



## GENETIC PROGRAMMING AND MULTIVARIATE ADAPTIVE REGRESSION SPLINES FOR PREDICTION OF BRIDGE RISKS AND COMPARISON OF PERFORMANCES

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### ABSTRACT

In this paper, two different data driven models, genetic programming (GP) and multivariate adoptive regression splines (MARS), have been adopted to create the models for prediction of bridge risk score. Input parameters of bridge risks consists of safe risk rating (SRR), functional risk rating (FRR), sustainability risk rating (SUR), environmental risk rating (ERR) and target output. The total dataset contains 66 bridges data in which 70% of dataset is taken as training and the remaining 30% is considered for testing dataset. The accuracy of the models are determined from the coefficient of determination ( $R^2$ ). If the  $R^2$  the testing model is close to the  $R^2$  value of the training model, that particular model is to be consider as robust model. The modeling mechanisms and performance is quite different for both the methods hence comparative study is carried out. Thus concluded robust models performance based on the  $R^2$  value, is checked with mathematical statistical equations. In this study both models were performed, examined and compared the results with mathematical methods successfully. From this work, it is found that both the proposed methods have good capability in predestining the results. Finally, the results reveals that genetic Programming is marginally outperforms over the MARS technique.

**Keywords:** bridge risks; genetic programming; multivariate adoptive regression splines; performance criteria.

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## 1. INTRODUCTION

Prediction of Bridge risk score plays a vital role in taking the remedial measures and maintenance criteria. So many input parameters are associated with the output makes the problem more complex and imports the nonlinearity. In that scenario, data-mining techniques can help the users to solve the problem and also determined the nonlinear relationship between the input variables and the expected output. The advantage of data mining techniques is that it can derive the initial constants from the data itself. There are many methods existing for prediction of unknowns. The application of different techniques and their potential in giving the best results are observed from the earlier works.

Lot of researches has been created in formation of robust models using different methods for gaining the accuracy in predictions. Pijush samui applied the MARS for prediction of elastic modulus of jointed rock mass and compared the MARS models with ANN models, concluded that MARS gives better models than ANN [1]. Samui & Kurup study about Multivariate adaptive regression splines (MARS) and least squares support vector machine (LSSVM) for OCR prediction revealed that LSSVM models performed a bit well over MARS [2]. Wang et. al. work on a comparison of neural network, evidential reasoning and multiple regression analysis in modeling bridge risks showed the ability of ANN in estimating the results [3]. Kaveh A successfully applied the ANN models in prediction of compressive strength of concrete. He showed the potential of ANN in predicting the values [4].

Yang C et al, considered the daily rainfall, potential evaporation, soil temperature, pesticide concentration as input parameters in preparing the models. They have tested the models using MARS and ANN methods and found that MARS gives low standard errors over ANN [5]. Hudaverdi T showed that the potential of MARS in estimating the intensity of dangerous ground vibrations induced during the blasting operations in quarries [6].

Genetic programming is another tool extensively used in many fields. Guven A reported that the usage of GP in prediction of scour depth gives satisfactory output than conventional regression analysis and showed that GP is an encouraging method to use in water resources engineering [7]. Baykasoglu A has successfully applied the GP in predicting the strength of limestone available at greater depths by taking the water absorption, ultrasonic pulse velocity, dry density, saturation density and bulk density as input variables [8]. Savic D et al, study showed that the potential of GP technique in modeling the runoff output by taking the rainfall data and evaporation data as input [9]. A part from this literature, researchers are extensively used the data-mining techniques in many engineering and technology fields [10-14]. Hence, it is the clear evidence for the potential of data mining tools in predicting the values with more accuracy.

Present study adopts two superior methods, genetic programming (GP) and MARS for predicting the bridge risks. GP and MARS models are performed on the dataset in modeling the bridge risks. For this study data set consists of safety risk rating, functional risk rating, serviceability risk rating and environmental risk ratings has been taken from the study of Wang et. al. The dataset is sub divided into training data set, testing data set to perform the models.

- Training data set- used to build the model for the given data.
- Testing data set- used to check the model performance.

The total data set consist of 66 bridge structures. Out of them 46 were taken for training

the models and 20 for testing the trained models performance. Performance criteria have been used to compare the performances of each model.

## 2. DETAILS OF GENETIC PROGRAMMING

GP is a branch of GA and it is one of the best evolutionary methods. GP is a biological evaluation process in nature which gives the relationship between the input and output elements. It is orthodox and optimization search space consists of chromosomes. GP is based on the Darwin law of survival of the fittest. Fitness function is the main objective function in GP. The population required for the GP is keeping in pool from the fitted parents through the operations of reproduction, mutation and crossover.

In GP models the mechanism is, first a random population is created in the pool. It assigns the fitness value for each one. After the fitness value of each is determined the selected parent elements with good fitness functions under goes through the reproduction operations such as mutation, crossover to create a new population in the pool. Similarly the same process will be continuing till the best fitted function is created. Finally after completion of this process a best suited equation will generate automatically.

In GP models, the training dataset and testing dataset contains the input variables SRR, FRR, SUR and ERR are represented as  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_3$  respectively for easy evaluation and assessment of results. The predicted output values corresponding to the target values are being extracted from the generated output. The relationship between the variable inputs and predicted output can be directly determined from GP. The general form of a GP expression is as shown in below equation (1).

$$y = a(x) + 2. \quad (1)$$

Here, 'y' is the predicted output, 'x' is the input variable and 'a' is constant.

GP is carried out using MATLAB software. The main advantage of GP model is it can generate the required equations directly. The performance of the models has been evaluated in terms of coefficient of determination value ( $R^2$ ). If  $R^2$  value is nearest to unity that model has to be taken as most effective model. Statistical evaluation criteria also determined to assess the predicted output values.

## 3. DETAILS OF MARS

MARS is a non-linear, non-parametric regression analysis that gives the relationship between input variable and output values (Friedman, 1991). MARS method is faster, easy and enables to give better outcomes. So MARS is widely used in various fields of engineering, science, medical and mathematics to get the relation between the input and output. The main advantage of this method is can give the best relation even the input parameters are large and more complex.

MARS is inspired by the divide and conquer method. Its mechanism is, the database is splits into number of splines, knots and basis functions (BF). Basis functions are the main

objectives in MARS. The number of knots and basis functions are automatically driven from the database. MARS contains forward and backward algorithms in which forward algorithm will select the maximum number of basis functions from the dataset and next backward algorithm will follow, check and delete the less effective basis function from dataset till the best set is obtained. After deletion the remaining basis functions will perform in the modeling process.

In this study MARS technique is adopted because of its flexibility and effectiveness in generating the output. The general form of the MARS expression is as shown in below equation (2).

$$y = \beta_0 + \sum_{j=1}^P \sum_{b=1}^B [\beta_{jb} (+) \max(0, x_j - H_{bj}) + \beta_{jb} (-) \max(0, H_{bj} - x_j)] \quad (2)$$

where 'P' is the predictor variable, ' $\beta_0$ ' is the initial coefficient, H are called 'hinges' or 'knots', " $\beta_{jb}$ " are the coefficient at ' $j^{\text{th}}$ ' basis function and ' $\max(0, x-H)$ ', ' $\max(0, H-x)$ ' are univariate functions.

Here also the input variables, target output, training data, testing data are same as used in the GP models. The best model was found from the correlation coefficient (R) value. The model with R value close to unity is the best model. For a good model the testing R is closer to the training R value. Statistical evaluation also performed to check the results. MARS is performed using MATLAB.

#### 4. RESULTS AND DISCUSSIONS

First GP is performed on the dataset. The training dataset is used to create the models then the created models are tested using testing dataset. The model with high coefficient of determination ( $R^2$ ) for both training and testing dataset is the robust model. The best model is obtained at population size of 700 and no. of generations 300. At this model, the best performance with minimum error is obtained. The predicted output values (y) are extracted from the generated graphs of GP models. Formula generated by GP model is given below equation (3). This formula is universal valid.

Simplified overall GP expression:

$$y = 0.05416 \text{ square}(- \exp(x_2) (x_1 - x_2)) - 77.66 \cos(x_1 \text{ square}(x_1)) + 2.001 (x_3 - 0.227) (x_1 - x_3)^2 + 3.603 x_2 \cos(x_1 - x_3) - 6.825 x_1 \sin(x_1) \text{ square}(x_1) - 2.504 \sin(x_1) \sin(x_2) (x_2 + x_3) (x_2 - x_3) + 89.84 \quad (3)$$

where, 'y' is the predicted risk score,  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$  are safety risk rating, functional risk rating, serviceability risk rating and environmental risk ratings respectively.

The training dataset predicted values were compared with testing dataset. In general testing  $R^2$  value is less than training  $R^2$  value. The model with testing value closer to training value is consider as robust model. Figs. 1 and 2 shows the training and testing performances of the robust GP models are given below:

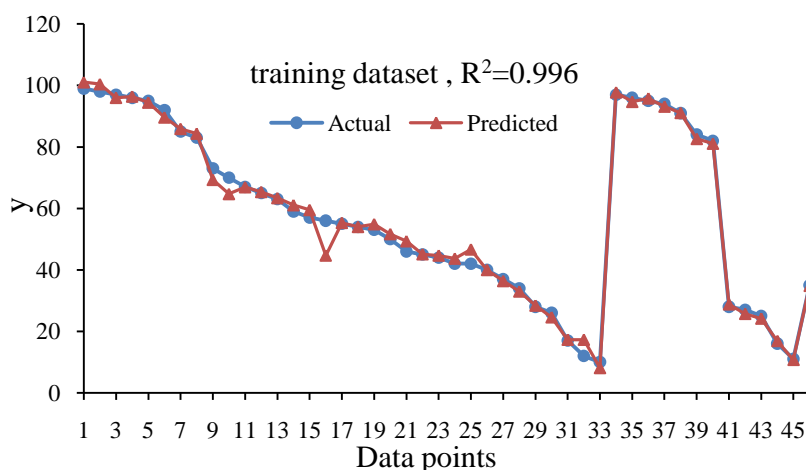


Figure 1. Performance of training dataset for GP

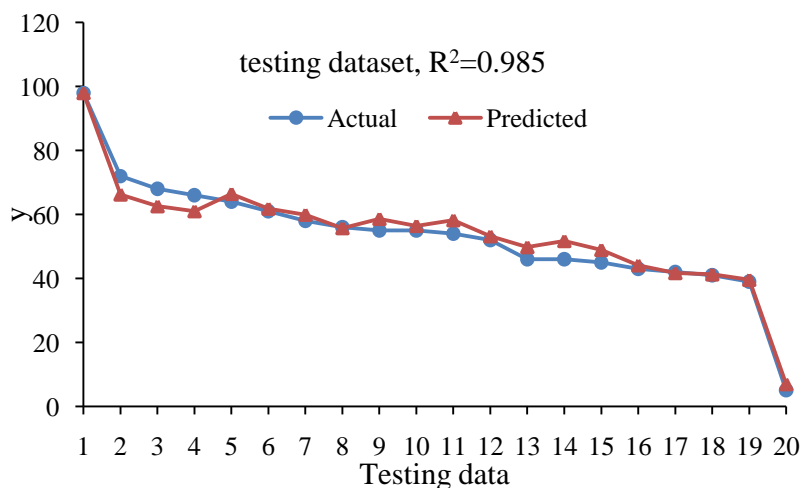


Figure 2. Performance of testing dataset for GP

For MARS model, correlation coefficient (R) has been used for performance evaluation. The R value is being determined by the below equation (4).

$$R = \frac{\sum_{i=1}^n (y_{ai} - y_{amean}) \sum_{i=1}^n (y_{pi} - y_{pmean})}{\sqrt{\sum_{i=1}^n (y_{ai} - y_{amean})^2} \sqrt{\sum_{i=1}^n (y_{pi} - y_{pmean})^2}} \tag{4}$$

Here,  $y_{ai}$  and  $y_{pi}$  are actual and predicted values respectively,  $y_{amean}$  and  $y_{pmean}$  are mean of actual and predicted values. In creating the robust MARS model, total 20 Basis Functions (BF) are carried in forward algorithm. Later backward algorithm carried to delete the less effective BFs. Six basis functions have been deleted by backward step. After deleting, the remaining BFs generated the required MARS model which is mentioned in Table 1 with their corresponding equations.

Table 1: Basis functions and their equations

Basis function	Equation
BF1	$\max(0, x1 - 1)$
BF2	$\max(0, 1 - x1)$
BF3	$\text{BF2} \times \max(0, x3 - 1)$
BF4	$\text{BF2} \times \max(0, 1 - x3)$
BF5	$\max(0, x2 - 2)$
BF6	$\max(0, 2 - x2) \times \max(0, 2 - x1)$
BF7	$\text{BF1} \times \max(0, x2 - 2)$
BF8	$\text{BF1} \times \max(0, 2 - x2)$
BF9	$\max(0, x1 - 2)$
BF10	$\max(0, x3 - 2)$
BF11	$\text{BF10} \times \max(0, x1 - 2)$
BF12	$\text{BF10} \times \max(0, 2 - x1)$
BF13	$\max(0, 2 - x2) \times \max(0, 1 - x3)$
BF14	$\max(0, x4 - 1)$

The equation (5) given below has been developed by the MARS model, used to determine the performances of training and testing dataset:

$$y = 50.446 + 0.670 \times \text{BF1} - 35.925 \times \text{BF2} + 12.142 \times \text{BF3} + 8.448 \times \text{BF4} + 41.463 \times \text{BF5} - 1.075 \times \text{BF6} - 20.042 \times \text{BF7} - 0.968 \times \text{BF8} + 42.00 \times \text{BF9} + 7.583 \times \text{BF10} - 6.735 \times \text{BF11} + 7.436 \times \text{BF12} - 3.208 \times \text{BF13} + 1.788 \times \text{BF14} \quad (5)$$

where 'y' is the predicted output, BF represents the basis functions given in Table 1.

The best model of MARS implies the above equation. This is the best fitted relation for the input variables and the expected output with minimum error. The performance of the training and testing dataset and the corresponding R values are mentioned in the Figs. 3 and 4.

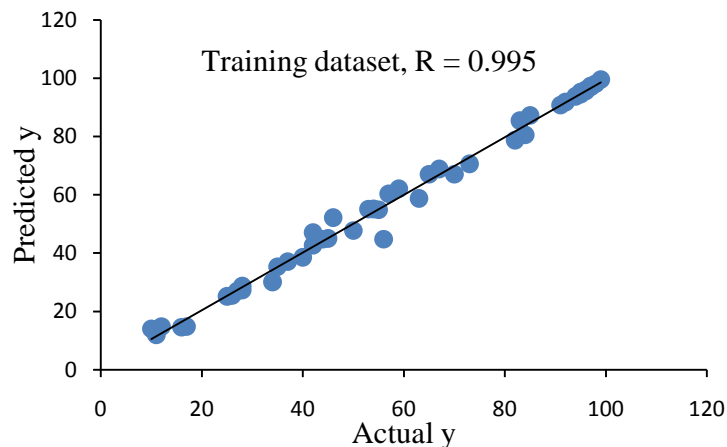


Figure 3. Performance of training dataset for MARS

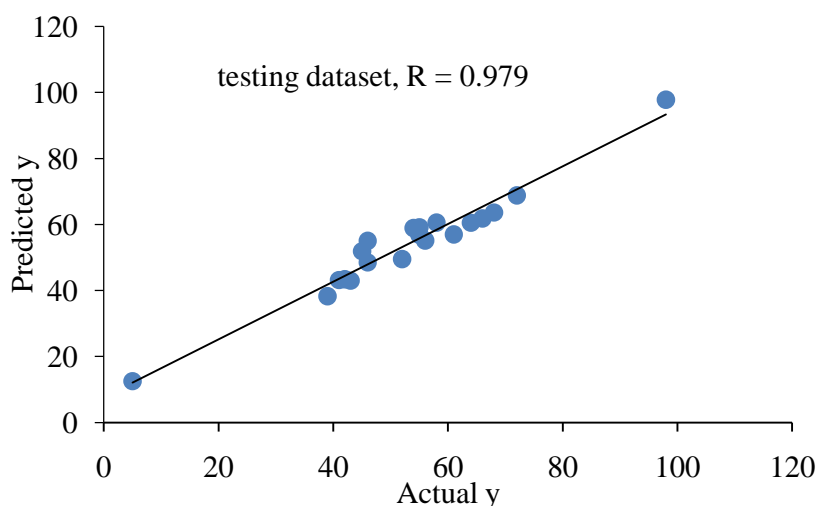


Figure 4. Performance of testing dataset for MARS

A comparative study also has been carried out between GP and MARS models to prove the accuracy. To compare the performances of GP and MARS models the following mathematical evaluation criteria has been adopted. The equations used are given below from (6-10).

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n (y_a - y_p)^2} \tag{6}$$

$$NS = 1 - \frac{\sum_{t=1}^n (y_a - y_p)^2}{\sum_{t=1}^n (y_a - y_{amean})^2} \tag{7}$$

$$WMAPE = \frac{\sum_{t=1}^n \left| \frac{y_a - y_p}{y_a} \right| * y_a}{\sum_{t=1}^n y_a} \tag{8}$$

$$VAF = \left( 1 - \frac{var(y_a - y_p)}{var(y_a)} \right) * 100 \tag{9}$$

$$R^2 = \frac{\sum_{t=1}^n (y_a - y_{amean})^2 - \sum_{t=1}^n (y_a - y_p)^2}{\sum_{t=1}^n (y_a - y_{amean})^2} \tag{10}$$

where, ‘RMSE’ represents root mean square error, ‘NS’ is Nash-Sutcliffe coefficient, ‘VAF’ is variance account factor, ‘WMAPE’ is weighted mean absolute percentage error, ‘R<sup>2</sup>’ is maximum determination coefficient, y<sub>a</sub> and y<sub>p</sub> are actual and predicted output, ‘n’ is number of data points.

The above equations are applied on the results of both genetic programming and MARS models. The below Table 2 represents the ‘testing’ performance of both the models to the mathematical equations.

Table 2: Performances criteria

	RMSE	NS	VAF	WMAPE	R <sup>2</sup>	R
GP	3.161	0.966	96.906	0.046	0.966	0.985
MARS	4.078	0.944	94.806	0.062	0.944	0.977

The models with less RMSE and high correlation coefficient (R) are considered as robust one has minimum error in the output. Fig. 2 shows the higher R<sup>2</sup> and closed to the training models. It means the GP models are performed well over the MARS models. The results of statistical methods have also proved that GP performance is better than the MARS in terms of less RMSE and high R<sup>2</sup> value. Hence, it is clear that GP outperforms over MARS model. The comparison of RMSE values of the proposed models and also with ANN is shown in the Fig. 5.

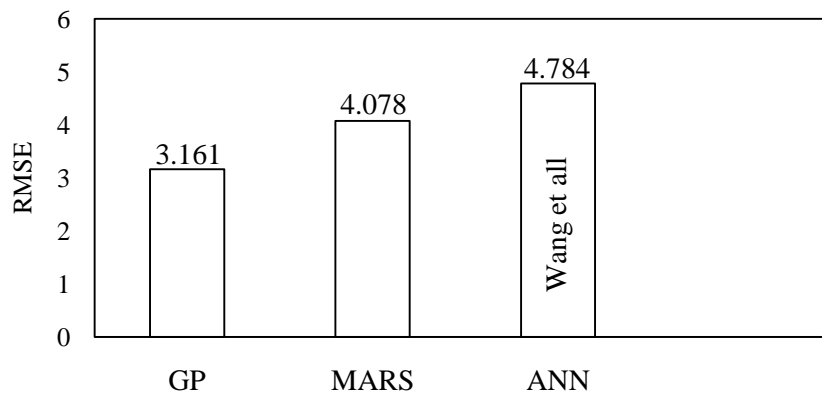


Figure 5. Comparison of RMSE for GP, MARS and ANN models

## 5. CONCLUSION

The objective of this paper was to apply genetic programming (GP) and multivariate adaptive regression splines (MARS) for prediction of bridge risks, and compare their performance with statistical methods. In GP model the robust model was generated at population size of 700 and generations of 300. At this particular models, the coefficient of determination values are 0.996 and 0.985 for training and testing models respectively. The MARS models were good at total 20 BFs and the correlation coefficient values are 0.995 and 0.979 for training and testing models respectively. Hence, it is observed that both the proposed methods are effective in developing the robust models. But the results demonstrate that GP performs is marginally better than MARS model. The statistical evaluation criteria results are also supporting the GP models with less RMSE value and high R<sup>2</sup> value over the MARS models. So it is established that the expression developed by the GP models showed good agreement between the input variables and predicted output. Hence, the users are suggested to use the developed GP formula in prediction of bridge risk score directly.



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