

UNCERTAINTY ANALYSIS OF STABILITY OF GRAVITY DAMS USING THE FUZZY SET THEORY

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ABSTRACT

This paper introduces a methodology for considering the uncertainties in stability analysis of gravity dams. For this purpose, a conceptual model based on the fuzzy set theory and Genetic Algorithm (GA) optimization is developed to be coupled to a gravity dam analysis model. The uncertainties are represented by the fuzzy numbers and the GA is used to estimate in what extent the input uncertainties affect the dam safety factors.

An example gravity dam is analyzed using the proposed approach. The results show that the crisp safety factors might be highly affected by the input uncertainties. For instance, $\pm 10\%$ uncertainty in the design parameters could result in about $(-346$ to $+ 146)\%$ uncertainty in the stability safety factors and $(-59$ to $+ 134)\%$ in the stress safety factor of the example dam.

Keywords: uncertainty; stability analysis; fuzzy set theory; gravity dam; safety factor.

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1. INTRODUCTION

Uncertainty is a general concept that reflects our lack of sureness about something or someone, ranging from just short of complete sureness to an almost complete lack of conviction about an outcome. There are different types of uncertainty, the majority of those can be categorized under two simple headings [1]; 1- natural variability (Aleatory) and 2- knowledge uncertainty (Epistemic). The uncertainties of basic information and design parameters play a significant role in performance and safety of important civil structures such as gravity dams. Hence, considering such uncertainties in analysis and design of dams is quite crucial. Several design codes have been prepared for handling common uncertainties

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arise from the material properties and design loads based on the engineering judgments and experiences. However, the effects of uncertainties on the structure performance cannot be explicitly evacuated using the common design criteria and models. The possibility of design or construction errors cannot be easily incorporated into the conventional simulation models. To investigate how input uncertainties are spread out over the system, more sophisticated models are required.

There are number of design regulations and standard references to evaluate the gravity dams stability. The well-known regulations have been issued by the U.S. Army Corps of Engineers (USACE), the U.S. Bureau of Reclamation (USBR) and the U.S. Federal Energy Regulatory Commission (FERC). These regulations suggest some safety factors for stability of concrete gravity dams.

Important parameters in evaluating a gravity dam safety factors include the geometry of dam body, material properties, loading conditions and geotechnical properties of the foundation. For the structural analysis of a dam, all the aforementioned parameters are assumed constant with certain or “crisp” values. However, in reality, they cannot be precisely quantified because of lack of information, imprecise measurements, and stochastic nature of some parameters. Therefore, they may introduce significant uncertainties to the dam simulations results. Besides, the inherent uncertainties associated with the design parameters, the mathematical models applied to simulate the dam are also including uncertainties. In practice, these uncertainties, in basic and design data as well as in the simulation model, are taken into account indirectly by using conservative safety factors. However, directly incorporating the uncertainties into the analysis of dam structure would result in more reliable designs [2].

Different approaches have been so far used for uncertainty analysis of engineering systems. These approaches vary from simple engineering judgments to the sophisticated statistical or intelligent models. Apart from how the uncertainty resources are identified and quantified in a system, there is a challenging issue to apply them to the system governing equations [3]. For important structures like dams, it is crucial to develop a mathematical model able to find out that in what extent the input uncertainties are spread out over the system and influence the responses. For a long time, the theory of statistics and probability was the predominant approach for handling uncertainties in simulations. The Monte Carlo simulation method has been one of the most popular approaches for representing and analyzing uncertainties in various engineering systems.

Another approach is the application of fuzzy set theory and fuzzy logic originally introduced by Zadeh [4]. Principles of the theory were then developed by him and his colleagues as well as by Mamdani and Assilian [5], Zimmermann [6] and Bit et al.[7]. The fuzzy set theory was initially intended to be an extension of dual logic and/or classical sets theory [8] however, during the last decades; the concept of fuzziness has been highly developed in the direction of a powerful ‘fuzzy’ mathematics.

At present, the fuzzy approach can, in some sense, be considered as the most general method for uncertainty analysis of engineering systems. In field of civil engineering, many investigations have so far exploited the fuzzy set theory for handling vague data in analysis and design of structures. In order to approach the subject of present investigation, a number of recent studies on application of fuzzy set theory to the uncertainty analysis of structures are briefly reviewed as follows.

Hanss and Turrin [9] stated that while, aleatory uncertainties can be successfully taken into account by the use of probability theory, the modeling of epistemic uncertainties still remains a challenging topic. They introduced an interdisciplinary methodology for modeling systems with inclusion of uncertainties – in particular of those of epistemic type. They provided an interface to couple the fuzzy set theory with analytical commercial software. They applied the method to two different illustrative examples including the simulation of automotive crash in structural dynamics and the simulation of landslide failure in geotechnical engineering. In both applications, epistemic uncertainties arise from lack of knowledge and simplifications in modeling were analyzed.

Valdebenito, Jensen [10] presented an approach for performing fuzzy analysis of linear structures subject to static loading. The approach was based on a non-linear approximation of responses taking into account the linearity of the displacements with respect to the loading condition. However, the nonlinear nature of displacements with respect to structural parameters were handled in the analysis by heuristic intervening variables. The uncertainties in both material properties and loadings were represented and analyzed by means of fuzzy set theory. To find the extreme values of fuzzy displacements, the fuzzy analysis was arranged as a simple mathematical optimization problem.

Balu and Rao [11] presented a model for estimating the bounds on structural reliability in presence of mixed uncertain (random and fuzzy) variables. For fuzzy analysis of the system, the nonlinear responses are considered as a linear function of input uncertainties. Accordingly, the bounds of each response is expressed as summation combination of the bounds of the input variables. This simplification resolved the need for optimization techniques.

Sieniawska, Zielichowski-Haber [12] introduced a numerical approach for the safety analysis of industrial roof and highway bridges. They considered uncertainties of the physical and geometrical parameters of the structure as well as the loading conditions using the fuzzy or random variables. They built a limited state function on the basis of the theory of the yield lines and the kinematic formulation of the shakedown problem. They showed that if all uncertainties appear in the limit state functions are treated as fuzzy variables, the dominant failure mechanism and the corresponding minimum load factor for the non-shakedown can be obtained by optimization.

The present study intends to use the fuzzy set theory for the stability analysis of gravity dams to evaluate how the input uncertainties affect the safety factors. For this purpose, a gravity dam simulation model is developed to calculate the stability and stress safety factors. All kinds of analysis variables are considered to have uncertainty. They are represented by fuzzy numbers and introduced to the model. Using the α – cut approach in the fuzzy set theory, the fuzzy variables are discretized in limited number of membership function values. To each α – cut a nonlinear optimization solver is twice applied to find the extreme values of the responses of interest, the safety factors, corresponding to the membership value in consideration. This procedure is applied to all α – cuts and finally, the fuzzy numbers of the responses are obtained. For optimization, a simple real genetic algorithm is utilized. The proposed scheme is applied against a case study and the obtained results are discussed.

2. STABILITY ANALYSIS OF GRAVITY DAMS

A concrete gravity dam is a solid structure made of concrete and constructed across a river. The section of a gravity dam is approximately triangular in shape as depicted in Fig. (1a). The section is so proportioned that, the dam body weight resists all kinds of forces acting on it. If the foundation is adequate and the dam is properly designed and constructed, it will be a permanent structure that requires little maintenance [13]. The most important part of the analysis and design of a dam are to identify critical forces and their combinations acting on the dam structure. The main forces are caused by (1) external water pressure, (2) temperature, (3) internal water pressure; i.e., pore pressure or uplift in the dam and foundation, (4) weight of the structure, (5) ice pressure, (6) silt pressure, (7) earthquake, and (8) forces from gates or other appurtenant structures. The dam weight and water pressure are directly calculated from the unit weight of the concrete and fluid respectively. However, the other forces e.g., the uplift, earthquake loads and silt and ice pressures, need decisions on the expected reliability and safety factors. For calculating these forces, special care must be taken to estimate the material properties and field data. For this purpose, engineering judgments and experiences are quite important.

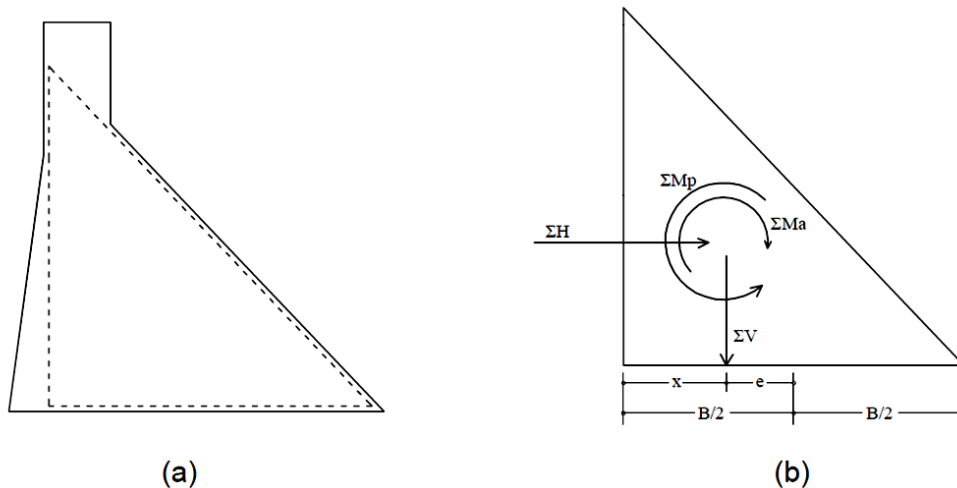


Figure 1. (a) Schematic of dam section, (b) the resultant forces and moments on dam body

According to the USACE (1995) provisions, there are seven critical loading conditions (Table 1) that must be taken into account for the stability analysis of concrete gravity dams. These conditions are classified into the usual, unusual and extreme loadings as presented in Table (1).

Table 1: USACE (1995) load conditions for stability analysis of gravity dams

	class	Earthquake	Headwater	Tailwater	Silt	Ice	Uplift
1	Unusual	-	-	-	-	-	-
2	Usual	-	Crest (top of gate)	Minimum	Yes	Yes	Yes
3	Unusual	-	SPF	Flood elevation	Yes	-	Yes
4	Extreme	OBE (upstream)	-	-	-	-	-

5	Unusual	direction) OBE (downstream direction)	Usual pool level	Minimum	Yes	-	Pre- earthquake level
6	Extreme	MCE(downstream direction)	Usual pool level	Minimum	Yes	-	Pre- earthquake level
7	Extreme	-	PMF	Flood elevation	Yes	-	Yes

SPF (Standard Project Flood), PMF (Probable Maximum Flood), OBE (Operating Basis Earthquake), MCE (Maximum Credible Earthquake)

Also, the stability and stress criterion for concrete gravity dams for each of the above loading conditions is presented in Table (2).

Table 2: USACE (1995) stability and stress criteria for concrete gravity dams

Loading condition	Resultant location at base	Minimum sliding safety factor	Concrete compressive stress
Usual	Middle 1/3	2	$0.3f'_c$
Unusual	Middle 1/2	1.7	$0.5f'_c$
Extreme	Within base	1.3	$0.9f'_c$

Note: f'_c is 1-year unconfined compressive strength of concrete. The sliding factors of safety (FS) are based on a comprehensive field investigation and testing program. Concrete allowable stresses are for static loading conditions.

The gravity dam stability analysis includes both overturning and sliding control. For each loading condition, the overturning stability is evaluated by applying all vertical (ΣV) and lateral forces and summing moments (ΣM) about the downstream toe (Fig. 1b). The resultant location along the base (x) is:

$$x = \frac{\Sigma M}{\Sigma V} \tag{1}$$

The eccentricity (e) of the resultant force is:

$$e = x - \frac{B}{2} \tag{2}$$

where, B is the length of the section base. According to the USACE (1995), the overturning should be controlled for the resultant locations in table (2). It is proved that, when the resultant of all forces acting above any horizontal plane through a dam intersects that plane outside the middle third, a non-compression zone will be resulted. For usual loading conditions, the resultant should always lie within the middle third of the base for no tension to develop in the concrete. For unusual loading conditions, the resultant must remain within the middle half of the base. Also, for the extreme loading conditions, the resultant

must remain sufficiently within the base to assure that the base pressures are within the prescribed limits. An eccentricity safety factor, FS_e , is defined as the following to control whether the resultant location in the base occurs in the aforementioned regions or not.

$$FS_e = \frac{\alpha \frac{B}{2} - e}{\alpha \frac{B}{2}} = \frac{\alpha B - 2e}{\alpha B} \quad (3)$$

where, α is the fraction introduced in table (2) to define the location of the resultant in the base. For usual loading conditions, α is $1/3$ and for unusual and extreme loading conditions is respectively $1/2$ and 1. When $FS_e > 0$ the above criteria are met for all different loading conditions otherwise, when $FS_e \leq 0$, the recommended criteria are violated.

Another important safety factor against the overturning is calculated by dividing the total passive moments ΣM_p by the total active moments ΣM_a about the toe (Fig. 1b) as the following.

$$FS_o = \frac{\Sigma M_p}{\Sigma M_a} \quad (4)$$

Obviously, the overturning safety factor FS_o must be greater than unity.

Another crucial measure is the sliding safety factor FS_s that is to evaluate the resistance of the dam body against the sliding forces. The multiple-wedge analysis [14] is used for analyzing the sliding phenomenon along the base and within the foundation. The sliding safety factor FS_s is defined as the ratio of the maximum resisting shear T_F and the applied shear T along the slip plane at the service loading conditions:

$$FS_s = \frac{T_F}{T} = \frac{N \tan(\varphi) + cL}{T} \quad (5a)$$

in which, N is the resultant of the forces normal to the sliding plane, φ is the soil foundation angle of internal friction, c is the cohesion intercept and L is the length of the base in compression for a unit strip of dam. This safety factor is used in this research. However, if dam is constructed on rock interface with interlock, the effect of interlock is taken into account for calculating FS_s as follows.

$$FS_s = \frac{T_F}{T} = \frac{N \tan(\varphi) + cL + r\tau\beta B}{T} \quad (5b)$$

where, r is the ratio of the average shear resistance to the ultimate shear resistance of concrete, τ is the unit ultimate shear resistance of concrete and rock, B is the length of the dam support and β , the foundation shear effective area ratio, is a coefficient representing the contribution of the rock foundation interface in bearing the shear stresses. The stability of gravity dams should be also controlled against the over stressing. Stress analysis is

performed to determine the distribution of stresses throughout the structure for static and dynamic loading conditions. The compressing stress in the toe is calculated using the composite stress equation defined for the unit width of the base as the following.

$$f = \frac{\Sigma V}{B} + \frac{MB/2}{I} \quad (6)$$

where, I is the base moment of inertia and, M is the moment of ΣV about the natural axis. M and I are calculated as follows.

$$I = \frac{B^3}{12} \quad (7)$$

$$M = (\Sigma V).e \quad (8)$$

Substituting equations (7) and (8) in (6), the compressing stress in the toe is obtained by the following equation.

$$f = \frac{\Sigma V}{B} \left(1 + \frac{6e}{B} \right) \quad (9)$$

There is an allowable compressive stress for each loading condition as presented in Table 2. On this basis, the overstressing safety factor FS_T for the compressive stress in the toe is calculated as follows.

$$FS_T = \frac{f_a}{f} \quad (10)$$

where, f_a is the allowable compressive stress in concrete for different loading conditions from Table 2. For a safe design FS_S must be greater than unity.

3. UNCERTAINTY ANALYSIS OF GRAVITY DAMS

A gravity dam analysis may include significant uncertainties due to the inaccuracy of material properties and loading estimations, mathematical simulations, construction mistakes and stochastic events during the operation. For a safe design or realistic assessment of an existing dam, considering these uncertainties in analyzes is very important to handle the risk of unforeseen conditions in practice.

Uncertainty can be separated into two broad categories of Aleatory and Epistemic [1]. By definition, aleatory means dependent on luck or chance. The source of the aleatory uncertainty is the natural variability arises from randomness events in the system parameters. For example, the inherent randomness in earthquake phenomenon is an aleatory type of uncertainty. Epistemic means dependent on human knowledge. The epistemic uncertainty can be reduced in theory by increasing the knowledge and information about the system. The uncertainties resulted from inaccurate statistical analysis, an imprecise

laboratory works and human measurements could be treated as epistemic type.

The introduced stability safety factors could appropriately represent the capacity of a gravity dam structure to resist the applied loads. However, due to different types of uncertainties the evaluated safety factors may be violated by significant uncertainties. In this condition, the dam safety factors could not be represented by certain crisp values. In other words, the analysis parameters including the material properties, the structure characteristics, the field coefficients and loading conditions introduce uncertainty to the dam and make the responses as well as the stability safety factors uncertain too. To have a reliable design, estimation of the uncertainty associated with the safety factors is quite important. This goal is followed herein by using the fuzzy set theory.

4. FUZZY ANALYSIS OF GRAVITY DAM

Fuzzy set theory is a mathematical method used to characterize and quantify uncertainty and imprecision in functional relationships. This theory is very useful in situations that there are insufficient data to characterize uncertainties by means and statistical tools. In traditional dual logic, for instance, a statement can be true or false and nothing in between. In the fuzzy set theory, an element can either belong to a set or not. The fuzzy logic has a simple basic idea: statements are not just 'true' or 'false' and partial truth is also accepted. Similarly, partial belonging to a set, called a fuzzy set, is possible.

In the fuzzy set theory, the uncertain parameters are represented by fuzzy numbers. A fuzzy number N is a set defined on the universe of real numbers $N \in R$. For each variable $x \in N$, $\mu_N \in [0,1]$ is called the grade of membership of x in N . If $\mu_N(x) = 0$ then, x is called 'not included' in the fuzzy set and if $\mu_N(x) = 1$, then x is called fully included and if $0 < \mu_N(x) < 1$, x is called the fuzzy member. A so-called α -cut operation, $\alpha \in \mu_N$, denoted by N_α is applied to the fuzzy numbers in such a way that, each N_α is a crisp interval defined as $[x_{a,\alpha}, x_{b,\alpha}]$ as shown in Fig. 2.

When $\alpha = 0$, the corresponding interval is called the 'support' indicated by the interval $[x_a, x_b]$. For $\alpha = 1$, if the membership function is triangular, the interval reduces to one crisp value only, x_c , that is, the 'most likely' value of N . This definition allows for identifying any crisp interval existing within the fuzzy set as a specific α -cut if the membership function μ_N is continuous and the fuzzy set is normalized and convex. The normalization condition implies that the maximum membership value is 1:

$$\exists x \in R, \mu_N(x) = 1 \quad (11)$$

The convexity condition indicates that two arbitrary α and α' intervals satisfy the following relation ($N_\alpha = [x_{a,\alpha}, x_{b,\alpha}]$):

$$\alpha' < \alpha \Rightarrow x_{a,\alpha'} < x_{a,\alpha}, x_{b,\alpha'} > x_{b,\alpha} \quad (12)$$

Two types of triangular and trapezoidal membership functions are very popular in engineering applications. In this study the triangular membership function is used for representing the gravity dam's uncertainties.

To introduce the uncertainty to the design parameters of a gravity dam it is considered that each parameter like x has a very likely crisp value x_c that includes at most $\pm\Delta x$ uncertainty so that, for a certain α -cut we have,

$$x_{a,\alpha} = x_c - \Delta x_\alpha \tag{13a}$$

$$x_{b,\alpha} = x_c + \Delta x_\alpha \tag{13b}$$

where, Δx_α is the uncertainty at cut α and $x_{a,\alpha}$ and $x_{b,\alpha}$ are the lower and upper bounds of x at cut α respectively.

Upon the above explanations, a fuzzy approach for uncertainty analysis of gravity dams is introduced as the following:

1. The crisp (most likely) values of input design variables (x_c) are determined.
2. The maximum uncertainty associated with each variable $\pm\Delta x$ is estimated.
3. To each uncertain variable a fuzzy number with continuous triangular membership function (Fig. 2) is introduced so that, the support of the fuzzy number is $2\Delta x$. Accordingly, a fuzzy number is defined by $A = [x_a, x_c, x_b]$ for x .
4. The membership functions are discretized by a limited number of α -cuts from 0 to 1. As a result, for each design variable at each α -cut, an interval is obtained from which, the variable can be quantified
5. As every input variable has an interval of possible values at a certain α -cut, every model response, safety factors here, would have an interval of probable values too. In fact, the responses intervals show that how the input uncertainties are spread out over the system. To find the responses intervals at each α -cut, two optimization problems are introduced to each safety factor. In other words, each extremity of a safety factor interval is obtained by using the optimization. For this purpose, the safety factor in consideration is defined as a nonlinear function $f(\vec{x})$ in which, \vec{x} is the vector of decision variables (input uncertainties). $f(\vec{x})$ is evaluated using the simulation model and for each α -cut, it is once maximized and once minimized by an optimization model.

To solve the raised nonlinear optimization problems, a simple real genetic algorithm based on Haupt and Haupt [15] is used in this study. The optimization is repeated for all α -cuts for all safety factors and, their corresponding fuzzy numbers are obtained. It is worth noting that, there are three loading conditions in the design process of a gravity dam and, for each condition four safety factors are evaluated. If the input fuzzy numbers are discretized by m α -cuts, the total number of required optimization for the analysis of each safety factor is $2m$.

5. EXAMPLE

To investigate the proposed model, it is applied to an example gravity dam designed based on the USACE (1995). The most likely design parameters are presented in Table 3. It is also assumed that the normal water level in the dam reservoir is equal to the standard project

flood level. Main parameters for this stability analysis could be categorized into four groups. 1) geometry parameters 2) material properties 3) loading variables and 4) Physical coefficients.

It is considered that all design parameters have $\pm 10\%$ uncertainty. Referring to Fig. 2, for each variable, $\Delta x = 0.1x_c$ at the support of the triangular fuzzy numbers. Table 3. also contains the extreme input uncertainties. Given the input fuzzy numbers; it is aimed at determining the output fuzzy numbers associated with the stability safety factors. For this purpose, a fuzzy simulation-optimization model is developed according to the proposed methodology. There are totally 23 fuzzy input variables in the simulation model and 4 fuzzy output variables to be obtained for each loading condition.

Table 3: Input fuzzy variables

Type	Variable	Unit (SI)	Uncertainty	Lower limit	Crisp value	Upper limit
Geometry	1 Base of main triangle	m	10%	36	40	44
	2 Height of main triangle	m	10%	40.5	45	49.5
	3 Height of upstream flare	m	10%	30.6	34	37.4
	4 Width of upstream flare	m	10%	0.9	1	1.1
	5 Height of freeboard	m	10%	1.71	1.9	2.09
	6 Width of freeboard	m	10%	4.86	5.4	5.94
Materials	7 Unit weight of concrete	KN/m ³	10%	21.24	23.6	25.96
	8 Unit weight of silt	KN/m ³	10%	9.9	11	12.1
	9 Compressive strength of concrete	KN/m ²	10%	21600	24000	26400
	10 Shear strength of concrete	KN/m ²	10%	6300	7000	7700
	11 Tan(Φ)	-	10%	0.63	0.7	0.77
Loading	12 Later pressure coefficient of silt	-	10%	0.9	1	1.1
	13 Standard project flood level	m	10%	40.5	45	49.5
	14 Probable maximum flood level	m	10%	54	60	66
	15 Downstream flood level	m	10%	18	20	22
	16 Downstream minimum level	m	10%	7.2	8	8.8
	17 Upstream saturated silt level	m	10%	22.5	25	27.5
	18 Operating basis earthquake coefficient	-	10%	0.09	0.1	0.11
	19 Maximum credible earthquake coefficient	-	10%	0.27	0.3	0.33
Coef.	20 Ice force per unit of length	KN/m	10%	171	190	209
	21 Efficiency coefficient of grout curtain	-	10%	0.603	0.67	0.737
	22 Ratio of average shear resistance to ultimate one	-	10%	0.45	0.5	0.55
	23 Foundation effective shear area ratio	-	10%	0.18	0.2	0.22

Every fuzzy variable is discretized with 11 α – cuts including $\alpha = 0$ (the support), 0.1, 0.2... 0.9, 1 (the crisp). To estimate each extreme value of each safety factor in each α – cut and for each loading condition, the GA is once applied to the problem. In the applied GA, the roulette- wheel method is used for the parent selection, the mutation ratio is assigned

5%, and the population size is considered 100. To analyze each safety factor in each loading condition, the GA was totally 20 times applied to the problem to estimate the corresponding α – cuts extreme values. Fig. 3. demonstrates the resulted fuzzy safety factors. Each type of safety factor is presented in a separate diagram for all possible loading conditions. Fig. 4. also presents the highest, lowest and the crisp values of safety factors at $\alpha = 0$.

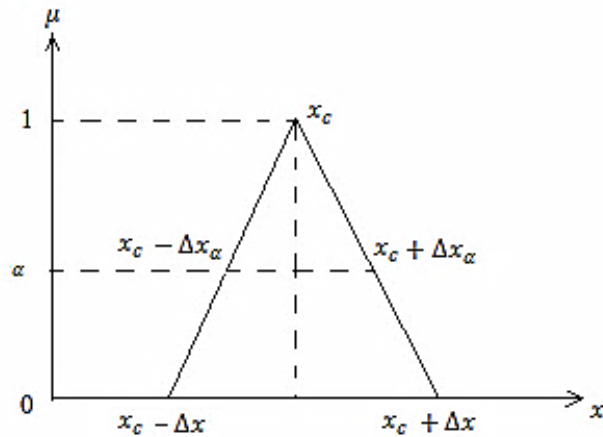


Figure 2. Triangular symmetric fuzzy number

Also, Table 4. reports the maximum uncertainty of each safety factor with its associated loading condition. It clearly shows that how the small $\pm 10\%$ input uncertainties are spread out over the system and lead to such large uncertainties in the responses.

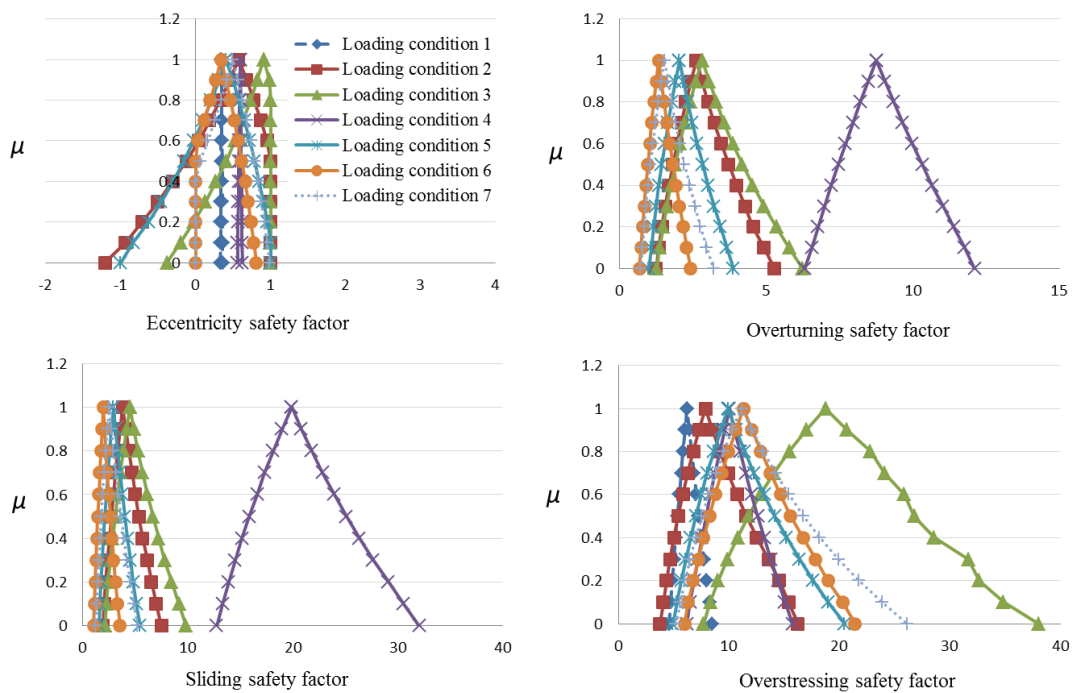


Figure 3. Fuzzy safety factors

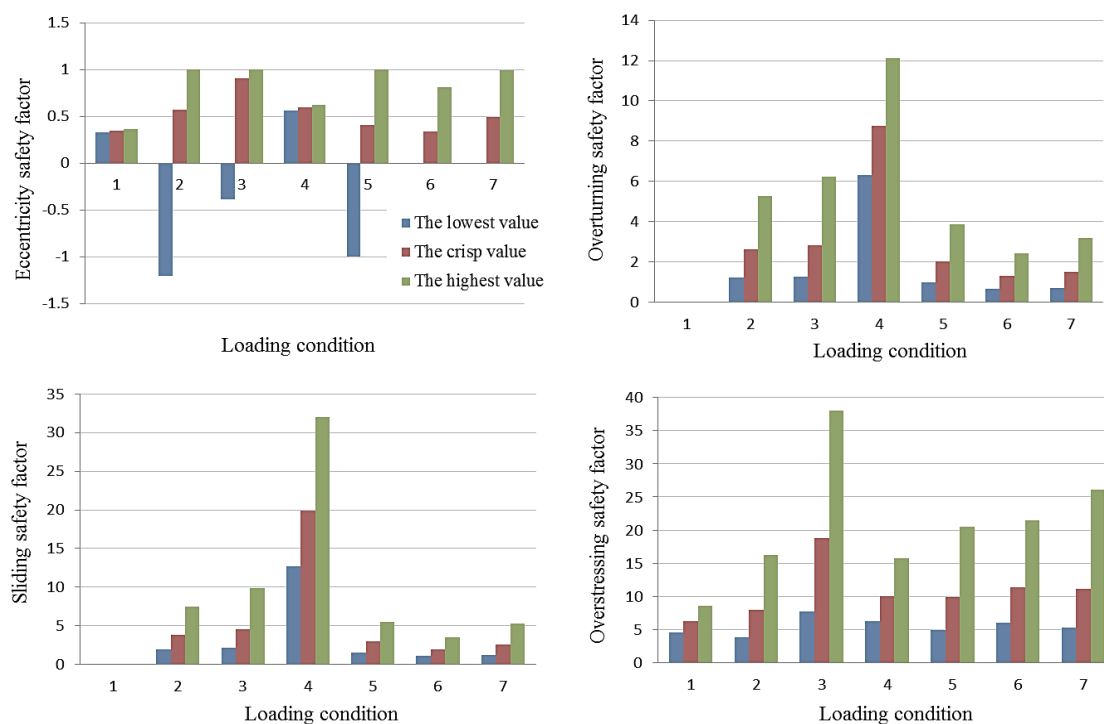


Figure 4. The highest, lowest and crisp values of analyzed safety factors

Table 4: The maximum uncertainties of safety factors

	Uncertainty (%)	Loading condition
Overturning safety factor	-55	3
	+122	3
Sliding safety factor	-53	3
	+118	3
Overstressing safety factor	-59	3
	+134	7
Eccentricity safety factor	-346	5
	+146	5

6. CONCLUSION

Dams are of the most important infrastructures in many countries. Safe design, construction and operation of dams must take into consideration all uncertainties to handle the risk of unforeseen conditions. Thus the uncertainty analysis of structural stability of dams is essential.

The main objective of the paper is to develop an approach which uses the fuzzy sets theory in the stability analysis of gravity dam to see how the input uncertainties spread on the safety factors. For this purpose, a conceptual model was introduced for uncertainty

analysis of the stability of gravity dams using the fuzzy sets theory and the GA optimization. The uncertainties are interpreted as fuzzy numbers with a symmetric triangular membership function. A limited number of α -cuts are considered to break the problem into a sequence of optimization sub-problems. For each α -cut the fuzzy input variables are introduced to the model in form of an interval. Consequently, the safety factor analysis is included an optimization problem in which the safety factor analysis model is the objective function and the bounds of decision variables are obtained from the intervals associated with each α -cut. A simple genetic algorithm is developed and coupled to the dam stability analysis model to find the extreme values of safety factors in each α -cut.

The model is applied to an example of gravity dam designed according to the USACE (1995). It was demonstrated that small uncertainties in the input variables of stability analysis of dam can lead to large uncertainties in the responses as well as in the safety factors. It was found that, when the effects of small uncertainties are superposed they can violate the design criteria recommended by the standard manuals. Accordingly, the system apparently safe in crisp input data may considerably fails when the data include uncertainties. Also, the results indicate that the dam safety factors do not behave monotonically with the input uncertainties. This means that to find the extreme values of the dam responses in fuzzy analysis, the use of optimization is inevitable.

ABBREVIATIONS

SPF	Standard Project Flood
OBE	Operating Basis Earthquake
MCE	Maximum Credible Earthquake
PMF	Probable Maximum Flood
FS	Factor of Safety

REFERENCES

1. Ayyub BM, MRH. *Probability, Statistics, and Reliability for Engineers and Scientists*, Ed. 2, Chapman Hall/CRC, Boca Raton, Fla, Chapman Hall/CRC, 2003
2. Clemens V, Verhoosel TPS, Steven J, Hulshoff Miguel A. Gutiérrez, Uncertainty and reliability analysis of fluid structure stability boundaries, *AIAA Journal* 2009; **47**(1): 91-104.
3. Haghghi A, Asl AZ. Uncertainty analysis of water supply networks using the fuzzy set theory and NSGA-II, *Engineering Applications of Artificial Intelligence* 2014; **32**: 270-82.
4. Zadeh LA. Fuzzy sets, *Information and Control* 1965; **8**: 338-53.
5. Mamdani EH, Assilian S. An experiment in linguistic synthesis with a fuzzy logic controller, *International Journal of Man-Machine Studies* 1975; **7**(1): 1-13.
6. zimmerman HJ. *Fuzzy Set Theory and Its Application*, Boston, Dordrecht, 1985.
7. Bit AK, Biswal MP, Alam SS. Fuzzy programming approach to multicriteria decision making transportation problem, *Fuzzy Sets Systems* 1992; **50**: 135-41.
8. Zimmermann HJ. Fuzzy set theory, *WIREs Computational Statistics* 2010; **2**: 317-32.

9. Hanss M, Turrin S. A fuzzy-based approach to comprehensive modeling and analysis of systems with epistemic uncertainties, *Structural Safety* 2010; **32**(6): 433-41.
10. Valdebenito M, et al. Approximation concepts for fuzzy structural analysis, in vulnerability, uncertainty, and risk, *American Society of Civil Engineers* 2014; 135-44.
11. Balu AS, Rao BN. Efficient assessment of structural reliability in presence of random and fuzzy uncertainties, *Journal of Mechanical Design, American Society of Mechanical Engineers* 2014; **136**(5): 051008.
12. Sieniawska R, Zielichowski-Haber W, Zukowki S. Safety estimation of the plate-girder structures with uncertain parameters from the shakedown theory point of view, in vulnerability, uncertainty, and risk, *American Society of Civil Engineers* 2014; 1283-92.
13. USBR. *DESIGN OF SMALL DAMS*, A Water Resources Technical Publication, 1987.
14. USACE. *Gravity Dam Design EM 1110-2-2200*, US Army Corps of Engineers: Washington, DC, 1995.
15. Haupt RL, Haupt SE. *Practical Genetic Algorithms*, 2nd ed, Hoboken, New Jersey, John Wiley & Sons, Inc, 2004.