



## DAMAGE DETECTION OF BRIDGE STRUCTURES IN TIME DOMAIN VIA ENHANCED COLLIDING BODIES OPTIMIZATION

M.A. Shayanfar<sup>\*,†</sup>, A. Kaveh, O. Eghlidos and B. Mirzaei  
*Department of Civil Engineering, Iran University of Science and Technology, Narmak,  
Tehran 16846-13114, Iran*

### ABSTRACT

In this paper, a method is presented for damage detection of bridges using the Enhanced Colliding Bodies Optimization (ECBO) utilizing time-domain responses. The finite element modeling of the structure is based on the equation of motion under the moving load, and the flexural stiffness of the structure is determined by the acceleration responses obtained via sensors placed in different places. Damage detection problem presented in this research is an inverse problem, which is optimized by the ECBO algorithm, and the damages in the structures are fully detected. Furthermore, for simulating the real situation, the effect of measured noises is considered on the structure, to obtain more accurate results.

**Keywords:** damage detection; bridge structures; ECBO meta-heuristic algorithm; time-domain; acceleration response.

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### 1. INTRODUCTION

One of the most unpleasant features of real systems is that they are prone to damages, dysfunction, and in general unwanted behavioral modes. This shows why there is a need for constant and precise monitoring of systems according to effective damage detection strategies. Especially for engineering systems which their complexity due to inevitable modern technology and also information and communication revolution, are constantly developing is mandatory. In design and function of engineering systems, damage detection plays an important role in control theory and its action.

Damage is an unwanted change in structural systems which can disrupt the behavior of the system in present or in the future. In other words, when a weakness appears in whole or one of the elements of a system due to input loads, this weakness is called "Damage".

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\*Corresponding author: Iran University of Science and Technology, Narmak, Tehran 16846-13114, Iran

†E-mail address: shayanfar@iust.ac.ir (M.A. Shayanfar)

Damage has influence on the equations governing the system. Therefore by comparing a healthy system with a damaged one, it is possible to achieve the location and the intensity of the damage occurred to the structure. This procedure is called "Damage Detection". From the intensity point of view, damage can be defined by its size and the time of its occurrence. From size point of view, damage commences from damages to elements and it outspreads to the whole structure. However, from time point of view, the time of damage expansion is considered (e.g. fatigue is a type of damage which occurs in a rather long time). Thus health monitoring of structures (especially infrastructures and important structures) has become very important during the recent years.

Depending on the response which is used for the health monitoring of the structure, this science is divided into two categories of static and dynamic. However, there is one more precise classification which is based on the domain (either time or frequency) and, the type of the input induced on the structure [1]. Damage detection using vibration data in frequency-domain is one of the important and vast topics in the science of health monitoring of structures. A very exhaustive review has been presented in [2-3] which defines the importance of the modal data for detecting damages. Using the modal data is one of the most common vibrational methods of detecting damages in frequency domain which has been used in many researches [4-13]. A problem which exists is that damages, especially small damages which are of more importance, affect higher modes of the structures and measuring them is more difficult. To solve these problems, researchers started using time-domain based damage detection. These methods need the forced vibration of the structural system, but in most real-life structures applying an artificial obligatory force is almost impossible. Hence, it is needed to consider the free vibration along with the response of the structure. Therefore, the detection of structural parameters under different operational and loading conditions like impact, moving loads, etc. have been considered by many researchers in the past years [14]. In [15] a finite element identification method for a moving load, passing a bridge, using a wavelet method has been presented. Zhu and Law in [16] have presented a method for damage detection in a simply supported concrete bridge under moving-vehicle load passing on it.

In this research, using the time-domain responses of bridge structure under moving loads, the problem of damage detection is defined as an inverse problem and by the help of optimization, location and intensity of the damage is detected in this structure.

## 2. THEORETICAL BACKGROUND

### 2.1 Finite element modeling of bridge vibration under moving loads

For a general finite element model of a linear elastic time-independent structure, the equation of motion is given by

$$[M]\{z_{,tt}\} + [C]\{z_{,t}\} + [K]\{z\} = [B]\{F\} \quad (1)$$

where  $[M]$  and  $[K]$  are mass and stiffness matrices and  $[C]$  is damping matrix.  $z_{,tt}$ ,  $z_{,t}$  and  $z$  are the respective acceleration, velocity, and displacement vectors for the whole structure,

respectively, and  $\{F\}$  is the vector of applied forces with matrix  $[B]$  mapping these forces to the associated DOFs of the structure.

A proportional damping is assumed to show the effects of damping ratio on the dynamic magnification factor. Rayleigh damping, in which the damping matrix is proportional to the combination of the mass and stiffness matrices, is used. Consider

$$[C] = a_0[M] + a_1[K] \quad (2)$$

where  $a_0$  and  $a_1$  are constants to be determined from two modal damping ratios. If a more accurate estimation of the actual damping is required, a more general form of Rayleigh damping, the Caughey damping model, can be adopted.

The dynamic responses of the structures can be obtained by direct numerical integration using Newmark method.

## 2.2 Objective function

The objective function used in this research is

$$F = 1 - |r| \quad (3)$$

Such that

$$r = \frac{Cov(R^*, R)}{S_R \cdot S_R} \quad (4)$$

where  $R^*$  and  $R$  are the experimental and numerical responses, respectively.  $Cov(i, j)$  is the covariance between data series  $i, j$  and is given by

$$Cov(i, j) = \frac{\sum(i - \bar{i})(j - \bar{j})}{n - 1} \quad (5)$$

where  $\bar{i}$  is the average of data series  $i$  and  $\bar{j}$  is the average of data series  $j$ .  $n$  is the number of data in each series.

Also  $S_i$  is the variance of data series  $i$  which is given by

$$S_i^2 = \frac{\sum(i - \bar{i})^2}{n - 1} \quad (6)$$

In fact the above formula is for measuring the amount of covariance between two series of experimental and numerical data which has a value between -1 and 1. As it can be seen above,  $|r|$  is used, for the response of the objective function to be between 0 and 1. When the result becomes 1, it shows a complete correlation and when it becomes 0 it shows that there is no correlation between two data series.

### 2.3 Damage index

In inverse problems of detecting damages it is assumed that stiffness matrix of the element is uniformly decreased with the damage, and if there will be a damage, the flexural stiffness  $EI_i$  of the  $i^{th}$  element of the beam, will become  $\beta_i EI_i$ . The small changes of stiffness in an element could be expressed as below

$$\Delta K_{bi} = (K_{bi} - \tilde{K}_{bi}) = (1 - \beta_i)K_{bi} \quad (7)$$

where  $K_{bi}$  and  $\tilde{K}_{bi}$  are the  $i^{th}$  element stiffness matrices of the undamaged and damaged beam, respectively.  $\Delta K_{bi}$  is the stiffness reduction of the element. A positive value of  $\beta_i \in [0, 1]$  will indicate a loss in element stiffness. The  $i^{th}$  element is undamaged when  $\beta_i = 1$  and the stiffness of the  $i^{th}$  element is completely lost when  $\beta_i = 0$ . The stiffness matrix of the damaged structure is the assemblage of the entire element stiffness matrices  $\tilde{K}_{bi}$ :

$$K_b = \sum_{i=1}^N A_i^T \tilde{K}_{bi} A_i = \sum_{i=1}^N \beta_i A_i^T K_{bi} A_i \quad (8)$$

where  $A_i$  is the extended matrix of element nodal displacement that facilitates assembling of global stiffness matrix from the constituent element stiffness matrix.

## 3. ENHANCED COLLIDING BODY OPTIMIZATION (ECBO)

Meta-heuristic algorithms try to solve optimization problems. The implementation of these algorithms can computationally be performed in a variety of ways. They often have many different variable representations and other settings that must be defined. These include the definition or representation of the solution, mechanisms for changing, developing, or producing new solutions to the problem under study, and methods for evaluating a solution's fitness or efficiency. Once a meta-heuristic algorithm is developed, a tuning process is often required to evaluate different experimental options and settings that can be manipulated by the user in order to optimize convergence behavior in terms of the algorithm's ability to find near optimal solution. A meta-heuristic algorithm is usually tuned for a specific set of problems. However, one of the nice features of efficient meta-heuristic algorithms is their applicability to a wide range of problems [17].

CBO algorithm is a meta-heuristic algorithm developed by Kaveh and Mahdavi [18]. In this algorithm, one object collides with another object and they move towards a minimum energy level. In this meta-heuristic algorithm, each solution candidate  $X_i$  containing a number of variables (i.e.  $X_i = \{X_{i,j}\}$ ) is considered as a colliding body (CB). The massed objects are composed of two main equal groups; i.e. stationary and moving objects, where the moving objects move to follow stationary objects and a collision occurs between pairs of objects. This is done for two purposes: (i) to improve the positions of moving objects and

(ii) to push stationary objects towards better positions. After the collision, new positions of colliding bodies are updated based on new velocity by using the collision laws.

The CBO procedure can briefly be outlined as follows:

1. The initial positions of CBs are determined with random initialization of a population of individuals in the search space:

$$x_i^0 = x_{min} + rand(x_{max} - x_{min}), \quad i = 1, 2, \dots, n \quad (9)$$

where  $x_i^0$  determines the initial value vector of the  $i^{th}$  CB.  $x_{min}$  and  $x_{max}$  are the minimum and the maximum allowable values vectors of variables; rand is a random number in the interval [0, 1]; and n is the number of CBs.

2. The magnitude of the body mass for each CB is defined as:

$$m_k = \frac{1}{\frac{fit(k)}{\sum_{i=1}^n \frac{1}{fit(i)}}}, \quad k = 1, 2, \dots, n \quad (10)$$

where fit (i) represents the objective function value of the agent i; n is the population size. It seems that a CB with good values exerts a larger mass than the bad ones. Also, for maximization, the objective function fit (i) will be replaced by  $\frac{1}{fit(i)}$ .

3. The arrangement of the CBs objective function values is performed in ascending order (Fig. 1). The sorted CBs are equally divided into two groups:

- The lower half of CBs (stationary CBs); These CBs are good agents which are stationary and the velocity of these bodies before collision is zero. Thus:

$$v_i = 0, \quad i = 1, \dots, \frac{n}{2} \quad (11)$$

- The upper half of CBs (moving CBs): These CBs move toward the lower half.

Then, according to Fig. 2b, the better and worse CBs, i.e. agents with upper fitness value, of each group will collide together. The change of the body position represents the velocity of these bodies before collision as:

$$v_i = x_i - x_{i-\frac{n}{2}}, \quad i = \frac{n}{2} + 1, \dots, n \quad (12)$$

where  $v_i$  and  $x_i$  are the velocity and position vector of the  $i^{th}$  CB in this group, respectively;  $x_{i-\frac{n}{2}}$  is the  $i^{th}$  CB pair position of  $x_i$  in the previous group.

4. After the collision, the velocities of the colliding bodies in each group are evaluated utilizing Eqs. (3) and (4), and the velocity before collision. The velocity of each moving CBs after the collision is obtained by:

$$\dot{v}_i = \frac{(m_i - \varepsilon m_{i-\frac{n}{2}}) v_i}{m_i + m_{i-\frac{n}{2}}}, \quad i = \frac{n}{2} + 1, \dots, n \quad (13)$$

where  $v_i$  and  $v'_i$  are the velocity of the  $i^{th}$  moving CB before and after the collision, respectively;  $m_i$  is mass of the  $i^{th}$  CB;  $m_{i-\frac{n}{2}}$  is mass of the  $i^{th}$  CB pair. Also, the velocity of each stationary CB after the collision is:

$$\dot{v}_i = \frac{(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}}) v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}}, \quad i = 1, \dots, \frac{n}{2} \quad (14)$$

where  $V'_i, V_i = \frac{n}{2}$  are the velocity of the  $i^{th}$  moving CB pair before and the  $i^{th}$  stationary CB after the collision, respectively;  $m_i$  is mass of the  $i^{th}$  CB;  $m_{i+\frac{n}{2}}$  is mass of the  $i^{th}$  moving CB pair;  $\varepsilon$  is the value of the COR parameter.

5. New positions of CBs are evaluated using the generated velocities after the collision in position of stationary CBs. The new positions of each moving CB is:

$$x_i^{new} = x_{i-\frac{n}{2}} + rand \circ \dot{v}_i, \quad i = \frac{n}{2} + 1, \dots, n \quad (15)$$

where  $x_i^{new}, x_i$  and  $v'_i$  are the new position, old position and the velocity after the collision of the  $i^{th}$  stationary CB, respectively.  $rand$  is a random vector uniformly distributed in the range (-1, 1) and the sign ‘ $\circ$ ’ denotes an element-by-element multiplication.

6. The optimization is repeated from Step 2 until a termination criterion, such as maximum iteration number, is satisfied. It should be noted that, a body's status (stationary or moving body) and its numbering are changed in two subsequent iterations.

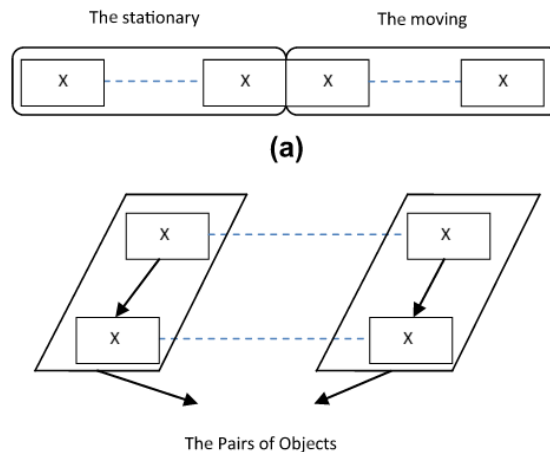


Figure 1. (a) CBs sorted in increasing order and (b) colliding object pairs

In order to enhance the function and accuracy of the responses obtained from CBO, the Enhanced Colliding Bodies Optimization (ECBO) algorithm has been created, which use a memory for saving best CBs and also use a mechanism for ignoring the local minimums [19,20]. Flowchart of this algorithm is shown Fig. 2.

### 4. NUMERICAL RESULTS

#### 4.1 One span bridge

In this example a bridge with fixed supports at its ends is shown in Fig. 3. For finite element modeling, the bridge is divided into 10 elements as is shown.

The bridge has a 10 meter span, with cross section area of  $A = 0.2 \times 0.2m^2$ , and it is composed of a material with elasticity module of  $E= 21Gpa$  and density of  $\rho = 2500 \frac{kg}{m^3}$ . The moving load is  $P=1000kg$  and its velocity is  $90 m/s$ . The Poisson ratio is assumed to be 0.05.

Five different damage scenarios have been considered for this bridge, and it is tried to find the exact location and intensity of the damage.

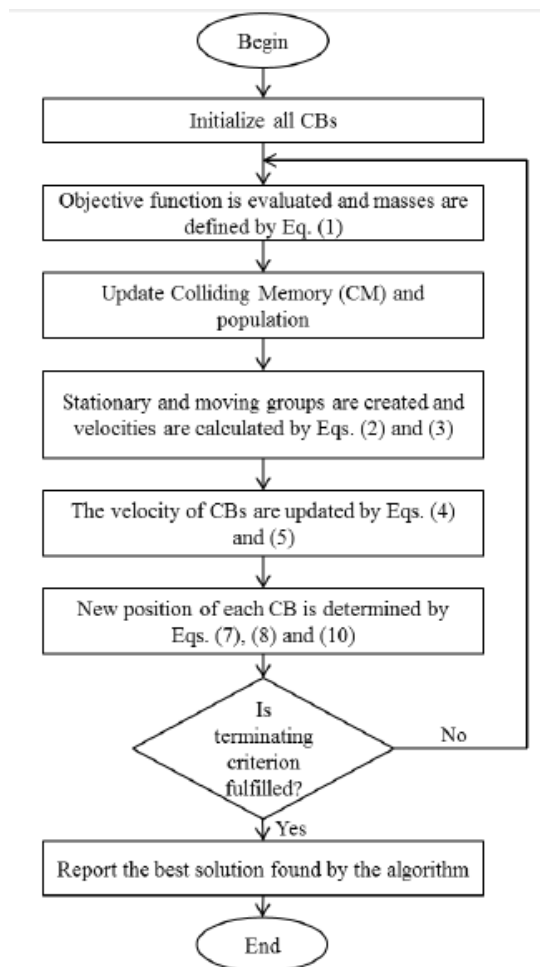


Figure 2. The flowchart of the CBO [18]

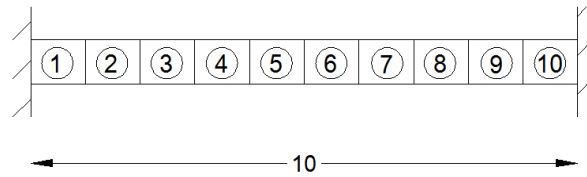


Figure 3. One span bridge

In the first scenario, it is assumed that the third element of the bridge is 15% damaged. The obtained results without the presence of noise are as shown in Fig. 4.

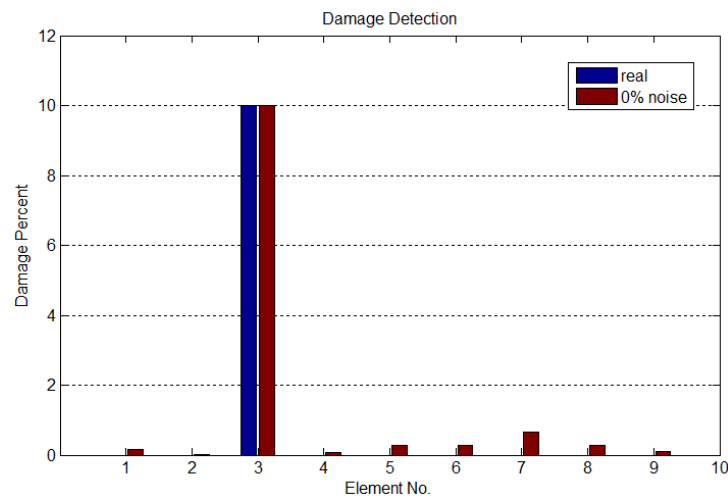


Figure 4. Damage detection results of a fixed supports bridge-first scenario

In the second scenario, it is assumed that the third element of the bridge is 15% damaged and the eighth element is 10% damaged. The obtained results without the presence of noise are as shown in Fig. 5.

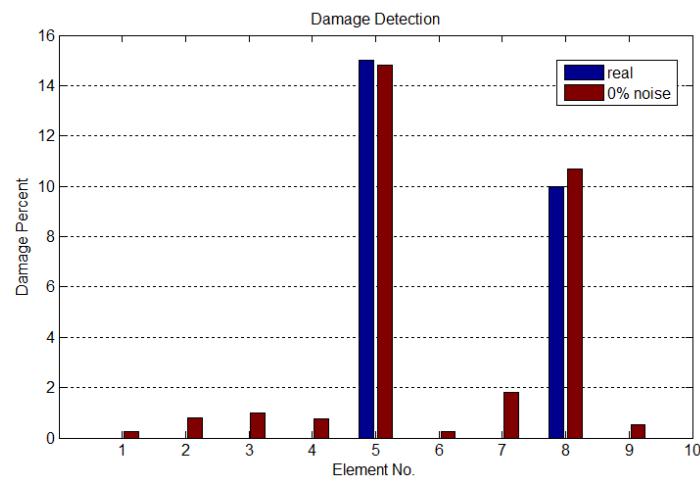


Figure 5. Damage detection results of a fixed supports bridge -second scenario



In the third scenario, it is assumed that each of the fourth and the seventh elements of the bridge are 15% damaged. The obtained results with 5% noise are as Fig. 6.

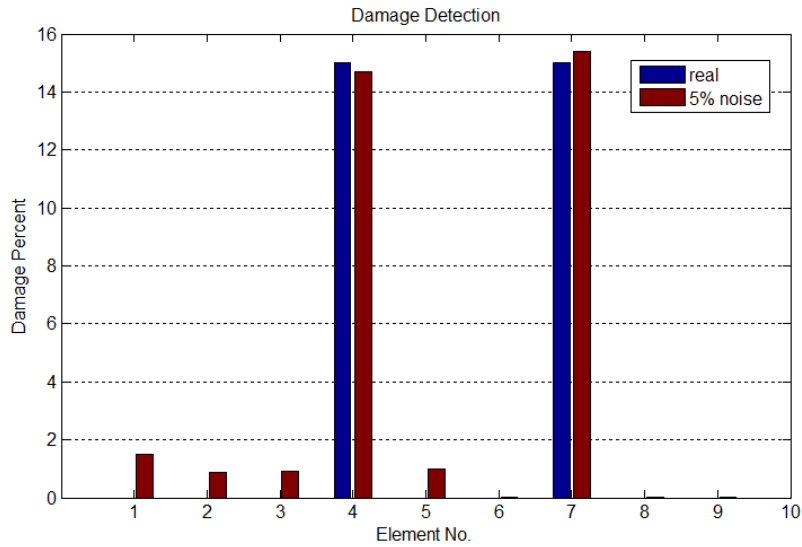


Figure 6. Damage detection results of a fixed supports bridge -third scenario

In the fourth scenario, it is assumed that the second element is 15%, the fourth element is 5% and the eighth element is 10% damaged. The obtained results with 5% noise are as Fig. 7.

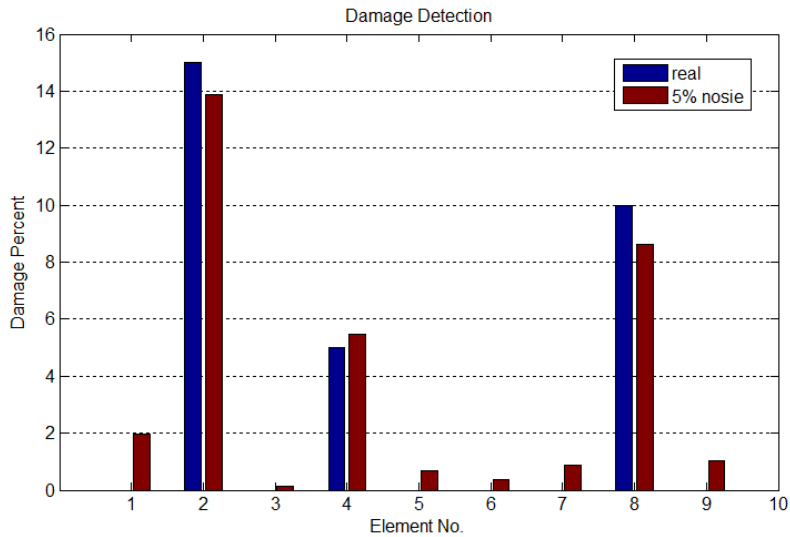


Figure 7. Damage detection results of a fixed supports bridge -fourth scenario

As the fifth and the last scenario, it is assumed that the third element is 15%, the fourth element is 20% and the seventh element is 10%, damaged. The obtained results with 10% noise are as Fig. 8.

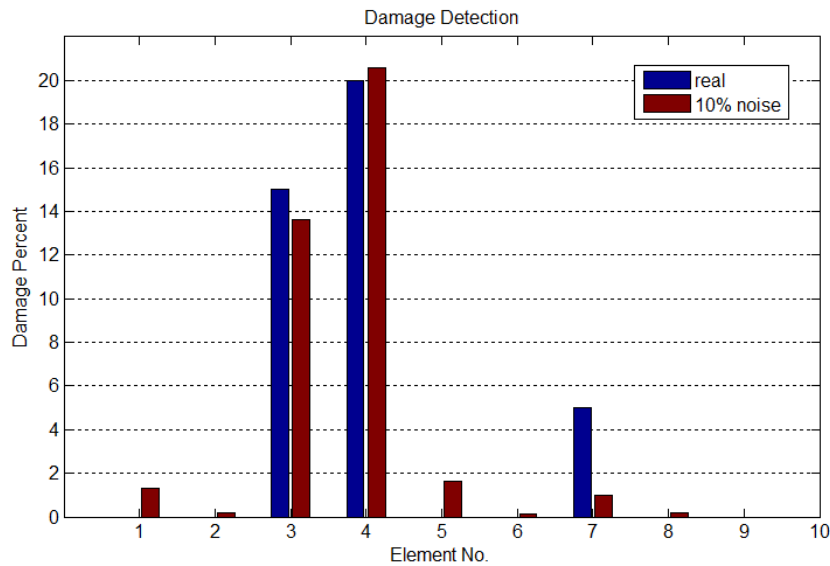


Figure 8. Damage detection results of a fixed supports bridge -fifth scenario

As it can be seen, despite high leveled multiple damage scenarios and a relatively high noise level the damage can be detected with a great accuracy.

#### 4.2 Three span bridge

In this example a bridge with fixed supports at its both ends is shown in the Fig. 9. For finite element modeling, this bridge is divided into 15 elements as is shown.

The structure is a 30 meter long, with three spans bridge, with cross section area of  $A=0.2 \times 0.2 \text{m}^2$ . The bridge structure is composed of a material with elasticity module of  $E=21 \text{Gpa}$  and density of  $\rho=2500 \text{ kg/m}^3$ . The moving load is  $P=1000 \text{Kg}$  with a velocity of  $90 \text{ m/s}$ . The Poisson coefficient is assumed to be  $0.05$ .

Six different damage scenarios are considered for this bridge, and it is tried to find the exact location and intensity of damage.

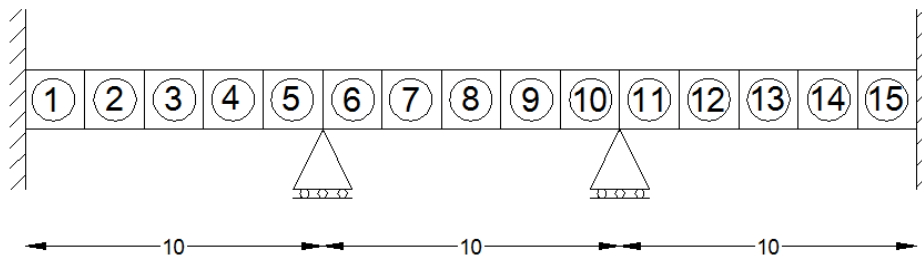


Figure 9. Three span bridge

In the first scenario, it is assumed that the third element is 15%, and the seventh element is 10% damaged. The obtained results without the presence of noise are as Fig. 10.

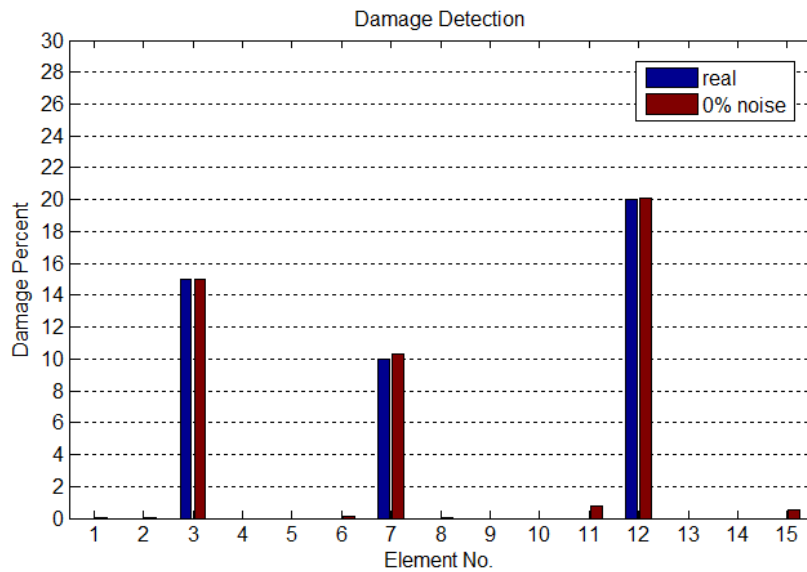


Figure 10. Results of the damage detection of a 3-span fixed-supported bridge (first scenario)

As it can be seen from Fig. 10, without the presence of the noise, in a multi damage scenario which is one of the worst cases, the algorithm gives results of a very high accuracy.

In the second scenario, it is assumed that the fifth element is 10% damaged. The obtained results with 3% noise are as Fig. 11.

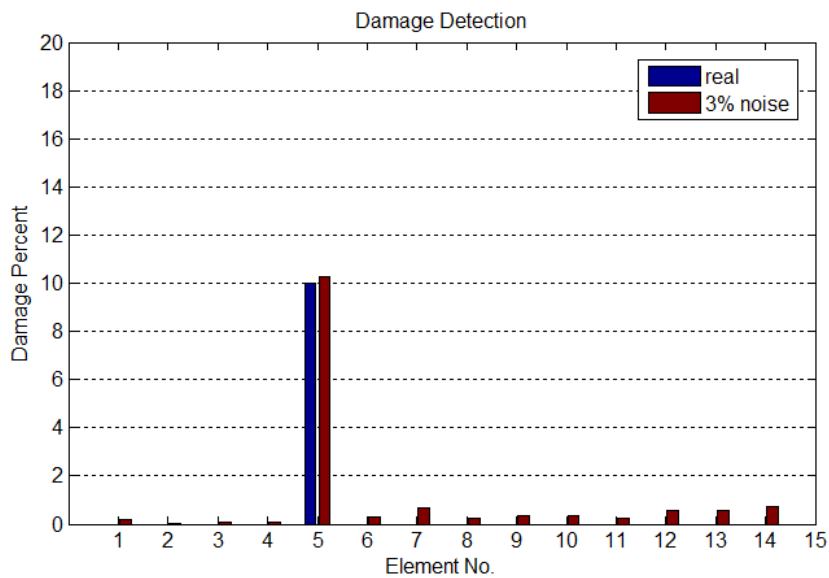


Figure 11. Results of the damage detection of a 3-span fixed-supported bridge (second scenario)

In the third scenario, like the second scenario it is assumed that the fifth element is 10% damaged, but a 5% noise is considered. The obtained results are shown in Fig. 12.

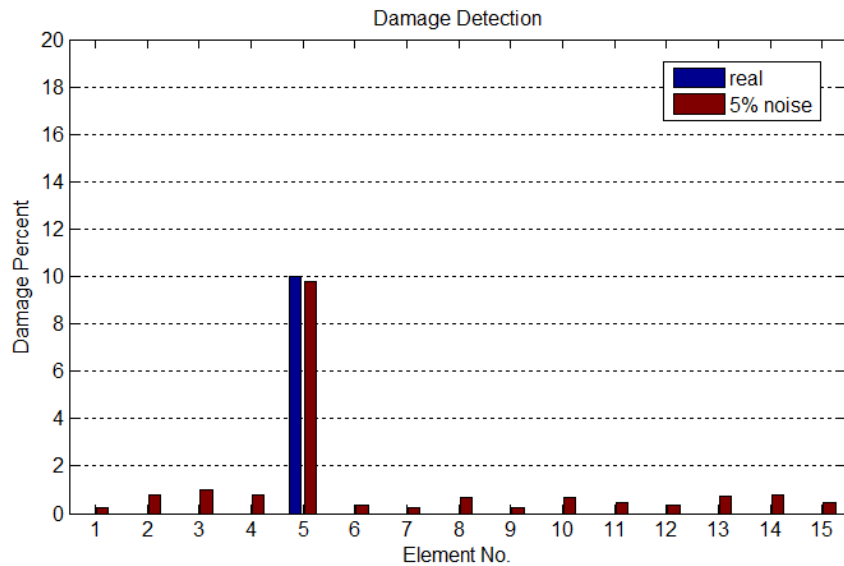


Figure 12. Results of the damage detection of a 3-span fixed-supported bridge (third scenario)

By comparing the two above scenarios, it could be said that presence of measurement noise does not affect the accuracy of the results.

In the fourth scenario, it is assumed that the third element is 15% and the seventh element is 10% damaged. The obtained results with 3% noise are as Fig. 13.

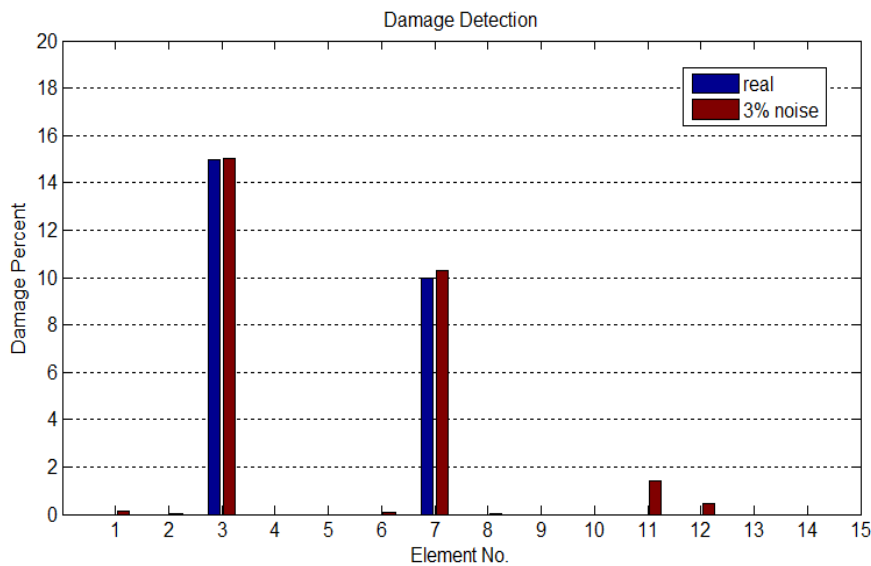


Figure 13. Results of the damage detection of a 3-spans fixed-supported bridge (fourth scenario)

In the fifth scenario, like the fourth scenario it is assumed that the third element is 15% and the seventh element is 10% damaged, and there is 5% noise. The obtained results are as shown in Fig. 14.

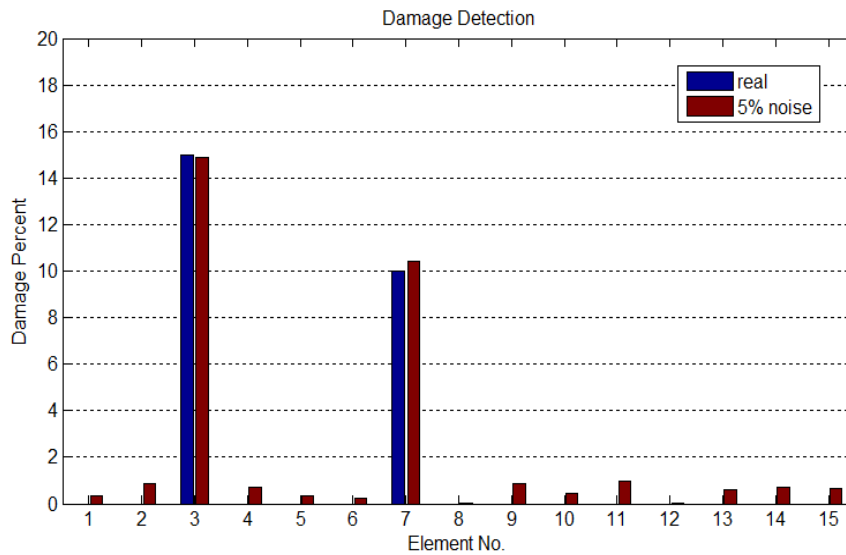


Figure 14. Results of the damage detection of a 3-span fixed-supported bridge (fifth scenario)

In the sixth scenario, it is assumed that the third element is 15%, the seventh element is 10% and the twelfth element is 20% damaged. The obtained results with 10% noise are as shown Fig. 15.

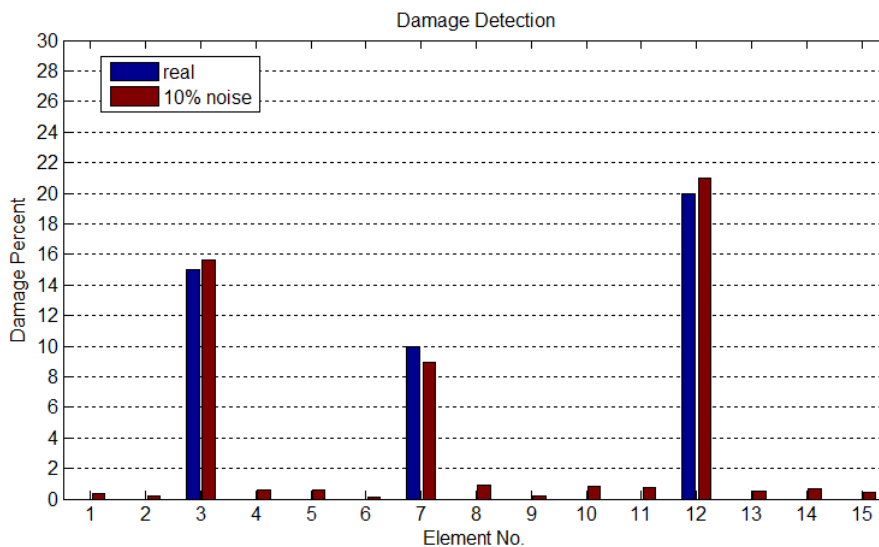


Figure 15. Results of the damage detection of a 3-span fixed-supported bridge (sixth scenario)

#### 4.3 Comparison

To show the efficiency of the proposed method, two damage detection scenarios for the mentioned three spans bridge via ECBO algorithm are compared with the results obtained from Genetic algorithm and CSS algorithm. All the properties of the bridge considered for this example are the same, except for the area of the cross section which is considered to be  $A = 0.3 \times 0.4 \text{ m}^2$ . The damage scenarios are shown in Table 1.

Table 1: The damage scenarios

Scenario	Number of element	Damage percent
1	2	30
	8	10
	11	15
	13	20
2	4	20
	7	10
	14	30

In the first scenario, 5% noise in the experimental responses is considered. The results for detecting the damage using the three mentioned algorithms are shown in Fig. 16.

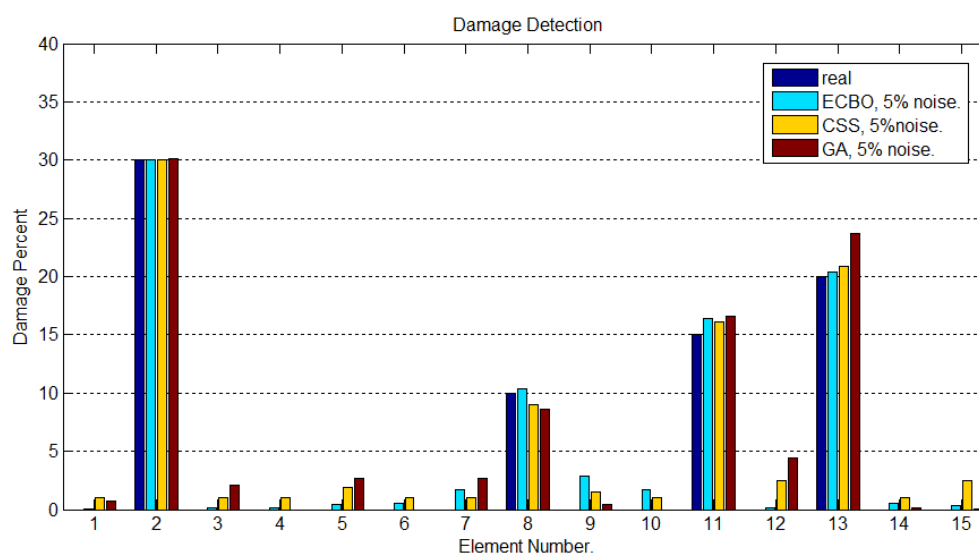


Figure 16. Results of comparison between the results obtained from GA, CSS and ECBO with 5% noise

As it can be seen from this figure, all the algorithms could detect the damage indicating the robustness of the model developed for this study and also the efficiency of the used cost function. However, the result of ECBO is more accurate than those of the other two algorithms.

In the second scenario, it has been considered that there is 5% noise in the experimental responses. The result for detecting the damage using the three mentioned algorithms is as follows:

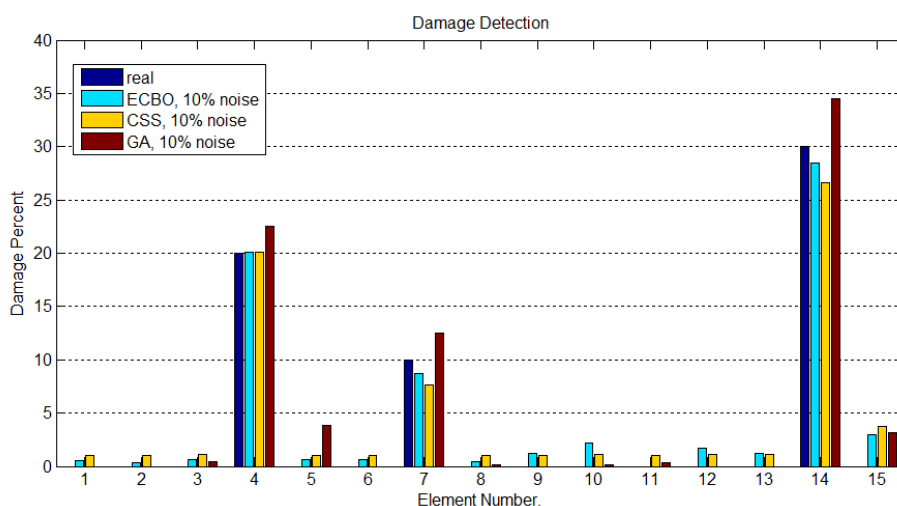


Figure 17. Results of comparison between results obtained from GA, ECBO, GA and CSS with 10% noise

As Fig. 17 shows, the result of applying ECBO is more precise than CSS, and the result of GA is less accurate than those of the other two algorithms.

## 5. CONCLUSION

In this paper, time-domain responses are used for damage detection of a bridge structure. The proposed method includes, measuring acceleration responses of the time-domain and also creating a finite element model of the structure, based on the equations of motion of the bridge under a moving load. Afterwards, an objective function for solving the inverse problem of damage detection is defined; and by the use of ECBO algorithm, the problem is solved. Hence, the location and the intensity of the damages are found. Two numerical examples were given to show the ability of the proposed algorithm in solving the problems with or without noise, and a comparison was made between the proposed algorithm (ECBO), GA and CSS. All the results demonstrate the efficiency and accuracy of the proposed method in detecting structural damage in bridge structures under moving load. However, by comparing the obtained results, the result of ECBO algorithm for proposed cost function was more accurate and less time consuming than GA and CSS.

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