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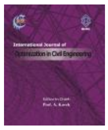

ACTIVE EARTH PRESSURE IN COHESIVE-FRICTIONAL SOILS USING FEM AND OPTIMIZATION

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ABSTRACT

Calculation of lateral earth pressure on retaining walls is one of the main issues in geotechnics. The upper and lower bound theorems of plasticity are used to analyze the stability of geotechnical structures include bearing capacity of foundations, lateral earth pressure on retaining walls and factor of safety of slopes. In this paper formulation of finite element limit analysis is introduced to determine plastic limit load in the perfect plastic materials. Elements with linear strain rates, which are used in the formulation, cause to eliminate the necessity of velocity discontinuities between the elements. Using non-linear programming based on second order cone programming (SOCP), which has good conformity with cone yield functions such as Mohr-Coulomb and Drucker-Prager, is another important advantage that remove the problem of using ordinary linear programming algorithms for yield functions such as divergent in the apexes. Finally, the optimization problem will be solved by mathematical method. The proposed method is used for calculating active earth pressure on retaining walls in cohesive-frictional soils. According to results of analysis, active earth force on retaining wall is decreased by increasing soil cohesion (C),wall inclination (α), friction angle between backfill and wall (δ) and friction angle of soil ($\emptyset$) wherein the force is increased by increasing surcharge on the backfill(q) and the backfill slope (β) . Mathematical method is used for obtaining accurate results in this research, however, heuristic methods are used when approximate solutions are sufficient.

Keywords: upper bound; finite element method (F.E.M); optimization; non-linear programming; active earth pressure ; cohesive-frictional soils.

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1. INTRODUCTION

The determination of earth pressure is an important issue in geotechnical engineering. Various methods have been developed for this purpose in the past years. Most of these methods can be classified into the following four categories:

(a) the limit equilibrium method, (b) the slip line method, (c) the limit analysis method, and (d) the finite element method[1]. Chen [2] once gave comprehensive introductions about the first three methods. Coulomb first studied the earth pressure problem by using the limit equilibrium method. He assumed an inclined plane failure surface in the soil behind the retaining wall and solved for the earth pressure by considering the equilibrium of the triangular wedge limited by the wall and the inclined failure surface. His hypothesis about the failure plane is relatively well verified for frictional soil in active state; it is not the case for cohesive soils. Mononobe and Matsuo [3] and Okabe [4] modified the Coulomb wedge method and proposed the well-known Mononobe–Okabe analysis of seismic lateral earth pressures, where the seismic effects are considered using a quasi-static inertia force whose magnitude is computed on the basis of the seismic coefficient concept. Janbu [5] proposed a generalized procedure for the limit equilibrium method with slices, to calculate earth pressure considering the inter-slice forces as well as their action points. Various limit equilibrium methods with slices for slope stability can be embraced in a general limit equilibrium method (GLE) with slices as proposed by Fredlund *et al*. [6]. Rahardjo and Fredlund [7] explored the possibilities of extending the general limit equilibrium method with slices to earth pressure problems considering the general direction of the inter-slice forces. The limit equilibrium method is the most commonly used method for estimating the earth pressure because of its simplicity.

The slip line method assumes that the sliding soil is in a plastic state completely, and the slip line field and the stress field are derived from the differential equilibrium equations, considering the failure condition and the boundary conditions. The classical Rankine's earth pressure theory can be regarded as a special case of the slip line method. Ko tter [8] was the first to derive the limit equilibrium equations along the slip line of cohesionless soil (Ko tter equations). Sokolovskii [9,10] developed the slip line method for rigid-plastic media with Mohr-Coulomb(M.C.) failure criterion under the plane strain hypothesis, and solved successfully a series of important plastic problems. In addition, he made a great contribution to the development of slip line method, which was then widely used to solve the earth pressure problems [11,12,13]. Serrano and Olalla developed a complete procedure for the slip line method with a nonlinear M.C. criterion [14]. They developed the theoretical hypothesis and the procedure to obtain the ultimate bearing capacity for weightless rock masses, based on the Hoek and Brown nonlinear failure criterion.

The limit analysis method assumes associated plastic flow for the soil, and by using the upper and lower bounds theory, it calculates the upper and lower limits of the ultimate load. Chen [1] once gave a comprehensive and profound description of the limit analysis method, and described how slope stability, bearing capacity and earth pressure problems can be formulated in a unified theoretical background of upper bound and lower bound analysis. Baker and Frydman [15] discussed the effects of nonlinearity of a general failure criterion on the upper bound solution. Zhang and Chen [16] proposed an effective solution procedure, called the inverse method, suitable for slope stability problems with a general nonlinear failure criterion. The computational procedure for the determination of the critical slip surface is usually complex [16]. Collins et al. [17] calculated the slope stability factor, for a nonlinear failure criterion, using an optimization method with the linear stability factors given by Chen [2]. However, in their calculation the nonlinear criterion is linearized by using the tangential of the nonlinear failure envelope. Lancellotta [18] provided an analytical solution for the active earth pressure coefficients, based on the lower bound theorem of plasticity. Yang et al. [19] proposed a generalized tangential technique to formulate the bearing capacity problem as a classical optimization problem in upper bound analysis. This technique is powerful for translating a nonlinear failure criterion to a linear one. Yang [20] employed this method to calculate active earth pressures by assuming the active earth pressure acting at the lower third-point of the wall for rotational failure mode. He investigated also the influence of the nonlinear failure parameters on the active earth pressure.

In numerical upper bound limit analysis, a key aspect is the efficient solution of the arising optimization problem. Linear programming (LP) has been used for a long time, but the need to replace the (invariably nonlinear) yield function by numerous linear inequality constraints means that the computational cost becomes prohibitive for large problems. During the last twenty years there has been considerable progress in the application of nonlinear programming (NLP), which allows the yield function to be treated in its native form [21].

The main difficulty in obtaining strict upper bounds via the finite element method is that the flow rule constraint can only be enforced at a finite number of points, yet it is required to hold throughout the discretized structure. Satisfying this requirement becomes especially difficult in the case of cohesive-frictional materials, where the only obvious solution is to use constant strain elements [21]. By using linear strain elements which is used in this paper, the difficulty is removed.

In this paper general formulation of finite element limit analysis is introduced to determine plastic limit load in the perfect plastic materials. Elements with linear strain rates, which are used in the formulation, cause to eliminate the necessity of velocity discontinuities between the elements. This is important, because in the problems with velocity discontinuities the accuracy of method is completely dependent on the position of velocity discontinuities; and inappropriate mesh will reduce the accuracy of method. Using non-linear programming based on second order cone programming (SOCP), which has good conformity with cone yield functions such as Mohr-Coulomb and Drucker-Prager, is another important advantage that remove the problem of using ordinary linear programming algorithms for yield functions such as divergent in the apexes. The proposed method is used for analysis and design of retaining walls in cohesive-frictional soils based on numerical upper bound limit analysis formulation .This is a novel method for calculating strict active earth pressure in this condition and sensitivity of calculated force is evaluated against backfill surcharge (q), soil cohesion (C), wall inclination (α), soil friction angle ($\emptyset$), backfill slope (β) and friction angle between backfill and wall (δ). Mathematical method is used for obtaining accurate results in this research, however, heuristic methods are used when approximate solutions are sufficient.

2. OPTIMIZATION PROBLEM

The optimization problem as shown in equation (1), has the form:

$$\begin{gathered} \min \quad C^T x \\ \text{S.t.} \quad Ax = b \\ x^T = [x_1^T \dots x_N^T] \end{gathered} \tag{1}$$

where $\mathbf{A} \in R^{m\times n}$,$\boldsymbol{b} \in R^m$,$\boldsymbol{c}$,$\boldsymbol{x} \in R^n$ and the sets C_i are second- order (or quadratic) cones of the form $\{\boldsymbol{x} \in R^d : \|x_{2:d}\| \le x_1, x_1 \ge 0\}$. For convenience we will employ the notation $(z, x) \in C$ as shorthand for $\|x\| \le z, z \ge 0$.

Consider a plane strain structure made of rigid- perfectly plastic material obeying the Mohr- Coulomb yield criterion (cohesion c , friction angle $\emptyset$). The structure is discretized into 6-node triangular finite elements with straight sides. For these elements it can be shown that if the (associated) flow rule is enforced at the three vertices, it will automatically be satisfied throughout the whole element. Upon applying the kinematic theorem, the arising SOCP optimization problem in the dual form ,according to (2) ,reads:

$$\begin{aligned} & Max\ \beta \\ s.t.\ & \sum_{i=1}^{NP}(A_i^e / 3)B_{m,i}\sigma_{m,i} + \sum_{i=1}^{NP}(A_i^e / 3)B_{d,i}s_i^{red} - \beta q = q_0 \\ & y_i + \sigma_{m,i}sin\emptyset = c.cos\emptyset \qquad \forall i \in \{1, \dots, NP\} \\ & y_i, S_i^{red} \in C_i \qquad \forall i \in \{1, \dots, NP\} \end{aligned} \tag{2}$$

where $\sigma_{m.i}$ and $s_i^{red} = [s_{xx.i} \;\; s_{xy,i}]^{\mathrm{T}}$ are the mean and deviatoric stresses at the ith flow rule point, A_i^e is the area of the element to which the ith flow rule point belongs, q and q_0 are load vectors, and the y_i are auxiliary variables. The matrices $B_{m,i}$ and $B_{d,i}$ incorporate the mean and deviatoric strain-displacement relations, according to (3): [21].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \boldsymbol{B}_{m,i}\boldsymbol{u} \text{ and } \left[\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \quad \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]^T = \boldsymbol{B}_{d,i}\boldsymbol{u} \quad at\ (x, y) = (x_i, y_i) \tag{3}$$

3. ACTIVE EARTH PRESSURE ON RETAINING WALLS

The determination of earth pressure is a very important issue to most geotechnical engineers. The main methods developed so far can be mainly classified into the following four categories: (1) the limit equilibrium method, (2) the slip line method, (3) the limit analysis method, (4) the numerical method.

Kaveh and Farhoudi [22] used Dolphin Echolocation optimization (DEO) as a newly developed meta-heuristic optimization method for design of cantilever retaining walls. They computed active and passive earth pressure according to Coulombs earth pressure theory and achieved better results and higher convergence rate in comparison with other methods.

Kaveh and Khayatazad [23] used Ray optimization method for optimal design of cantilever retaining walls. They utilized a pseudo-dynamic approach for optimal design in seismic condition.

Kaveh and Shakouri Mahmud Abadi [24] introduced a harmony search based algorithms for the optimum cost design of reinforced concrete cantilever retaining walls. They calculated active and passive earth pressure according to Coulombs theory.

Ghanbari and Ahmadabadi [25] proposed a formulation for determination of active earth pressure in inclined walls considering limit equilibrium and horizontal slice methods. Their results have a well accommodation with known methods in vertical walls, however, active earth pressure on inclined walls is smaller in comparison with vertical ones thus designing an inclined retaining wall is not economical by methods developed for vertical walls based on their results.

Vieira [26] proposed a simplified approach to estimate the resultant force should be provided by a retention system for the equilibrium of unstable slopes.

Fattahi [27] used hybrid harmony search (HS) with support vector machine (SVM) for the prediction of slope stability state, in which HS was used to determine the optimized free parameters of the SVM. The results obtained indicate that the SVM-HS model can be used for the prediction of slope stability state for circular failure , therefore , the SVM combined with HS is a powerful tool for modeling some problems involved in civil engineering.

Fattahi [28] introduced a new approach to prediction of earthquake induced displacements of slope (EIDS) using hybrid support vector regression (SVR) with particle swarm optimization (PSO). The results shows that the SVR-PSO model can be used successfully for prediction of earthquake induced displacements of slopes.

In this paper, an effective and accurate method is proposed for analyzing and designing the retaining walls in cohesive-frictional soils based on the above-mentioned upper bound finite element formulation.

Two-dimensional problems of the plane strain, herein exemplified, follow the Mohr-Coulomb criterion. The program has been written in MATLAB, which creates the geometry, formulates optimization, and solves the problem. The interior-point algorithm is used for solving the optimization problem and fmincon solver from the MATLAB optimization toolbox is used, too. In order to verify the method, at first, a benchmark problem is solved and the results are compared using the well-known methods, then the main problem will be solved.

3.1 Active earth pressure on retaining walls in cohesionless soils

In order to verify the program, active earth pressure on the retaining wall in cohesionless soil is calculated using Coulomb limit equilibrium method, the proposed upper bound method and method proposed by Vieira. Finally, the results are compared.

For analyzing the upper bound of the results, a mesh of triangles, composed of 256 triangular six-node elements were compared with the Coulomb limit equilibrium and Vieira proposed method and the results were summarized in Table 1. Clearly, the results of the proposed upper bound are very close to those obtained using the Coulomb and Vieira proposed methods.

Table 1: Active earth force on retaining wall in cohesionless soil (kN/m)

$\alpha(deg.)$	H (m.)	$\beta(deg.)$	Ɣ(kN/m^3)	q(kN/m^2)	C(kN/m^2)	Ø(deg.)	$\delta(deg.)$	Active force based on Coulomb method	Active force based on Proposed upper bound method	Active force based on Vieira proposed method	differences with Coulomb proposed method %	differences with Vieira proposed method %
90°	5	0	18	10	0	20	0	134.83	134.83	134.66	0	0.13
90°	5	0	18	10	0	20	5	127.83	127.97	128.81	0.11	0.7
90°	5	0	18	10	0	20	10	122.85	123.40	124.09	0.45	0.56
90°	5	0	18	10	0	20	15	119.46	120.69	120.75	1.03	0.05
90°	5	0	18	10	0	30	0	91.67	91.67	91.54	0	0.14
90°	5	0	18	10	0	30	10	84.22	85.00	85.69	0.93	0.81
90°	5	0	18	10	0	30	15	82.88	83.28	83.74	0.48	0.55
90°	5	0	18	10	0	30	20	81.76	82.46	82.63	0.86	0.21

3.2 Active earth pressure on retaining wall in cohesive-frictional soils

Proposed upper bound method provides a new solution for calculating the active earth force in all conditions. For evaluating the effect of each parameter on the resultant active force on the retaining wall, all parameters are changed and the results are shown in Figs. 1-5.

In the following subjects all of physical, mechanical and geometrical parameters are defined as follow:

H_1: Height of backfill
α: Wall inclination angle
β: Backfill surface angle
q: Backfill surcharge
C: Cohesion of soil
Φ:Friction angle of soil
δ: Friction angle between soi l and wall
γ: Unit weight of soil

3.2.1 Effect of wall inclination on active earth force

Effect of wall inclination (α) on active earth force is shown in Fig. 1. In this figure other parameters are:

C=0, $\beta = 0$, δ=2φ/3, $\gamma = 18\ kN/m^3$, q=0, H=4m

Active earth force on retaining wall is decreased by increasing wall inclination (α) for a given value of soil friction angle (φ) as shown in Fig. 1.

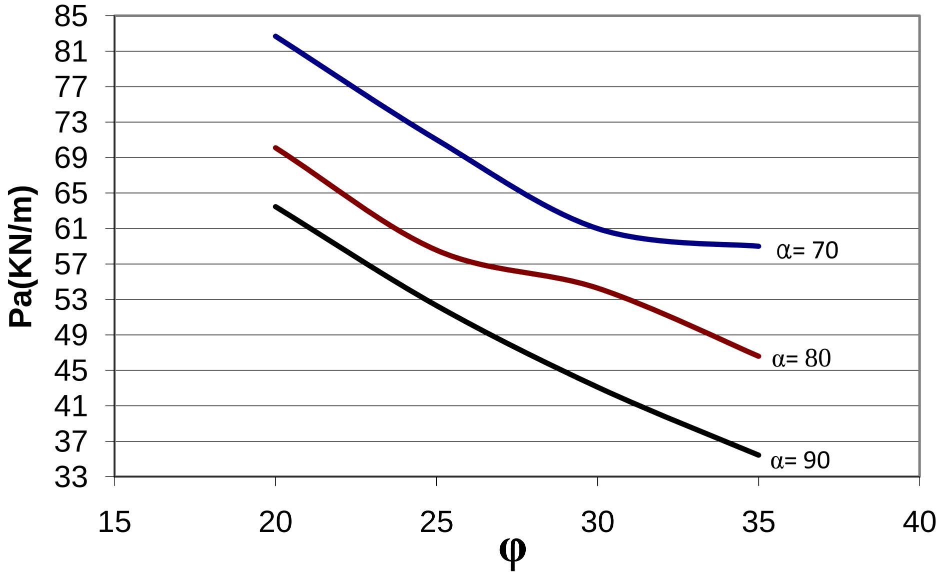

Figure 1. wall inclination (α) effect on active earth force for different friction angle (φ) values

3.2.2 Effect of soil cohesion on active earth force

Effect of soil cohesion (C) is shown in Fig. 2. In this figure other parameters are: $\alpha = 90°$, $\beta = 0$, δ=2φ/3, $\gamma = 18\ kN/m^3$, q=0, H=4m.

It is clear that soil cohesion (C) have a significant effect on active earth force, wherein the force is decreased by increasing the soil cohesion.

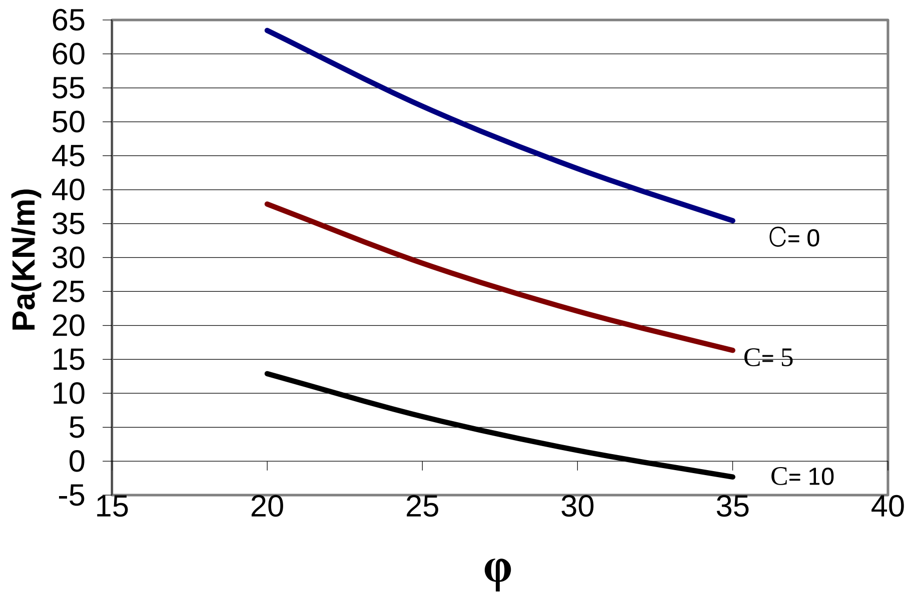

Figure 2. Soil cohesion (C) effect on active earth force for different friction angle (φ) values

3.2.3 Effect of backfill slope (β) on active earth force

Effect of backfill slope (β) is shown in Fig. 3. In this figure other parameters are:

$\alpha = 90°$, $C = 0$, δ=2φ/3, $\gamma = 18\ kN/m^3$, q=0, H=4m

Active earth force on retaining wall is increased by increasing backfill surface slope (β) for a given value of soil friction angle (φ) as shown in Fig. 3.

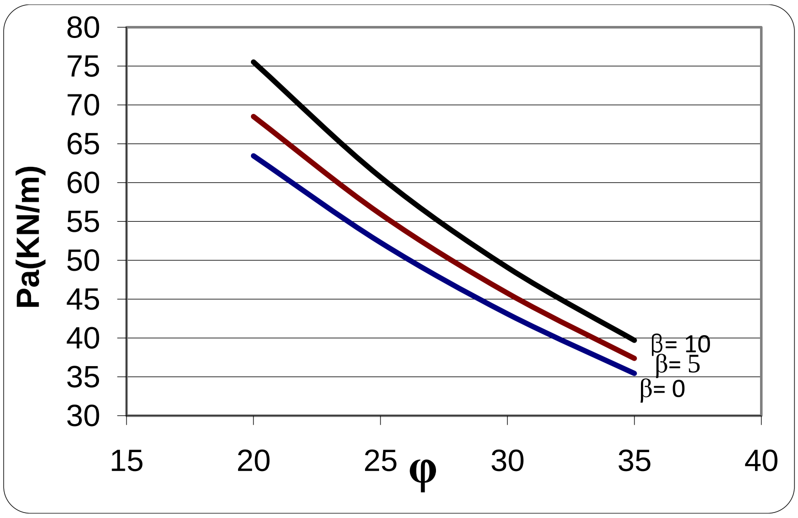

Figure 3. Backfill slope (β) effect on active earth force for different friction angle (φ) values

3.2.4 Effect of backfill surcharge (q) on active earth force

Effect of backfill surcharge (q) is shown in Fig. 4. In this figure other parameters are:

$\alpha = 90°$, $C = 0$, δ=2φ/3, $\gamma = 18\ kN/m^3$, $\beta = 0$, H=4m

It is clear that the backfill surcharge (q) have a significant effect on active earth force, wherein the force is increased by increasing the backfill surcharge (q).

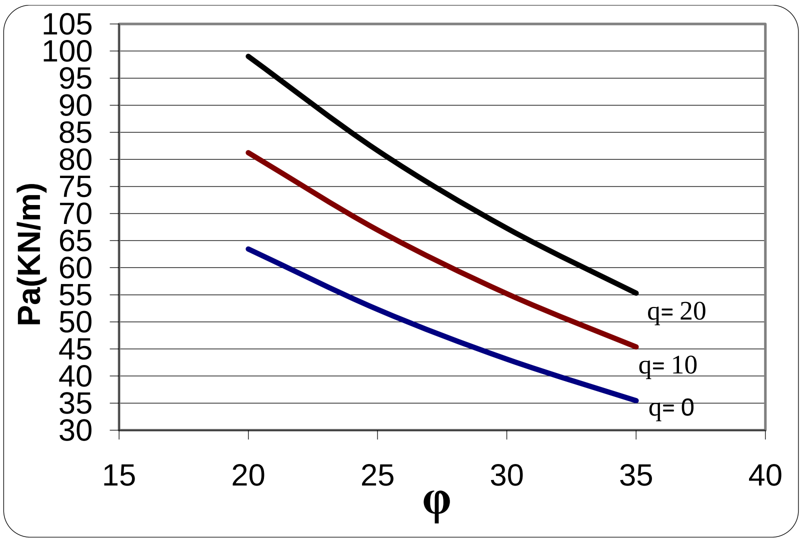

Figure 4. Backfill surcharge (q) effect on active earth force for different friction angle (φ) values

3.2. 5. Effect of friction angle between soil and wall(δ) on active earth force

Effect of friction angle between soil and wall(δ) on active earth force is shown in Fig. 5. In this figure other parameters are:

$\alpha = 90°$, $C = 0$, q=0, $\gamma = 18\ kN/m^3$, $\beta = 0$, H=4m

According to Fig. 5 friction angle between soil and wall (δ) is another parameter affect the active force wherein the force is decreased by increasing δ.

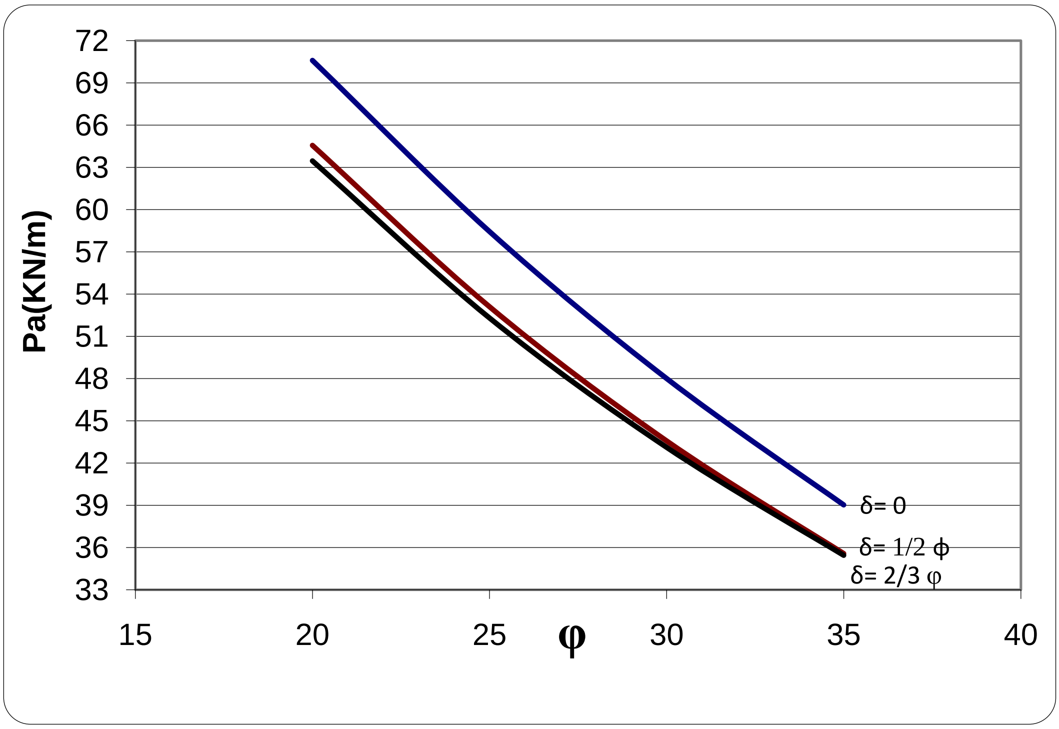

Figure 5. Soil-wall friction angle (δ) effect on active earth force for different friction angle (φ) values

4. DISCUSSION

As shown in Figs. 1-5, the results are summarized as follow:

1. Active earth force on retaining wall is decreased by increasing soil friction angle (φ) in all cases.
2. Active earth force on retaining wall is decreased by increasing wall inclination (α) for a given value of soil friction angle (φ).
3. Cohesion of backfill soil(C) is another parameter affect the active force wherein the force is decreased by increasing C.
4. Active earth force on retaining wall is increased by increasing backfill surface slope (β) for a given value of soil friction angle (φ).
5. Backfill Surcharge(q) acting on the surface have a significant effect on active force wherein the force is increased by increasing q.

6. The friction angle between backfill and wall (δ) is another parameter affect the active force wherein the force is decreased by increasing δ.

5. CONCLUSION

An accurate prediction of collapse load is one of the main challenges in geotechnical engineering problems. In this paper general formulation of finite element limit analysis was introduced to determine plastic limit load in the perfect plastic materials. Elements with linear strain rates, which were used in the formulation, cause to eliminate the necessity of velocity discontinuities between the elements. This is important, because in the problems with velocity discontinuities the accuracy of method is completely dependent on the position of velocity discontinuities; and inappropriate mesh will reduce the accuracy of method. Using non-linear programming based on second order cone programming (SOCP), which has good conformity with cone yield functions such as Mohr-Coulomb and Drucker-Prager, is another important advantage that remove the problem of using ordinary linear programming algorithms for yield functions such as divergent in the apexes. The results obtained from examples demonstrate that the proposed method is highly effective and precise for analyzing the earth pressure in soils.

According to results of analysis, active earth force on retaining wall is decreased by increasing soil cohesion (C), wall inclination (α), friction angle between backfill and wall (δ) and friction angle of soil $(\emptyset)$ wherein the force is increased by increasing surcharge on the backfill (q) and the backfill slope (β).

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