OPTIMUM DESIGN OF GRILLAGE SYSTEMS USING CBO AND ECBO ALGORITHMS

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ABSTRACT

Grillages are widely used in various structures. In this research, the Colliding Bodies Optimization (CBO) and Enhanced Colliding Bodies Optimization (ECBO) algorithms are used to obtain the optimum design of irregular grillage systems. The purpose of this research is to minimize the weight of the structure while satisfying the design constraints. The design variables are considered to be the cross-sectional properties of the beams and the design constraints are employed from LRFD-AISC. In addition, optimum design of grillages is performed for two cases: (i) without considering the warping effect, and (ii) with considering the warping effect. Also, several examples are presented to show the effect of different spacing and various boundary conditions. Finally, the results show that warping effect, beam spacing and boundary conditions have significant effects on the optimum design of grillages.

Keywords: irregular grillage systems; optimal design; CBO algorithm; enhanced CBO algorithm; warping.

1. INTRODUCTION

Grillage systems are extensively used in different structures such as bridge decks, ship hulls, decks, airplane wings, building floors, overhead water tanks slabs and specifically in the roof of big areas where no columns are used. Grillage systems have some advantages over other types of roof systems, including: (i) it is possible to build more beautiful structures using grillage systems, (ii) these are very efficient in transferring concentrated loads and in having the entire structure to participate in the load carrying action [1].

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Depending on the type of the structure, grillage systems can be regular or irregular. Regular grillages are frequently used in different type of structures. However, the performance limitations of building a structure sometimes make the designer to model the systems in irregular form. Utilization and optimization of the irregular grillages seems to be necessary if, for instance, there is opening in part of a grillage structure, the loads applied on the grillage are agglomerated in a specific area or when the boundary conditions do not allow arranging a fulcrum. In order to optimize an irregular grillage system, it is important to use a method which can solve the optimization problem precisely and in a reasonable time. For this reason, meta-heuristic algorithms are employed to find desirable regions in the search space in an affordable time [1]. Meta-heuristic algorithms are more suitable than conventional methods for structural optimum design due to their capability of exploring and finding promising regions in the search space in an affordable time [2]. Meta-heuristic algorithms tend to perform well for most engineering optimization problems. This is because these methods refrain from simplifying or making assumptions about the original form [3]. Different meta-heuristics algorithms have been used for structural optimization which the followings have been used more frequently: Genetic Algorithm (GA) which was introduced by Holland [4] is one of the most well-known algorithms that is applied in different problems. This algorithm was inspired by Darwin theory and it is based on the principle of the survival and reproduction of the superior type. Simulated Annealing (SA) was proposed by Metropolis et al. in 1953; then, in 1983, Kirkpatrick et al. applied it to optimization problems [5]. This algorithm is generally based on the similarity between cooling the molten solids and solving combinatorial optimization problems. Ant Colony Optimization (ACO) presented by Dorigo et al. [6] is another population-based optimization technique which simulates the behavior of the ants when they try to find the shortest route from nest to food and vice versa. Particle Swarm Optimization (PSO) is a very well-known and commonly used optimization algorithm proposed by Eberhart and Kennedy [7] and it is based on the social behavior of birds. Democratic Particle Swarm Optimization (DPSO) was proposed by Kaveh and Zolghadr in order to improve the exploration capabilities of the PSO and thus to address the problem of premature convergence. As the name suggests, in the Democratic PSO all eligible particles have the right to be involved in decision making [8]. Harmony Search (HS) is another powerful optimization method given by Geem et al. This method imitates natural musical performance routines that come to musician mind when they search a better state of harmony [9]. The big bang-big crunch algorithm (BB-BC) introduced by Erol and Eksin [10] is based on big bang-big crunch theory which is one of the universe evolution theories. The Standard Charged System Search (CSS) algorithm and Enhanced Charged System Search (ECSS) introduced by Kaveh and Talatahari [11] is inspired by the electrostatics laws in physics and the motion laws from the Newtonian mechanics. These algorithms are powerful and efficient methods in structural optimization. Dolphin Echolocation (DE) proposed by Kaveh and Farhoudi [12] imitates the behavior of the dolphins when they trace their hunt. The Colliding Bodies Optimization (CBO) introduced by Kaveh and Mahdavi is a new and simple optimization algorithm is based on one-dimensional collisions between bodies, with each agent solution being considered as an object or body with mass [13]. In this technique, one object collides with other object and they move towards a minimum energy level. The CBO is simple in concept, does not depend on any internal parameter, and does not use memory for saving the best-so-far solutions

[14]. The Enhanced Colliding Bodies Optimization (ECBO) introduced by Kaveh and Ilchi Ghazaan [14] is improved version of Standard CBO which saves some best solutions obtained so far in a memory and uses a mechanism to escape from local minima.

The purpose of an optimization study is minimizing or maximizing the values of some selected variables. Cross-sectional properties of beams are one of the effective variables in designing grillage systems because they are correlated with the weight of the structure and by reducing the cross-sectional areas the weight of the grillage is reduced. In this context, the response of the system to external loading must be within the criteria defined by LRFD-AISC code [15].

Analysis of grillage systems can be performed with or without considering the warping effect. Since warping plays an important role in the analysis of the grillage systems and makes the optimum design more realistic, it is recommended to consider it in the analysis [1].

In this paper, the optimum design of grillage systems is carried out. The CBO and ECBO algorithms are utilized as meta-heuristic algorithms for optimization process and their capability are compared. Cross-sectional areas of the beams are selected as design variables and the weight of the structure is considered as the objective function. For design constraints, including displacement and stress limitations, the criteria defined by LRFD-AISC code [15] are used. Analysis has been done by stiffness method in two cases: (i) without considering the warping effect, and (ii) with considering the warping effect. In addition, the impacts of using different beam spacing and various boundary conditions are investigated.

The remainder of this paper is organized as follow: In Section 2, the optimization algorithms are presented. Objective function and design constraints are proposed in Section 3. In Section 4, three examples are studied. Finally, in Section 5, some concluding remarks are provided.

2. OPTIMIZATION ALGORITHMS

This section describes two algorithms used in this paper. Firstly, the standard CBO is explained and then the ECBO is introduced.

2.1 Colliding bodies optimization (CBO)

The Colliding Bodies Optimization is a new meta-heuristic algorithm which was developed by Kaveh and Mahdavi [13]. In this algorithm, each solution candidate $\left(X_i \right)$ containing a number of variables $\left(X_i = \left\{ x_{i,j} \right\} \right)$ is considered as a colliding body (CB). The massed objects are composed of two main equal groups; i.e. stationary and moving objects, where the moving objects move to follow stationary objects and a collision occurs between pairs of objects. This is done for two purposes: (i) to improve the positions of moving objects and (ii) to push stationary objects towards better positions. After the collision, new positions of colliding bodies are updated based on new velocity by using the collision laws.

The CBO procedure can briefly be outlined as follows:

Step 1: Initialization

The initial positions of CBs are determined with random initialization of a population of

individuals in the search space:

$$x_i^0 = x_{min} + rand(x_{max} - x_{min}) \qquad\qquad i = 1, 2, ..., n \qquad\qquad (1)$$

where x_i^0 determines the initial value vector of the ith CB. x_{min} and x_{max} are the minimum and the maximum allowable values vectors of variables; *rand* is a random number in the interval $[0,1]$; and n is the number of CBs.

Step 2: Defining mass
The magnitude of the body mass for each CB is defined as:

$$m_k = \frac{\dfrac{1}{fit(k)}}{\displaystyle\sum_{i=1}^{n} \dfrac{1}{fit(i)}} \qquad\qquad k = 1, 2, ..., n \qquad\qquad (2)$$

where $fit(i)$ represents the objective function value of the agent i; n is the population size. It seems that a CB with good values exerts a larger mass than the bad ones. Also, for maximization, the objective function $fit(i)$ will be replaced by $\dfrac{1}{fit(i)}$.

Step 3: Creating groups & Criteria before the collision
The arrangement of the CBs objective function values is performed in ascending order (Fig. 1a). The sorted CBs are equally divided into two groups:
- The lower half of CBs (stationary CBs): These CBs are good agents which are stationary and the velocity of these bodies before collision is zero. Thus:

$$v_i = 0 \qquad\qquad i = 1, 2, ..., \frac{n}{2} \qquad\qquad (3)$$

- The upper half of CBs (moving CBs): These CBs move toward the lower half. Then, according to Fig. 1b, the better and worse CBs, i.e. agents with upper fitness value, of each group will collide together. The change of the body position represents the velocity of these bodies before collision as:

$$v_i = x_i - x_{i-\frac{n}{2}} \qquad\qquad i = \frac{n}{2} + 1, ..., n \qquad\qquad (4)$$

Where, v_i and x_i are the velocity and position vector of the ith CB in this group, respectively; $x_{i-\frac{n}{2}}$ is the ith CB pair position of x_i in the previous group.
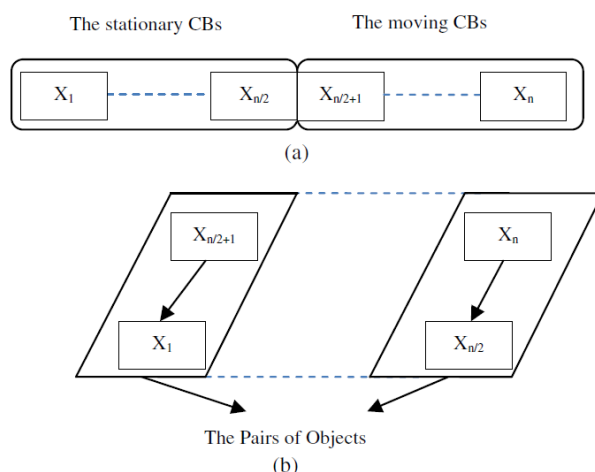
Figure 1. (a) CBs sorted in increasing order (b) colliding object pairs [13]

Step 4: Criteria after the collision

After the collision, the velocities of the colliding bodies in each group are evaluated. The velocity of each stationary CB after the collision is:

$$v'_i = \frac{\left(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}} \right) v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}} \qquad i = 1,...,\frac{n}{2} \tag{5}$$

where $v_{i+\frac{n}{2}}$ and v'_i are the velocity of the i th moving CB pair before and the i th stationary CB after the collision, respectively; m_i is mass of the i th CB; $m_{i+\frac{n}{2}}$ is mass of the i th moving CB pair. Also, the velocity of each moving CBs after the collision is obtained by:

$$v'_i = \frac{\left(m_i - \varepsilon m_{i-\frac{n}{2}} \right) v_i}{m_i + m_{i-\frac{n}{2}}} \qquad i = \frac{n}{2}+1,...,n \tag{6}$$

where v_i and v'_i are the velocity of the i th moving CB before and after the collision, respectively; m_i is mass of the i th CB; $m_{i-\frac{n}{2}}$ is mass of the i th CB pair. ε is the coefficient of restitution (COR) that decreases linearly from unit to zero. Thus, it is stated as:

$$\varepsilon = 1 - \frac{iter}{iter_{max}} \tag{7}$$

where *iter* is the current iteration number and $iter_{max}$ is the total number of iteration for optimization process.

Step 5: Updating CBs

New positions of CBs are evaluated using the generated velocities after the collision in position of stationary CBs. The new positions of stationary CBs are obtained by:

$$x_i^{new} = x_i + rand \circ v'_i \qquad\qquad i = 1,...,\frac{n}{2} \qquad\qquad (8)$$

where x_i^{new} , x_i and v'_i are the new position, old position and the velocity after the collision of the *i* th stationary CB, respectively. Also, The new positions of each moving CB is:

$$x_i^{new} = x_{i-\frac{n}{2}} + rand \circ v'_i \qquad\qquad i = \frac{n}{2}+1,...,n \qquad\qquad (9)$$

where x_i^{new} and v'_i are the new position and the velocity after the collision of the *i* th moving CB, respectively; $x_{i-\frac{n}{2}}$ is the old position of *i* th stationary CB pair. *rand* is a random vector uniformly distributed in the range $(-1,1)$ and the sign ''$\circ$'' denotes an element-by-element multiplication.

Step 6: Terminal condition check

The optimization is repeated from Step 2 until a termination criterion, such as maximum iteration number, is satisfied. It should be noted that, a body's status (stationary or moving body) and its numbering are changed in two subsequent iterations.

2.2 Enhanced colliding bodies optimization (ECBO)

The Enhanced Colliding Bodies Optimization (ECBO) is a recent meta-heuristic algorithm that was introduced by Kaveh and Ilchi Ghazaan [14]. This algorithm is a modified version of the CBO, which improves the CBO to get faster and more reliable solutions. The introduction of memory can increase the convergence speed of ECBO with respect to standard CBO. Furthermore, changing some components of colliding bodies will help ECBO to escape from local minima.

The ECBO procedure can briefly be outlined as follows:

Step 1: Initialization

The initial positions of all CBs are determined randomly in an m-dimensional search space.

$$x_i^0 = x_{min} + rand \circ (x_{max} - x_{min}) \qquad\qquad i = 1,2,...,n \qquad\qquad (10)$$

where x_i^0 is the initial solution vector of the ith CB. Here, $x_{\min}$ and $x_{\max}$ are the bounds of design variables; $rand$ is a random vector which each component is in the interval [0, 1]; n is the number of CBs.

Step 2: Defining mass

The value of mass for each CB is evaluated according to Eq.(2).

Step 3: Saving

Considering a memory which saves some historically best CB vectors and their related mass and objective function values can improve the algorithm performance without increasing the computational cost. For that purpose, a Colliding Memory (CM) is utilized to save a number of the best-so-far solutions. Therefore in this step, the solution vectors saved in CM are added to the population, and the same numbers of current worst CBs are deleted. Finally, CBs are sorted according to their masses in a decreasing order.

Step 4: Creating groups

CBs are divided into two equal groups: (i) stationary group and (ii) moving group. The pairs of CBs are defined according to Fig. 1.

Step 5: Criteria before the collision

The velocity of stationary bodies before collision is zero (Eq. (3)). Moving objects move toward stationary objects and their velocities before collision are calculated by Eq. (4).

Step 6: Criteria after the collision

The velocities of stationary and moving bodies are evaluated using Eqs. (5) and (6), respectively.

Step 7: Updating CBs

The new position of each CB is calculated by Eqs. (8) and (9).

Step 8: Escape from local optima

Meta-heuristic algorithms should have the ability to escape from the trap when agents get close to a local optimum. In ECBO, a parameter like Pro within (0, 1) is introduced and it is specified whether a component of each CB must be changed or not. For each colliding body Pro is compared with rn_i $(i = 1, 2, ..., n)$ which is a random number uniformly distributed within (0, 1). If $rn_i < Pro$, one dimension of the ith CB is selected randomly and its value is regenerated as follows:

$$x_{ij} = x_{j,\min} + rand \circ (x_{j,\max} - x_{j,\min}) \qquad\qquad i = 1, 2, ..., n \qquad\qquad (11)$$

where x_{ij} is the jth variable of the ith CB. $x_{j,\min}$ and $x_{j,\max}$ respectively, are the lower and upper bounds of the jth variable.

In order to protect the structures of CBs, only one dimension is changed. This mechanism provides opportunities for the CBs to move all over the search space thus providing better diversity.

Step 9: Terminal condition check

The optimization process is terminated after a fixed number of iterations. If this criterion is not satisfied go to Step 2 for a new round of iteration.

3. OPTIMUM DESIGN OF GRILLAGE SYSTEMS

3.1 Objective function
The optimum design of a grillage system is to reach a set of design variables which are the cross-sectional areas corresponding to minimum weight of the structure satisfying the behavioral and performance limitations which are implemented from the Load and Resistance Factor Design, American Institute of Steel Construction (LRFD-AISC) [15]. This can be expressed as:

$$
\begin{aligned}
&find \quad A = [A_1, A_2, ..., A_{ng}] \\
&A_i \in D_i \\
&to\ minimize \quad W(A) = \sum_{i=1}^{nm} \gamma_i . A_i . l_i
\end{aligned}
\tag{12}
$$

Where A is the set of design variables (the cross section areas of the beams); ng is the number of member groups; D_i is the allowable set of values for the design variable; A_i which is the set of 273 W-Sections as given in LRFD-AISC [15]. $W(A)$ is the total weight of the grillage system; nm is the number of all elements in the structure; γ_i is the material density of member i and l_i is the length of member i.

3.2 Design constraints to satisfy LRFD-AISC
According to LRFD-AISC conditions [15], for designing a grillage system, displacement and strength constraints must be considered as follow:

3.2.1 Maximum displacement constraint

$$
\frac{\delta_i}{\delta_i^u} \leq 1 \qquad\qquad i = 1, 2, ..., n_j
\tag{13}
$$

Where δ_i is the displacement of joint i and δ_i^u is its upper bound.

3.2.2 The strength constraints without the effect of warping

$$
\frac{M_{u,i}}{\phi_b . M_{n,i}} \leq 1 \qquad\qquad i = 1, 2, ..., nm
\tag{14}
$$

$$
\frac{V_{u,i}}{\phi_v . V_{n,i}} \leq 1 \qquad\qquad i = 1, 2, ..., nm
\tag{15}
$$

Where $M_{u,i}$ is the required flexural strength in member i; $M_{n,i}$ denotes the nominal flexural strength; ϕ_b is flexural resistance reduction factor which is equal to 0.9; $V_{u,i}$ is the factored service load shear for member i; $V_{n,i}$ is the nominal strength in shear; and ϕ_v represents the resistance factor for shear given as 0.9.

According to LRFD-AISC, the nominal flexural strength for a rolled compact section is computed as follow:

$$M_n = \begin{cases} M_p = Z_x F_y \leq 1.5\, S_x F_y & \lambda \leq \lambda_p \\[2mm] M_p - (M_p - M_r)\dfrac{\lambda - \lambda_r}{\lambda_r - \lambda_p} & \lambda_p < \lambda \leq \lambda_r \\[2mm] M_{cr} = S_x F_{cr} \leq M_p & \lambda > \lambda_r \end{cases} \tag{16}$$

Where M_p is the plastic moment; Z_x is the plastic section modulus; S_x is the section modulus; M_{cr} is the buckling moment; F_{cr} is the critical stress and M_r is the limiting buckling moment, given as:

$$M_r = \left(F_y - F_r\right)S_x \tag{17}$$

Where F_r is the compressive residual stress in the flange, which is given as $69\,MPa$ for rolled shapes in the code.

In the above equation, $\lambda = \dfrac{b_f}{2t_f}$ for I-shaped member flanges, in which b_f and t_f are the width and the thickness of the flange; $\lambda = \dfrac{h}{t_w}$ for a beam web, in which $h = d - 2k$ plus allowance for undersize inside fillet at compression flange for rolled I-shaped sections; d is the depth of the section; k is the distance from the outer face of the flange to the web toe of the fillet; t_w is the web thickness. λ_r and λ_p are given in table LRFD-B5.1 of the code as:

$$\lambda_r = \begin{cases} 0.38\sqrt{\dfrac{E}{F_y}} & \rightarrow \quad \textit{for compresion flang} \\[4mm] 3.76\sqrt{\dfrac{E}{F_y - F_r}} & \rightarrow \quad \textit{for the web} \end{cases} \tag{18}$$

$$\lambda_p = \begin{cases} 0.83\sqrt{\dfrac{E}{F_y - F_r}} & \rightarrow \quad \textit{for compresion flang} \\[4mm] 5.70\sqrt{\dfrac{E}{F_y}} & \rightarrow \quad \textit{for the web} \end{cases} \tag{19}$$

Where E is the modulus of elasticity and F_y is the yield stress of steel. It is apparent that M_n is computed for the flange and for the web separately by using the corresponding λ values. The nominal moment strength of the section is the smallest of these values.

The nominal shear strength of a rolled compact and non-compact W-section is computed from the data given in LRFDAISCF2.2 as follows:

$$V_n = \begin{cases} 0.6F_{yw}A_w & \dfrac{h}{t_w} \le 2.45\sqrt{\dfrac{E}{F_{yw}}} \\[4mm] 1.47F_{yw}A_w\dfrac{t_w}{h}\sqrt{\dfrac{E}{F_{yw}}} & 2.45\sqrt{\dfrac{E}{F_{yw}}} < \dfrac{h}{t_w} \le 3.07\sqrt{\dfrac{E}{F_{yw}}} \\[4mm] A_w\dfrac{4.52Et_w^2}{h^2} & 3.07\sqrt{\dfrac{E}{F_{yw}}} < \dfrac{h}{t_w} \le 260 \end{cases} \tag{20}$$

3.2.3 The strength constraints considering the effect of warping

For a steel grillage system with its members rigidly connected to each other, bending and torsional moments develop at their ends due to external loading and it causes these thin-walled elements warp. If the warping is restrained, it causes large values of normal stresses in the section. Hence, it becomes necessary to consider the effect of warping in the analysis of grillage systems [16, 17, 18].

According to LRFD-AISC [15], when the effect of warping is included, we utilize the following strength constraint instead of Eq. (14):

$$\pm\frac{\sigma_{bx}}{\phi_b F_{cr}} \pm \frac{\sigma_{by}}{0.9F_y} \pm \frac{\sigma_w}{0.9F_y} \le 1 \qquad i = 1, 2, ..., n \tag{21}$$

In which F_{cr} is the critical flexible stress; σ_b is the normal stress due to bending about either the x-axis or the y-axis and σ_w is the warping normal stress that is computed as follow:

$$\sigma_w = \frac{M_w.w}{I_w} \tag{22}$$

Where w is the warping function and I_w is the warping moment of inertia. Other constraints are the same as the grillage system without warping.

Here, direct stiffness method is used to analyze grillage systems. For a grillage system without considering the effect of warping, we can use a 6x6 element stiffness matrix in which there are three degrees of freedom for each node as given in detail in Ref. [19].

If the effect of warping is considered, the rate of warping will be added to the displacement matrix and then the number of degree of freedom will be four. The corresponding matrix is given in details in Ref. [19].

4. DESIGN EXAMPLES

In this section, several examples are optimized utilizing the CBO and ECBO algorithms to show the influence of different conditions of a grillage system, i.e. different spacing, boundary conditions and number of elements, on the weight of the structure. Examples are extracted from Ref. [1]. All grillages covered a distinct area of $225 \, m^2 \, \left(15m \times 15m\right)$ with an evenly distributed load of $15 \, kN/m^2$ (the total load of $3375 \, kN$). Each example has been done in two separate cases: (i) without considering the warping effect, (ii) with considering the warping effect.

The assumptions used in the examples are as follow: The yield stress of materials is $250 \, MPa$, the modulus of elasticity and the shear modulus are taken as $205 \, kN/mm^2$ and $81 \, kN/mm^2$, respectively. The discrete set from which the design algorithm selects the sectional designations for grillage members is considered to be the set of 273 W-sections as given in LRFD-AISC. The maximum vertical displacement for each node is up to $25 \, mm$. The grillage systems are optimized by the CBO and ECBO algorithms. A population of 20 CBs is selected in these algorithms. The maximum number of iterations is assumed to be 250. Four groups are allocated to longitudinal and transversal beams; group 1 and group 2 are assigned to outer and inner longitudinal beams respectively, while group 3 and group 4 are assigned to outer and inner transversal beams respectively. The algorithms are coded in MATLAB software and the grillage systems are analyzed using the direct stiffness method.

4.1 Example 1

In this example a grillage system with five bays in each direction is considered to cover a district area and the general model of this grillage is shown in Fig. 1. It is assumed that the total external load ($3375 \, kN$) is exerted to the 16 joints of the grillage system as point loads. Therefore, every node carries a point load of $210.9375 \, kN$. This grillage is optimized for two cases: (i) in Case 1 all the supports and elements of the grillage are considered, (ii) in Case 2, four supports (1, 4, 29 and 32) and the related elements are neglected. The results are shown in Table 1 and Table 2 for these two cases, respectively.
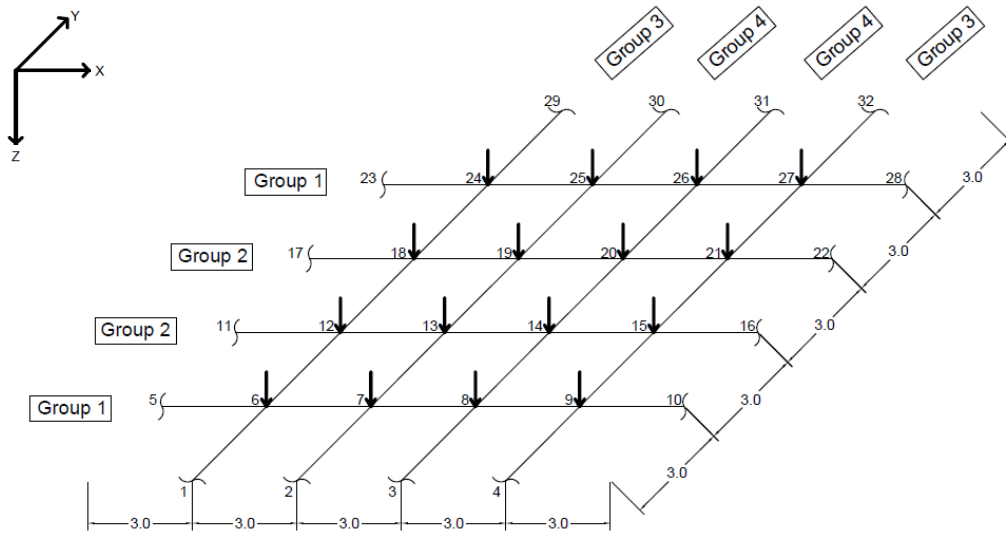
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Figure 2. A general model of 40-member grillage system

Table 1: Case 1, A regular 40-member grillage system which has 4 supports in each side

Search Method		CBO		ECBO	
Support Type	Group	Wihout Warping	With Warping	Wihout Warping	With Warping
Fixed	Group 1	W460X52	W530X66	W360X57.8	W100X19.3
	Group 2	W310X86	W530X109	W530X74	W610X113
	Group 3	W200X15	W200X26.6	W100X19.3	W360X79
	Group 4	W840X193	W920X238	W840X193	W840X226
	Weight (kg)	10827.90554	13755.69423	10774.9994	13707.52704
	Δ_{max} (mm)	21	19.3	21	19.5
	Maximum Strength Ratio	0.9484	0.9442	0.9912	0.9829
Hinged	Group 1	W100X19.3	W610X101	W250X17.9	W690X125
	Group 2	W610X101	W530X85	W610X125	W530X101
	Group 3	W530X82	W100X19.3	W530X72	W100X19.3
	Group 4	W1100X499	W1100X499	W1100X499	W1100X499
	Weight (kg)	22254.1396	23206.67612	22068.02549	23043.99955
	Δ_{max} (mm)	24	24.8	23.9	24.8
	Maximum Strength Ratio	0.7307	0.9298	0.7383	0.9686

Table 2: Case 2, A 36-member grillage system by removing 4 supports and related elements in Fig. 2

Search Method		CBO		ECBO	
Support Type	Group	Wihout Warping	With Warping	Wihout Warping	With Warping
Fixed	Group 1	W530X66	W100X19.3	W530X66	W100X19.3
	Group 2	W460X74	W690X192	W760X147	W690X170
	Group 3	W250X22.3	W690X170	W250X17.9	W690X152
	Group 4	W920X201	W840X176	W690X125	W840X193
	Weight (kg)	10631.4	14679	10462.2	14297.37565
	Δ_{max} (mm)	17.4	15	17.6	14.5
	Maximum Strength Ratio	0.8939	0.899	0.9355	0.9375
Hinged	Group 1	W530X72	W410X100	W530X66	W460X74
	Group 2	W610X125	W610X153	W1000X222	W610X140
	Group 3	W150X13.5	W100X19.3	W130X28.1	W310X79
	Group 4	W1100X499	W1100X499	W1100X390	W1100X499
	Weight (kg)	21279.40037	22907.4	20845.8	22812
	Δ_{max} (mm)	25	24	24.7	24.5
	Maximum Strength Ratio	0.704	0.9511	0.7785	0.8782

From Table 1, it can be clearly seen that the warping has a substantial effect on the whole weight of the grillage. The optimization results also reveal that by using hinged support the total weight of grillage is almost doubled. Furthermore, the results obtained using ECBO algorithm for both fixed and hinged supports are better compared to the results using the CBO algorithm.

The optimization results of the grillage system for Case 2 are given in Table 2. Comparing the results of Case 1 from Table 1 to results of Case 2 from Table 2, indicates that ignoring a limited number of the supports and the corresponding elements has insignificant effects on the weight of the grillage. This issue can be stated that removing of some elements of structure naturally reduces the structural weight; but, due to the hardness reduction of nodes and consequently smaller entries of stiffness matrix, displacements and stresses in elements will be increased. Thus, the strong sections will be needed to satisfy the constraints and this will be caused that the weight loss from the elimination of the element is neutralized. The convergence histories of two algorithms for both cases with considering the warping effect are depicted in Figs. 3-6. Based on Figs. 3-6 and the results summarized in Tables 1 and 2, it can be concluded that ECBO gives better results in comparison to the CBO algorithm.
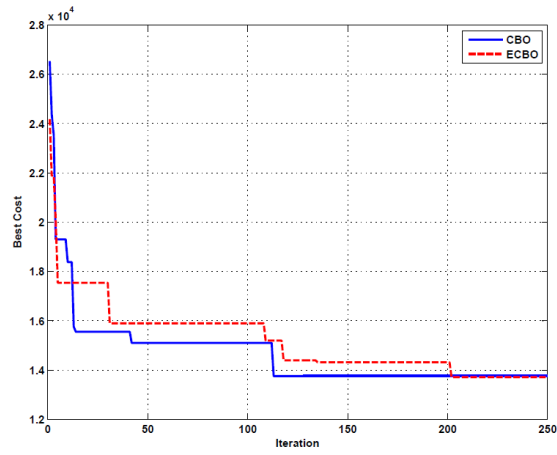
Figure 3. Convergence curve of the 40-member grillage system considering warping and fixed supports
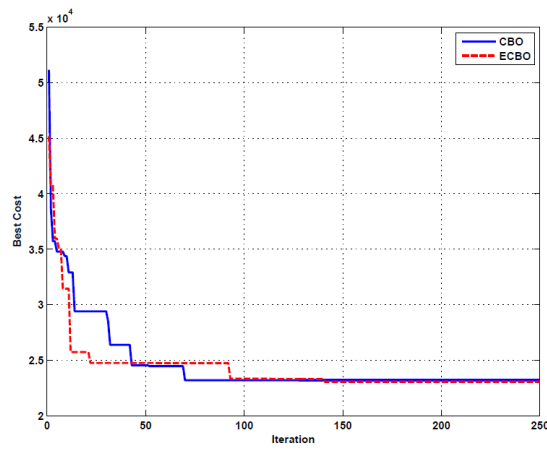

Figure 4. Convergence curve of the 40-member grillage system considering warping and hinged supports
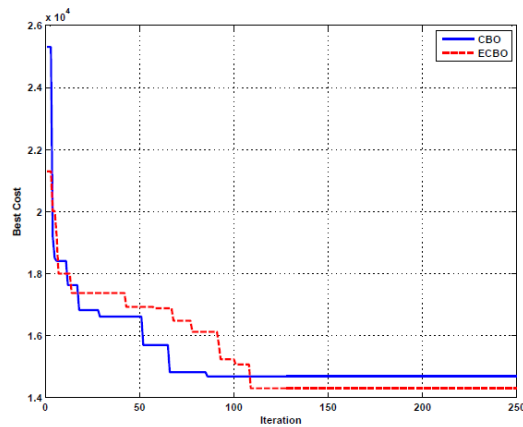

Figure 5. Convergence curve of the 36-member grillage system considering warping and fixed supports
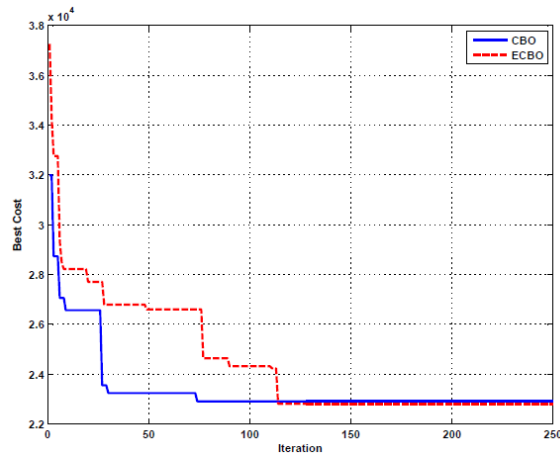
Figure 6. Convergence curve of the 36-member grillage system considering warping and hinged supports

4.2 Example 2

In this example a 50-member grillage system is considered to cover a district area and the general model of this grillage is depicted in Fig. 7. As can be seen from the figure, the upper and lower supports in Fig. 2 are replaced by beams of $15\,m$ length with two supports at their ends. Therefore, the number of supports reduced to 12 while the number of beam elements increased to 50. The number of free nodes is 24; hence a concentrated load of $140.625\,kN$ is applied on each free node. This grillage is optimized for two cases: (i) in Case 1 the grillage is considered as a regular structure with equal beam spacing in both direction as shown in Fig. 7, (ii) in Case 2, the beam spacing in one direction are changed; this means that, the elements of group 3 are closer to the supports as much as $1\,m$ and the elements of group 4 are closer to the supports as much as $0.5\,m$. Thus, the beam spacing in longitudinal direction are as follow: $2, 3.5, 4, 3.5, 2\,m$. The results are shown in Table 3 and Table 4 for these two cases, respectively.
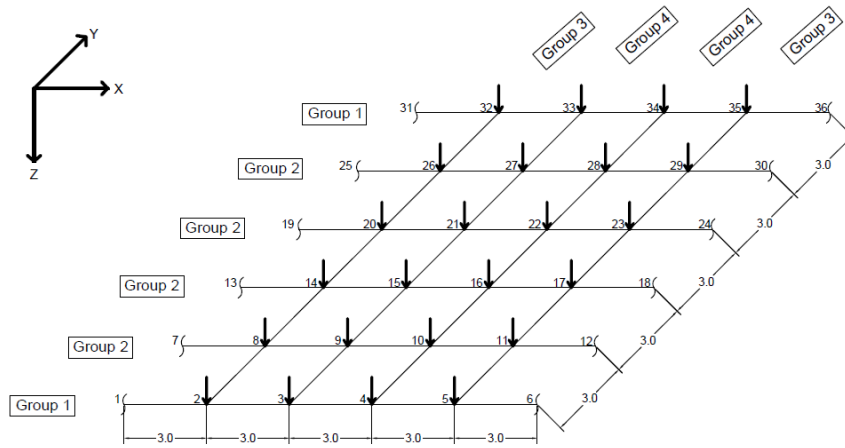

Figure 7. A general model of 50-member grillage system

Table 3: Case 1, A regular 50-member grillage system with end bearings in 2 sides of it

Search Method		CBO		ECBO	
Support Type	Group	Wihout Warping	With Warping	Wihout Warping	With Warping
Fixed	Group 1	W690X140	W690X192	W690X140	W690X217
	Group 2	W690X140	W760X161	W690X140	W690X140
	Group 3	W150X18	W150X13	W100X19.3	W100X19.3
	Group 4	W310X28.3	W100X19.3	W100X19.3	W130X23.8
	Weight (kg)	13989	16389	13758	16203
	Δ_{max} (mm)	24.1	23.4	23.2	23.8
	Maximum Strength Ratio	0.8184	0.262	0.8224	0.256
Hinged	Group 1	W1000X296	W1000X321	W1000X314	W1000X371
	Group 2	W1100X343	W1000X321	W1000X321	W1000X321
	Group 3	W150X13	W100X19.3	W100X19.3	W360X64
	Group 4	W150X13	W840X176	W200X26.6	W310X74
	Weight (kg)	30240	34749	30057	34530
	Δ_{max} (mm)	22.5	23.4	23.5	23.8
	Maximum Strength Ratio	0.3949	0.125	0.3961	0.12

Table 4: Case 2, An irregular 50-member grillage system with end bearings in 2 sides of it with different beam spacing

Support Type	Group	Wihout Warping	With Warping	Wihout Warping	With Warping
Fixed	Group 1	W610X125	W690X140	W690X125	W840X210
	Group 2	W690X125	W760X147	W690X125	W690X140
	Group 3	W150X13	W530X82	W150X13.5	W100X19.3
	Group 4	W100X19.3	W100X19.3	W150X13	W100X19.3
	Weight (kg)	12219	16059	12045	15858
	Δ_{max} (mm)	24.3	20.3	24.2	20.5
	Maximum Strength Ratio	0.8151	0.9365	0.8831	0.8131
Hinged	Group 1	W1000X272	W1000X249	W1000X272	W1000X296
	Group 2	W1000X272	W1000X321	W1000X272	W1000X296
	Group 3	W100X19.3	W100X19.3	W150X18	W310X67
	Group 4	W200X15	W530X74	W150X13	W100X19.3
	Weight (kg)	25509	29529	25410	29229
	Δ_{max} (mm)	22.7	23.7	23.7	24.8
	Maximum Strength Ratio	0.3691	0.4223	0.3691	0.4305

By comparing the results provided in Table 1 and Table 3, it can be obviously seen that the weight of the grillage for both fixed and hinged supports and also the maximum displacement for fixed supports are considerably increased. The results indicate that the grillage weights increase of about 20 to 30 % if there is no possibility to place supports in 4 sides of the grillage.

The optimization results of the grillage system for Case 2 are given in Table 4. Comparing the results of Case 1 from Table 3 to results of Case 2 from Table 4, demonstrates that the beam spacing has significant effect on the total weight of the grillage. This comparison shows a reduction in the weight of the grillage. For instance, for fixed supports without considering warping, the grillage weight was $13758\,kg$ while it is decreased to $12045\,kg$ with a small change in the beam spacing. These results for the hinged supports, indicate a reduction of nearly 15% in the grillage weight. The convergence histories of two algorithms for both cases with considering the warping effect are depicted in Figs. 8-11. Similar to Example 1, it can be concluded that the ECBO algorithm performs better optimization than CBO.
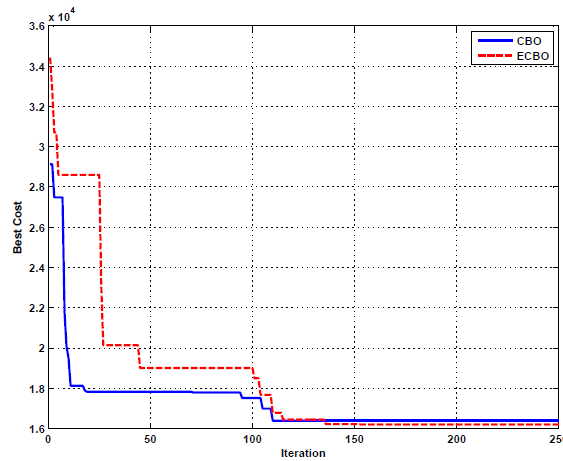

Figure 8. Convergence curve of the 50-member grillage system considering warping and fixed supports
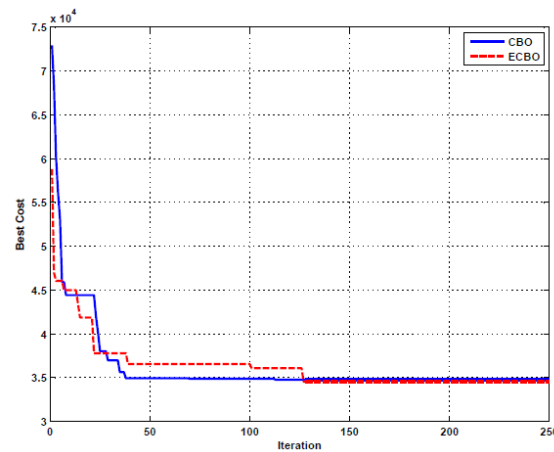

Figure 9. Convergence curve of the 50-member grillage system considering warping and hinged supports
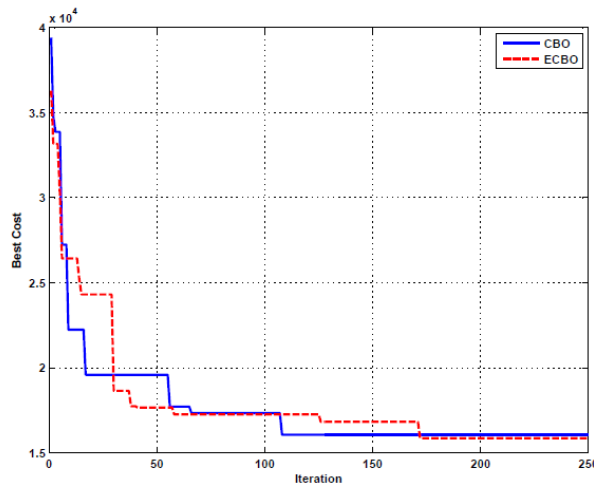
Figure 10. Convergence curve for the 50-member grillage system considering warping, fixed supports and different spacing
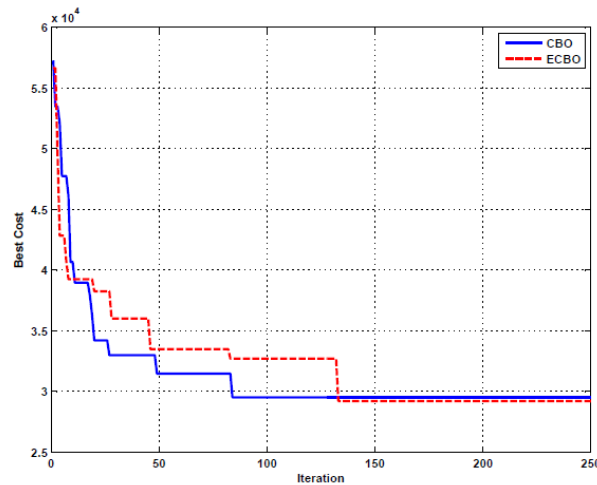

Figure 11. Convergence curve of the 50-member grillage system considering warping, hinged supports and different spacing

4.3 Example 3

In this example, to investigate the effect of beam spacing on weight and maximum displacement of the grillage structure, the beam spacing in the transversal direction is reduced to the half of the distance and also in longitudinal direction, the number of bays is increased from 5 to 6 so that the number of grillage elements is raised to 104, Fig. 12. Similar to the previous examples, the distributed load is assumed to be fixed of $15\,kN/m^2$. Since the area of the grillage is constant, subsequently a point load of $75\,kN$ is applied on each node. This grillage is optimized for two cases: (i) in Case 1 all elements of the grillage are considered, (ii) in Case 2, four elements (elements between node 14,15; 17,18; 56,57;

59,60) are removed. The results are shown in Table 1 and Table 2 for these two cases, respectively.
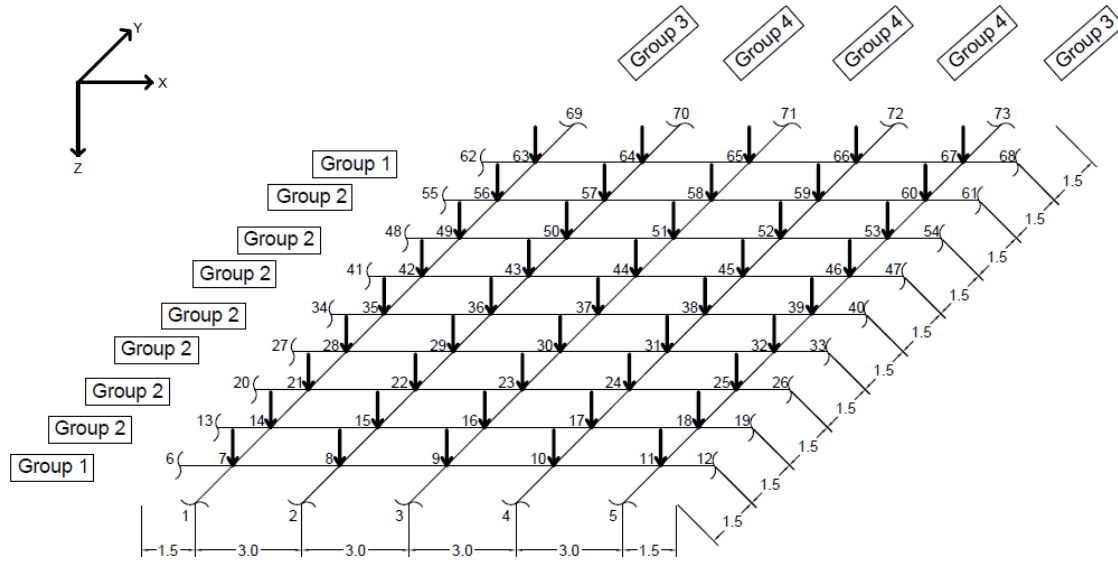

Figure 12. A general model of 104-member grillage system

Table 5: A 104-member grillage with decreasing beam spacing in one direction

Search Method		CBO		ECBO	
Support Type	Group	Wihout Warping	With Warping	Wihout Warping	With Warping
Fixed	Group 1	W200X15	W250X38.5	W150X13	W150X13
	Group 2	W310X38.7	W200X41.7	W310X38.7	W360X39
	Group 3	W150X13	W150X13	W200X15	W250X28.4
	Group 4	W760X147	W760X185	W760X134	W690X192
	Weight (kg)	11518.5	14248.5	10933.5	13977
	Δ_{max} (mm)	23.4	14.4	23.4	14.3
	Maximum Strength Ratio	0.9468	0.8912	0.9569	0.9375
Hinged	Group 1	W100X19.3	W100X19.3	W250X22.3	W100X19.3
	Group 2	W310X32.7	W250X22.3	W360X32.9	W410X60
	Group 3	W150X29.8	W1000X249	W100X19.3	W530X109
	Group 4	W1100X343	W1100X343	W1100X343	W1100X343
	Weight (kg)	20341.5	25825.5	20137.5	25584
	Δ_{max} (mm)	24.4	19.3	23.9	19.8
	Maximum Strength Ratio	0.9778	0.8498	0.8543	0.8831

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Table 6: A 100-member grillage system by eliminating 4 elements in Fig. 12

Search Method		CBO		ECBO	
Support Type	Group	Wihout Warping	With Warping	Wihout Warping	With Warping
Fixed	Group 1	W200X19.3	W150X29.8	W100X19.3	W100X19.3
	Group 2	W200X41.7	W460X52	W310X38.7	W460X60
	Group 3	W150X29.8	W310X44.5	W200X26.6	W200X35.9
	Group 4	W760X134	W760X147	W760X134	W690X140
	Weight (kg)	11381.1	13680	11006.1	13536
	Δ_{max} (mm)	23.1	16.9	23	21.1
	Maximum Strength Ratio	0.9793	0.9917	0.9855	0.9929
Hinged	Group 1	W100X19.3	W100X19.3	W150X13	W410X75
	Group 2	W310X38.7	W310X44.5	W310X38.7	W460X52
	Group 3	W410X38.8	W610X92	W410X38.8	W310X74
	Group 4	W1100X343	W1100X390	W1100X343	W1100X343
	Weight (kg)	20777.1	25027.5	20588.1	24741
	Δ_{max} (mm)	23.2	22.7	23	22.6
	Maximum Strength Ratio	0.9918	0.8498	0.9852	0.794

Comparison between results of Case 1 of this example in Table 5 to Case 1 of example 1 in Table 1, shows negligible changes in the weight of the grillage when the number of elements in one direction is increased. However, considering groups 1-4 in Table 5, indicates that the cross-sections selected for group 2 (internal vertical beams) have smaller depth while the cross-sections selected for group 4 (inner transversal beams) have large depth. This issue can be caused by increasing the number of elements in transversal direction. As a result, it can be concluded that by increasing the number of elements in both directions, the thickness of a grillage can be smaller.

The optimization results of the grillage for Case 2 are given in Table 6. This results show that by removing some elements of structure, the changes in weight and a maximum displacement of structures are insignificant. On the other hand, there are two ways for creating openings in the grillage system, when it is necessary: (i) changing beam spacing equal to dimensions of the opening, and (ii) removing some elements to reach to a desirable size. In the previous example, it was found that changing beam spacing can be caused considerable changes in the weight of the grillage. But, according to results of Case 2 of this example in Table 6, it can be said that there are negligible changes in the weight of the grillage by elimination of some elements. Therefore, it is reasonable to create openings in the grillages by removing some elements, while stability is preserved. The convergence histories of two algorithms for both cases with considering the warping effect are depicted in Figs. 13-16. Similar to previous examples, it can be concluded that the ECBO algorithm performs better optimization than CBO.
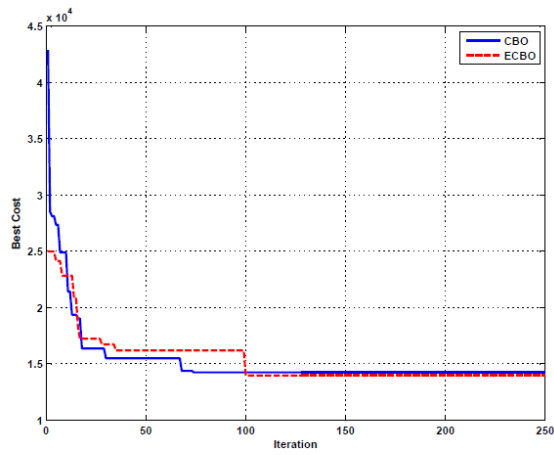
Figure 13. Convergence curve for the 104-member grillage system considering warping and fixed supports
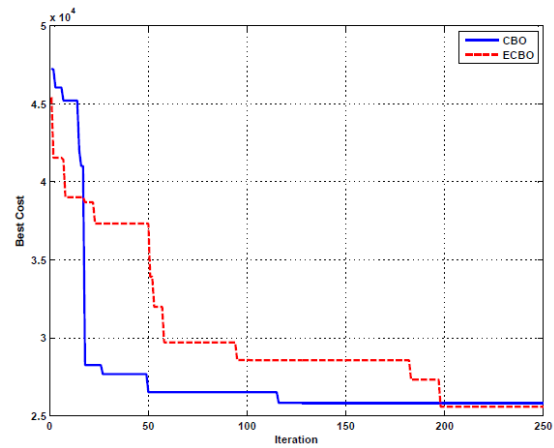

Figure 14. Convergence curve for the 104-member grillage system considering warping and hinged supports
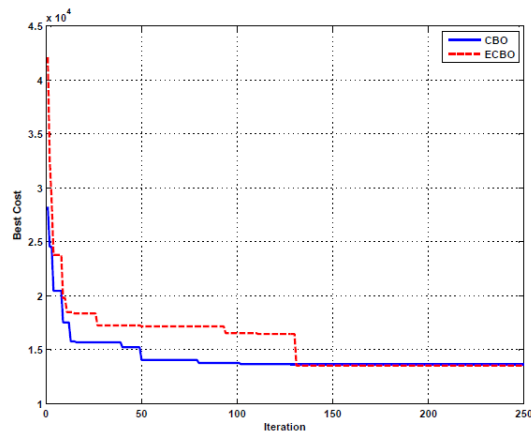

Figure 15. Convergence curve for 100-member grillage system considering warping and fixed supports
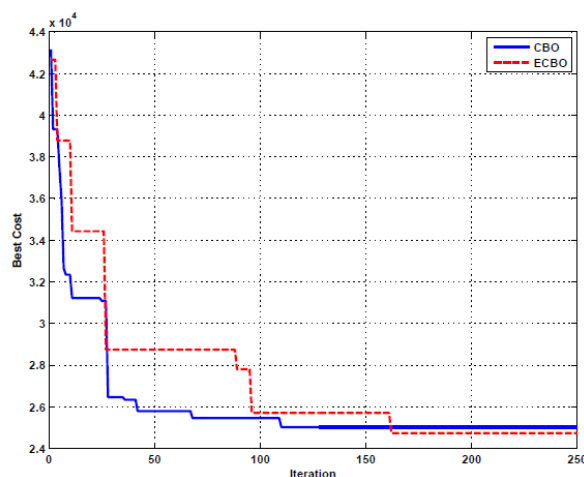
Figure 16. Convergence curve for the 100-member grillage system considering warping and hinged supports

5. CONCLUSION

In this paper, the optimization of grillages with different boundary conditions and beam spacing is performed using the CBO and ECBO algorithms in two cases: (i) without considering the warping effect, and (ii) with considering the warping effect. The results show that the ECBO algorithm presents better solutions for the optimization of grillages compared to the CBO. However, because the work of meta-heuristic algorithms are based on random search, it can not be said with certainty that the results of the ECBO is better than CBO in all cases; but in general, the ECBO algorithm has priority to CBO algorithm in terms of reliability, accuracy and speed of convergence and it can be said that the probability of finding more optimal solution by ECBO is stronger than CBO. It is worth mentioning, if the results of this paper compare to the results in Ref. [1], it can be seen that the ECSS algorithm gives better solutions than ECBO algorithm. This can be caused by high precision, strong exploration and effective exploitation of ECSS algorithm; however, the disadvantage of this algorithm is the complex structure, various parameters and adjustment of these parameters to achieve the acceptable performance of the algorithm. While, the CBO and ECBO algorithms are simple and are not related to any internal parameter and it can be said that the advantage of these algorithms is that they do not need input parameters and tuning and they work with less computational effort and time. Despite this simply, the results have acceptable quality and it can be claimed that these algorithms have good ability in solving optimization problems.

Furthermore, the warping effect on the weight of grillage systems is investigated and the results show that when the warping effect is considered, the weight of the grillages increase significantly and analysis becomes more reliable and real. In addition, another important issue that is discussed in this paper is irregular grillages with different boundary conditions and beam spacing. One of the results that can be cited is the use of fixed supports instead of hinged supports which can greatly reduce the weight of the grillages by almost half.

Also, replacing supports of the two opposite sides of the grillage by longitudinal beams with two supports at their ends, increases the weight of grillages by about 20-30%. The results indicate that ignoring a limited number of the supports and the corresponding elements has insignificant effects on the weight of the grillage. In contrast, beam spacing has significant effect on the total weight of the grillage; as by a small change in beam spacing, the weight of the grillage can be reduced of about 15%. Although decrease of beam spacing (or increase the number of elements) in one direction causes negligible changes in the weight of the grillage, depth of selected cross-sections in the same direction will be reduced. Moreover, the results show that if the grillages need a specified opening, it can be achieved by removing a number of elements instead of changing beam spacing. It should be mentioned that the removal of elements must be such that the structure stability is preserved.

Since the analysis and design of grillages is very extensive, to deal with all of them in the form of a paper is not possible; therefore, some further investigation can be noted as follow:

- Increase the number of grouping.

- Use a richer list of available sections and reinforced sections for design variables in order to increase the search space to find more efficiently answers.

- Optimization of grillage systems with more than one variable; for example both cross-section and beam spacing can be considered as design variables.

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