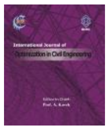
INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING
Int. J. Optim. Civil Eng., 2017; 7(2):231-240



OPTIMAL DESIGN OF SHELLS WITH STEP-FUNCTION DISTRIBUTION OF THICKNESS

L. Stupishin[*, †], K. Nikitin, A. Kolesnikov and F. Altuhov
Urban, road Construction and Structural Mechanics Department, South-West State University, Kursk, Russian Federation

ABSTRACT

The paper is concerned with a methodology of optimal design of shells of minimum weight with strength, stability and strain constraints. Stress and strain state of the shell is determined by Galerkin method in the mixed finite element formulation within the geometrically nonlinear theory. The analysis of the effectiveness of different optimization algorithms to solve the set problem is given. The results of solving test problems are presented.

Keywords: optimization; variable thickness; ribbed shells; nonlinearity; Galerkin method; mixed finite element.

Received: 20 August 2016; Accepted: 14 October 2016

1. INTRODUCTION

A methodology of optimal design of shells with step-function distribution of thickness, e.g., ribbed shells, shells with a discrete inner layer, etc. is suggested in the paper. If solving these problems directly, then the number of design parameters for the step-variable shell thickness may be sufficiently large. Therefore, these problems are usually solved in two stages [1].

At the first stage the shell is considered at global level as structurally-orthotropic. By solving the optimization problem at this stage as the design parameters are considered general dimensional parameters of shell (e.g. a shell shape parameters) and its reduced stiffness parameters (e.g., reduced thickness).

At the second stage the problem of optimization is considered at local level. At this stage, detailed dimensional parameters of shell are considered as design parameters. These are the parameters that describe the location and size of the ribs, the reinforcement parameters, etc.

[*]Corresponding author: Urban, road Construction and Structural Mechanics Department, South-West State University, Kursk, Russian Federation
[†]E-mail address: lusgsh@yandex.ru (L. Stupishin)

The solution to this problem is based on the results of the first stage calculations.

A methodology of solving problem of the first stage for ribbed shells working in the geometrically nonlinear deformation stage is presented in this paper.

Stress-strain state of such shells was determined using the Galerkin method [2] in mixed finite element formulation [3, 4]. This method makes it possible to refine solution, if necessary in a particular area by reducing the finite element mesh. Using the Galerkin method in mixed finite element formulation allows to do without the construction procedure of the functional of problem and to solve equations of problem in the form in which they are written. The authors were obtained in an explicit form of the matrix and vectors for the finite element shell. Their advantage is that the numerical integration is not used, which improves the accuracy of calculations. Another of the advantages of using the mixed formulation (as indicated by, e.g., Streng [3]) - reduction of the condition number of the matrix of the finite element, in comparison with the classical method of finite elements in displacements, which also positively effect on the accuracy of the calculation results. The methodology for determining the stress-strain state is described in detail in [4-6].

2. DESCRIPTION OF THE METHODOLOGY

The problem of optimal design of the shell was considered as the problem of shell volume minimization with strength, stability and displacement constraints. Such parameters as shell shape parameters, thickness distribution, etc. can be selected as the design variables.

Constraints on stability, displacement, and strength are written as follows:

$$p - p_{c\kappa}(\bar{x}) \geq 0\,,\; w_{\max}(\bar{x}) - w_u \leq 0\,,\; \sigma_{\max}(\bar{x}) - R \leq 0\,, \tag{1}$$

where p is the load imposed on the shell; $p_{c\kappa}(\bar{x})$ is the critical load; $w_{\max}(\bar{x})$ is the maximum deflection of the shell; w_u is the ultimate permissible deflection; $\sigma_{\max}(\bar{x})$ is the maximum equivalent stresses in the shell; R is the ultimate allowable stress; $\bar{x}$ is the vector of the design variables.

The values $p_{c\kappa}(\bar{x}), w_{\max}(\bar{x})$ and $\sigma_{\max}(\bar{x})$ are determined at each step of the optimization algorithm using the Galerkin method in mixed finite element formulation [4 - 6].

The problem of optimal design with constraints is reduced to the problem of unconstrained minimization using the penalty functions method. Penalty function takes the form proposed by Mischke [7]. As a result, the original optimization problem is replaced by a series of M successive minimization problems with the growing value of the penalty parameter r_k :

$$\bar{V}_k(\bar{x}) = V(\bar{x}) + r_k d \sum_{i=1}^{N} b_i \cdot (\bar{x}_c - \bar{x})^2 + s \to \min\,, \tag{2}$$

where, $V(\bar{x})$ is the objective function of the problem with constraints; $\bar{x}_c$ is the vector of design parameters values corresponding to any point in design space, where all constraints

$g_i(\bar{x})$ hold (in the domain of feasible solutions); N is the number of constraints; d is the scale multiplier to reduce functions $V(\bar{x})$ and $g_i(\bar{x})$ to same order; b_i is the parameter equal to 1 if the constraint $g_i(\bar{x})$ is satisfied and equal to 0 if it is not satisfied; r_k is the penalty parameter for the k^{th} minimization problem (iteration) selected from the following sequence: 1, 100, 10,000, 1,000,000; s is the parameter used in some problems to define a barrier on the boundary of feasible solutions (s=0 within the domain of feasible solutions and s=const>0 outside this domain); $k=1.M$ – number of iteration; M is the number of iterations.

Problem (1) is solved by classical algorithms of unconstrained minimization (methods of steepest descent, grid search, conjugate gradients, etc.).

3. TEST PROBLEMS

To check the validity of the results obtained using the developed method and to compare the effectiveness of different algorithms of unconstrained optimization, three test problems of optimal design of a shallow axisymmetric shell of revolution were solved. Shell dimensions: the shell base radius a=3 [m], rising height f_0=0.45 [m]; shell material characteristics: $E=2.1\cdot10^{11}$[Pa], ν=0.3, R=210 [MPa]; ultimate strain w_u=0.01 [m]. The shell is closed at the top. The shell is fixed rigidly to the support. The load is distributed uniformly on the shell; its intensity is p =10 [kPa]. The shell was divided evenly into 20 finite elements along the generatrix. The position the of the finite elements nodes along the shell generatrix was calculated by formula: $f(\rho)=f_0\rho^Z$, where Z is the shell shape parameter, ρ is the radial coordinate.

The first test problem is to determine the shape and thickness of the shell of minimum weight (design variables are Z and h) under the assumption that the shell thickness is the same over its entire surface.

The second test problem is to determine the optimum thickness distribution for a minimal weight shell (design variables are h and $h1$) under the assumption that the thickness varies linearly from the centre to the edges and does not change in the circumferential direction.

In all cases, the objective function is as follows:

$$V=2\pi\sum_{i=1}^{N}\int_{-1}^{1}h_i(\rho)\rho\sqrt{1+\left(\frac{df(\rho)}{d\rho}\right)^2}\,d\xi\,. \tag{3}$$

Shell stresses are defined by the following formula:

$$\sigma=\sqrt{\sigma_\alpha{}^2+\sigma_\beta{}^2-\sigma_\alpha\sigma_\beta+3\tau^2}\,, \tag{4}$$

where $\sigma_\alpha=\frac{N_\alpha}{h}+\frac{6M_\alpha}{h^2}$, $\sigma_\beta=\frac{N_\beta}{h}+\frac{6M_\beta}{h^2}$, $\tau=\frac{S}{h}+\frac{6H}{h^2}$, $h=h(\alpha,\beta)$ is the piecewise continuous function of curvilinear coordinates [8], which describes a ribbed shell.

4. CHECKING THE RESULTS

The diagrams of changes of objective and penalty functions depending on the node parameter values were plotted (Figs. 1-5 for the 1st problem, Fig. 6 for the 2nd problem and Figs. 7, 8 for the 3rd problem).

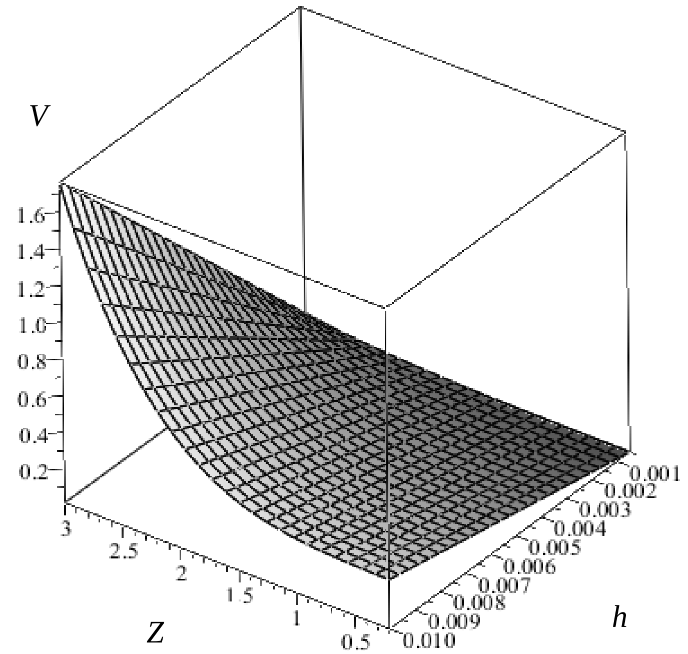


Figure 1. Dependence of shell volume change on parameters *Z* and *h*

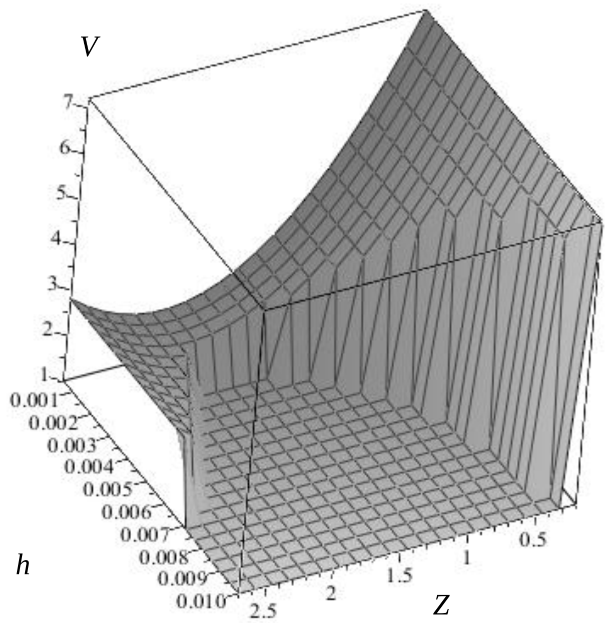


Figure 2. Dependence of penalty function for stability constraints on parameters *Z* and *h*

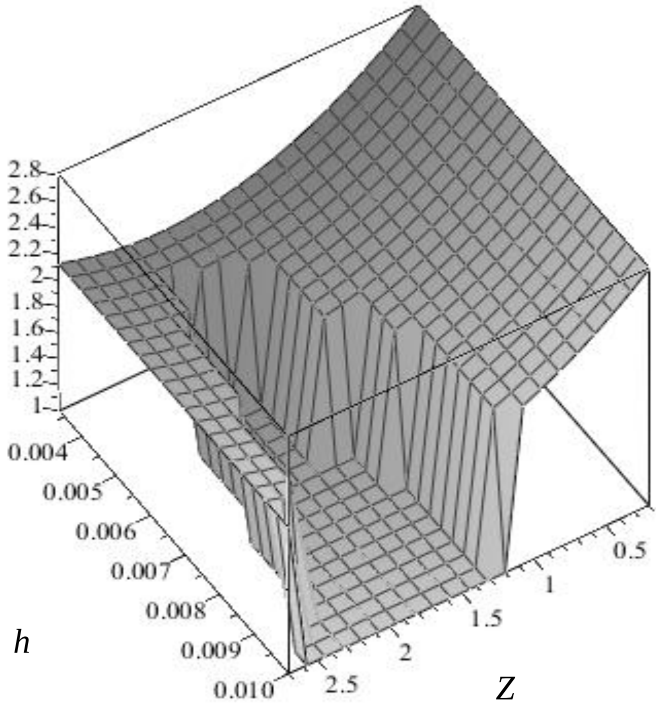


Figure 3. Dependence of penalty function for strength constraints on parameters Z и h

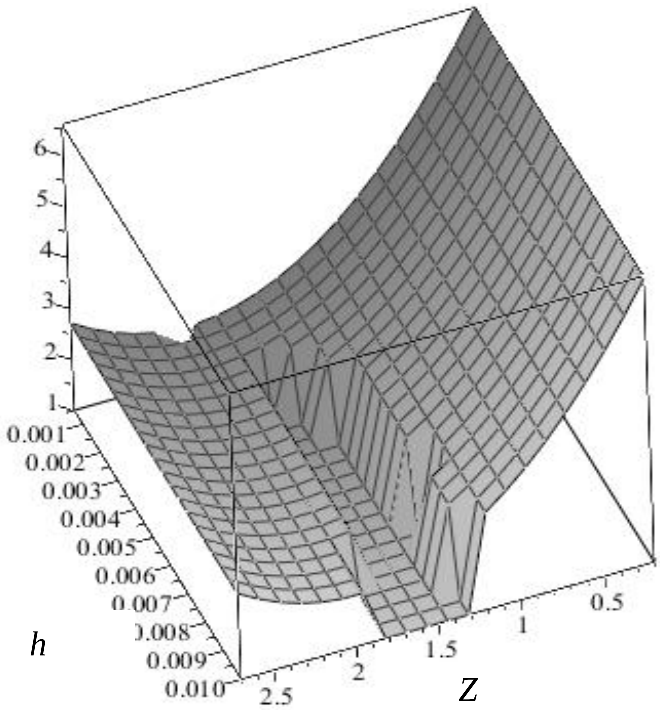


Figure 4. Dependence of penalty function for strain constraints on parameters Z и h (wu =0.0001)

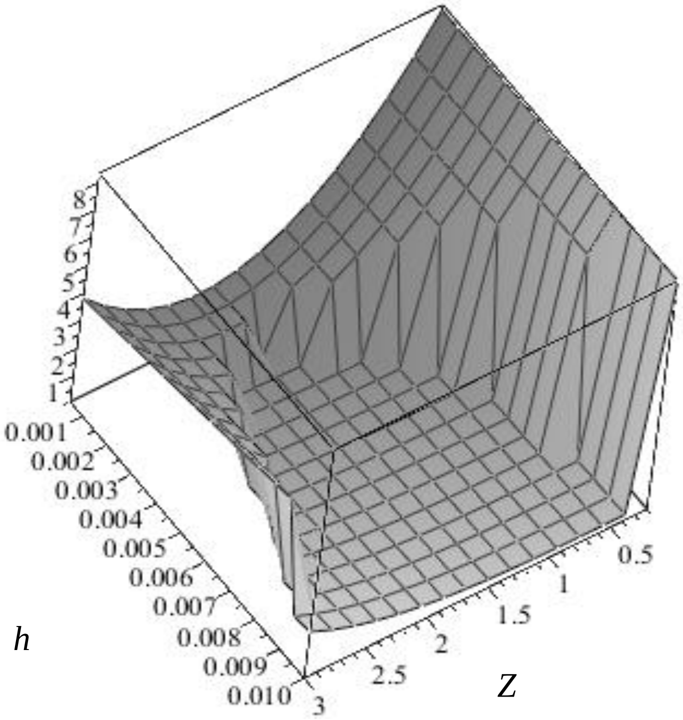


Figure 5. Dependence of problem 1 objective function on parameters Z and h (when s≠0)

For 1st problem the boundary of permissible values with stability, strain and strength constraints has the form of a parabola with the vertex at the values Z = 1.5 – 2 (Figs. 1-5). Taking into account the descending direction of the volume function, it is possible to determine visually the position of the objective function minimum on the boundary of the domain of permissible values when Z = 1 - 1.7 (see Fig. 5).

The results of the 1st optimization problem solution using the developed method are the values of design variables Z = 1.65, h = 0.00173 and the objective function value of 0.0075. The reliability of these results is confirmed by visual analysis (Fig. 5).

For 2nd problem the boundary of permissible values has the form close to a straight line (Fig. 6). Taking into account the descending direction of the volume function the minimum of the objective function is on the boundary of the search domain with the minimum values of $h1$ (Fig. 6).

When studying the shape of the boundary of the permissible values domain with regards to stability, minor irregularities associated with the inaccuracy of critical load determination algorithm were found. They can cause difficulties when determining an optimal solution. Therefore, to obtain reliable results search for an optimal solution should be started from several points, and then the best solution should be selected.

The results of solving 2nd problem using the developed optimization method are the values of design variables $h1$ = 0.0005, h = 0.00756 when the value of the objective function is 1.07. This solution is consistent with a shell whose thickness decreases towards the centre. The validity of these results can be confirmed by the visual analysis of the diagram of the objective function changes (Fig. 6).

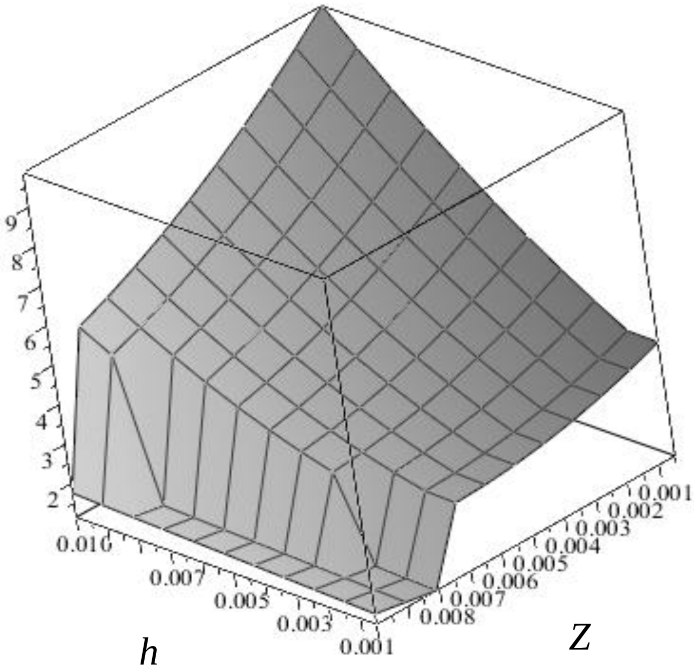


Figure 6. Dependence of problem 2 objective function on parameters *h1* and *h* (when *s*≠0)

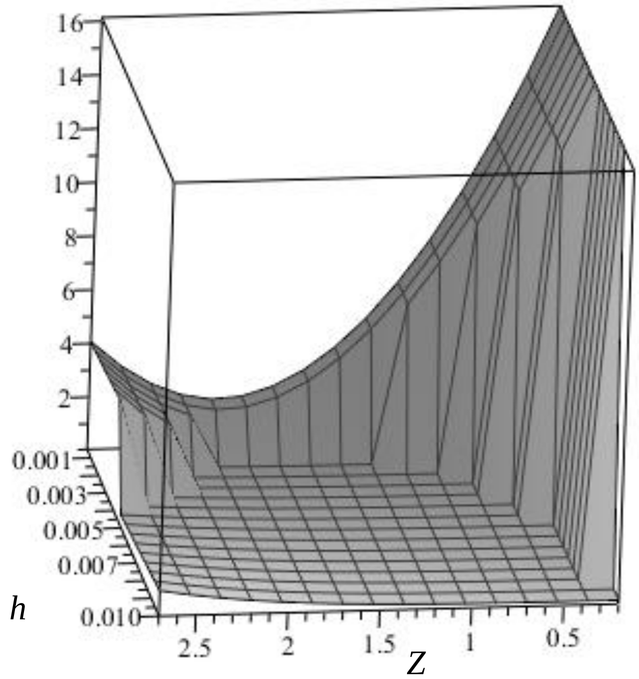


Figure 7. Dependence of problem 3 objective function on parameters *Z* and *h*,
when *h1*=0.001

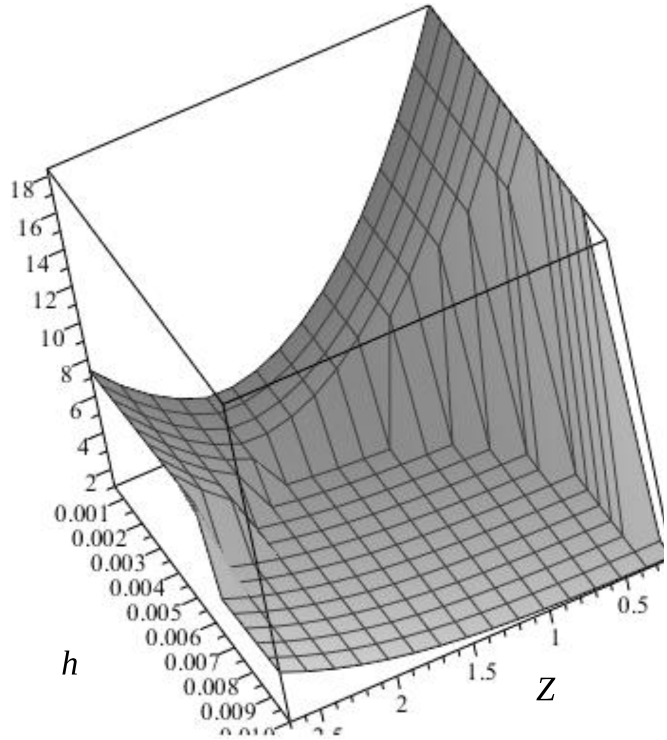


Figure 8. Dependence of problem 3 objective function on parameters *Z* and *h*, when *h1*=0.1

The objective function for the 3rd problem cannot be represented in one diagram, as it was for the other two problems due to larger number of design variables. For the visual analysis was a series of diagrams for different values of the variable h1. Two drawings from this series are shown in Figs. 7 and 8.

The result of solving 3rd problem are the values of the design variable h=0.0024, h1=0.001, Z=1.65 when the value of the objective function is 0.007. The validity of these results can be confirmed by the visual analysis of the series of diagrams.

5. ANALYSIS OF EFFICIENCY OF OPTIMIZATION ALGORITHMS

A comparative analysis of the effectiveness of different optimization algorithms used to first solve problem have been carried out ([9-11]). Such methods as the grid (multidimensional continuous search) with the exclusion of domains, the steepest descent method with a variable step, a method based on a combination of the gradient method and the random search method [12], [13] have been considered

Grid search method with the exclusion of domains was used with the objective function (1) without iterative refinement with the constant penalty parameter r_k =1,000,000 and parameter s, which is equal to the mean of function $V(\bar{x})$ beyond the boundaries of feasible solutions. Search domain is divided into 4 parts along each of the coordinates.

Grid search method with the exclusion of domains has proved to be a reliable one for searching an optimal solution; it also requires a small number of calculations (384 calculations of the objective function were done in the problem considered) when searching is carried put with a small number of design variables. However, with an increase in the number of design variables a significant increase in the number of calculations of the objective function should be expected which hinders the use of this method in such cases.

Application of the steepest descent method with a variable step and parameter s=0 in (2) (objective function without a penalty barrier on the boundary of feasible solutions), has shown that in most cases it does not lead to finding an optimal solution. This is due to a ravine surface shape of the objective function (1). The algorithm usually gets caught in an endless loop on the boundary of permissible values domain.

The method, based on a combination of gradient and random search, proved to be a reliable one when solving problems when in expression (1) s=0. To obtain sufficiently accurate results 412 calculations of the objective function were done. This value is higher than the one, which was done for the grid search method with the exclusion of domains. However, with an increase in the number of design variables, in contrast to the grid search method, there is no significant increase in the number of calculations of the objective function. Therefore, it can be recommended for solving problems with the use of the developed optimal design method with a large number of design variables.

6. CONCLUSIONS

The developed method by the example of the decision of test problems has demonstrated the possibility and accuracy of finding the optimal solution. The selected algorithm based on a combination of gradient and random search, allows to effectively search the optimal solution for a large number of design parameters.

This work was supported by the grant of the President of the Russian Federation № MK-9203.2016.8

REFERENCES

1. New directions in optimum structural design / ed. by E. Atrek, R.H. Gallagher, K.M. Ragsdell, O.C. Zienkiewicz, J. Wiley and Sons, Chichester, New York, Brisbane, Toronto, Singapore, 1984.
2. C.A.J. Fletcher Computational Galerkin methods, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1984.
3. Strang G, Fix G. *An Analysis of the Finite Element Method*, Wellesley-Cambridge Press, 2008.
4. Stupishin LU, Nikitin KE. Mixed finite element for geometrically nonlinear orthotropic shallow shells of revolution, *Appl Mech Mater* 2014; **919-921:** 1299-1302.
5. Stupishin LU, Nikitin KE. Mixed finite element of geometrically nonlinear shallow shells of revolution, *Appl Mech Mater* 2014; **501-504**: 514-7.

6. Stupishin LU, Nikitin KE. Numerical research methodology of free oscillations of geometrically nonlinear shell using the mixed finite element method, *Appl Mech Mater* 2014, **988**: 338-41.
7. Misvhke CR. *An Introduction to Computer-Aided Design*, Englewood Cliffs, N.J, Prentice-Hall, 1968.
8. Karpov VV, Ignatev OV, Salnikov AU. Nonlinear mathematical model of deformation of shells of varying thickness and algorithms for their research, ASV Publ, Moscow, 2002.
9. Kaveh A. *Optimal Structural Analysis*, John Wiley (RSP), 2nd edition, Somerset, UK, 2006.
10. Andreev V, Avershyev A. Stationary problems of moisture-elasticity for inhomogeneous thick-walled shells, *Adv Mater Res* 2013; **671-674**: 571-5.
11. Floudas CA, Pardalos PM. *Encyclopedia of Optimization*, Second Edition, Springer Publ, 2009.
12. Stupishin LU, Kolesnikov AG. Reconstruction of shallow shells for increase bearing capacities and operating characteristics, *Appl Mech Mater* 2014; **580-583**: 3062-5.
13. Skokov VA. Some computational experience of solving nonlinear programming problems, *Mathemat Meth Study Econom Probl* 1977; **7**: 51-48.