigger system. In addition, over the intermediate range of η , rigidity of the outrigger has little effect on optimum location of this system. In addition, the graphs, which presented in this paper can be used as a useful tool to determine the optimum location of belt truss and outrigger system.

REFERENCES

1. Kamgar R, Saadatpour MM. A simple mathematical model for free vibration analysis of combined system consisting of framed tube, shear core, belt truss and outrigger system with geometrical discontinuities, *Appl Math Model* 2012; **36**(10): 4918-30.
2. Kamgar R, Rahgozar R. A simple approximate method for free vibration analysis of framed tube structures, *Struct Des Tall Spec Build* 2013; **22**(2): 217-34.
3. Heidari A, Rahgozar R, Kamgar R. Free vibration analysis of tall building with geometrical discontinuities, *Asian J Civil Eng* 2014; **15**(1): 119-34.
4. Rahgozar R, Ahmadi AR, Sharifi YA. A simple mathematical model for approximate analysis of tall buildings, *Appl Math Model* 2010; **34**(9): 2437-51.
5. Stanford Smith B, Salim I. Parameter study of outrigger-braced tall building structures, *J Struct Div, ASCE* 1981; **107**(10): 2001-14.
6. Taranath BS. *Structural analysis and design of tall buildings: Steel and composite construction*, Taylor & Francis, CRC Press, New York, 2012.
7. Stafford Smith B, Coull A. *Tall building structures: Analysis and design*, John Wiley & Sons, New York, 1991.
8. Taranath BS. *Steel, concrete, and composite design of tall buildings*, McGraw-Hill, New York, 1998.
9. Wu JR, Li QS. Structural performance of multi-outrigger braced tall buildings, *Struct Des Tall Spec Build* 2003; **12**(2): 155-76.

10. Hoenderkamp JCD. Shear wall with outrigger trusses on wall and column foundations, *Struct Des Tall Spec Build* 2004; **13**(1): 73-87.
11. Hoenderkamp JCD. Second outrigger at optimum location on high rise shear wall, *Struct Des Tall Spec Build* 2008; **17**(3): 619-34.
12. Raj Kiran Nanduri PMB, Suresh B, Ihtesham Hussain MD. Optimum position of outrigger system for high-rise reinforced concrete buildings under wind and earthquake loadings, *Am J Eng Res* 2013; **2**(8): 76-89.
13. Assadi Zeidabadi N, Mirtalae K, Mobasher B. Optimized use of the outrigger system to stiffen the coupled shear walls in tall buildings, *Struct Des Tall Spec Build* 2004; **13**(1): 9-27.
14. Stanford Smith B, Salim I. Formulae for optimum drift resistance of outrigger braced tall building structures, *Comput Struct* 1983; **17**(1): 45-50.
15. Rutenberg A, Tal D. Lateral load response of belted tall building structures, *Eng Struct* 1987; **9**(1): 53-67.
16. Zhang J, Zhang ZX, Zhao Wg, Zhu Hp, Zhou CS. Safety analysis of optimal outriggers location in high-rise building structures, *J Zhejiang Univer Sci A* 2007; **8**(2): 264-9.
17. Bayati Z, Mahdikhani M, Rahaei A. Optimized use of multi-outriggers system to stiffen tall buildings, *The 14th World Conference on Earthquake Engineering*, Beijing, China, 2008.
18. Gerasimidis S, Efthymiou E, Baniotopoulos CC. Optimum outrigger locations of high-rise steel buildings for wind loading, *EACWE 5*, Florence, Italy, 2009.
19. Rahgozar R, Sharifi Y. An approximate analysis of combined system of framed tube, shear core and belt truss in high-rise buildings, *Struct Des Tall Spec Build* 2009; **18**(6): 607-24.
20. Fawzia S, Fatima T. Deflection control in composite building by using belt truss and outriggers systems, *Proceedings of the 2010 World Academy of Science, Engineering and Technology Conference*, 2010, pp. 646-51.
21. Fawzia S, Nasir A, Fatima T. Study of the effectiveness of outrigger system for high-rise composite buildings for cyclonic region, *International Conference on Electrical, Computer, Electronics and Communication Engineering*, 2011, pp. 937-45.
22. Taranath BS. Optimum belt truss locations for high-rise structures, *Struct Eng* 1975; **53**(8): 345-7.
23. McNabb JW, Muvdi BB. Drift reduction factors for belted high-rise structures, *Eng J AISC* 1975; **12**(3): 88-91.
24. Lee KK, Loo YC, Guan H. Simple analysis of framed-tube structures with multiple internal tubes, *J Struct Eng, ASCE* 2001; **127**(4): 450-60.