



OPTIMAL DESIGN OF CANTILEVER RETAINING WALL USING DIFFERENTIAL EVOLUTION ALGORITHM

V. Nandha Kumar and C.R. Suribabu^{*,†}

School of Civil Engineering, SASTRA University, Thanjavur- 613401

ABSTRACT

Optimal design of cantilever reinforced concrete retaining wall can lead considerable cost saving if its involvement in hill road formation and railway line formation is significant. A study of weight reduction optimization of reinforced cantilever retaining wall subjected to a sloped backfill using Differential Evolution Algorithm (DEA) is carried out in the present research. The retaining wall carrying a sloped backfill is investigated manually and the problem is solved using the algorithm and results were compared. The Indian Standard design philosophy is followed throughout the research. The design variables, constraint equations were determined and optimized with DEA. The single objective constrained optimization problem deals with seven design variables of cantilever retaining wall in which four design variables constitutes to geometric dimensions and remaining three variables constitutes to the reinforcement steel area. Ten different constraints are considered and each of it deals with ten failure modes of retaining wall. Further, a sensitivity analysis is carried out by varying the parameters namely, height of the stem and thickness of stem at top, both of it being a constant design variable in the normal optimization problem. Results show that about 15% weight reduction is achieved while comparing with manual solution.

Keywords: retaining wall; weight reduction; differential evolution algorithm; structural optimization.

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1. INTRODUCTION

The designers often tend to use the previous experiences and thumb rules for designing any structure. This may cause unnecessary increase in dimensions and other design parameters. In current scenario, optimal design of each and every structure has been given prime importance due to the shortage in materials and increasing effects in

*Corresponding author: School of Civil Engineering, SASTRA University, Thanjavur- 613401

†E-mail address: suribabu@civil.sastra.edu (C.R. Suribabu)

environment in production of materials.

Retaining walls are used for retaining soil in hilly areas along the roadside to prevent landslide and to provide proper drainage to the roads. Among the retaining walls, reinforced cantilever retaining walls are widely in usage for retaining soil of height within 6 m. These retaining walls are constructed over several meter length. Hence, reduction of dimensions for each material length might bring drastic reduction in overall weight of the structure. This weight reduction can be achieved by optimal design of cantilever retaining wall.

Weight reduction optimization of cantilever retaining wall is handled in the present research. In the case of retaining wall, the dimensional reduction is very wide since it spans in all three dimensions and has more scope. Moreover, the steel area reinforcement is spanned in all the three directions; hence, reduction of at least 100 mm^2 might bring a drastic weight reduction in entire structure. These dimensions and steel reinforcement are considered as the key design variables in the optimal design. The optimal design is carried out by the Differential Evolution Algorithm (DEA). DEA has been proved as most the efficient in arriving to the global optimum solution quickly. The mathematical relation between the constraints, random value generation for design variables, manipulation of objective function, and execution of differential evolution algorithm are done in C++ programming with the help of predefined library functions.

2. LITERATURE REVIEW

The optimal design of retaining wall has been experimented by many researchers (Saribas and Erbatur [1], Ceranic et al.[2], Yepes et al.[3], Sivakumar babu, and Basha, [4], Ahmadi and Varae.H, [5], Kaveh and Abadi, [6], Talatahari and Sheikholeslami, [7]) with main focus being weight reduction, therein cost reduction. Sivakumar babu and Basha [4] pointed out that the reduction in the cross sectional area of retaining wall structure through optimization approach brings more economical design. The reduction in cross section obviously brings the reduced volume of concrete and therein cost reduction can be found out. The general three phases considered in the optimum design of any structure are: structural modeling, optimum design modeling, and the optimization algorithm [5]. For the optimum design of retaining wall, one has to study the problem parameters in depth, so as to decide on design parameters, design variables, constraints, and the objective function. Bearing capacity of soil under the toe region and the shear strength of critical section in the toe region are the key design controlling factors (Kaveh, and Soleimani, [8]. Camp and Akin [9] optimized retaining walls using big bang-big crunch algorithm and shown that solution of design is capable of satisfying safety, stability and material constraints. Keveh and Farhoudi [10] proposed Dolphin Echolocation optimization model for optimal design of cantilever retaining walls. The differential evolution has been chosen in order to experiment with all possible design variable combinations since it is significantly faster and robust for solving numerical optimization problems [11]. The computational model of differential evolution has been thoroughly discussed in Suribabu [12] which was kept as a model to code the C++ algorithm. Sivakumar Babu and Bhasha [4] used reliability index to address the

uncertainties in soil, concrete, steel, wall proportions and safety for optimal design of cantilever reinforced concrete retaining wall. Ahmadi and Varae, [5] used particle swarm algorithm to optimize the cantilever reinforced retaining wall and it is indicated that there is 12% reduction in concrete volume and 6% for reinforcement while minimization of cost as objective function. Recently, Manas Ranjan Das et al [13] formulated multiobjective model taking both the cost and factor of safety as trade-off in developing Pareto-optimal set for dimensioning the reinforced cement concrete cantilever retaining wall using elitist non-dominated sorting GA (NSGA-II). The present work considers the same numerical problem to optimize the retaining wall using Differential Evolution with aim of exploring global optimal solution to the problem. The design variable constraints are taken from Bowles [14] book of Foundation Analysis. The Indian standard design of retaining wall from Sushil Kumar's book, "Treasure of R.C.C design" [15] is followed in the research.

3. METHODOLOGY

Identification of design variables, mathematical formulation of constraints and manipulation of objective function are the three main steps involved in optimization of any structural component. This optimum cantilever retaining wall contains seven different design variables and ten failure mode constraints and a single weight reduction objective function. The entire design considers the Rankine's theory of assumptions.

4. DESIGN VARIABLES

The design variables chosen for the formulation are related to the cross-sectional dimensions of the wall and various reinforcing steel areas. These include the following: the first four design variables are related to the geometry of the cross section, and the last three consider various steel areas. The height of the stem and the stem thickness at the top are included in the design parameters. Design parameters are pre-defined at the beginning of the structural optimization process. Other design parameters include some soil properties, loading characteristics. The design variables are plotted in the Fig. 1.

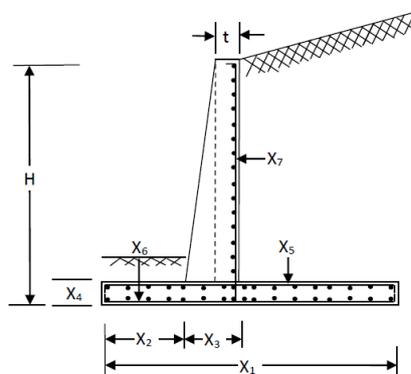


Figure 1 Design variables and parameters of retaining wall

The two parameters in the considered problem are t – Thickness of the stem at top and H – Height of the stem. These two parameters remain constant throughout the problem.

5. CONSTRAINTS

From reference with Bowles [14], the design constraints are classified as geotechnical and structural requirements that summarized in the following sections. These requirements represent the failure modes as a function of the design variables. Retaining wall design is mainly based on the failure modes since the wall design results in low values for different variables, the safety of the wall must be ensured. The failure modes include stability, overturning, sliding, bearing capacity, shear and moment capacity of stem, heel and toe slab. The mathematical relation for design constraints between the design variables for a cantilever retaining wall with a flat backfill is discussed in Sivakumar Babu and Basha [4]. The important mathematical relations required for constructing the constraint equations are discussed below:

6. LOAD CALCULATION

The load calculation forms the base for entire design of the retaining wall. The different loads acting on the retaining wall are shown in Fig. 2. The mathematical relation for finding the loads acting is given as follows:

$$W_1 = \gamma_c(H - t)S \quad (1)$$

$$W_2 = \frac{1}{2}\gamma_c(H - t)b \quad (2)$$

$$W_3 = \gamma_c tB \quad (3)$$

$$W_4 = \frac{1}{2}(H_1 + H_2)\gamma_1 L_h \quad (4)$$

$$P_a = \frac{1}{2}\gamma_1 TH^2 K_a \quad (5)$$

$$\sum V = W_1 + W_2 + W_3 + W_4 + (P_a \sin \alpha) \quad (6)$$

where W_1 - the load exerted on the soil by rectangular section of stem slab,

W_2 - the load exerted on the soil by triangular section of stem slab,

W_3 - the load exerted on the soil by total base slab,

W_4 - the load exerted on the wall by trapezoidal backfill portion, and

P_a - the active earth pressure acting on the wall.

The total load acting or sum of all loads $\sum V$ is given by equation 6.

γ_c - unit weight of concrete,

H - height of the wall on toe side of slab,

t - thickness of the base slab (X_4),

b - width of the triangular section at bottom of stem slab,

B - total width of the base slab (X_1),

γ_1 - unit weight of the backfill soil,
 L_h - length of the heel slab (X_1 - X_2 - X_3),
 TH - Total height of the wall on the heel side including the backfill portion,
 K_a – coefficient of active earth pressure,
 α or β - backfill slope,
 H_1 - stem height,
 H_2 - height of stem inclusive of backfill slope height,
 S - thickness of stem at top.

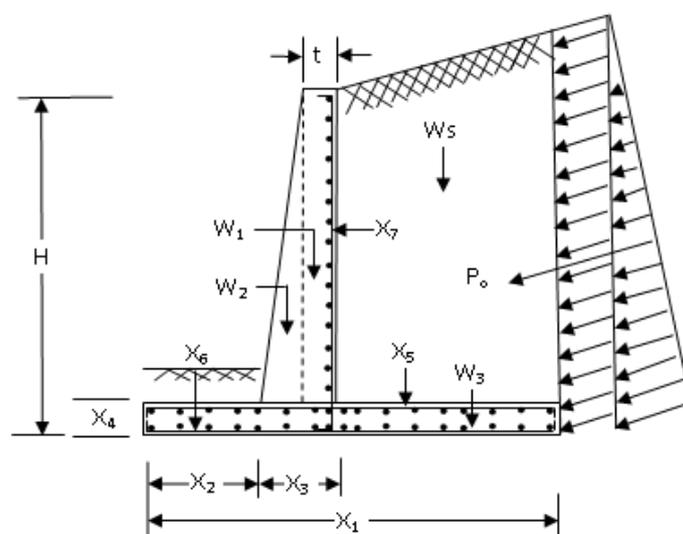


Figure 2. Loads acting on the retaining wall

7. MOMENT CALCULATION

The moment acting on each section due to each load is calculated by multiplying the load with the center of gravity distance. M_1 to M_4 are the moment with respect to the loads W_1 to W_4 , M_5 is the overturning moment about the toe point due to horizontal component of coulomb active earth pressure, and M_6 is the resisting moment about the toe point vertical component of coulomb active earth pressure.

$$M_1 = W_1(L_t + b + \frac{S}{2}) \quad (7)$$

$$M_2 = W_2(L_t + \frac{2}{3}b) \quad (8)$$

$$M_3 = W_3 \frac{B}{2} \quad (9)$$

$$M_4 = W_4((L_t + b + S) + (\frac{L_h}{3}) + (\frac{H_1 + 2H_2}{H_1 + 2H_2})) \quad (10)$$

$$M_5 = P_a \frac{TH}{3} \cos \alpha \quad (11)$$

$$M_6 = P_a B \sin \alpha \quad (12)$$

$$\sum M_r = M_1 + M_2 + M_3 + M_4 + M_5 + M_6 \quad (13)$$

$$\sum M_o = M_5 \quad (14)$$

L_t - Length of the toe slab or toe projection (X_2).

$\sum M_r$ - Total resisting moment,

$\sum M_o$ - total overturning moment.

The constraint equations for sliding and overturning are given as:

$$FS_{ot} = \frac{\sum M_r}{\sum M_o} \quad (15)$$

$$\sum F_r = \sum V \mu \quad (16)$$

$$\sum F_d = P_a \cos \alpha \quad (17)$$

$$FS_{sli} = \frac{\sum F_r}{\sum F_d} \quad (18)$$

FS_{ot} - Factor of safety against overturning,

FS_{sli} - Factor of safety against sliding,

μ - Co-efficient of friction between concrete and soil.

The equations for eccentricity and maximum soil intensity to be acted on soil are given as:

$$e = \frac{B}{2} - \frac{\sum M_r - \sum M_o}{\sum V} \quad (19)$$

$$q_{max} = \left(\frac{\sum V}{B} \right) \left(1 + \frac{6e}{B} \right) \quad (20)$$

e - Eccentricity,

q_{max} - maximum soil intensity to be acted on soil.

The moment and shear constraints are same as that of followed in IS-456, 2000 [16] working stress method. The mathematical relation and values for constraints are tabulated in Table 2.

Table 1: Design Variables of Retaining wall

| Design variable | Definition |
|-----------------|---|
| X_1 | Total base width |
| X_2 | Toe projection |
| X_3 | Stem thickness at the bottom |
| X_4 | Thickness of the base slab |
| X_5 | Horizontal steel area of the heel per unit length of wall |
| X_6 | Horizontal steel area of the toe per unit length of wall |
| X_7 | Steel area of stem per unit length of wall |

Table 2: Constraints of Retaining wall

| Inequality constraint no. | Failure mode | Condition |
|---------------------------|--------------------------|-------------------------------|
| 1 | Overtuning Failure | $FS_{ot} \geq 2$ |
| 2 | Sliding Failure | $FS_{sli} \geq 1.5$ |
| 3 | Eccentricity Failure | $e \leq B/6$ |
| 4 | Bearing capacity Failure | $Q_{max} \leq Q_u$ |
| 5 | Stem Shear Failure | $\tau_{vstem} < \tau_{cstem}$ |
| 6 | Stem Moment Failure | $Mr_{stem} \geq Mu_{stem}$ |
| 7 | Toe Shear Failure | $\tau_{vtoe} < \tau_{cto}$ |
| 8 | Toe Moment Failure | $Mr_{toe} \geq Mu_{toe}$ |
| 9 | Heel Shear Failure | $\tau_{vheel} < \tau_{cheel}$ |
| 10 | Heel Moment Failure | $Mr_{heel} \geq Mu_{heel}$ |

8. OBJECTIVE FUNCTION

The objective function to minimize the weight is defined as:

$$f(x) = W_{st} + W_c \quad (21)$$

where ,

W_{st} is the weight of reinforcement per unit length of the wall and

W_c is the concrete weight used in unit length of wall.

The weight reduction is done for one meter length of wall. The weight reduction optimization indirectly produces a less cost structure. The weight of the retaining wall is expressed in kN.

9. PENALTY ALLOCATION

The objective function is directly calculated with design variables only if it satisfies the ten different failure modes. If the variables violate the constraints, the penalty is counted for each violation and it is imposed on the objective function. For each constraint violation, a penalty of 200 kN is added to the objective function in order to get least preference in subsequent selection process of optimization.

10. ALGORITHM FOR SHEAR STRESS CALCULATION

Calculation of shear stress (τ_c) plays an important role in satisfying the constraint conditions of retaining wall. The shear stress calculation is involved in three constraints; shear capacity

of stem, toe and heel slab. Shear stress calculation deals with the percentage of steel area (pst) and grade of concrete. The entire design procedure of retaining wall is represented as a flow chart Fig. 3.

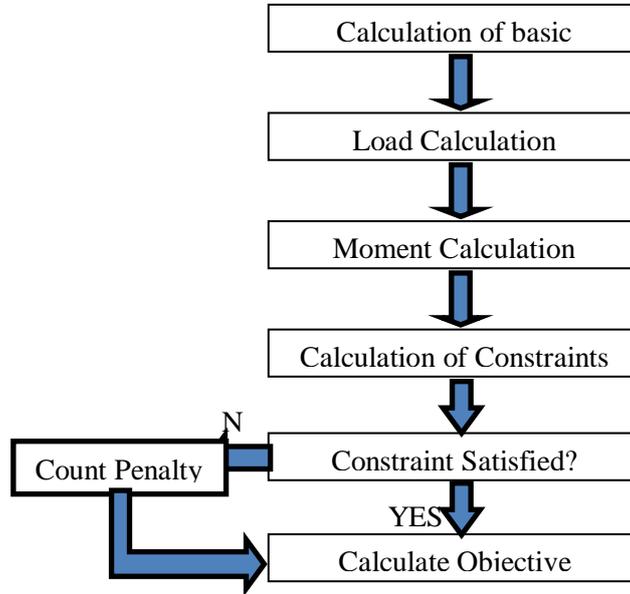


Figure 3. Design of Retaining wall

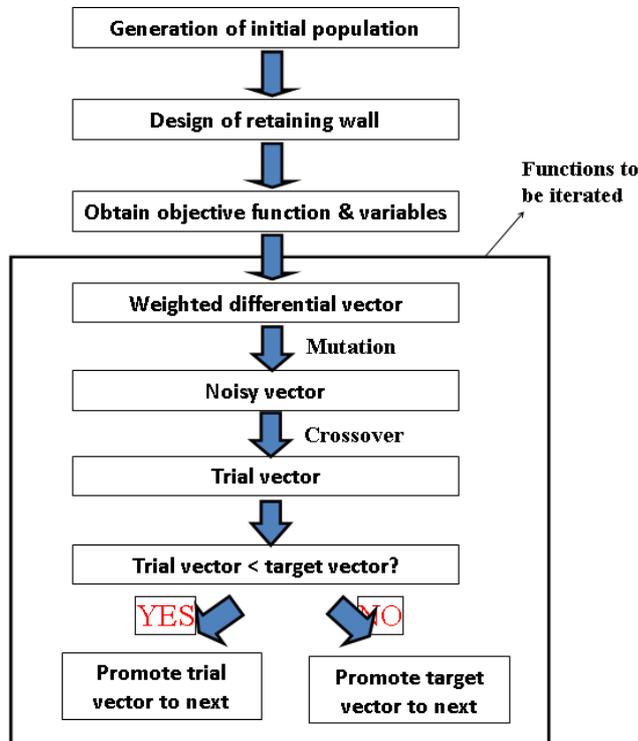


Figure 4. Optimization model based on DEA

The model for optimal design of retaining wall is shown in Fig. 4. The initial population of design variables are generated using random functions for the upper bound and lower bound limits. The initial population is passed into the retaining wall design function. The objective function of the selected design variables inclusive of the penalty is obtained and they are tabulated in a two dimensional array with the design variables. The tabulated values are passed into the DEA function and the optimal values are obtained.

11. DIFFERENTIAL EVOLUTION ALGORITHM

Differential evolution is a randomized population based search algorithm. After generating initial population, the following steps are carried out:

- Three individuals or vectors are selected randomly from population and differences of variables of two vectors are calculated and it is multiplied by weighing factor called mutation (0.8).
- The resulting weighted variables are added with the corresponding variables of third vector and it is called noisy vector.
- A vector called trial vector is created by doing crossover between noisy vector and a target vector selected from the population; the objective function of both are compared; and the vector which is having lesser weight is taken to the next generation.
- The above procedure is repeated many times with different vectors of the population and a new population is obtained for next generation.
- The entire procedure is repeated for a number of generations. The variable values of the best vector of the last generation are taken as the solution to the problem.

11. DESIGN OF RETAINING WALL

11.1 Problem

The cantilever retaining wall design problem is chosen from SushilKumar's [15] "Treasure of R.C.C design". The problem considered is:

Design a R.C.C retaining wall to retain an embankment 4 m high above ground level with given data for cohesionless soil: Backfill slope- 15° , Unit weight of retained soil – 18 kN/m^3 , Angle of repose - 30° , Depth of soil in front of wall – 1 m, Permissible capacity of soil – 160 kN/m^3 , Co-efficient of friction between concrete and soil- 0.62. Adopt M15 grade concrete and mild steel reinforcement. The detailed design parameters and problem is given in Table 3.

11.2 Initial population

According to Bowles [14], the initial population for the design variables is generated by calculating the lower bound and upper bound values. Table 4 shows the upper bound and lower bound values for the given problem. For each design variables, the design parameters are substituted and the upper bound and lower bound values are found. The design parameters are H = Height of stem, t -thickness of stem at top, ρ_{\min} = Minimum steel ratio,

ρ_{\max} = Maximum steel ratio, m = sum of clear concrete cover and half of diameter of reinforcing bars.

Table 3: Design parameters and variables

| Input Parameter | Unit | Symbol | Value |
|--|--------------------|------------------|-------|
| Height of stem | m | H | 4.5 |
| Top thickness of stem | m | T | 0.2 |
| Yield Strength of reinforcing steel | MPa | f_y | 140 |
| Compressive Strength of concrete | MPa | f_c | 15 |
| Concrete Cover | cm | d_c | 7 |
| Max Steel Percentage | - | ρ_{\max} | 0.016 |
| Min Steel Percentage | - | ρ_{\min} | 0.003 |
| Diameter of Bar | cm | d_{bar} | 1.6 |
| Surcharge Load | kPa | q_s | 0 |
| Backfill Slope | degree | α | 15 |
| Internal Friction Angle of Retained Soil | degree | ϕ | 30 |
| Internal Friction Angle of Base Soil | degree | ϕ' | 0 |
| Unit Weight of Retained Soil | kN /m ³ | γ_s | 18 |
| Unit Weight of Base Soil | kN /m ³ | γ_s' | 18.5 |
| Unit Weight of Concrete | kN /m ³ | γ_c | 25 |
| Cohesion of Base Soil | kPa | c | 125 |
| Depth of Soil in Front of Wall | m | D | 1 |
| Permissible Capacity of Soil | kN /m ² | Q_{\max} | 160 |
| Friction Coefficient Between Concrete and Soil | - | μ | 0.62 |
| Unit Weight of Steel | kN /m ³ | w_{st} | 7850 |

Table 4: Upper and lower bound limit for design variables

| VARIABLE | LOWER BOUND | X_{\min} | UPPER BOUND | X_{\max} |
|----------------------------|--|------------|--|------------|
| X_1 (m) | $0.4H(12/11)$ | 1.96 | $(0.7H)/0.9$ | 3.5 |
| X_2 (m) | $[0.4H(12/11)]/3$ | 0.65 | $[(0.7H)/0.9]/3$ | 1.17 |
| X_3 (m) | t | 0.2 | $[H/0.9]/10$ | 0.5 |
| X_4 (m) | $[H(12/11)]/12$ | 0.41 | $[H/0.9]/10$ | 0.5 |
| X_5 (mm ² /m) | $10000\rho_{\min} \times (t-0.01\text{m})$ | 1053 | $10000\rho_{\max} \times (x_{3\max}-0.01\text{m})$ | 7072 |
| X_6 (mm ² /m) | $10000\rho_{\min} \times (x_{4\min}-0.01\text{m})$ | 1053 | $10000\rho_{\max} \times (x_{4\max}-0.01\text{m})$ | 7072 |
| X_7 (mm ² /m) | $10000\rho_{\min} \times (x_{4\min}-0.01\text{m})$ | 426 | $10000\rho_{\max} \times (x_{4\max}-0.01\text{m})$ | 7072 |

11.3 Manual solution to problem

The considered problem was solved manually and the results are tabulated in Table 5. While solving the problem manually the ten failure mode constraints have also been taken into account. The manual solution obviously did not yield any penalty and the objective function is calculated using equation 21. The weight of the retaining wall designed manually work out as 109.663 kN. The weight reduction of the retaining wall can be achieved by finding the mathematical relation between the design constraints with respect to the design variables. These constraints can be substituted with the initial population and objective functions of the considered set of design variables can be found through any optimization procedure. In the

present research differential evolution algorithm is used.

Table 5: Manual solution for design variables

| Design variable | Unit | Solution | Definition of variable |
|-----------------|--------------------|----------|---|
| X ₁ | M | 3.0 | Total base width |
| X ₂ | M | 1.0 | Toe projection |
| X ₃ | M | 0.4 | Stem thickness at the bottom |
| X ₄ | m | 0.5 | Thickness of the base slab |
| X ₅ | mm ² /m | 1082 | Horizontal steel area of the heel per unit length of wall |
| X ₆ | mm ² /m | 1349 | Horizontal steel area of the toe per unit length of wall |
| X ₇ | mm ² /m | 2478 | Steel area of stem per unit length of wall |

11.4 Initial population for sensitivity analysis

The two constant design parameters, thickness of stem at top and height of the stem are varied for 0.2, 0.25, 0.3 and 3.5, 4.5, 5.5 respectively. The initial population to be generated within upper and lower bound for each of the combination for all seven variables is given in Table 6, Table 7, and Table 8.

Table 6: Upper and lower bound for design variables for thickness of stem at top = 0.2 m

| Thickness of stem at top=0.2 | | | | | |
|------------------------------|---------|------------|---------|------------|---------|
| Height=3.5 | | Height=4.5 | | Height=5.5 | |
| Lower | Upper | Lower | Upper | Lower | Upper |
| 1.53 | 2.72 | 1.96 | 3.50 | 2.40 | 4.28 |
| 0.51 | 0.91 | 0.65 | 1.17 | 0.80 | 1.43 |
| 0.20 | 0.39 | 0.20 | 0.50 | 0.20 | 0.61 |
| 0.32 | 0.39 | 0.41 | 0.50 | 0.50 | 0.61 |
| 778.00 | 5294.22 | 1050.27 | 7072.00 | 1323.00 | 8849.78 |
| 778.00 | 5294.22 | 1050.27 | 7072.00 | 1323.00 | 8849.78 |
| 426.00 | 5294.22 | 426.00 | 7072.00 | 426.00 | 8849.78 |

Table 7: Upper and lower bound for design variables for thickness of stem at top = 0.25 m

| Thickness of stem at top=0.25 | | | | | |
|-------------------------------|---------|--------------|---------|--------------|---------|
| Height = 3.5 | | Height = 4.5 | | Height = 5.5 | |
| Lower | Upper | Lower | Upper | Lower | Upper |
| 1.53 | 2.72 | 1.96 | 3.50 | 2.40 | 4.28 |
| 0.51 | 0.91 | 0.65 | 1.17 | 0.80 | 1.43 |
| 0.25 | 0.39 | 0.25 | 0.50 | 0.25 | 0.61 |
| 0.32 | 0.39 | 0.41 | 0.50 | 0.50 | 0.61 |
| 778.00 | 5294.22 | 1050.00 | 7072.00 | 1323.00 | 8849.78 |
| 778.00 | 5294.22 | 1050.00 | 7072.00 | 1323.00 | 8849.78 |
| 576.00 | 5294.22 | 576.00 | 7072.00 | 576.00 | 8849.78 |

Table 8: Upper and lower bound for design variables for thickness of stem at top = 0.3 m

| Thickness of stem at top=0.3 | | | | | |
|------------------------------|---------|------------|---------|------------|---------|
| Height=3.5 | | Height=4.5 | | Height=5.5 | |
| Lower | Upper | Lower | Upper | Lower | Upper |
| 1.53 | 2.72 | 1.96 | 3.50 | 2.40 | 4.28 |
| 0.51 | 0.91 | 0.65 | 1.17 | 0.80 | 1.43 |
| 0.30 | 0.39 | 0.30 | 0.50 | 0.30 | 0.61 |
| 0.32 | 0.39 | 0.41 | 0.50 | 0.50 | 0.61 |
| 778.00 | 5294.22 | 1050.00 | 7072.00 | 1323.00 | 8849.78 |
| 778.00 | 5294.22 | 1050.00 | 7072.00 | 1323.00 | 8849.78 |
| 726.00 | 5294.22 | 726.00 | 7072.00 | 726.00 | 8849.78 |

11.5 DEA parameters

The initial population of each variable is set as 100, the maximum number of generations permitted in the present research is set as 1000, the mutation factor is considered in the analysis is 0.8, and the crossover component is set as 0.5. The noisy vector created is passed into a limiting function which sets the variable values into bound if it goes out of the bound when mutation is performed. Ten trial runs are carried for each case by changing the random seed in order to locate least cost solution.

12. RESULTS AND DISCUSSION

The mathematical relation between the failure modes with respect to the design variables, objective function equations, initial population of design variables between upper bound and lower bound values are coded in C++ program. The results have been tabulated in Table 9 along with results obtained using manual computation and also solution presented by Ahamdi and Varee [5] using Particle Swarm Algorithm.

Table 9: Optimized solution using DEA

| Details | Manual solution | PSO (Ahmadi-Nedushan. B., and H.Varaee, 2009) | DEA |
|-------------------------------------|-----------------|---|----------|
| WEIGHT (kN) | 109.663 | 103.11 | 94.9255 |
| X ₁ (m) | 3 | 2.45 | 2.40368 |
| X ₂ (m) | 1 | 1.17 | 0.841223 |
| X ₃ (m) | 0.4 | 0.44 | 0.462555 |
| X ₄ (m) | 0.5 | 0.43 | 0.41 |
| X ₅ (mm ² /m) | 1082 | 1110 | 1053 |
| X ₆ (mm ² /m) | 1349 | 1110 | 1053 |
| X ₇ (mm ² /m) | 2478 | 2919 | 2018.43 |

The optimized result obtained based on DEA shows that material saving of 15.525% while comparing with manual solution and material saving of 8.622% is obtained when comparing solution based on PSO approach. It clearly represents the weight reduction difference between each solution. The least value in each generation is segregated separately and they are plotted in Fig. 5. The Fig. 5 clearly represents that the DEA traces the local optimum solution in 126th generation and global optimum solution in 869th generation. This shows that DEA is very effective in tracing the best solution very quickly and convergence of optimal solution is appreciable. Out of ten trial runs the approximate time for compilation of the code has been found out. The average of these ten trial runs is found to be seven seconds. This shows that DEA is very quick in calculation and simple for manipulation.

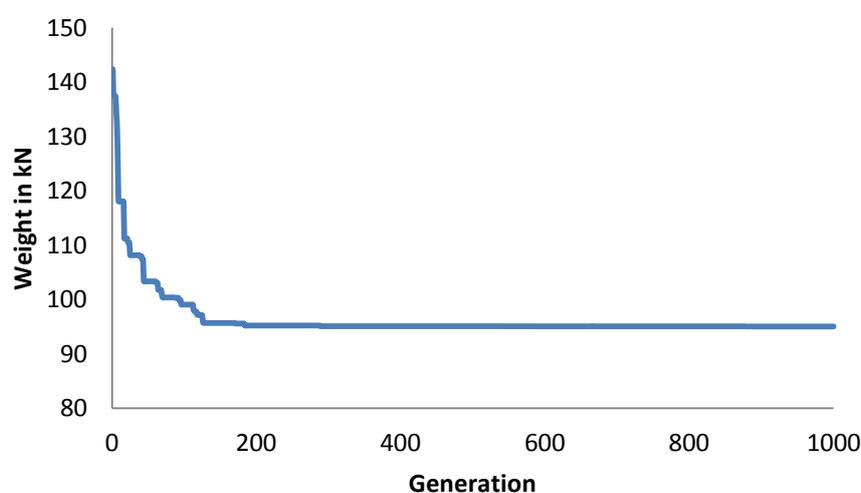


Figure 5. Evolution of solution using DEA

12.1 Sensitivity analysis

The design parameters considered in the present study cover a wide range of parameters that are related to loading, geometry, soil properties, code specifications, unit cost, and other characteristics of construction materials. Sensitivity of the optimum solution to changes in these parameters is an important issue as far as practical design concerned. The analysis of results includes the sensitivities of the optimum weight as objective functions and the optimum values of the seven design variables. As a representative of such analyses, results concerned with the sensitivity of optimum solutions with respect to height and top thickness of stem, surcharge load, backfill slope, internal friction angle of retained soil and the yield strength of reinforcing steel are reported. Sensitivities of the objective functions are explained for all design parameters considered. The thickness of stem at top is varied for 0.2 m, 0.25 m, 0.3 m and height of the stem at top is varied for 3.5 m, 4.5 m, 5.5 m and its results are tabulated in Table 10-12 respectively. For each combination, the lower and upper bound values are changed in original code for generating the initial population. Repeated application of same algorithm with different bounds, solution is generated.

Table 10: Analysis for stem thickness at top= 0.2m

| Height (m) | 3.5 | 4.5 | 5.5 |
|-------------------------------------|-------|-------|--------|
| Weight (kN) | 72.54 | 94.92 | 127.41 |
| X ₁ (m) | 1.96 | 2.4 | 2.98 |
| X ₂ (m) | 1.04 | 0.84 | 1.17 |
| X ₃ (m) | 0.36 | 0.46 | 0.5 |
| X ₄ (m) | 0.41 | 0.41 | 0.41 |
| X ₅ (mm ² /m) | 1053 | 1053 | 1415 |
| X ₆ (mm ² /m) | 1053 | 1053 | 1410 |
| X ₇ (mm ² /m) | 1408 | 2018 | 3296 |

Table 11: Analysis for stem thickness at top = 0.25m

| Height(m) | 3.5 | 4.5 | 5.5 |
|-------------------------------------|-------|-------|--------|
| WEIGHT(kN) | 74.78 | 97.79 | 130.38 |
| X ₁ (m) | 1.96 | 2.4 | 2.95 |
| X ₂ (m)s | 0.87 | 0.89 | 1.17 |
| X ₃ (m) | 0.31 | 0.43 | 0.49 |
| X ₄ (m) | 0.41 | 0.41 | 0.41 |
| X ₅ (mm ² /m) | 1053 | 1053 | 1169 |
| X ₆ (mm ² /m) | 1053 | 1053 | 1053 |
| X ₇ (mm ² /m) | 1406 | 1931 | 2588 |

Table 12: Analysis for stem thickness at top = 0.3m

| Height(m) | 3.5 | 4.5 | 5.5 |
|-------------------------------------|-------|--------|--------|
| WEIGHT(kN) | 77.44 | 100.69 | 141.45 |
| X ₁ (m) | 2.23 | 2.41 | 3.5 |
| X ₂ (m) | 0.65 | 0.78 | 1.17 |
| X ₃ (m) | 0.25 | 0.36 | 0.49 |
| X ₄ (m) | 0.36 | 0.41 | 0.5 |
| X ₅ (mm ² /m) | 1053 | 1053 | 1084 |
| X ₆ (mm ² /m) | 1053 | 1053 | 1486 |
| X ₇ (mm ² /m) | 1418 | 2013 | 2892 |

The thickness of base slab, weight, length of toe slab, width of base slab, thickness of base slab increases as the height increases. The thickness of stem at bottom remains constant for all the stem heights. For heights 3.5 m and 4.5 m the steel area for toe and heel slab remain constant being the lower bound value. The steel area for stem slab only increases for all the heights due to the increase of stem height. As in the case of thickness of stem at top = 0.2 m the X₁, X₂, X₃ value increases as the height increases. The thickness of stem at bottom remains constant as the height increases remaining to be the lower bound value. For heights 3.5 m and 4.5 m the steel area for toe and heel slab remain constant being the lower

bound value. The steel area for stem slab only increases for all the heights due to the increase of stem height. The steel area of heel slab remains constant for all the heights of stem.

For top width 0.3 m case, the variable value inclination for first three variables occurs in this case also. The thickness of base slab at bottom does not remain the same in this case. For height 5.5 m and thickness of stem at top 0.3 m, X_1 , X_2 , X_3 , X_4 values have the upper bound values. For heights 3.5 m and 4.5 m the steel area for toe and heel slab remain constant being the lower bound value. The steel area for stem slab only increases for all the heights due to the increase of stem height.

13. CONCLUDING REMARKS

Weight reduction of cantilever retaining wall is achieved successfully by structural optimization. The considered problem was solved manually and compared with the optimization results. Material saving of 15.525% is obtained by comparing manual solution and material saving of 8.622% is obtained by comparing PSO solution. A detailed sensitivity analysis is done by varying the stem height and stem thickness at top. The thickness of base slab, weight, length of toe slab, width of base slab, thickness of base slab increases as the height increases. These results can be interpolated for any cantilever retaining wall construction with respect to its weight constraint. From sensitivity analysis, the change in values of variable with respect to increase in height of stem and thickness of stem at top is observed carefully. The thickness of stem at bottom remains constant for all the stem heights for thickness of stem at top 0.2 m and 0.25 m. For height 5.5 m and thickness of stem at top 0.3 m, X_1 , X_2 , X_3 , X_4 values have the upper bound values. For heights 3.5 m and 4.5 m the steel area for toe and heel slab remain constant being the lower bound value for all the three cases. The convergence of DEA is quick as the best value is traced at 126th generation (average of 10 trial runs). The computational time taken for population size of 100 and 1000 generations is approximately 7 seconds (average of 10 trial runs in Computer processor: Intel(R) core(TM) i5-2450M CPU@2.50 GHz RAM 4.00 GB). Design of structure without considering the seismic and traffic loads are the limitation of the research which will be taken as the future area of study.

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