ANN FOR CORRELATION BETWEEN SHEAR WAVE VELOCITY OF SOIL AND SOME GEOTECHNICAL PARAMETERS

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ABSTRACT

Shear wave velocity ($V_s$) is known as one of the fundamental material parameters which is useful in dynamic analysis. It is especially used to determine the dynamic shear modulus of the soil layers. Nowadays, several empirical equations have been presented to estimate the shear wave velocity based on the results from Standard Penetration Test (SPT) and soil type. Most of these equations result in different estimation of $V_s$ for the same soils. In some cases a divergence of up to 100% has been reported. In the following study, having used the field study results of Urmia City and Artificial Neural Networks, a new correlation between $V_s$ and several simple geotechnical parameters (i.e. Modified SPT value number ($N_{60}$), Effective overburden stress, percentage of passing from Sieve #200 ($F_c$), plastic modulus (PI) and mean grain size ($d_{50}$)) is presented. Using sensitivity analysis it is been shown that the effect of PI in $V_s$ prediction is more than that of $N_{60}$ in over consolidated clays. It is also observed that $F_c$ has a high influence on evaluation of shear wave velocity of silty soils.

KEY WORDS: shear wave velocity ($V_s$); geotechnical parameters; artificial neural networks; sensitivity analysis.

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1. INTRODUCTION

Maximum shear modulus ($G_{\text{max}}$) and shear wave velocity ($V_s$) are two essential parameters employed to perform dynamic analysis. Shear modulus of the soil highly depends on strain levels. Maximum shear modulus is the equivalent term for the shear modulus of a soil when undergoes strain levels equal or less than $10^{-3}$. Having been determined, $G_{\text{max}}$ can be used to obtain shear modulus of any soil for different strain levels by using the $G/G_{\text{max}}$ plots. $V_s$ and $G_{\text{max}}$ are used in soil classification, liquefaction potential and soil-structure interaction analysis [1].

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Vs is generally determined by seismic field measurements. Since these field experiments are performed in low strain levels, the results of these experiments can be used to determine G_max. Given the unit mass and Vs of a soil, G_max can be obtained using the following equation:

\[ G_{\text{max}} = \rho V_s^2 \]  

Vs and G_max are obtained by low strain laboratory measurements on undisturbed soil samples. Resonant column tests and bending element experiments are common laboratory tests for determination of low strain parameters. Cyclic triaxial apparatus combined with exact measurement of axial strains has also been used for this purpose.

Soil shear wave velocity can be also measured using geophysical methods such as Cross-Hole (CHT), Down-Hole (DHT), seismic cone penetration tests (SCPTs), Micro-tremor, wave propagation analysis at several stations (MASW) and spectral analysis of surface waves (SASW).

The effect of sampling disturbance on stiffness of samples is remarkable for low strain laboratory tests in which the weak boundaries between soil particles were broken during the process of sampling. Thus, undisturbed soil sampling on coarse material is not possible unless expensive freezing methods are used [2].

Compared to laboratory methods, the most important advantage of field measurement methods is that they naturally cause less disturbance on soil. However, various limitations such as space limitations, cost considerations, high noise levels (which is important in urban areas), leave these methods impractical.

Although exact determination of soil shear wave velocity using aforementioned methods is possible, they are generally expensive and in some of the projects may not be economically reasonable. Thus, the general trend is to determine the amount of Vs using indirect methods.

Many efforts have been recently made by researchers to develop new relations between the shear wave velocity of soil and geotechnical parameters. Recent studies focused on finding relationship between Vs and parameters like SPT, effective overburden stress, the percentage of fine grains, depth and tip resistance in cone penetration test [3-6].

Several studies revealed that effective overburden stress and porosity play a pivotal role in magnitude of G_max. Geological age also has an important influence on estimation of G_max. Pre-consolidation ratio of the soil, on the other hand, has a little effect on G_max. There are different opinions about effects of Plasticity Index (PI) on maximum shear modulus of the soil. Some of the studies show direct relationship between G_max and PI variations and some of them revealed reverse relationship [7].

Hardin and Drnevich [8] showed that G_max and Vs are firstly dependent on unit weight, porosity and effective stress. Soil type, age and cementation impose a little effect on these parameters.

Dobry and Vucetic [9] showed that with an increase in vertical effective stress, age, cementation and pre-consolidation value, Vs also increases. A reverse relationship governs the correlation between Vs and porosity.

In the following study, using the field measurements of Urmia City, it is tried to find a
correlation between shear wave velocity and SPT number \( (N_{60}) \), effective overburden pressure \( (\sigma') \), fine grain percentage \( (F_c) \), Plasticity Index \( (P_I) \) and average grading size \( (d_{50}) \). In order to perform the analysis, artificial neural network is used. Using sensitivity analysis, effect of each parameter on \( V_s \) is also evaluated.

2. REVIEW OF THE PROPOSED RELATIONSHIP

Several correlation relationships have been presented to correlate SPT and \( V_s \). The initial relationships were based on field data which use SPT number value \( (N) \). This value should be modified in order to consider the energy, length of the probe and internal diameter of the sampler.

Different methods have been used to measure \( V_s \) which includes Cross-Hole test \( (CHT) \), Down-Hole test \( (DHT) \), seismic cone penetration tests \( (SCPTs) \) and spectral analysis of surface waves \( (SASW) \). SASW, for instance, employs short frequencies to measure shear wave velocity. The result of such an experiment is mean velocity for a large volume of soil. CHT and DHT, on the other hand, use high frequency waves which results in the mean velocity of a small volume of soil and thus, provide high resolution results equal to results of laboratory scale tests.

Most of the common functions in recent studies are in the form of \( V_s = A.N^B \), where \( A \) and \( B \) are obtained by statistical regression of a set of data. The value of \( N \) is not corrected for overburden pressure but is modified for the energy, length of the probe and internal diameter of the sampler. Table 1 represents some of the important correlation relationships for different soil types.

Seed and Idriss [11] presented equation that correlates shear wave velocity to SPT number corrected for depth of soil. Seed et al. [12] also presented a correlation for \( V_s \) and SPT number based on equations presented for \( G_{\text{max}} \).

Using statistical analysis, Lee [14] presented different combinations of regression models based on SPT number, depth, Effective stress and type of soil.

Table 1: Empirical relationships for soil shear wave velocity \( (V_s) \) estimation based on SPT number \( (N) \)

<table>
<thead>
<tr>
<th>References</th>
<th>Soil type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Imai [10]</td>
<td>( V_s = 91N^{0.337} )</td>
</tr>
<tr>
<td>2 Seed and Idriss [11]</td>
<td>( V_s = 61.4N^{0.5} )</td>
</tr>
<tr>
<td>3 Seed et al. [12]</td>
<td>( V_s = 116(N+.318)^{0.202} )</td>
</tr>
<tr>
<td>4 Jinan [13]</td>
<td>( V_s = 90.9(D+.62)^{0.212} )</td>
</tr>
<tr>
<td>5 Lee [14]</td>
<td>( V_s = 51.5N^{0.516} )</td>
</tr>
<tr>
<td>6 Hasançebi and Ulusay [4]</td>
<td>( V_s = 90N^{0.309} )</td>
</tr>
<tr>
<td>7 Iyisan [3]</td>
<td>( V_s = 90N^{0.319} )</td>
</tr>
<tr>
<td></td>
<td>( V_s = 90N^{0.269} )</td>
</tr>
</tbody>
</table>
Iyisan [3] proposed several equations for $V_s$ and represented an evaluation for the effect of several parameters such as SPT number ($N$), vertical effective stress ($\sigma'$), mean grain diameter $d_50$, modified SPT number ($N_{1-60}$), tip resistance in cone penetration test ($q_c$) and depth of the sedimentary layer ($H$) on shear wave velocity of the soil.

Hasançebi and Ulusay [4] presented several correlation relations by using 97 sets of data gathered from North West of Turkey. They obtained different equation for clayey and sandy soils. Using datasets gathered from seismic micro zonation studies in India, Anbazhagan and Sitharam [15] presented an equation to determine shear wave velocity of the soil based on Modified standard penetration test ($N_{1-60}$).

Using statistical regression analysis, Brandenburg et al. [6] presented an equation to estimate $V_s$ for soils under Caltrans bridges. They gathered datasets from 79 logs in 21 bridges and express $\ln(V_s)$ as a function of SPT number, overburden effective stress for sandy, silty and clayey soils.

Using datasets gathered from Taiwan, Kuo et al. [16] presented an equations to determine $V_s$ based on SPT number and depth of the sample. Using datasets from different zones of the word and employing polynomial neural networks, Ghorbani et al. [2] presented the following equation to evaluate shear wave velocity:

$$V_s = 3.02 + 1.8839Y_2 + 0.9307Y_3 + 0.3368Y_2^2 + 0.35324Y_3^2 - 0.68995Y_2Y_3$$

where:

$$Y_3 = -157.27 - 1.184\sigma' + 3.3944Y_1 - 0.00198\sigma'^2 - 0.0089Y_1^2 + 0.0086\sigma_Y Y_1$$

$$Y_2 = 1.62 + 0.933Y_1 + 0.551N_{1-60} - 0.0003\sigma_Y Y_1^2 + 0.00372N_{1-60}^2 - 0.0039\sigma_Y Y_{1-60}$$

$$Y_1 = 106.27 + 2.34N_{1-60} + 0.48\sigma' - 0.021N_{1-60}^2 + 0.00052\sigma'^2 - 0.00204N_{1-60}$$
3. NEURAL NETWORKS

Neural Network is one of the major branches in artificial intelligence. Neural network is a data analysis system based on a mathematical model of the human brain's nerve fibers. Neural network is primarily trained with processing large data sets. Based on a proper training, neural networks are able to provide fairly accurate output for a data set. A network has three main parts: the transition function, the network structure and the learning law, that are defined separately based on type of the defined problem [17].

Generally, the Multilayer Perceptron Neural Network (MLPNN) is a common network for engineering purposes. However, other Neural Networks such as GRNN and RBFNN are used to solve various problems.

The performance of a neural network is evaluated by correlation coefficient (r), mean absolute error (MAE) and root mean square error (RMSE). The correlation coefficient is expressed as follows:

\[ r = \frac{\sum_{i=1}^{n} (O_i - \bar{O})(T_i - \bar{T})}{\sqrt{\sum_{i=1}^{n} (O_i - \bar{O})^2} \sqrt{\sum_{i=1}^{n} (T_i - \bar{T})^2}} \]  

(3)

The mean absolute error and root mean square error are defined as follows:

\[ MAE = \frac{\sum_{i=1}^{n} |T_i - O_i|}{n} \]  

(4)

\[ RMSE = \sqrt{\frac{\sum_{i=1}^{n} (T_i - O_i)^2}{n}} \]  

(5)

where \( T_i \) and \( O_i \) are the target output and the output calculated by the neural network, respectively.

4. MULTILAYER PERCEPTRON NEURAL NETWORKS (MLPNN)

In a MLPN networks, input data are linked to weights matrix from the first layer that in turn has been linked to second layer weights matrix as well. The weights matrix from the hidden layer is linked to the output layer data. Hidden layers are defined as a black box that modifies input data to obtain output data. Standard BP is a gradient descent algorithm, in which the network weights are moved along the negative of the gradient of the performance function. There are a number of variations on the basic algorithm that are based on other standard optimization techniques. In this study, Levenberg-Marquardt (LM) [18] algorithm...
is employed. The basic BP algorithm adjusts the weights in the steepest descent direction.

5. RBF NEURAL NETWORKS

RBF neural networks are widely used in the field of structural engineering due to their fast training, generality and simplicity. RBF neural networks are two layers feed forward networks. The hidden layer consists of RBF neurons with Gaussian activation functions. The outputs of RBF neurons have significant responses to the inputs only over a range of values of input data called the receptive field.

The numerical results of many scientific and engineering applications indicate that RBF networks are very good tools for interpolation and also their training is very fast.

6. GENERALIZED REGRESSION NEURAL NETWORKS

Generalized Regression Neural Network (GRNN) is developed by Specht which falls in the category of radial neural networks [19]. This network consists of four layers; 1- the input layer, 2-pattern layer, 3- summation layer and 4- output layer. Parameters such as learning rate and momentum is not required for this type of neural networks. GRNN networks have decent performance for discrete and continuous data collection, and the rate of training process is relatively high.

7. DATA DIVISION

In this study, a total of 191 data sets are used which are collected from field studies in Urmia City. Table 2 shows some of the statistical indices for datasets. As it is seen from Table 2, the rang for N60, σ', Fc, PI, d₅₀ and Vs, are 8-66, 21.8-331.6, 0-0.98, 0-27.7, 0-5, 82-566, respectively.

<table>
<thead>
<tr>
<th>variable</th>
<th>max</th>
<th>min</th>
<th>mean</th>
<th>std</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₆₀</td>
<td>66</td>
<td>8</td>
<td>30.14</td>
<td>11.91</td>
</tr>
<tr>
<td>σ'(kPa)</td>
<td>331.6</td>
<td>21.80</td>
<td>101.73</td>
<td>70.56</td>
</tr>
<tr>
<td>Fc</td>
<td>0.98</td>
<td>0</td>
<td>0.8</td>
<td>0.21</td>
</tr>
<tr>
<td>PI</td>
<td>27.7</td>
<td>0</td>
<td>4.78</td>
<td>8.02</td>
</tr>
<tr>
<td>d₅₀(mm)</td>
<td>5</td>
<td>0</td>
<td>0.34</td>
<td>1.29</td>
</tr>
<tr>
<td>Vₛ(m/s)</td>
<td>566</td>
<td>82</td>
<td>375.12</td>
<td>117.37</td>
</tr>
</tbody>
</table>

There are several ways for data division. In random data division sometimes some of the test data are missed in training process and there might be an inconsistency between training and test data.
The effect of four different types of data classification in results of neural network model is studied by Shahin et al. [20]. Corresponding neural network was designed to evaluate the settlements of shallow foundations on granular soils. The four division methods were: 1- random data division, 2- Statistical Classification, 3- Self-organized mapping (SOM) and 4- Fuzzy clustering.

The results showed that similarity of statistical characteristics of the training and testing datasets has a remarkable impact on the results obtained from artificial neural network. The results also showed that comparing to other methods, the fuzzy clustering method is more effective in optimum performance of artificial neural networks. To achieve the accurate results, Fuzzy clustering is also used in this study for data clustering.

Datasets are clustered into 16 subsets by using subtractive fuzzy clustering method with effect (Subtractive Clustering) and with a radius of influence equal to 0.4. Then, 155 (80%) and 36 (20%) of data sets are used for training and test, respectively. The software MATLAB 8 (R2012b) [21] is used for data clustering and artificial neural network training.

8. SENSITIVITY ANALYSIS

Using a complicated process, ANN works as a black box that correlates the input data to the outputs. Instant understanding of the determination process for network weights and neuron values of hidden layers of datasets is not possible. Therefore, in a neural network analysis quick perception of the impacts of each independent variable on dependent one is not easily possible [22]. Different methods have been proposed to describe the operation of neural networks including input neurons, hidden-layers neurons and output neurons. These methods combined with neural networks and sensitivity analysis is performed using the following two methods:

1) Analysis based on weight values
2) Sensitivity analysis method (PaD)

Different empirical studies have shown that analysis based on weight values is not efficient to determine the effect of input variables on the output variables [23]. In the present study, the PaD method is used to perform sensitivity analysis.

9. PARTIAL DERIVATION METHOD (PAD)

Two important results are obtained by using PaD method: 1- Output variables profile for partial changes of each input variable and 2- classification of the relative impact of each variable on neural network outputs. To obtain the output variables profile with respect to small changes of input variable, one can calculate the partial differential of ANN output with respect to inputs. PaD method produce more stable results comparing to other methods (e.g. weights method) [23].

PaD method depend on the input $x_i$, the output $y_k$, the connection weights between inputs and hidden modes $w_{ij}$, the connection weights between hidden and output nodes $u_{jk}$, and the activation function.

Sensitivity analysis for neural networks is expressed as follows:
\[
\frac{\partial y_k}{\partial x_i} = f'(net_k) \sum_{j=1}^{n} u_{kj} f'(net_j) w_{ij}
\]  
\hspace{1cm} (6)

where \( f'(net_k) \) and \( f'(net_j) \) are the derivatives of the activation function of hidden nodes and output nodes, respectively. In PaD method, the relative importance of each input variable with respect to network output is calculated by SSD index defined as follows:

\[
SSD = \sum_{j=1}^{n} \left( \frac{\partial y_i}{\partial x_j} \right)^2
\]  
\hspace{1cm} (7)

This sensitivity is calculated for each taken sample from model database (records) and \( n \)-dimensional space of variables. SSD index indicates the relative importance of contribution of each input variable in calculating the output variable. The higher the value of SSD, the higher the effect of the variable in model outputs.

10. RESULTS AND DISCUSSIONS

Three multilayer perceptron neural network methods (e.g. MLP, RBF and GRNN) are used for training. For initial comparisons of the models and to evaluate them, a parameter called coefficient of correlation is used. This parameter is one of the most precise and widely used index in evaluation of neural networks performance. This index indicates the level of correlation between two variables.

In next step, all networks that have a correlation higher than 90% are chosen and the other networks are removed. Table 3 and table 4 shows 4 MLP and 3 RBF and GRNN networks that have a correlation higher than 90%. Criterion such as R, MAE and RMSE are used to evaluate the performance of these 7 networks. As it is shown, GRNN results in the lowest error and consequently best solution. This evaluation parameters are \([R=0.99, \text{MAE}=9.52 \text{ m/s and RMSE}= 16.27] \) and \([R=0.95, \text{MAE}=27.71 \text{ m/s and RMSE}= 37.25]\) for training set and testing set, respectively.

Fig. 1 shows the graphs of correlation equations for \( V_s \) and \( N \) that are presented in Table 1. Although some of the graphs represent relatively appropriate fit, there is a large difference in the wide range of values of \( V_s \) obtained for an \( N \) value. Therefore, the exact percentages of differences that caused by the diversity of soil types, errors in field measurements and effects of disregarding the other parameters, remained unclear.

<table>
<thead>
<tr>
<th>Number of network</th>
<th>Training type</th>
<th>Number of middle layers</th>
<th>Number of neurons</th>
<th>Activation function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BP(trainbr)</td>
<td>1</td>
<td>50</td>
<td>tansig-tansig</td>
</tr>
<tr>
<td>2</td>
<td>BP(trainbr)</td>
<td>1</td>
<td>40</td>
<td>tansig-purelin</td>
</tr>
<tr>
<td>3</td>
<td>BP(trainbr)</td>
<td>2</td>
<td>30-30</td>
<td>tansig-tansig-tansig</td>
</tr>
<tr>
<td>4</td>
<td>BP(trainlm)</td>
<td>2</td>
<td>50-50</td>
<td>tansig-tansig-tansig</td>
</tr>
</tbody>
</table>
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Table 4: Specifications of successful RBF and GRNN neural networks

<table>
<thead>
<tr>
<th>Number of network</th>
<th>Training type</th>
<th>Spread</th>
<th>Number of neurons</th>
<th>Activation function</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>RBF</td>
<td>0.16</td>
<td>155</td>
<td>radbas-purelin</td>
</tr>
<tr>
<td>6</td>
<td>GRNN</td>
<td>0.25</td>
<td>155</td>
<td>tribas-purelin</td>
</tr>
<tr>
<td>7</td>
<td>GRNN</td>
<td>0.11</td>
<td>155</td>
<td>radbas-purelin</td>
</tr>
</tbody>
</table>

Table 5: Evaluation of successful neural networks

<table>
<thead>
<tr>
<th>Number of network</th>
<th>R (train)</th>
<th>R (test)</th>
<th>MAE (train)</th>
<th>RMSE (train)</th>
<th>MAE (test)</th>
<th>RMSE (test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.97</td>
<td>0.9</td>
<td>21.05</td>
<td>29.24</td>
<td>39.01</td>
<td>52.52</td>
</tr>
<tr>
<td>2</td>
<td>0.96</td>
<td>0.9</td>
<td>23.24</td>
<td>32.42</td>
<td>39.58</td>
<td>52.01</td>
</tr>
<tr>
<td>3</td>
<td>0.94</td>
<td>0.9</td>
<td>31.04</td>
<td>40.06</td>
<td>40.39</td>
<td>51.08</td>
</tr>
<tr>
<td>4</td>
<td>0.94</td>
<td>0.9</td>
<td>29.89</td>
<td>39.67</td>
<td>40.76</td>
<td>49.73</td>
</tr>
<tr>
<td>5</td>
<td>0.99</td>
<td>0.93</td>
<td>3.38</td>
<td>7.60</td>
<td>30.77</td>
<td>43.16</td>
</tr>
<tr>
<td>6</td>
<td>0.98</td>
<td>0.94</td>
<td>16.50</td>
<td>25.97</td>
<td>30.25</td>
<td>39.43</td>
</tr>
<tr>
<td>7</td>
<td>0.99</td>
<td>0.95</td>
<td>9.52</td>
<td>16.27</td>
<td>27.71</td>
<td>37.25</td>
</tr>
</tbody>
</table>

Fig. 2 represents the evaluation of the $V_s$-N correlation equations for two samples of boreholes. For comparison purposes some relationships that are only depth-dependent. This comparison shows that GRNN results in the most logical results which are very close to real values.

Figure 1. Used data versus Empirical correlation relationships between $V_s$ and N
Fig. 3 shows the relative importance of input values by using PaD method and SSD parameter. Regarding this figure, one can surmise that $d_{50}$ has negligible effects on sandy and silty soils and almost no effect on clayey soils. $N_{60}$ is the most important parameter that tremendously affects the shear wave velocity of the silty and sandy soils. Shear wave velocity of normally consolidated clays and silty soils are mostly influenced by contribution percentage of $N_{60}$. $F_c$ and $\sigma'$ are of the secondary importance. $F_c$ does not play a significant role for sandy soils. For the case of overconsolidated clays, PI is the most important parameter. The effect of PI is almost twice the influence of the $N_{60}$. However, $N_{60}$ has a significant effect on output results. Rate of influence for Both $F_c$ and $\sigma'$ is around 5%.
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11. CONCLUSIONS

In this study shear wave velocity \( (V_s) \) of the soil and some primary geotechnical parameters that were gathered from field studies in Urmia city have been used to represent a model to correlate \( V_s \) to simple geotechnical parameters. This model is based on artificial neural networks. It has been shown that GRNN has an appropriate performance for both the training and test datasets. Recent relationships do not consider the effects of geotechnical parameters such as \( \sigma' \), \( F_c \), PI and \( d_{50} \), and only evaluate \( V_s \) based on \( N_{60} \) values, while the effect of some of the geotechnical parameters is significant. The effect of PI in overconsolidated clays by far greater than that of \( N_{60} \). In the evaluation of \( V_s \), \( \sigma' \) is an
important factor for all type of soils and Fc is determinant factor for normally consolidated clays and silty soils. Therefore, in order to come up with an accurate estimation of $V_s$, in addition to $N_60$, other geotechnical parameters should be considered. While recent studies proposed different relationships for each soil type, in this study a unique relationship was obtained for all soil types.

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