FORM FINDING FOR RECTILINEAR ORTHOGONAL BUILDINGS THROUGH CHARGED SYSTEM SEARCH ALGORITHM

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ABSTRACT

Preliminary layout design of buildings has a substantial effect on the ultimate design of structural components and accordingly influences the construction cost. Exploring structurally efficient forms and shapes during the conceptual design stage of a project can also facilitate the optimum integrated design of buildings. This paper presents an automated method of determining column layout design of rectilinear orthogonal building frames using Charged System Search (CSS) algorithm. The layout design problem is presented as a combinatorial optimization problem named multi-dimensional knapsack problem by setting some constraints to the problem, where the minimum cost and maximum plan regularity are the objectives. The efficiency and robustness of CSS to solve the combinatorial optimization problem are demonstrated through a numerical design problem. The results of the algorithm are compared to those of an ant colony algorithm in order to validate the solution.

Keywords: preliminary layout; building design; column layout; orthogonal plan; charged system search; knapsack problem.

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1. INTRODUCTION

The effect of preliminary layout design of buildings, as the starting point of a comprehensive design procedure, on final outcomes of the phase of detailed design is significant. Optimization in the phase of detailed design without considering the effect of the preliminary layout design will not lead to a globally optimum solution. In a comprehensive structural optimization procedure consequently, the influence of preliminary layout design on the objectives needs to be considered in parallel to the other architectural and structural requirements.

In order to achieve effective early stage decision-making on preliminary design, designers must be better positioned to select the optimum forms among various concepts that satisfy architectural and structural requirements. To that end, a designer needs to be aware of the effects of shape variables on the performance of buildings. The early stage of design process therefore, requires tools that provide a practical mechanism to compare the performance of alternative designs.

Given that the number of feasible options for preliminary stages of design is extremely high, unless a practical approach is adopted, it is almost impossible to ever gain anything more than a satisfying solution for layout design. Yet, in the literature, the layout optimization of structures is mostly confined to the weight minimization of structures, and other objectives such as cost are rarely taken into account in this phase of design [1-2]. Recently, in order to assist designers with decision making in the highly convoluted search space of conceptual design, different techniques have been developed and discussed in the literature [3, 4].

In practice, the structural optimum geometric design consists of selecting the best combination of a finite number of structural elements and available parameters. It gives the optimum design procedure a combinatorial nature. Discrete structural optimization methods, such as combinatorial optimization, in which a set of variables are selected from a pre-defined list, are becoming effective tools in this area [5-8]. Combinatorial optimization problems with the help of graph theory methods are also demonstrated to be strong methods of simulating geometry and topology-related optimization problem such as conceptual design optimization of buildings [9-10]. To solve these kinds of complex problems, which are mostly classic NP-hard problems [11-13], a variety of modern heuristic algorithms such as genetic algorithms [14-20], simulated annealing [21], ant algorithms [22-23] and particle swarm optimization [24] have been developed. These techniques have gained significant attention as a result of their great potential for solving complex engineering problems.

In layout optimization of buildings, it is of great importance how to efficiently represent the structure’s layout so that it could account for all the relevant objectives. In a recent study by Sharafi et al. [25], the conceptual optimum design of rectilinear, orthogonal building frames was represented as a bi-objective knapsack problem, considering building plan, the number and the size of unsupported spans variables. For this purpose, an ant colony optimization (ACO) algorithm was performed to solve the multi-objective optimization problem.

This paper presents a preliminary layout design of orthogonal buildings through recently...
developed charged system search (CSS) algorithm which aims to determine the optimum forms for conceptual design in terms of cost and plan regularity [26]. The CSS algorithm, inspired by the Coulomb and Gauss laws known from electrostatics and the laws of motion from Newtonian mechanics, has successfully been applied to various engineering optimization problems and is classified as a multi-agent approach [26, 27]. Using the robustness of CSS, a numerical bi-objective problem of rectilinear building frames is formulated and the results of the optimization problem are compared with those of an ACO algorithm.

### 2. MULTIDIMENSIONAL KNAPSACK PROBLEM

Given a set of items, each with a weight and a benefit, the knapsack problem is to determine the number of items to include in a collection so that the total weight is less than or equal to a given limit and the total benefit is as large as possible. Each item consumes a known amount of resources and contributes a known benefit. Items are to be selected in a way that maximizes the total benefit without exceeding a given amount of resources. An important generalization of the knapsack problem is the multidimensional knapsack problem (MKP), in which multiple resource constraints are considered. Given a set of \( n \) items and a set of \( m \) knapsacks with a limited capacity of \( a_j \) each (\( m \leq n \)), the multiple knapsack problem is the problem of selecting \( m \) subsets of items so that the total profit of the selected items is a maximum. The MPK problem can be mathematically formulated as follows:

\[
\begin{align*}
\text{max } & \ f = \sum_{j=1}^{m} \sum_{i=1}^{n} b_{ij} y_{ij} \\
\text{subject to } & \ \sum_{i=1}^{n} r_{ij} y_{ij} \leq a_j \quad j \in \{1, 2, ..., m\} \\
y_{ij} & = \begin{cases} 
1 & \text{if Item } i \text{ is chosen for knapsack } j \\
0 & \text{if Item } i \text{ is not chosen for knapsack } j
\end{cases}
\end{align*}
\]

where \( b_{ij} \) and \( r_{ij} \) respectively indicate the profit and the weight of Item \( i \) when selected for Knapsack \( j \).

### 3. STATEMENT OF THE PROBLEM OF LAYOUT DESIGN

The column layout of a building having a rectilinear, orthogonal floor plan can be described by an arrangement of sub-rectangles that completely cover the entire floor area, as illustrated in Fig. 1. In order to design a building having a rectilinear floor plan of total area \( A \), every arrangement of sub-rectangles that provides the total area \( A \) and satisfies the constraints is a potential solution. Each potential solution represents both a column layout design and a rectilinear pattern for the floor plan. Selecting the optimum set of sub-rectangles and their optimum arrangement, among the sets of potential solutions, results in the optimum layout design of the building.
The column layout is therefore, characterized as creating any feasible sub-rectangles arrangement of various lengths and widths in a way that thoroughly cover the floor plan of area $A$ and meet the constraints. Considering the schematic floor plan in Fig. 1, whose length and width are $L_x$ and $L_y$ ($L_x \geq L_y$), the objective is a representation of column layout configuration which leads to a maximum profit for the corresponding combinatorial optimization problem. In order to apply a knapsack problem to the layout optimization problem, some types of new restraints should be defined. This procedure has comprehensively been described in [25] and will be summarized here.

Consider the multi-story building frame shown in Fig. 1, corresponding to the rectilinear floor plan there. The problem is: what is the rectilinear, orthogonal column layout that results in the maximum profit? The profit can be defined as the minimum cost, minimum displacements, minimum energy consumption, or any other objectives or a combination of them. A rectilinear, orthogonal floor pattern consists of a number of sub-rectangles having various lengths and widths that meet the geometric constraints, such as maximum and minimum unsupported spans. Each sub-rectangle in the plan identifies the floor (or slab) and the location of the columns on its corners. The optimization problem is to find a set of sub-rectangles with a total area $A$ and an appropriate configuration that maximizes the profit (objective function) and meet the constraints.

Primarily, the sub-rectangles are situated on the floor plan with their four sides parallel to the $x$ and $y$ axis while there is no overlap for any of them. The edges shared by the adjacent rectangles have the same length which will lead to a grid pattern placement of columns. Also, the columns are considered to be positioned on the rectangles’ corners and not along any of the edges.

![Figure 1. Subdivided Rectangular Plan](image)

The total area of a rectilinear shape is discretized to a set of rectangles of the allowable dimensions bounded by the minimum and maximum permissible spans. Spans normally vary discretely by a certain size, called accuracy in the present work. This problem can be categorized as a simple knapsack problem aiming to determine the number and lengths of spans in each direction.

Now, given $m = NS_{y_{\text{max}}}$ knapsacks with a limited capacity of $a_j = L_x$ and a limited
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The problem of layout optimization becomes one of finding $m$ subsets of sub-rectangles that has the maximum profit, as illustrated in Fig. 2. In other words, the longer side of the enclosing rectangle ($L_y$) is divided into $m = N_{S_{y,\text{max}}}$ segments; each treated as a knapsack with a limited capacity and a limited number of rooms to be covered by a set of sub-rectangles. Each sub-rectangle is treated as an item that is selected from the set of sub-rectangles, whose dimensions meet the geometric constraints. Since there may be multiple sub-rectangles having the same dimensions in a knapsack, and given that some constraints are applied on both weights (capacity) and numbers, the problem is treated as a multi-constraint multiple knapsack problem.

![Figure 2. The knapsack model for column layout optimization](image)

4. BI-OBJECTIVE OPTIMIZATION PROBLEM

A multi-objective optimization problem is the problem of finding a vector of decision variables which satisfies constraints and optimizes the vector function whose elements represent some objective functions, which are usually in conflict with each other. In the phase of preliminary design, objectives like costs, rigidity for buildings under lateral loads or plan regularity could be considered along with the architectural requirements. In this study, two objectives are considered to formulate a bi-objective optimization problem: cost and plan regularity. It should be noted that for specific instances where the minimum required regularity, such as the maximum allowable eccentricity between the center of mass and rigidity is already predetermined, plan regularity is considered as a state variable and the maximum allowable eccentricity is considered as a behavioral constraint. In such cases, the problem is reduced to a single objective problem.

In a multi-story reinforced concrete (RC) framed building with a rectilinear floor plan, as shown in Fig. 1, the building is formed of a set of beams, columns and slabs. It is assumed that all the stories have the same floor plan. Any variation in the building's plan layout will
lead to a variation in members’ cross-sectional design, and consequently variation in cost and regularity, which are defined as the profit. Sharafi et al. [28] formulated a cost optimization problem for multi-story RC framed buildings, where if the ultimate strengths of the $i^{th}$ beam section in shear, positive and negative flexures are $V_{u_i}^{(b)}$, $M_{u_i}^{+(b)}$ and $M_{u_i}^{-(b)}$; the axial compression capacity, bending moment capacity and shear capacity of the $i^{th}$ column section are denoted by $N_{u_i}^{(c)}$, $M_{u_i}^{(c)}$ and $V_{u_i}^{(c)}$ and bending moment capacity of the $i^{th}$ slab section is denoted $M_{u_i}^{(s)}$ respectively, then the variation in cost of the beam, column and slab sections can be represented by Equations (10) through (12).

$$
\Delta C_i^{(b)} = c_1 \Delta M_{u_i}^{+(b)} + c_2 \Delta M_{u_i}^{-(b)} + c_3 \Delta V_{u_i}^{(b)}
$$

$$
\Delta C_i^{(c)} = c_4 \Delta N_{u_i}^{(c)} + c_5 \Delta M_{u_i}^{(c)} + c_6 \Delta V_{u_i}^{(c)}
$$

$$
\Delta C_i^{(s)} = c_7 \Delta M_{u_i}^{(s)}
$$

in which the cost function $c_1$ through $c_7$ has been defined by Sharafi et al [28, 29].

In rather tall buildings with large weight-to-base size ratio, the horizontal movement of the floors under lateral loads is considerable. Buildings with simple geometry in plan perform well during strong lateral loads. In the elastic analysis of structures, an important criterion for building regularity in plan is the approximate symmetry of lateral stiffness and mass with respect to two orthogonal horizontal axes. Irregularity in plan is often measured in terms of the static eccentricity between the center of mass (CM) of a floor and the center of rigidity (CR) of a story that can be obtained from Equations (13) and (14), in which $X$ and $Y$ are the coordinates of the element, $E I_x$ and $E I_y$ denote the section rigidities for bending within a vertical plane parallel to the horizontal directions $x$ or $y$ respectively, and $M$ is the mass of each element.

$$
X_{CR} = \frac{\sum X E I_y}{\sum E I_y}
$$

$$
Y_{CR} = \frac{\sum Y E I_x}{\sum E I_x}
$$

$$
X_{CM} = \frac{\sum X M}{\sum M}
$$

$$
Y_{CM} = \frac{\sum Y M}{\sum M}
$$

In order to achieve the maximum plan regularity, the eccentricity between center of mass and the center of rigidity must be minimized. Minimizing this eccentricity is the second objective of the optimization problem in this study.
5. CHARGED SYSTEM SEARCH ALGORITHM

In the CSS [30, 31], each solution candidate \( X_i \), containing a number of decision variables (i.e. \( X_i = \{x_{ij}\} \)) is considered as a charged particle (CP), which is affected by the electrical fields of the other CPs. The quantity of the resultant force is determined by using the electrostatics laws, and the amount of the movement is determined using the Newtonian mechanics laws. A CP with good results must exert a stronger force than the bad one, so the amount of the charge will be defined considering the objective function value. CSS consists of a number of CPs while each has a charge of magnitude \( q_i \), defined considering the quality of its solution as follows:

\[
q_i = \frac{\text{fit}(i) - \text{fitworst}}{\text{fitbest} - \text{fitworst}} \quad i = 1, 2, \ldots, N \tag{15}
\]

where \( \text{fitbest} \) and \( \text{fitworst} \) are the best and worst fitness of all the particles; \( \text{fit}(i) \) serves as the agent \( i \) fitness, and \( N \) is the total number CPs. The separation distance \( r_{ij} \) between two charged particles is expressed as follows:

\[
r_{ij} = \frac{\|X_i - X_j\|}{\|(X_i + X_j)/2 - X_{best}\| + \epsilon} \tag{16}
\]

where \( X_i \) and \( X_j \) are the positions of the \( i \)th and \( j \)th CPs, \( X_{best} \) is the best current CP location and \( \epsilon \) is a small positive number to avoid singularities. In the search space, the initial positions of CPs are determined randomly while their initial velocities are considered to be zero. In order to enhance the exploitation ability of the algorithm, the electric forces between any two CPs are assumed to be attractive. However, in noisy domain search space, where achieving a complete search before result convergence is essential, adding repelling force to the algorithm may boost the performance. Good CPs can attract the other agents and bad CPs repel the others, according to the following probability function:

\[
C_{ij} \propto \text{rank}(CP_j)^{c_{ij}} \begin{cases} 
0 \leq c_{ij} \leq +1 & \text{if the CP is above average} \\
-1 \leq c_{ij} < 0 & \text{if the CP is below average}
\end{cases} \tag{17}
\]

where \( C_{ij} \) is a coefficient determining the type and the degree of each CP impact on the other agents with regards to their fitness and regardless of their charges. The value of the resultant electrical force affecting a CP is measured using Equation 18.

\[
F_j = q_j \sum_{i, i \neq j} \left( \frac{q_i}{a^2} r_{ij} l_1 + \frac{q_i}{r_{ij}} l_2 \right) c_{ij} (X_i - X_j) \quad \begin{cases} 
 j = 1, 2, \ldots, N \\
 i_1 = 1, i_2 = 0 \Rightarrow r_{ij} < a \\
 i_1 = 0, i_2 = 1 \Rightarrow r_{ij} \geq a
\end{cases} \tag{18}
\]
where $F_j$ is the resultant force affecting the $j$th CP, as shown in Fig. 3. In this method, each CP is recognized as a charged sphere with radius $a$ having a uniform volume charge density. In this research, “$a$” is set to unity.

![Diagram showing the resultant force on a CP]

Figure 3. Determining the resultant electrical force acting on a CP [26]

The new position and velocity of each CP is determined according to the above-mentioned governing laws of physics as:

$$X_{j,new} = rand_1 \cdot k_a \cdot \frac{F_j}{m_j} \cdot \Delta t^2 + rand_2 \cdot k_v \cdot V_{j,old} \cdot \Delta t + X_{j,old}$$  \hspace{1cm} (19)

$$V_{j,new} = \frac{X_{j,new} - X_{j,old}}{\Delta t}$$  \hspace{1cm} (20)

where $k_a$ is the acceleration coefficient; $k_v$ is the velocity coefficient to control the previous velocity effects; and $rand_1$ and $rand_2$ are two random digits uniformly distributed in the range of (0,1). $m_j$ is the mass of the CPs which is equal to $q_j$ in this study. $\Delta t$ is the time step and is set to one. Fig. 4 depicts a CP movement to its new position following this rule.

Charged memory (CM) is utilized to save a number of so far solutions. Here, the CM size is assumed to be $m$. The vectors stored in the CM can affect the CPs that may result in an increase in the computational cost. Therefore it is assumed that the same number of the worst particles cannot attract others. The agents violating the limits of the variables are regenerated using the harmony search-based handling approach as described in [32]. The minimum number of iteration is considered as the terminating criterion.

The general form of the geometric layout optimization problem for RC rectilinear buildings represented by a bi-objective knapsack problem can be solved using a CSS algorithm. First, the mechanical properties of the materials and their relative costs, the building’s layout constraints such as the maximum dimensions, the required area, the maximum and minimum spans and the desired accuracy of the solution are defined. Having
the required information, the number of items \( n \), the number of knapsacks \( m \) and the required capacity \( a \) are determined. The number of CPs, i.e. candidate agents, is set to \( C=am \). Using a larger number of CPs may result in more accurate results, but it significantly increases the computational time. On the other hand, using a smaller number of them may lead to undesirable results. The considered number of CPs is capable of keeping the balance at a moderate level.

![Figure 4. The movement of a CP to the new position [26]](image)

Each probable selection of item \( i \) for knapsack \( j \) is considered to be a potential solution which is called an agent. In CSS these agents are regarded as CPs. In fact, each solution candidate \( X_i \) containing a number of decision variables \( x_{ij} \), is considered to be a charged particle and each \( x_{ij} \) presents the item selected for the knapsack. Thus a solution candidate \( X_i \) which represents the position of CP \( i \), contains \( n \) arrays \( x_{ij} (j=1,2,...,n) \) which stand for the selected items. Therefore, the amount of each position element \( x_{ij} \) of solution candidates is proportional to the probability of item \( i \) to be selected for knapsack \( j \). Then, candidate solution \( X_i \) which are located in their positions are presented. The initial velocity for all CPs is considered to be zero.

The magnitude of charge for each CP is calculated using Eq. (15). For this purpose, the objective functions, i.e. cost and eccentricity, for each agent are calculated. After the objective functions are calculated, they are put in order and the best and the worst ones are saved. This will help the algorithm to judge better in the next steps. Then the magnitude of charge for each CP is obtained through the Eq. (15). The separation distances between CPs are calculated. Having the \( X_i \) for all the CPs, the separation distance between them are calculated using the Eq. (16), and the type and the degree of influence of each CP on the other agents are determined. The value of the resultant electrical force affecting a CP is determined using the Eq. (18). New position and velocity of each CP is determined, which shows the new probability of each item to be selected for each knapsack. Then, the agents violating the limits of the variables are regenerated using the harmony search-based handling.
approach. The best so far solutions are saved and the maximum number of iterations is considered as the terminating criterion.

6. NUMERICAL EXAMPLE

An eight-storey East-West oriented reinforced concrete frame, located in Wollongong, Australia, as shown in Fig. 5(a), is considered. The aim is to determine an optimum rectilinear plan layout for the building under wind loading. The total required area is 3200 m² ±2% equivalent to 400 m² ±2% for each story. The dimensions of the rectangular building envelope, which is oriented in the principal directions x and y, are considered to be $L_x=24$ m and $L_y=30$ m. The permissible spans are defined within the bounds of $l_{max}=6.0$ m and $l_{max}=3.0$ m respectively. The height of the building is 24.0 m (3.0 m height for each story). The live load is 5.0 kN/m² and the dead load, excluding the self-weight of members, is 2.5 kN/m². The average unit price for concrete is assumed to be 55 units/m³, and 3900 units/m³ for reinforcing steel. The average unit price for the formwork is 20 units/m². The other design parameters used in this example are the characteristic tensile strength of steel reinforcement $f_y=460$ N/mm², the characteristic strength of concrete $f'_c=35$ N/mm², and the cover of the steel bars 25 mm. The effect of the wind loading is simulated by two uniformly distributed windward and leeward impulse loads as shown in Fig. 5.

![Windward and leeward impulse loads](image)

Figure 5. An eight-story building under uniformly distributed wind loads and corresponding knapsacks

Considering the building envelope dimensions and the limitations on the span lengths, 10 knapsacks of capacity 8 each, were formed to represent the problem ($m=10$, $n=8$). The accuracy of $\varepsilon=0.20$ m, results in 16 possible options for rectangles’ dimensions, which is equivalent to a set of 256 items (rectangles). Therefore, the problem of an optimum rectilinear shape of the building’s layout turns to the multiple knapsack problem of selecting eight rectangles from the set of items, such that the total area of rectangles does not exceed 400 m² ±2% and the profit is a maximum. As a primary design for proposed CSS algorithm, a rectangular layout of 4 identical spans in each direction is considered. The initial solution...
and the knapsacks corresponding to the problem and the primary solution are displayed in Fig. 5(b). It should be noted that every initial design based on a preliminary judgment of the designer and/or using approximate designs, which meet the design standard requirements, can be used as the initial design and as the starting point of the optimization process. The cost per unit of area for the primary design, which can also be used for validating the solutions obtained from the algorithm, is equal to 787 units. The plan irregularity, which is measured in terms of the static eccentricity between the CM of a floor and the CR of a story, equals zero. However, in order to obtain a real number for plan irregularity benefit, and considering the construction errors, a minimum irregularity of 0.05 m is considered for the plans.

The optimum preliminary layouts resulting from the CSS algorithm are shown in Table 1. This example is solved through an Ant Colony Algorithm in [25]. The results of CSS algorithm and those of ACO are compared in Table 1 for the three best solutions.

Table 1: Results of CSS and ACO algorithms

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<th>Optimum Rectilinear Shapes</th>
<th>Spans (m)</th>
<th>Objectives Values</th>
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7. CONCLUSIONS

In a comprehensive structural optimization procedure, the influence of conceptual design on the objectives needs to be considered in parallel to the architectural requirements, as any optimization in the phase of detailed design without considering the effect of the conceptual design will not lead to a globally optimum solution. Conceptual design optimization of a rectilinear building frame can be treated as a combinatorial optimization, where architectural, structural, and other constraints are represented by a multi-objective knapsack problem.

This study employs CSS algorithm for preliminary layout optimization of rectilinear orthogonal frames. The aim is to achieve an optimum plan layout of rectilinear shape for frames, by minimizing cost and plan irregularity. The numerical example demonstrates the robustness of the approach and shows that the methodology with the help of the proposed CSS algorithm can be easily carried out to simplify the computer-aided preliminary layout design of rectilinear frames. Results of CSS algorithm closely agree with those of obtained from a previously developed ACO algorithm.

REFERENCES