



## A NEW DAMAGE INDEX FOR STRUCTURAL DAMAGE IDENTIFICATION BY MEANS OF WAVELET RESIDUAL FORCE

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### ABSTRACT

In this paper a new method is presented for structural damage identification. First, the damaged structure is excited by short duration impact acceleration and then, the recorded structural displacement time history responses under free vibration conditions are analyzed by Continuous Wavelet Transform (CWT) and Wavelet Residual Force (WRF) is calculated. Finally, an effective damage-sensitive index is proposed to localize structural damage with a high level of accuracy. The presented method is applied to three numerical examples, namely a fifteen-story shear frame, a concrete cantilever beam and a four-story, two-bay plane steel frame, under different damage patterns, to detect structural damage either in free noise or noisy states. In addition, some comparative studies are carried out to compare the presented index with other relative indices. Obtained results, not only illustrate the good performance of the presented approach for damage identification in engineering structures, but also introduce it as a stable and viable strategy especially when the input data are contaminated with different levels of random noises.

**Keywords:** structural damage identification; displacement time history response; continuous wavelet transform (CWT); wavelet residual force (WRF); damage index.

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### 1. INTRODUCTION

Structural damage identification, as the most important part of Structural Health Monitoring (SHM), is concentrated on identifying damages by analyzing structural feedbacks.

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Generally, structural damage causes changes in structural physical properties, mainly stiffness and damping, at damaged elements. Recently, vibration investigation of a damaged structure has attracted more attention as an approach for fault diagnosis. Damage in a structure induces a local variation in flexibility which causes changes in the dynamic behavior of the structure, and from this change the damage features such as damage location and extent can be determined [1, 2]. In a general point of view, damage localization methods employing vibrational feedbacks can be divided into two major groups: modal data-based methods and time history response processing-based strategies. Modal data-based approaches receive natural frequencies and/or mode shape vectors as input data and identify structural damage features via direct index-based methods [3–9] or iterative model-based algorithms [10–18]. A modified version of Modal Residual Force (MRF) for damage prognosis in structures is presented by Ge et al. [4] to detect non-proportional fault in steel frames. Sung et al. [19] detected damages in beams by proposing a new version of modal flexibility-based methods employing angular velocities measured from gyroscopes as input data. Ghodrati Amiri et al. [8] employed B-spline wavelet to detect cracks in Euler beams by analyzing mode shape vectors in a reference-free scheme. For localizing damages in linear-shaped structures, Nazari et al. [20] suggested a new damage index by applying Grey System Theory concepts on the curvature of diagonal members of the flexibility matrix. Zare Hosseinzadeh et al. [13] defined damage detection problem as an inverse optimization problem using modal data and solved it with Cuckoo Optimization Algorithm. Kaveh and Mahdavi [18] proposed optimization-based approach for damage identification in trusses and illustrated that the Enhanced Colliding Bodies Optimization algorithm performs better than Colliding Bodies Optimization in finding global extremums of the solution domain.

Although these methods perform well, but from practical point of view, it is more preferred and realistic that damage indices become formulated by considering recorded time histories as input data. This is because of some complex calculations in preprocessing recorded time history responses for extracting modal data. Moreover, in most cases, modal data-based damage indices are accurate only when some of the higher modes' data are also extracted, however, in the mentioned preprocessing approaches, it seems difficult to obtain such data with an acceptable accuracy. Different time history response processing-based strategies are developed to overcome the presented disadvantages in the modal data-based methods [21–29]. Zhu and Law [23] detected cracks in the bridge beam by applying continuous wavelet transform on the operational deflection time history response recorded at a single measuring point. Yinfeng et al. [26] proposed a damage index to detect structural damage based on the empirical mode decomposition employing vector autoregressive moving average. This algorithm captures abrupt changes in the energy distribution of structural responses at high frequencies. Recently, Yang and Nagarajaiah [28], based on the wavelet transform and the independent component analysis, developed a free-baseline and output-only damage identification method to estimate damage location and the instant of damage occurrence.

This paper is aimed at presenting a novel and practical method for damage identification by considering time history responses as input data. Wavelet Residual Force (WRF) is calculated as a sensitive parameter utilized in the proposed multi-scale damage index by considering only the recorded displacement time history responses as input data. Fast speed analysis, high level of sensitivity to different damage cases, and direct analysis of recorded

time history responses without any requirement to preprocessing of recorded data are some of the important advantages of this method which can strengthen its applicability in real on-line SHM programs. The applicability of the presented method is demonstrated by applying it in the simulated different single and multiple damage patterns on three numerical examples of engineering structures. In addition, different challenges such as presence of different levels of noises in the input data are studied to investigate the robustness of the method in practical cases. Finally, the stability of the presented damage index is compared with two other related indices by carrying out a comparative study.

The paper is organized as follows. The overview of the CWT is presented in Section 2. Then, the details of the proposed damage identification method are described in Section 3. Section 4 introduces the numerical examples and presents the obtained results and finally, the paper ends with some conclusion remarks which are presented in section 5.

## 2. CONTINUOUS WAVELET TRANSFORM (CWT)

Generally, wavelet transforms have developed as a mathematical tool for multi-resolution decomposition of signals. They have potential applications in many fields of signal processing that require variable time–frequency localization. In this section the basic concepts of the Continuous Wavelet Transform (CWT) are briefly summarized.

In mathematics, a continuous-time function can be divided into wavelets by a CWT. Unlike Fourier transform, the CWT possesses the ability to implement a multi-resolution analysis by adopting a flexible time–frequency window. For any square-integrable function  $y(t)$  in the time domain  $(-\infty, +\infty)$ , the definition of CWT is [30]:

$$\mathbf{W}_\psi^y(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} \mathbf{y}(t) \bar{\psi}\left(\frac{t-b}{a}\right) dt ; a > 0, b \in \mathbb{R} \quad (1)$$

where  $a$  and  $b$  denote the parameters of scaling and translation, respectively. The translation parameter indicates the location of the moving wavelet window in the wavelet transform while the scale parameter indicates the width of the wavelet window.  $\psi(t)$  is a mother wavelet which comprises wavelet coefficients and the bar shows its complex conjugation. It should be noted that the wavelet transform of mother wavelet needs to satisfy below condition in the frequency domain:

$$\int_{-\infty}^{+\infty} \frac{|\Psi(w)|^2}{|w|} dw \leq \infty \quad (2)$$

where  $\Psi(w)$  is Fourier transform of  $\psi(t)$ . Readers can find more details about CWT in [30].

With respect to structural damage condition assessment and structural health monitoring, wavelet analyses can be used to detect instantaneous changes or onset time in structural properties by monitoring on-line responses.

### 3. PROPOSED METHOD

This section introduces the details of the proposed method for structural damage identification. Generally, Wavelet Residual Force (WRF) can be defined similar to the Modal Residual Force (MRF). The main difference between WRF and MRF can be explained by considering the input data: although the MRF uses modal data for finding non-absorbed forces in the damaged elements, the WRF employs the structural time history responses for this purpose. To generate WRF, first, the Continuous Wavelet Transform (CWT) should be applied to the both sides of structural motion equation. It is more preferred that above mentioned CWTs can be calculated by analyzing only one type of the time history responses instead of studying all types of them. In this regards, Yan et al. [25] developed WRF concepts using recorded acceleration time history responses from an undamped structural system and then, by considering some simplifications, utilized it for damped structures. In the present paper, to propose an effective damage-sensitive index, an accurate approach for WRF calculation in real damped structural systems using only the displacement time history responses is developed. In the following, not only the premise rules as well as the main concepts of the WRF are presented, but also the proposed WRF-based damage index is introduced.

The free vibration second-order differential equations for a linear time-invariant structural system with  $N$  degrees of freedom (DOFs) can be written as below:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{0} \quad (3)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are the global mass, damping and stiffness matrices, respectively. In addition,  $\ddot{\mathbf{x}}(t)$ ,  $\dot{\mathbf{x}}(t)$  and  $\mathbf{x}(t)$  are the acceleration, velocity and displacement time history responses, respectively. If the sampling time consists of  $N_t$  time steps, for instance, the displacement time history responses can be presented as follow:

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \\ \vdots \\ \mathbf{x}_N(t) \end{bmatrix} = \begin{bmatrix} \{x_1(t_1), x_1(t_2), \dots, x_1(t_{N_t})\} \\ \{x_2(t_1), x_2(t_2), \dots, x_2(t_{N_t})\} \\ \vdots \\ \{x_N(t_1), x_N(t_2), \dots, x_N(t_{N_t})\} \end{bmatrix}_{(N \times N_t)} \quad (4)$$

As mentioned before, to calculate WRF, at the first stage, the CWT is applied to the both sides of Eq. (3) by selecting  $\psi_{a,b}(t)$  as the wavelet basis. So, this equation can be rewritten as:

$$\mathbf{M}\mathbf{W}_{\psi}^{\ddot{\mathbf{x}}}(a,b) + \mathbf{C}\mathbf{W}_{\psi}^{\dot{\mathbf{x}}}(a,b) + \mathbf{K}\mathbf{W}_{\psi}^{\mathbf{x}}(a,b) = \mathbf{0} \quad (5)$$

whereas  $\mathbf{W}_{\psi}^{\ddot{\mathbf{x}}}(a,b)$ ,  $\mathbf{W}_{\psi}^{\dot{\mathbf{x}}}(a,b)$  and  $\mathbf{W}_{\psi}^{\mathbf{x}}(a,b)$  are the CWT of the acceleration, velocity and

displacement time histories, respectively. In general, damage is defined as some deterioration in the structural physical properties. Here, it is assumed that damage causes some reduction in the stiffness matrix of the damaged elements. So, the global stiffness matrix of a damaged structure can be written as below:

$$\mathbf{K}^d = \mathbf{K}^u - \Delta\mathbf{K} \quad (6)$$

in which, the subscripts ‘*d*’ and ‘*u*’ are referred to the damaged and undamaged structures. Using this damage model, Eq. (5) can be constructed for the healthy and damaged structures as followings:

$$\mathbf{M}\mathbf{W}_{\psi}^{\ddot{x}^u}(a,b) + \mathbf{C}\mathbf{W}_{\psi}^{\dot{x}^u}(a,b) + \mathbf{K}^u \mathbf{W}_{\psi}^{x^u}(a,b) = \mathbf{0} \quad (7)$$

$$\mathbf{M}\mathbf{W}_{\psi}^{\ddot{x}^d}(a,b) + \mathbf{C}\mathbf{W}_{\psi}^{\dot{x}^d}(a,b) + \mathbf{K}^u \mathbf{W}_{\psi}^{x^d}(a,b) - \Delta\mathbf{K}\mathbf{W}_{\psi}^{x^d}(a,b) = \mathbf{0} \quad (8)$$

The later equation can be rewritten as:

$$\mathbf{M}\mathbf{W}_{\psi}^{\ddot{x}^d}(a,b) + \mathbf{C}\mathbf{W}_{\psi}^{\dot{x}^d}(a,b) + \mathbf{K}^u \mathbf{W}_{\psi}^{x^d}(a,b) = \Delta\mathbf{K}\mathbf{W}_{\psi}^{x^d}(a,b) \quad (9)$$

By comparing Eqs. (9) and (7), it can be concluded that damage occurrence causes some extra unabsorbed force in the structure which are revealed in the right side of Eq. (9). Therefore, the right side of Eq. (9) can be defined as WRF. From theoretical viewpoint, since  $\Delta\mathbf{K}$  is unknown for the monitored structure, the WRF should be achieved by considering the left side of Eq. (9). Therefore, the WRF is defined as a matrix:

$$\mathbf{WRF}(a,b) = \mathbf{M}\mathbf{W}_{\psi}^{\ddot{x}^d}(a,b) + \mathbf{C}\mathbf{W}_{\psi}^{\dot{x}^d}(a,b) + \mathbf{K}^u \mathbf{W}_{\psi}^{x^d}(a,b) \quad (10)$$

By inspecting Eq. (10), it can be seen that all types of structural time history responses should be available for calculation of WRF. Although this is true from mathematical concepts, from practical viewpoint it is more preferred that the WRF can be calculated using only a single type of the structural time history responses. Based on the mathematical relation between different time history responses of a structural system, the following equation is considered as the main relation for replacing the input signal with its different derivatives in the presence of CWT [31]:

$$\mathbf{W}_{\psi}^{x^{[n]}}(a,b) = \frac{(-1)^n}{a^n} \mathbf{W}_{\psi^{[n]}}^x(a,b) \quad (11)$$

where superscript [*n*] is a sign for denoting *n*-th derivative. Using Eq. (11), it is possible to rewrite Eq. (10) in a new format in which only the displacement time history responses are needed as the input signals:

$$\mathbf{WRF}(a,b) = \frac{1}{a^2} \mathbf{M} \mathbf{W}_{\psi}^{\mathbf{x}^d}(a,b) - \frac{1}{a} \mathbf{C} \mathbf{W}_{\psi}^{\mathbf{x}^d}(a,b) + \mathbf{K}^u \mathbf{W}_{\psi}^{\mathbf{x}^d}(a,b) \quad (12)$$

Where

$$\mathbf{W}_{\psi}^{\mathbf{x}}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} \mathbf{x}(t) \bar{\psi} \left( \frac{t-b}{a} \right) dt \quad (13)$$

$$\mathbf{W}_{\dot{\psi}}^{\mathbf{x}}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} \mathbf{x}(t) \bar{\dot{\psi}} \left( \frac{t-b}{a} \right) dt \quad (14)$$

$$\mathbf{W}_{\ddot{\psi}}^{\mathbf{x}}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} \mathbf{x}(t) \bar{\ddot{\psi}} \left( \frac{t-b}{a} \right) dt \quad (15)$$

By concentrating on Eqs. (13) to (15), it is obvious that the mother wavelet should be selected in a way that its derivatives can be defined as wavelet basis, too. By considering this fact, here the Gaussian family wavelet is utilized for CWT. The mother wavelets of this family can be presented as below:

$$\psi(t) = (-1)^n \frac{d^n}{dt^n} e^{-\left(\frac{t^2}{2}\right)}, \quad n \in \mathbb{N} \quad (16)$$

In this paper, the first derivative of Gaussian family wavelet is considered as  $\psi(t)$ . After constructing WRF matrix, the proposed damage index can be achieved as following. To convert WRF matrix to a vector which can perform such as MRF, for the  $i$ -th DOF with a known scale 'a', the normalized WRF-based damage-sensitive index (WRFDI) is defined as:

$$WRFDI_a(i) = \frac{\sum_b |\mathbf{WRF}_i(a,b)|}{\max(|\mathbf{WRF}_i(a,b)|)} \quad (17)$$

The performance of this vector can be expressed as below:

*“The  $i$ -th element of the monitored structure will be considered as a damaged element if calculated WRFDI for its free DOFs are distinguishably bigger than zero”.*

Although WRFDI can be an effective damage index, it is possible that the uncertainties such as noises, which are unavoidable in the real SHM programs, influence the efficient performance of this index. To present a proper damage index with high level of accuracy in detecting damaged members, the multi-scale WRFDI for the  $i$ -the DOF, can be defined as:

$$MWRFDI(i) = \prod_a WRFDI_a(i) \quad (18)$$

Since the WRFDI are normalized and have a value between 0 and 1, by utilizing MWRFDI not only the performance of the presented WRF-based index can be promoted in

the noisy states, but also it can perform well because of magnifying the index values. Similar to the WRFDI, the MWRFDI can detect damaged elements by reporting non-zero values in those DOFs which are related to the free DOFs of the damaged element.

#### 4. NUMERICAL STUDIES

This section is devoted for validating the applicability of the proposed damage identification method by studying different damage patterns on three numerical examples. In addition, to evaluate the robustness of the method in a real SHM program, the effects of feeding noisy input data are investigated. Moreover, a comparative study is carried out for assessing the advantages of the presented damage index in comparison with other related indices.

##### 4.1 Fifteen-story shear frame

In the first example, a fifteen-story shear frame, similar to that had been studied by Zare Hosseinzadeh et al. [11], is considered. Table 1 summarizes its material properties. In addition, damping ratio for all modes is  $\xi=5\%$ . The presented method is applied to detect the simulated three damage patterns which are described in Table 2. Although the first damage pattern simulates a single damage case, the second and third patterns consist of multiple damage cases. After simulating damage patterns in the workspace of MATLAB software, the presented method is applied to prepare appropriate data for calculating the suggested damage index. As mentioned before, the presented method is based of analyzing the structural displacement time history responses in the free vibration condition. To record such responses in a real SHM program, exciting the structure with a short duration impact force can be considered as a suitable practical way. In such a case, it is obvious that the free vibration responses will be achieved after ending the external force. Due to this fact, in this paper the free vibration responses are extracted by exciting the damaged structure with acceleration time history of a short duration impact load. Fig. 1 shows two different short duration impact acceleration (IA) which are used for exciting damaged structure. In the present example, although for damage patterns 1 and 3, IA-1 is utilized for structural excitation; in damage pattern 2, IA-2 is employed for this purpose. Since in practical investigations, it is possible that the recorded data are contaminated with different levels of noises from different sources, it is more desirable that the applicability of a new damage index is inspected not only in a noise free state, but also in a situation in which the input data are polluted with some artificial random noises. In this paper, this issue is considered by adding Gaussian white noises with the mean values of zero and the Root Mean Square (RMS) of the noises are equal to  $n\%$  of the RMS of the corresponding response. In this section,  $n$  is equal to 0 for ideal state (noise free case) and 3 and 5 for noisy states. In addition, six different values are considered for parameter ' $a$ ': 5, 10, 12, 15, 20, and 22.

The obtained results for the WRFDI<sub>a</sub> and MWRFDI are shown in Figs. 2-4 for the studied damage patterns. It should be noticed that each floor of the shear frame is considered as a free node and is numbered. So, for instance,  $i$ -th story is located between  $(i-1)$ -th and  $i$ -th floors or nodes. By inspecting these figures, this conclusion can be drawn that the WRFDI<sub>a</sub> detects structural damages properly when the input data are ideal. However, in the noisy states, it is possible that this index performs not well. For instance, consider Fig. 4(c)

which shows  $WRFDI_a$  for damage pattern 3, with 5% noise in input data. It can be seen that if  $a=15$  or 20, distinguishable differences cannot be found between calculated damage indices for related nodes to the healthy and damaged stories. However, MWRFDI can detect damages with high level of accuracy either the ideal input data are fed or not. Therefore, as an important result, MWRFDI reveals very low sensitivity to noise as well as very high sensitivity to damage and it is because of the presented strategy in extracting this index from  $WRFDI_a$ . The last conclusion can be made on the employed impact accelerations to excite structure. Since only the free vibration responses are important for calculating the presented damage index; as it is expected, it can be seen the method performs properly when each of introduced impact accelerations are utilized for exciting the monitored structure.

Table 1: Physical properties of the 15-story shear frame

Story number	Mass (ton)	Stiffness (MN/m)
1~5	50	8.5
6~10	50	7.0
11~15	40	5.5

Table 2: Simulated damage patterns in the 15-story shear frame

Pattern Number	Damage Scenario		Acceleration Type
	Story	Damage (%)	
1	5	10	IA-1
2	6, and 12	5, and 10	IA-2
3	4, 8, and 13	20, 20, and 10	IA-1

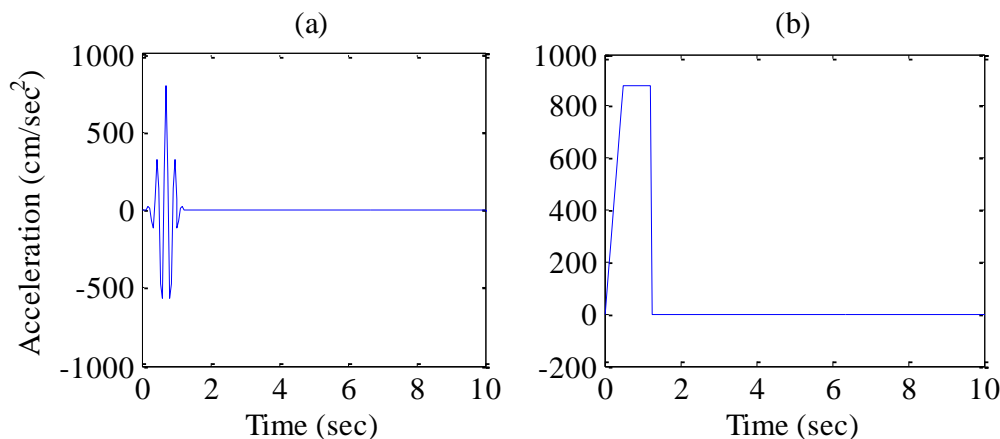


Figure 1. Short impact acceleration time history for exciting damaged structure: (a) IA-1, and (b) IA-2



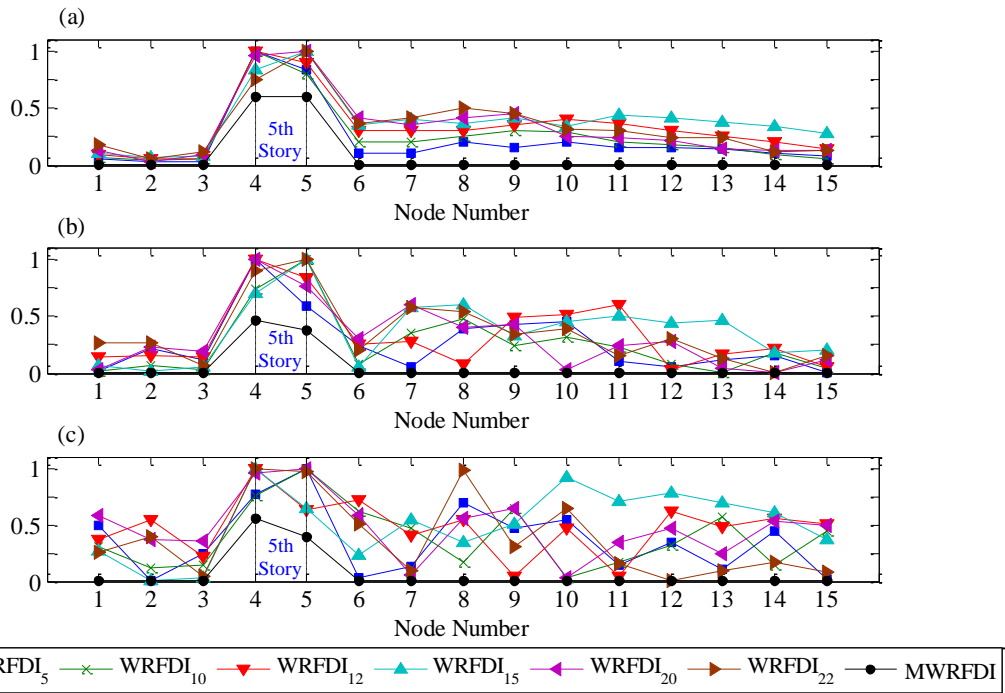


Figure 2. Damage identification results for damage pattern 1 in the 15-story shear frame for: (a) 0% noise, (b) 3% noise, and (c) 5% noise

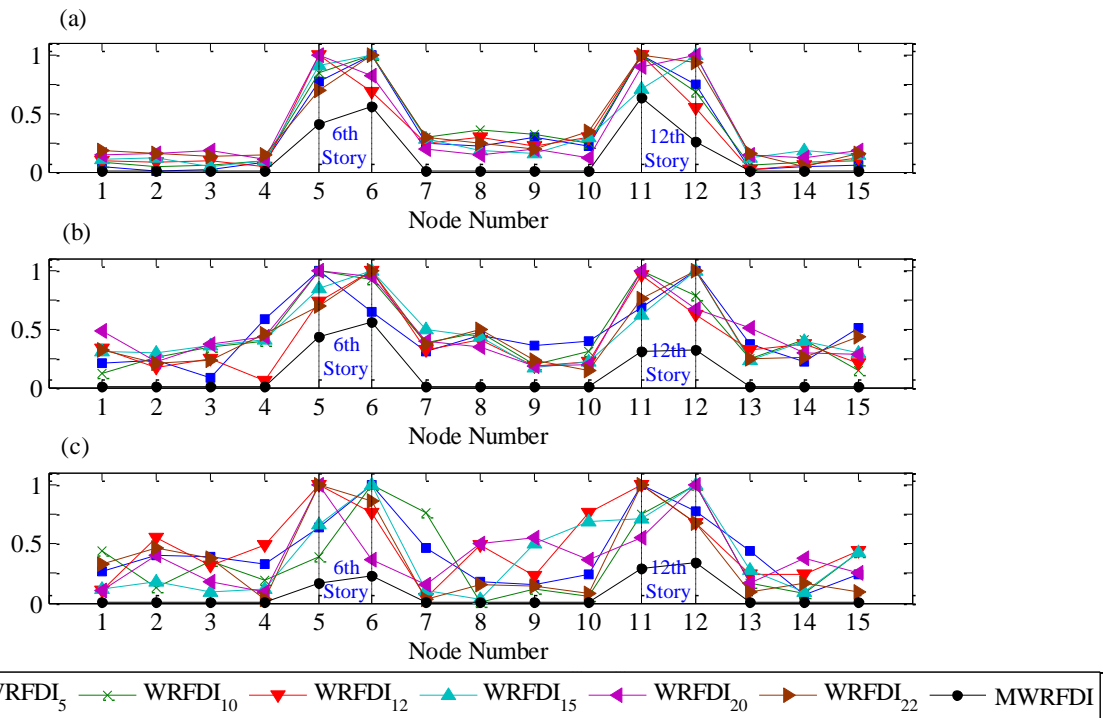


Figure 3. Damage identification results for damage pattern 2 in the 15-story shear frame for: (a) 0% noise, (b) 3% noise, and (c) 5% noise

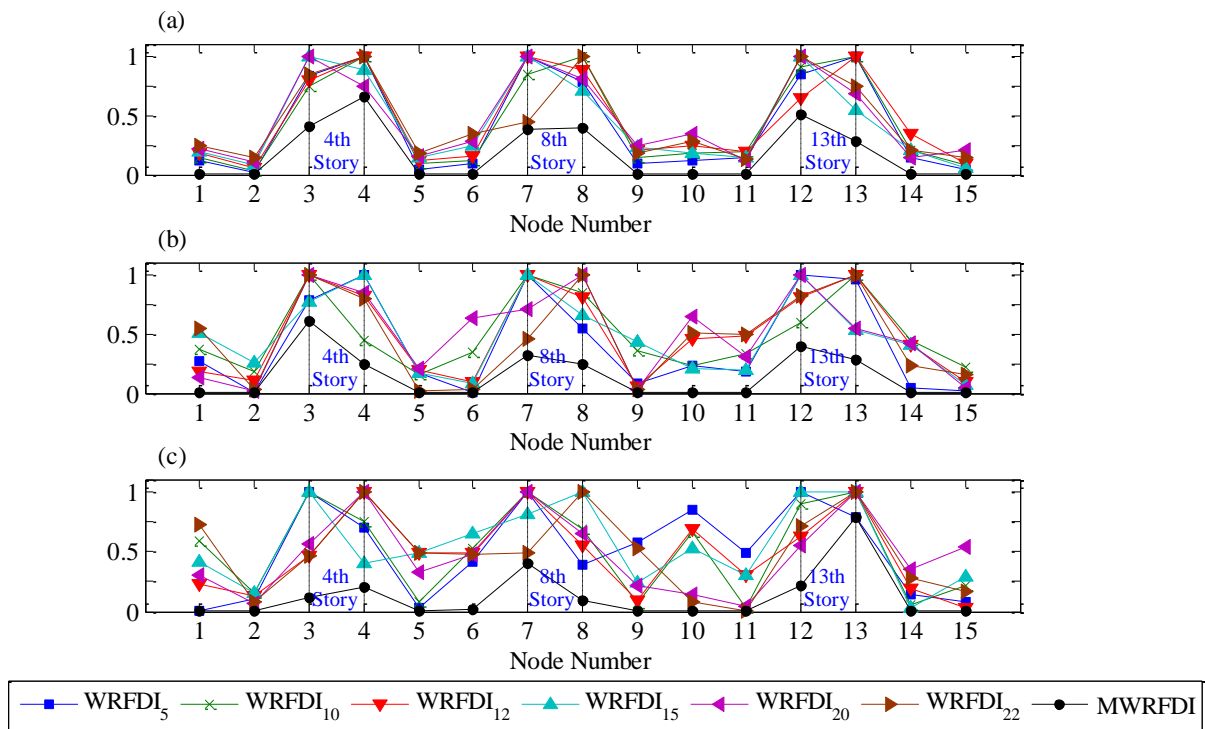


Figure 4. Damage identification results for damage pattern 3 in the 15-story shear frame for: (a) 0% noise, (b) 3% noise, and (c) 5% noise

#### 4.2 Concrete cantilever beam

As the second example, a concrete cantilever beam, which is shown in Fig. 5, is considered. The finite element model of this beam consists of 20 elements and each free node has two types of DOFs (translational and rotational). Therefore, this beam consists of 40 DOFs. The Young's modulus and mass density of beam are considered as  $E=25$  GPa and  $\rho=2500$  kg/m<sup>3</sup>, respectively. Moreover, damping ratio for all modes, the cross sectional area and the moment of inertia for all elements are equal to  $\zeta=5\%$   $A=0.35$  m<sup>2</sup> and  $I=0.01429$  m<sup>4</sup>, respectively. It is worth noting that these parameters are selected by satisfying those conditions which are considered in designing concrete structures as well as some recommendations from [32].

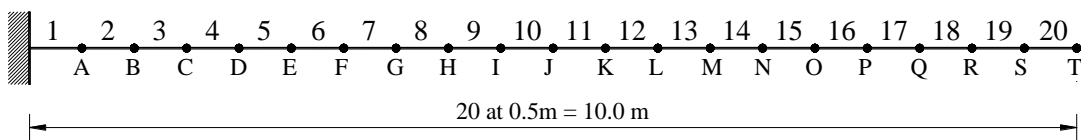


Figure 5. Finite element model of the concrete cantilever beam

Details of two different damage patterns which are simulated in this example are listed in Table 3. After extracting the proposed damage index, the results are shown separately for each type of DOFs to compare their applicability and stability in detecting damages. In this example, four different values are considered for parameter 'a': 5, 10, 15, and 20. The

obtained results for free noise state as well as noisy state are shown in Figs. 6 and 7 for damage patterns 1 and 2, respectively. It should be noticed that the noisy input data are generated with the presented instruction in the previous example and  $n=6$ . As it can be seen, the method is able to detect single and multiple damage cases with high level of accuracy and this conclusion is right whether the ideal input data (i.e. noise free data) are fed or not. In addition, from DOF-type viewpoint, although both types can detect damages without any false-negative and/or -positive results, overall, the translational DOFs performs better than rotational ones. This issue can be seen obviously by inspecting obtained results from damage pattern 2, when noisy input data are employed for damage identification. Similar to the previous example, it is concluded that the utilized impact acceleration doesn't have any impacts on the reliability of the method and so, it can be indicated that the method is independent from the properties of the utilized impact acceleration for exciting damaged structure and the only important thing is recording structural responses under free vibration scheme.

Table 3: Details of the simulated damage patterns in the concrete cantilever beam

Pattern Number	Damage Scenario		Acceleration Type	Related Nodes
	Element	Damage (%)		
1	10	10	IA-1	I-J
2	4, 12, and 17	10, 5, and 15	IA-2	C-D, K-L, and P-Q

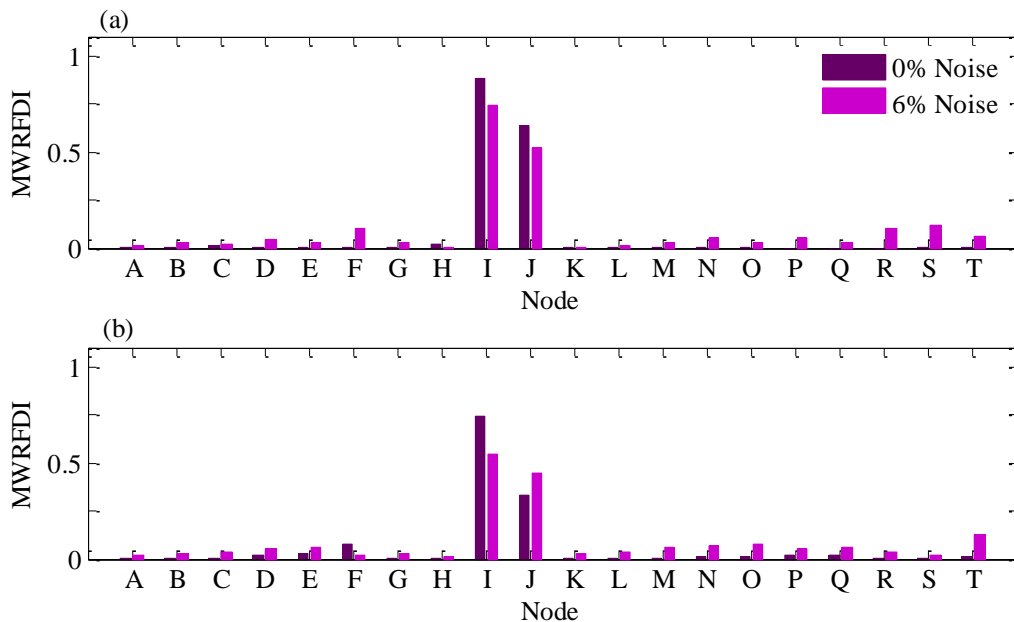


Figure 6. Damage identification results for damage pattern 1 in the concrete cantilever beam: (a) translational DOFs, and (b) rotational DOFs

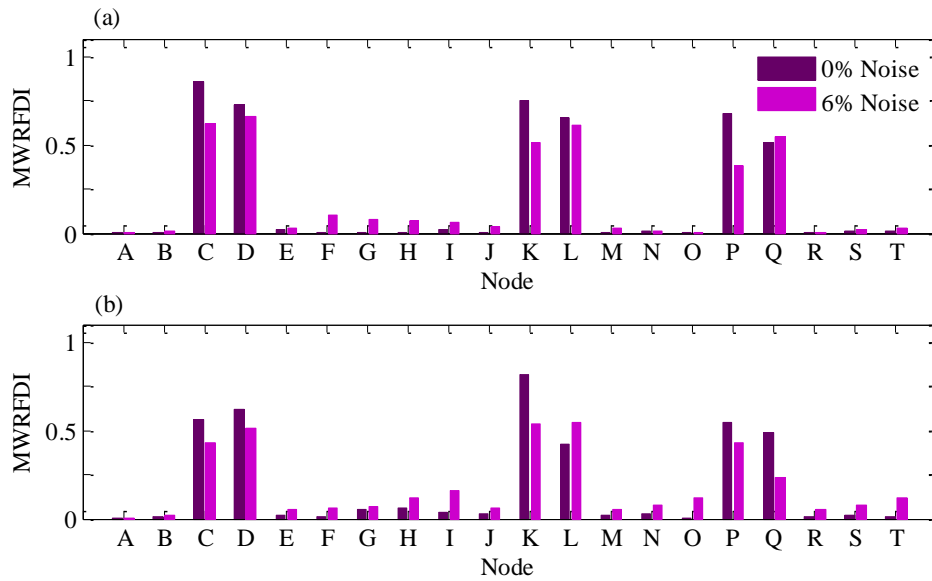


Figure 7. Damage identification results for damage pattern 2 in the concrete cantilever beam: (a) translational DOFs, and (b) rotational DOFs.

This section ends with presenting a comparative study on the robustness of the proposed method in damage localization by applying two other similar methods and comparing obtained results. In this regards, a Modified version of the Modal Residual Force Damage Index (MMRFDI) and a Flexibility-based Damage Index (FDI) is considered for identifying damage pattern 1 in the noisy state. MMRFDI which is presented by Ge et al. [4], is a multi-mode damage index and its input data are natural frequencies and mode shape vectors. Moreover, FDI, which has proposed by Nazari et al. [20], presents an index for damage localization in linear-shaped structures by applying Grey System Theory (GST) on the curvature of the diagonal members of flexibility matrix. It should be noted that MMRFDI and FDI perform similar to MWRFDI for detecting damaged elements.

To calculate MMRFDI and FDI, modal data are required. These data are extracted from recorded noisy time histories by means of the presented parameter identification approach in [33]. The obtained results for a condition in which the first four modes' data are employed for calculating MMRFDI and FDI, are shown in Fig. 8. In this figure, index values are normalized to 1.0. Based on the presented details in Table 3, it is expected that nodes I and J reveal non-zero values, however, the obtained MMRFDI in the nodes I, J and K are distinguishably bigger than zero and it is possible that the 11th element is reported as a damaged element, mistakenly. Similar situation exists for the FDI, and although it has detected damaged element correctly, because of reporting non-zero values for other nodes related to undamaged elements; some false-positive results can be obtained, too. It should be noted that in this investigation, both of the extracted natural frequencies and mode shape vectors from noisy time history responses, consist of considerable levels of noises. By considering this fact and comparing obtained results with those from MWRFDI, it can be concluded that MWRFDI has low sensitivity to noise in comparison with MMRFDI and FDI.

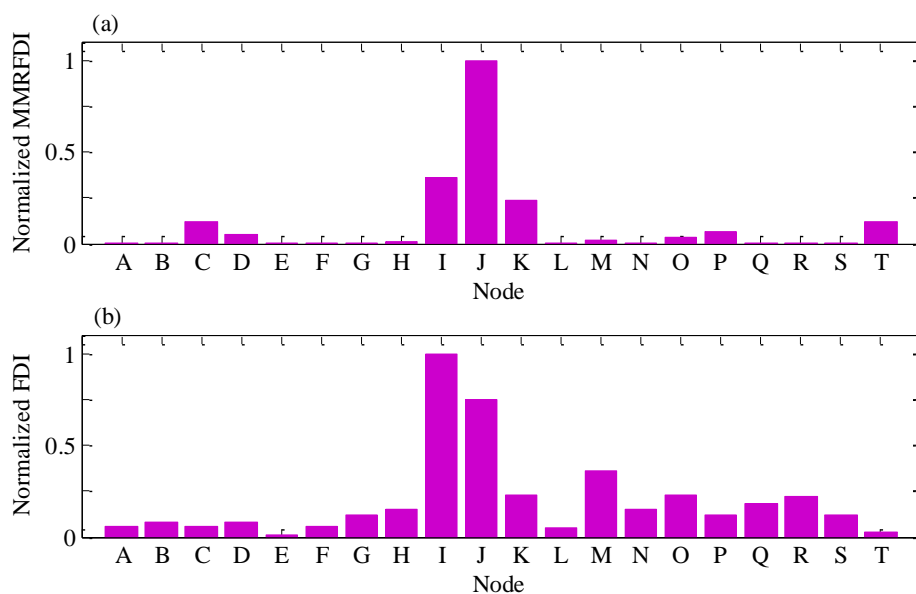


Figure 8. Damage identification results for damage pattern 1 in the concrete cantilever beam by means of noisy modal data using: (a) MMRFDI [4] (translational DOFs), and (b) FDI [20]

#### 4.3 Plane steel frame

In the last example, the presented method is applied for the damage identification in a four-story, two-bay plane steel frame (Fig. 9). The finite element model of this frame consists of 20 elements and 12 free nodes with three types of DOFs for each node (two translational in vertical and horizontal directions and one rotational). The module of elasticity and mass density for all elements are  $E=200 \text{ GPa}$  and  $\rho=7850 \text{ kg/m}^3$ . The mass per unit length, inertial moment, and cross sectional area are equal to  $m=117.7 \text{ kg/m}$ ,  $I=3.3 \times 10^{-4} \text{ m}^4$ , and  $A=0.0150 \text{ m}^2$  for columns, and  $m=1129.32 \text{ kg/m}$ ,  $I=3.69 \times 10^{-4} \text{ m}^4$ , and  $A=0.0152 \text{ m}^2$  for beams, respectively. In addition, damping ratio for all modes is equal to  $\zeta=5\%$ .

Three different damage patterns, which are described with more details in Table 4, are considered and simulated in the workspace of MATLAB software to generate time history responses. Then, the proposed method is applied to calculate the suggested damage index. Similar to the previous examples, investigations are carried out under free noise ( $n=0$ ) and noisy states ( $n=4$  and  $8$ ). In addition, five different values are considered for parameter ' $a$ ': 5, 10, 15, 20, and 25. The obtained results for three different types of DOFs are shown in Figs. 10, 11 and 12 for damage patterns 1, 2 and 3, respectively. By inspecting these figures, those nodes which reveal distinguishable non-zero values, will be considered as related nodes for damaged elements. For instance, consider obtained results for the third damage pattern, when the vertical-translational DOFs with  $n=8$  is available (Fig. 12(a)). As it can be seen, nodes: C, D, G, K and L, have non-zero values, so these nodes are considered as nodes related to the damaged elements. By inspecting Fig. 9, it can be concluded that elements 3 (node C), 7 (nodes D and G), and 20 (nodes K and L) are damaged. Such interpretation can be expressed in other cases.

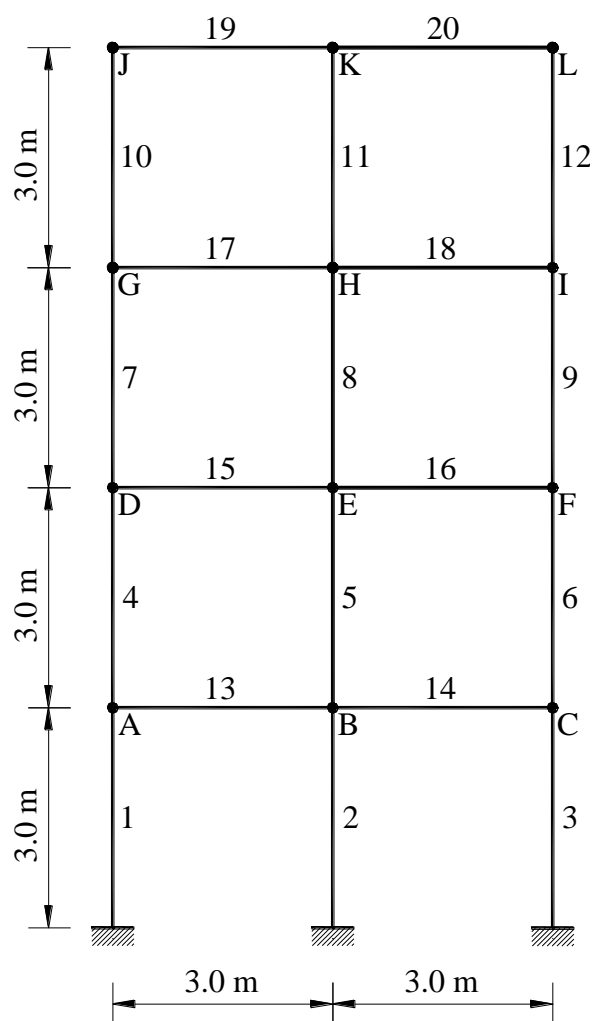


Figure 9. Finite element model of plane steel frame

Table 4: Details of the simulated damage patterns in the plane steel frame

Pattern Number	Damage Scenario		Acceleration Type	Related Nodes
	Element	Damage (%)		
1	8	25	IA-2	E-H
2	1, and 16	15, and 10	IA-1	A, and E-F
3	3, 7, and 20	20, 15, and 10	IA-2	C, D-G, and K-L

Finally, it is concluded that all obtained results emphasize the applicability of the presented method for damage identification in noisy states as well as free noise state, without any dependence on the impact acceleration which is utilized for exciting damaged structure. In addition, it is worth noting that although by inspecting all three kinds of DOFs damaged elements can be localized, the obtained results from translational DOFs are more distinguishable than those coming from rotational ones.

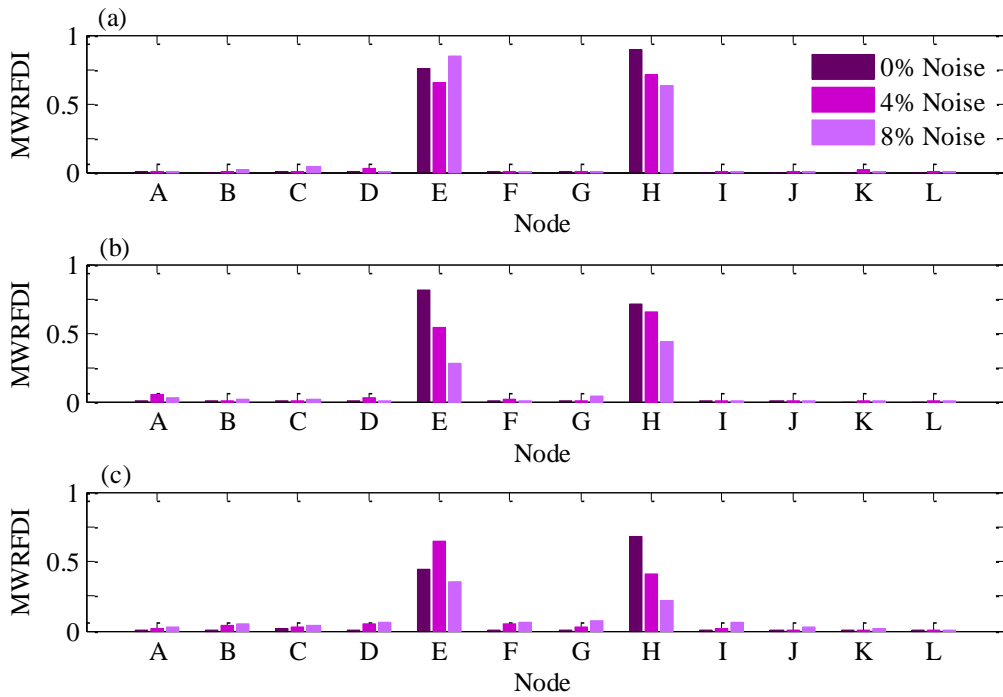


Figure 10. Damage identification results for damage pattern 1 in the plane steel frame: (a) vertical-translational DOFs, and (b) horizontal-translational DOFs, and (c) rotational DOFs

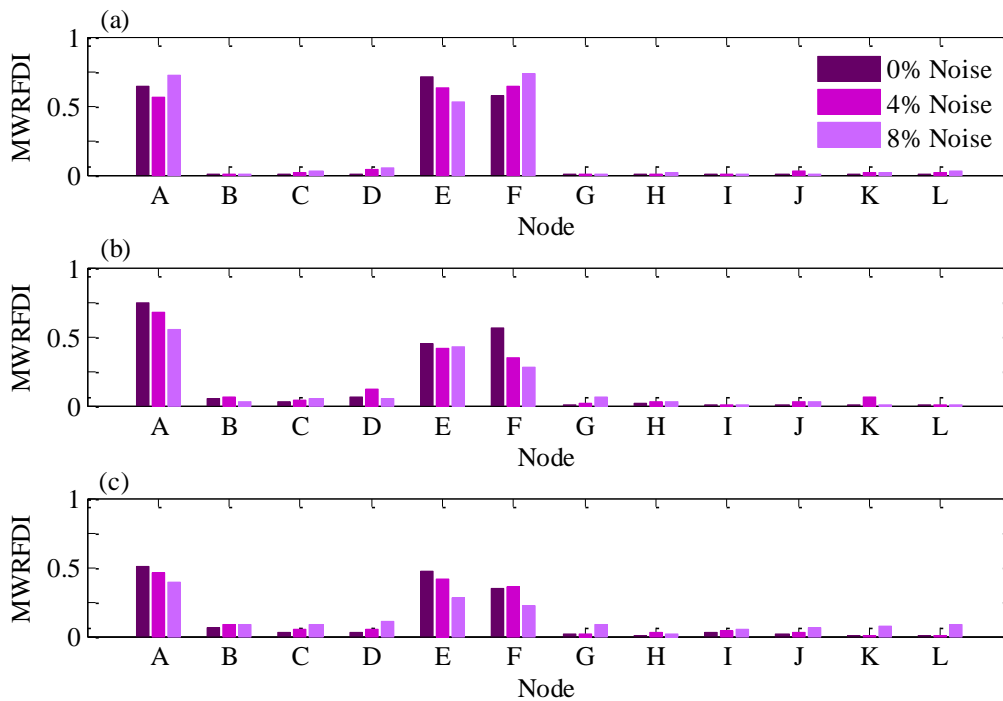


Figure 11. Damage identification results for damage pattern 2 in the plane steel frame: (a) vertical-translational DOFs, and (b) horizontal-translational DOFs, and (c) rotational DOFs

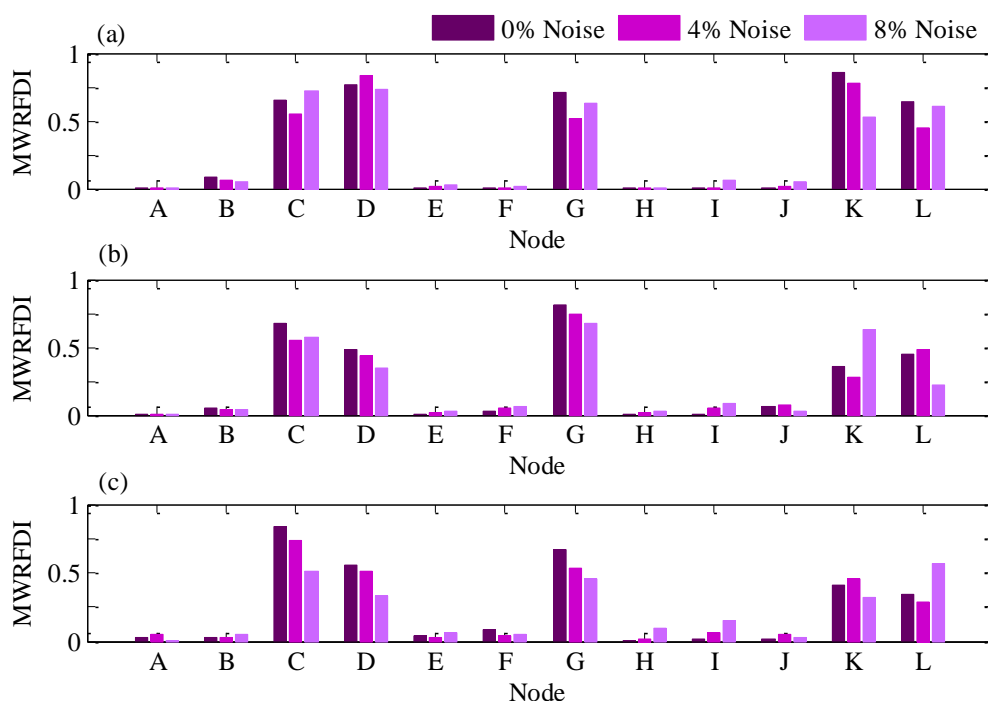


Figure 12. Damage identification results for damage pattern 3 in the plane steel frame: (a) vertical-translational DOFs, and (b) horizontal-translational DOFs, and (c) rotational DOFs

## 5. CONCLUSIONS

The main objective of this paper was concentrated on presenting an effective method for damage identification by analyzing displacement time history responses under free vibration scheme. Using Continuous Wavelet Transform (CWT), Wavelet Residual Force (WRF) was introduced and then, a multi-scale damage-sensitive index was suggested for damage localization. The applicability of the method was demonstrated by studying different damage patterns on three numerical examples of engineering structures. Moreover, some studies were carried out to investigate the robustness of the method in such a condition in which the input data are contaminated with different levels of random noises. The obtained results illustrated the good and viable applicability of the method for identifying different damage scenarios with single and/or multiple damage cases which was able to localize damages with high level of accuracy. In addition, based on the obtained results from comparative studies, it was concluded that the presented method had low sensitivity to noise in comparison with modal-data based damage indices.

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