

## OPTIMAL DESIGN OF WATER DISTRIBUTION SYSTEM USING CENTRAL FORCE OPTIMIZATION AND DIFFERENTIAL EVOLUTION

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### ABSTRACT

For any agency dealing with the design of the water distribution network, an economic design will be an objective. In this research, Central Force Optimization (CFO) and Differential Evolution (DE) algorithm were used to optimize Ismail Abad water Distribution network. Optimization of the network has been evaluated by developing an optimization model based on CFO and DE algorithm in MATLAB and the dynamic connection with EPANET software for network hydraulic calculation. Conclusions show CFO runtime is less than DE. While optimization of CFO (737,924 \$) and DE (737,920 \$) are %1.61 and %1.57 more than the absolute optimum that determined by the MILP method (726,463 \$), respectively.

**Keywords:** central force optimization; differential evolution algorithm; optimization; distribution systems; epanet.

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### 1. INTRODUCTION

A significant portion of the water supply system cost is relevant to the water distribution network [1]. Thus, finding the optimal cost for implementation of these networks is economically helpful in the water supply systems design. Schaake and Lai for the first time presented a linear programming solution to optimize pipe diameters of the water distribution network (WDN) of New York City [2].

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In the past few decades, many researchers focused on finding the optimal cost of pipe-sizing of water distribution networks (WDNs) by utilizing the meta-heuristic optimization algorithms [3, 4]. They have shown that these methods were successful in solving a single objective WDN problem. But as mentioned in the previous researches, a WDN design with the least pipe-sizing cost could not have the sufficient surplus energy which is required to meet the future demands or to overcome the failure conditions during the operational period of WDN [5]. The researches relevant to this issue emphasized to consider the WDN problem as a multi-objective problem with minimizing the cost function and maximizing the benefit function of WDN. So far, various contractual indices have been defined for benefit function.

In the past few decades, many researchers focused on finding the optimal cost of pipe-sizing of water distribution networks (WDNs) by utilizing the meta-heuristic optimization algorithms. Application of the genetic algorithm [6, 7], colliding bodies optimization [8], the modified genetic algorithm [9], the tug of war Optimization [10], the simulated annealing algorithm [11], the shuffled leapfrog algorithm [12], ant colony optimization [13], charged system search algorithm [14], novel cellular automata [15], the particle swarm algorithm [16], and the differential evolution algorithm [17] for optimal design of water distribution systems are some of them. Also Sheikholeslami and Kaveh [18] studied on vulnerability assessment of water distribution networks by using graph theory method.

These researches showed that these algorithms were successful in finding the most economic cost of water networks. But, due to their stochastic nature there was no guarantee that the global optimum was found. Also, since they are stochastic optimization techniques, the solution found in each run was not always the same and therefore, several runs were necessary to ensure that the solutions identified as good quality.

The present paper is focused on implementation of the DE algorithm and Central Force Optimization (CFO) for optimal design and rehabilitation of water distribution networks. Use of addition, subtraction and component swapping are the distinguishing features of DE and CFO that successively update the population of solution vectors, until the population hopefully converges to an optimal solution.

Recently a new approach to the optimization has been introduced, called central force optimization. Central Force Optimization (CFO) is a new nature-inspired, gradient-like meta-heuristic for multidimensional search and optimization [19]. Since this physical law in which masses move towards the gravitational field is deterministic, CFO's equations are inherently deterministic. Therefore, in CFO, multiple runs for the sake of finding its performance are not required and every CFO run with the same setup will result the same [20]. This algorithm has been successfully applied to a variety of problems, among them: antenna optimization [21]; drinking water distribution networks [3] and improve the global search ability of Standard CFO [22]. These researches have shown high efficiency of this optimization method and also in some of them, the researchers have enhanced CFO performance through creating some modifications in the method.

In the recent past, DE algorithm was used to optimize the water pumping system [23], multi-objective reservoir system operation [24] and irrigation system planning [25].

In this paper, CFO and DE algorithm is developed to obtain the optimum pipe size and inlet pressure head that produce the least cost design of networks by [26]. The hydraulic analysis of the network is based on continuity at nodes and Hazen-Williams formula for

head loss calculations by using link between EPANET and MATLAB Software [27]. The results of this investigation compared with absolute optimization is obtained by mixed integer linear programming (MILP) model that is presented by Shahinezhad [26].

## 2. MATERIAL AND METHODS

### 2.1 EPANET model

EPANET developed by the U.S. Environmental Protection Agency's Water Supply and Water Resources Division and can perform hydraulic analysis of unlimited network size and complexity (looped systems, etc.). They also provide EPANET Programmer's Toolkit that is a dynamic link library (DLL) of functions which allows developers to customize EPANET's computational engine for the user's specific needs. This model computes hydraulic performance (pressure in the nodes, flow and head-loss in the pipe) for a given layout and nodal demands. It can analyze the performance of the system and can be used to design system components to meet distribution requirements.

The basic hydraulic equations involved in EPANET are the mass and energy conservation [36]. The law of mass conservation states that the rate of storage in a system is equal to the difference between the inflow and outflow to the system. For each junction, the conservation of mass can be expressed as:

$$\sum Q_{in} - \sum Q_{out} = Q_{ext} \quad (1)$$

where  $Q_{in}$  and  $Q_{out}$  are the inflows and outflows of the node; and  $Q_{ext}$  is the external demand.

Conservation of energy states that the total head loss within a loop must be equal to zero. For each closed loop, the conservation of energy can be expressed as:

$$\sum_{i \in \text{loop } l} \Delta H_i = 0 \quad \forall l \in N_L \quad (2)$$

where  $\Delta H_i$  is head loss in the pipe  $i$  and  $N_L$  is total number of loops in the system.

The head loss in the pipe  $i$ th which is located between junction  $j^{\text{th}}$  and  $k^{\text{th}}$  is equal the difference between nodal head at both ends:

$$\Delta H_i = H_j - H_k \quad (3)$$

EPANET computes friction head loss using the Hazen-Williams, Darcy-Weisbach, or Chezy-Manning formulas.

Herein, the Hazen-Williams (HW) equation is used to approximate the head loss and can be described as:

$$h_f = \omega \frac{L_i}{C_i^\alpha D_i^\beta} Q_i^\alpha \quad (4)$$

Here  $\alpha = 1.85$ ,  $\beta = 4.87$ ,  $Q_i$  is the pipe flow ( $\text{m}^3/\text{s}$ ),  $C_i$  is the Hazen-Williams roughness coefficient which ranges from 150 for smooth-walled pipes to as low as 80 for old, corroded cast iron pipes,  $D_i$  is pipe diameter (m), and  $L_i$  is pipe length (m).

$\omega$  is a dimensionless conversion factor whose numerical value depends on the units used which in this research the Hazen-Williams formula in EPANET has an  $\omega$  value of 10.5879; therefore 10.5879 as well as 10.5088 were chosen as the  $\omega$  values here. Also all the researches in the literature that used in this study to compare the results have the same  $\omega$  value. Higher  $\omega$  values require larger diameters to deliver the same amount of water, because these can violate the minimum pressure requirements, while the lower  $\omega$  values may just meet the constraint. Thus, higher  $\omega$  values eventually require more expensive water network designs.

## 2.2 Objective function and constraints

On the other hand, two parameters are crucial in any optimization problem, 1- objective function and 2- constraints; these parameters need to be specified.

The purpose of this research is also to minimize the economic costs of Ismail Abad water Distribution network, while all administrative and technical constraints are considered. The total annual cost of a pressurized branched network system can be introduced as:

$$f(D_i) = \sum_{i=1}^{NP} (L_i \cdot CP_i \cdot CRF) + \sum_{I=1}^{NPU} (C_{PU I} \cdot CRF) + C_{en} \cdot H_{PI} \quad (5)$$

where,  $L_N$  = length of pipe number N, N = subscript representing pipe number in the network,  $CP_N$  = unit length cost of pipe N, which is a function of pipe diameter, NP = Number of pipes,  $C_{PU I}$  = cost of the Ith pump which is a function of the total power of the pump required, NPU = Number of pumps in the network system,  $C_{en}$  = annual energy cost per unit head,

The annual energy cost per unit head of the pump can be expressed as:

$$C_{en} = \frac{C_{fu} \cdot Q_s \cdot Q_t \cdot EAE}{102\eta_e} \quad (6)$$

In which,  $C_{fu}$  is the fuel cost (\$/kWh);  $O_t$  is the number of annual system operating in hours; EAE is the equivalent annualized escalating energy cost factor;  $\eta_e$  is the overall pump efficiency in fraction.

$$EAE = \frac{(1+e)^y - (1+r)^y}{(1+e) - (1+r)} \left[ \frac{r}{(1+r)^y - 1} \right] \quad (7)$$

In which,  $e$  is the decimal equivalent annual rate of energy escalation;  $y$  is the life time of the design in years, and  $r$  is the decimal equivalent annual interest rate. HPI = total dynamic head of the  $i$ th pump, CRF=capital return factor which is calculated as below:

$$\text{CRF} = \frac{r(1+r)^y}{(1+r)^y - 1} \quad (8)$$

$$\text{CPU}_i = P_i \cdot K \quad (9)$$

where,  $P_i$ =total power of the  $i$ th pump and  $K$ = pump station cost per unit total power (\$/KW).

Multiplying the terms of the first summation of equation (1) by zero-unity variables such as  $X_{NJ}$ , and adding for all commercially available pipes yields:

$$f(D_i) = \sum_{i=1}^{NP} \sum_{j=1}^{ND} (L_i \cdot CP_{ij} \cdot \text{CRF} \cdot X_{ij}) + \sum_{I=1}^{NPU} (\text{CPU}_I \cdot \text{CRF}) + C_{en} \cdot H_{PI} \quad (10)$$

ND = number of commercially available pipe Diameter,

The objective function is to be minimized under the design and hydraulic constraints that should be respected by CFO algorithm for reaching optimum solution for design cost of network. The pipe diameter bounds as a design constraint and the velocity bounds in each pipe and the pressure bounds in each node of network are given respectively as:

#### Pressure constraint

Minimum Allowable pressure head required for each node is considered to be 50 m.

#### Velocity constraint

In order to prevent sediment deposition in low flow velocities and avoid water hammer at high velocities, minimum and maximum allowable flow velocities in pipes are considered to be 0.7 m/s and 2m/s, respectively.

### 2.3 Central force optimization (CFO) algorithm

Central Force Optimization, by contrast, is completely deterministic because it is based on the metaphor of gravitational kinematics, the branch of physics that computes the motion of masses moving solely under the influence of gravity. It comprises two simple deterministic equations based on the metaphor of gravitational kinematics. Because gravity is deterministic, so CFO's equations are inherently deterministic. CFO's deterministic nature is a major distinction setting which contrast it from other swarm intelligence (SI) algorithms such as Particle Swarm Optimization (PSO) and ant colony optimization (ACO), which are fundamentally stochastic. Their equations are formulated in terms of random variables, and removing randomness causes these algorithms to fail completely. While every ACO or PSO run is defaulted with the same sets of run parameters, the models generate an entirely different solution in each trial. But in CFO which is inherently deterministic, it is not necessary to characterize its performance statistically by making multiple runs and every CFO run with the same parameters leads to the same result.

CFO flies a set of probes through the space over a set of discrete time steps. A decision space is defined by  $x_i^{\min} \leq x_i \leq x_i^{\max}$ ,  $i=1, \dots, N_d$  where  $x_i$ 's are decision variables and  $N_d$  is the number of dimension in decision space depending on the problem. The position vector  $\vec{R}_j^p$  specifies the location of each probe at each time step. Probe  $p$ 's position vector at step  $j$  in  $N_d$  dimensioned problem is given by:

$$\vec{R}_j^p = \sum_{i=1}^{N_d} x_i^{p,j} \hat{e}_i \quad (11)$$

Here  $x_i^{p,j}$  is the coordinate of the probe  $p$  at the time  $j$  and  $\hat{e}_i$  is the unit vector along the  $x_i$ -axis. The indexes  $j$ ,  $0 \leq j \leq N_t$  and  $p$ ,  $1 \leq p \leq N_p$  respectively, are the iteration number and probe number, with  $N_t$  and  $N_p$  being the corresponding total numbers of time steps and probes.

In CFO, each decision variable in objective function is a coordinate axis. The initial probes can be distributed via below five methods:

- Place the probes uniformly along each coordinate axes.
  - Place the probes uniformly on a 2D grid (for 2-dimensional objective functions)
  - Place the probes uniformly slightly off decision space diagonal
  - Place the probes randomly in decision space
  - User-defined method to distribute the probes in decision space
- Decision space diagonal length (DSDL) is defined as follow:

$$DSDL = \sqrt{\sum_{i=1}^{N_d} (R_{j-1}^{k,i} - R_{j-1}^{p,i})^2} \quad (12)$$

In metaphorical CFO space each of the  $N_p$  probes experiences an acceleration created by the gravitational pull of masses in decision space which is the first of CFO's two equations of motion. The total acceleration experienced by Probe  $p$ , which produced by each of other probes on it at step  $j-1$  is given by:

$$\vec{a}_{j-1}^p = G \sum_{\substack{k=1 \\ k \neq p}}^{N_p} U(M_{j-1}^k - M_{j-1}^p) \cdot (M_{j-1}^k - M_{j-1}^p)^\alpha \frac{(\vec{R}_{j-1}^k - \vec{R}_{j-1}^p)}{\|\vec{R}_{j-1}^k - \vec{R}_{j-1}^p\|^\beta} \quad (13)$$

Here  $M_{j-1}^p = f(x_1^{p,j-1}, x_2^{p,j-1}, \dots, x_{N_d}^{p,j-1})$  is the fitness at time step  $j-1$  at probe  $p$ 's location. Each of the other probes also has associated with it the fitness  $M_{j-1}^k, k=1, 2, \dots, p-1, p+1, \dots, N_p$ . Also  $\alpha$ ,  $\beta$  and  $G$  are the CFO constants which usually are equal to 2 as are in this study [19]. Also  $U$  is the Unit Step function which in maximizing the objective function is given by:

$$U(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{Otherwise} \end{cases} \quad (14)$$

And  $\|\vec{R}_{j-1}^k - \vec{R}_{j-1}^p\|$  is the distance between the position of probes  $p$  and  $k$  which is given by:

$$\|\vec{R}_{j-1}^k - \vec{R}_{j-1}^p\| = \sqrt{\sum_{m=1}^{N_d} (\mathbf{R}_{j-1}^{k,m} - \mathbf{R}_{j-1}^{p,m})^2} \quad (15)$$

The acceleration  $\vec{a}_{j-1}^p$  causes probe  $p$  to move from position  $\vec{R}_{j-1}^p$  at step  $j-1$  to position  $\vec{R}_j^p$  at step  $j$  according to the below trajectory equation which is CFO's second equation of motion:

$$\vec{R}_j^p = \vec{R}_{j-1}^p + \vec{V}_{j-1}^p \Delta t + \frac{1}{2} \vec{a}_{j-1}^p \Delta t^2, \quad j \geq 1 \quad (16)$$

where  $\vec{V}_{j-1}^p$  is probe  $p$ 's velocity at the end of time  $j-1$  and  $\Delta t$  is the time step increment.

Here for simplicity,  $\vec{V}_{j-1}^p$  and  $\Delta t$  were considered to be zero and unity, respectively.

Movement of each probe is restricted to the bounded feasible region. However, when some of probes fly out, a retrieving mechanism as below scheme is used to act on them and reposition them in feasible region:

$$\text{if } \vec{R}_{j,i}^p < x_i^{\min} \text{ then } \vec{R}_{j,i}^p = x_i^{\min} + F_{\text{rep}} (x_i^{\min} - \vec{R}_{j-1,i}^p) \quad (17)$$

$$\text{if } \vec{R}_{j,i}^p > x_i^{\max} \text{ then } \vec{R}_{j,i}^p = x_i^{\max} - F_{\text{rep}} (\vec{R}_{j-1,i}^p - x_i^{\max}) \quad (18)$$

Here  $F_{\text{rep}}$  is the repositioning factor and  $0 \leq F_{\text{rep}} \leq 1$ .

In this study as Formato proposed, at each step the current and previous 4 fitness values were stored in a 5-element array. The  $F_{\text{rep}}$  parameter started at a value of 0.5 and was incremented by 0.005 whenever the absolute value of the difference between 5th array element and the average value of elements 3, 4, and 5 differed by less than 0.0005 (Fitness tolerance). If incrementing  $F_{\text{rep}}$  in this manner resulted in  $F_{\text{rep}} \geq 1$ , then it was reset to the starting value, and this procedure was repeated with the then current probe distribution. Fig. 1 shows the flowchart of CFO algorithm.

To evaluate CFO method for solving water network problem Ismail Abad network has been selected. CFO has been run to optimize the design cost of Ismail Abad network. The CFO variables and parameters have been set into the model. The values of  $x_i^{\max}$  and  $x_i^{\min}$  are the maximum and minimum existing commercial diameters which have the same value in

all dimensions in this problem.

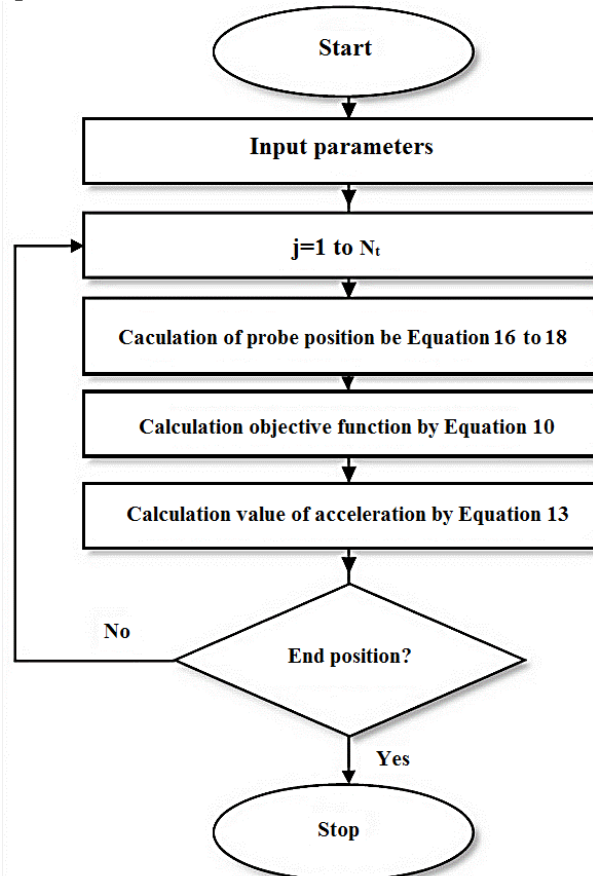


Figure 1. CFO algorithm

Therefore initial probes distribution using the above mentioned (a) and (c) methods make an improper probe distribution. In this study to overcome this challenge, a user-defined initial probes distribution used. In this regard commercial diameters in water distribution system considered as a permutation and generate probes via displacement diameter values in this permutation. This method has been used to place 42 probes ( $N_p=42$ ) in decision space and their initial acceleration vectors have been set to zero.

Given the poor performance of CFO to optimize these networks, applying a few modifications on this method seemed required to have proper optimization in this kind of problems.

The performance of the CFO algorithm has been evaluated in Ismail Abad networks. In this regards, CFO code was developed in MATLAB program and EPANET has been linked via the EPANET Toolkit. As to define the setting of the algorithm, the values of  $\alpha$ ,  $\beta$  and  $G$  constant parameters,  $N_p$  and the initial acceleration vectors have been set same as in CFO and then applied the above mentioned User-defined probe distribution method.

After first iteration of CFO, the new diameters, via a subscript which coded in Visual Basic program, replaced into the input file of EPANET to simulate the considered network. The constraints of the conservation of mass and energy (Eq. 1 and 2) are satisfied by



EPANET. To check hydraulic constraints, another subscript code which was developed in Visual Basic program used to read output file of EPANET and specified hydraulic characteristics of the network.

Then the velocity in each pipe and the pressure in each node compared with the constraints of these parameters and computed penalty values. So the objective function (Eq. 10) evaluated for each probe and these fitness values compared with each other and the least value replaced to the best cost obtained have ever seen.

To prevent disregarding of global best solution value have ever obtained (which happens in CFO), if the new fitness of the best probe become higher than previous one, it is ordered to keep unchanged. Thereafter CFO computed acceleration for each probe (Eq. 13). But since in this research minimizing an objective function is the main purpose, so  $U$  rectified as below formulation to pull and attract the probes out by other probes with lower mass while the previous studies [20, 21] have applied a negative sign on objective function.

$$U(x) = \begin{cases} 0 & \text{if } x \geq 0 \\ 1 & \text{Otherwise} \end{cases} \quad (19)$$

Since the acceleration values are big in the water distribution system problem and made all probes throw out of the decision space, the normalization method is applied to reduce them. The normalization method was not applied for zero acceleration values. The maximum and minimum bounds for normalization depend on the existing commercial diameters in the specified water distribution system.

After normalizing accelerations with regards to the obtained acceleration values, probe positions have changed and then new diameters obtained (Eq. 16). The errant probes repositioned and returned to the decision space via mentioned method. The input file for EPANET updated using these new diameters. This process is continued until the number of iterations is reach to the maximum which in this research is 3500 ( $N_t = 3500$ ).

During running this process when most probes trapped in the local optimum solution and consequently acceleration values became zero, mutation operator act on the probes. In order that some probes mutated to the new positions which produced via applying Swap, Insertion and Reversion operators on the specified matrix. This matrix has obtained by replication the existing commercial diameters matrix sized by pipes quantity in the network. The mutated rate is considered as 15% herein.

#### 2.4 Differential evolution (DE) algorithm

Differential Evolution (DE) algorithm is a branch of evolutionary programming developed by Rainer Storn and Kenneth Price for optimization problems over continuous domains [28, 29]. In DE, each variable's value is represented by a real number. The advantages of DE are its simple structure, ease of use, speed and robustness. DE is one of the best genetic type algorithms for solving problems with the real valued variables. Differential Evolution is a design tool of great utility that is immediately accessible for practical applications. DE has been used in several science and engineering applications to discover effective solutions to nearly intractable problems without appealing to expert knowledge or complex design algorithms. Differential Evolution uses mutation as a search mechanism and selection to

direct the search toward the prospective regions in the feasible region. Genetic Algorithms generate a sequence of populations by using selection mechanisms. Genetic Algorithms use crossover and mutation as search mechanisms. The principal difference between Genetic Algorithms and Differential Evolution is that Genetic Algorithms rely on crossover, a mechanism of probabilistic and useful exchange of information among solutions to locate better solutions, while evolutionary strategies use mutation as the primary search mechanism.

Differential Evolution (DE) is a parallel direct search method which utilizes NP D-dimensional parameter vectors.

$$x_{i,G}, \quad i=1,2,\dots, NP \quad (20)$$

As a population for each generation G. NP does not change during the minimization process. The initial vector population is chosen randomly and should cover the entire parameter space. As a rule, we will assume a uniform probability distribution for all random decisions unless otherwise stated. In case a preliminary solution is available, the initial population might be generated by adding normally distributed random deviations to the nominal solution  $x_{nom,0}$ .

DE generates new parameter vectors by adding the weighted difference between two population vectors to a third vector. Let this operation be called mutation. The mutated vector's parameters are then mixed with the parameters of another predetermined vector, the target vector, to yield the so-called trial vector. Parameter mixing is often referred to as "crossover" in the ES-community and will be explained later in more detail. If the trial vector yields a lower cost function value than the target vector, the trial vector replaces the target vector in the following generation. This last operation is called selection. Each population vector has to serve once as the target vector so that NP competitions take place in one generation. More specifically DE's basic strategy can be described as follows:

### Mutation

For each target vector  $x_{i,G}$ ,  $i=1,2,\dots, NP$ , a mutant vector is generated according to:

$$V_{i,G+1} = x_{r1,G} + F \times (x_{r2,G} - x_{r3,G}) \quad (21)$$

With random indexes  $r_1, r_2, r_3 \in \{1, 2 \dots NP\}$  integer, mutually different and  $F > 0$ . The randomly chosen integers  $r_1, r_2$  and  $r_3$  are also chosen to be different from the running index  $i$ , so that NP must be greater or equal to four to allow for this condition. F is a real and constant factor  $\in [0, 2]$  which controls the amplification of the differential variation  $(x_{r2,G} - x_{r3,G})$ . Fig.2 shows a two-dimensional example that illustrates the different vectors which play a part in the generation of  $V_{i,G+1}$ .

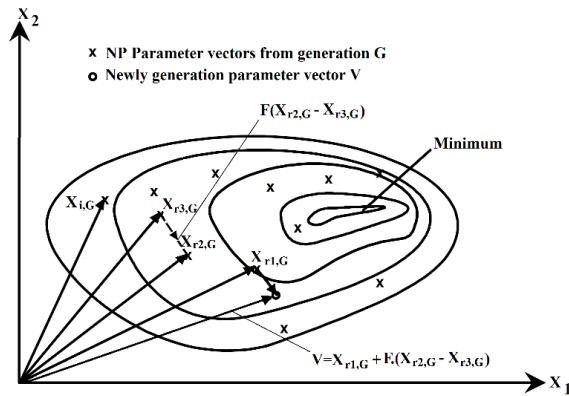


Figure 2. An example of a two-dimensional cost function showing its contour lines and the process for generating  $V_{i,G+1}$

**Crossover**

In order to increase the diversity of the perturbed parameter vectors, crossover is introduced. To this end, the trial vector:

$$u_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \dots, u_{Di,G+1}) \tag{22}$$

Is formed, where:

$$u_{ji,G+1} = \begin{cases} V_{ji,G+1} & \text{if randb(j)} \leq CR \text{ or } j = \text{ranbr}(i) \\ x_{ji,G} & \text{otherwise} \end{cases} \tag{23}$$

$j=1,2,\dots,D.$

In Equation (23), randb(j) is the jth evaluation of a uniform random number generator with outcome  $\in [0; 1]$ . CR is the crossover constant  $\in [0; 1]$  which has to be determined by the user. ranbr(i) is a randomly chosen index  $\in 1, 2, \dots, D$  which ensures that  $u_{i,G+1}$  gets at least one parameter from  $V_{i,G+1}$ .

**Selection**

To decide whether or not it should become a member of generation G+1, the trial vector  $u_{i,G+1}$  is compared to the target vector  $x_{i,G}$  using the greedy criterion. If vector  $u_{i,G+1}$  yields a smaller cost function value than  $x_{i,G}$ , then  $x_{i,G+1}$  is set to  $u_{i,G+1}$ ; otherwise, the old value  $x_{i,G}$  is retained.

$$x_{ji,G+1} = \begin{cases} u_{ji,G+1} & \text{if } f(u_{i,G+1}) \leq f(x_{i,G}) \\ x_{ji,G} & \text{if otherwise} \end{cases} \tag{24}$$

Finally, this process continues to reach new generations to the number of NP. Then the same process is repeated to reach termination condition.

Fig. 3 schematically overview of differential evolution algorithm for numerical model, the entire above process is specified numerically in this figure.

2.5 Mixed integer linear programming (MILP)

In general, an optimization problem either linear or nonlinear consists of an objective function which is subjected to some constraints. The classical linear optimization method may results in a branch which consists of many pipe sizes. In practice, this is considered as a strong weak point. On the other hand, linear optimization methods yields pipe sizes which are not commercially available. This leads to choose the pipe size close to that obtained by optimization. Consequently, the hydraulic conditions and cost of the network system will be different from that obtained by the optimization technique which means that the design is not optimum any more. The developed model guarantees obtaining the global optimum of pressurized branched networks.

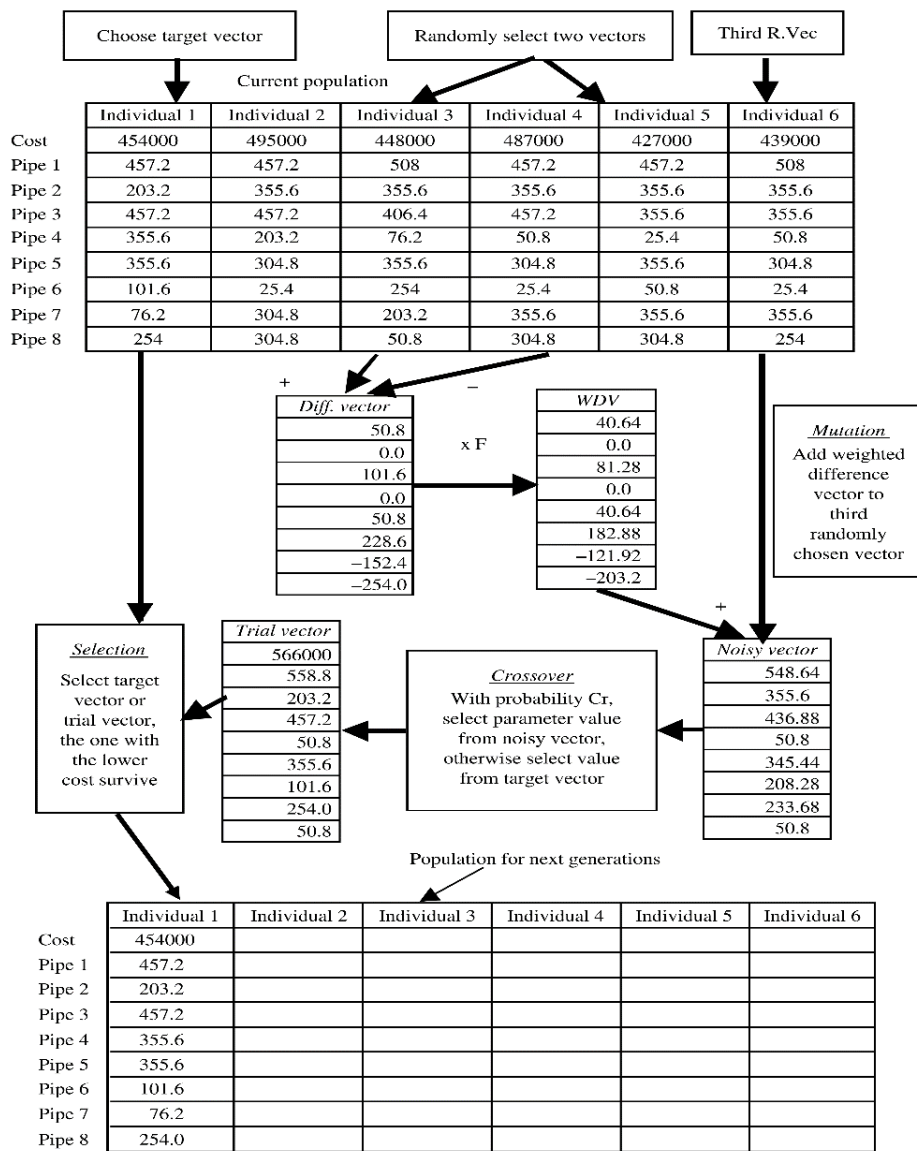


Figure 3. Computational module for differential evolution algorithm

Shahinezhad [26] to ensure of performance the model, MILP model was used for Ismail Abad branch network. This study showed that MILP method, with the above objective function is the ability to provide absolute optimum for branch network.

In this study, the CFO algorithm was implemented, in order to derive the optimal of water distribution networks by using CFO algorithm.

### 3. RESULTS AND DISCUSSION

#### 3.1 Case study

The Ismail Abad network is located in 7 kilometers North West of Noorabad city in Lorestan province. Land area of this project is 1000 ha. Fig. 4 depicts the schematic network of Ismael Abad. This network consists of 18 pipes and 19 nodes are. In Table 1, the hydraulic details and arrangement of pipes for water distribution networks Ismael Abad is presented.

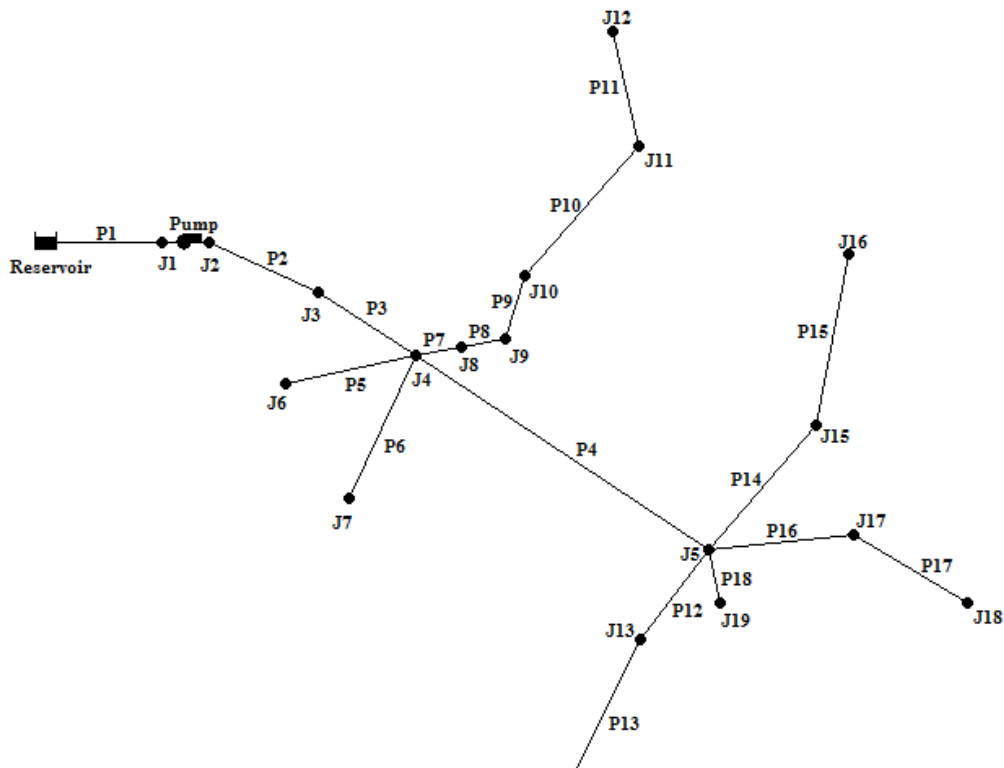


Figure 4. Ismael Abad water distribution network

This project consists of two kinds of steel pipe. Polyethylene pipe material is used for pipe sizes equal or less than 500mm and GRP for greater sizes. Pipe specifications are given in Table 2.

Table 1: Main and sub main pipe line data of Ismail Abad Network

Pipe	Pipe No.	Length (m)	Discharge (L/s)	Beginning Elevation (m)	End Elevation (m)
Res.-J1	P1	-	-	-	-
J2-J3	P2	558	856.56	1791	1816.54
J3-J4	P3	558	856.56	1816.54	1842.08
J4-J5	P4	1430	429.8	1842.08	1847.57
J4-J6	P5	955	52.9	1842.08	1838.71
J4-J7	P6	1100	128.94	1842.08	1856.52
J4-J8	P7	200	244.92	1842.08	1847.05
J8-J9	P8	201	190.34	1847.05	1846.32
J9-J10	P9	390	128.94	1846.32	1841.18
J10-J11	P10	806	58.33	1841.18	1811.32
J11-J12	P11	575	21.49	1811.32	1810.94
J5-J13	P12	550	165.8	1847.57	1853.21
J13-J14	P13	700	132	1853.21	1861.89
J5-J15	P14	670	98.24	1847.57	1821.48
J15-J16	P15	840	33.77	1821.48	1814.43
J5-J17	P16	720	119.73	1847.57	1826.47
J17-J18	P17	660	49.12	1826.47	1847.95
J5-J19	P18	110	46.05	1847.57	1847.57

Table 2: Pipe specifications data of Ismail Abad Network

No.	Material	Internal diameter (mm)	Outer diameter (mm)	Cost (\$/m)
1	PE80	93.8	110	5.895
2	PE80	106.6	125	7.895
3	PE80	119.4	140	9.495
4	PE80	136.4	160	12.375
5	PE80	153.4	180	15.705
6	PE80	170.6	200	19.305
7	PE80	191.8	225	24.525
8	PE80	213.2	250	30.150
9	PE80	238.8	280	37.800
10	PE80	268.6	319	47.700
11	PE80	302.8	355	60.525
12	PE80	341.2	400	76.725
13	PE80	383.8	450	97.200
14	PE80	426.4	500	108.820
15	GRP	600.0	600	111.323
16	GRP	700.0	700	137.997
17	GRP	800.0	800	170.633
18	GRP	900.0	900	204.289

### 3.2 CFO algorithm result

The convergence behavior of the CFO method to optimize pipe-sizing in Ismail Abad network has shown in Fig. 5.

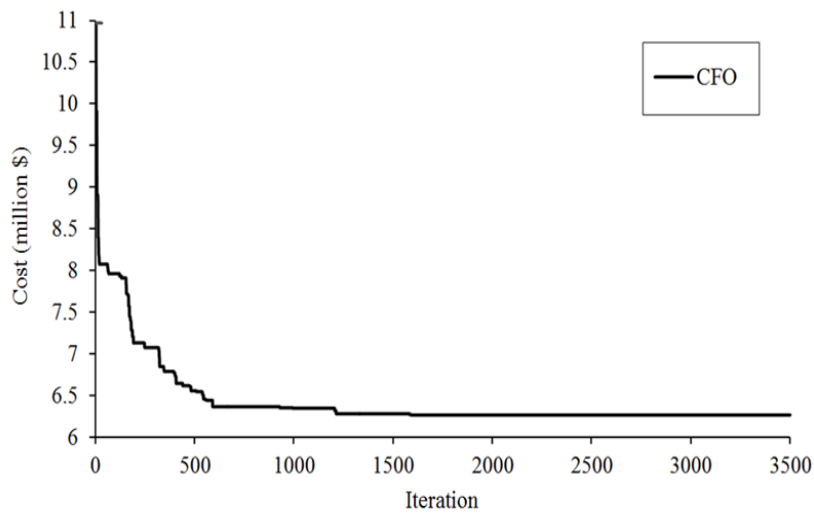


Figure 5. Convergence behavior of CFO for Ismail Abad network

Using CFO model the best cost obtained after 66,822 evaluation is 737,924\$ which is fairly close to the best solution obtained by MILP technique. The algorithm makes relationship between EPANET and MATLAB software to optimize the water distribution network. The combination of optimum pipe diameter is shown in Table 3.

This combination of optimal diameter is the best diameter to have the optimal costs. According to these network diameters, hydraulic conditions in Tables 4 and 5 are for pipes and nodes.

Table 3. Optimum pipe diameter in central force optimization

Pipe	No. Pipe	Optimum Diameter (inch)	Internal Optimum Diameter (mm)	Outer Optimum Diameter (mm)
Res.-J1	P1	10.575	268.6	315
J2-J3	P2	31.496	800	800
J3-J4	P3	31.496	800	800
J4-J5	P4	23.622	600	600
J4-J6	P5	8.394	213.2	250
J4-J7	P6	11.921	302.8	355
J4-J8	P7	16.787	426.4	500
J8-J9	P8	15.110	383.8	450
J9-J10	P9	11.921	302.8	355
J10-J11	P10	8.394	213.2	250
J11-J12	P11	4.701	119.4	140
J5-J13	P12	15.110	383.8	450
J13-J14	P13	11.921	302.8	355
J5-J15	P14	10.575	268.6	315
J15-J16	P15	6.039	153.4	180
J5-J17	P16	11.921	302.8	355
J17-J18	P17	7.551	191.8	225
J5-J19	P18	7.551	191.8	225
<b>Optimal cost (\$)</b>			<b>737924</b>	
<b>Runtime (s)</b>			<b>8936</b>	

Table 4. Hydraulic conditions optimal diameters in pipes

Pipe	No. Pipe	Optimum Diameter (mm)	Discharge (L/s)	Velocity (m/s)	Losses in 1000 (m)
Res.-J1	P1	315	-	-	-
J2-J3	P2	800	856.56	1.70	0.72
J3-J4	P3	800	856.56	1.70	0.72
J4-J5	P4	600	429.8	1.52	0.81
J4-J6	P5	250	54.9	1.73	5.34
J4-J7	P6	355	128.94	1.79	2.45
J4-J8	P7	500	244.92	1.72	1.62
J8-J9	P8	450	190.34	1.65	1.69
J9-J10	P9	355	128.94	1.79	2.61
J10-J11	P10	250	58.33	1.63	3.32
J11-J12	P11	140	21.49	1.92	8.80
J5-J13	P12	450	165.8	1.43	1.31
J13-J14	P13	355	132	1.83	2.73
J5-J15	P14	315	98.24	1.73	2.84
J15-J16	P15	180	33.77	1.83	6.00
J5-J17	P16	355	119.73	1.66	2.28
J17-J18	P17	225	49.12	1.70	4.04
J5-J19	P18	225	46.05	1.59	3.59

Due to the hydraulic conditions in the pipes, it can be seen from Table 4, each pipe is in standard conditions and velocity in each pipe is in permitted range. Table 5 shows pressure in each node in permitted range. So it can be said in this optimized network the constraint of pressure and velocity is considered.

Table 5: Hydraulic conditions optimal diameters in nodes

No. Node	Discharge (L/s)	Hydraulic Elevation (m)	Pressure (m-water)
Res.	-856.69	1789.00	0.00
J1	0.00	1788.71	-1.29
J2	0.00	1926.02	134.69
J3	0.00	1924.71	109.44
J4	0.00	1920.27	80.98
J5	0.00	1919.57	73.19
J6	52.90	1906.13	71.73
J7	128.94	1914.55	60.20
J8	54.58	1922.32	80.93
J9	61.40	1921.21	77.91
J10	70.61	1917.86	79.37
J11	36.84	1909.08	100.03
J12	21.49	1892.48	84.97
J13	33.80	1917.20	65.51
J14	132.01	1910.93	50.48
J15	64.57	1913.33	91.90
J16	33.77	1896.79	83.39
J17	70.61	1914.18	88.77
J18	49.12	1905.42	59.08
J19	46.05	1918.27	70.25



### 3.3 DE algorithm result

#### *F and CR Factor*

In the first step, to obtain the best conditions for algorithm that provide the most optimum and do not face local optimum problem, 18 combinations of different modes for the coefficients F and CR were examined. The results are shown in Table 6.

The Results show that median values for the coefficients of F and Cr provide the optimum situation and cause DE algorithm not to be trapped in local optimum. The most optimal answer for coefficients are 0.6 and 0.5 for F and Cr coefficients, respectively. These values matched with the results of Suribabu [18].

Scale factor (F) can increase the accuracy of the search. The smaller coefficient, the shorter steps needs to be taken for an accurate research. But the problem is that the algorithm may be trapped in local optimum and it cannot be withdrawn. On the other hand, the higher value of F, the more area will be searched, but the best optimum situation may not be obtained.

Table 6: Study F and CR

No. Combination	F	Cr	Optimal cost (\$)
1	F=0.1	Cr =0.1	115427393
2		Cr =0.3	832628
3		Cr =0.4	768561
4	F=0.5	Cr =0.5	758917
5		Cr =0.6	740000
6		Cr =0.3	738039
7	F=0.6	Cr =0.4	737931
8		Cr =0.5	737920
9		Cr =0.6	737992
10		Cr =0.3	737924
11	F=0.7	Cr =0.4	737988
12		Cr =0.5	740588
13		Cr =0.6	758028
14		Cr =0.3	786416
15	F=0.8	Cr =0.4	824850
16		Cr =0.5	832628
17		Cr =0.6	833455
18	F=1	Cr =1	55293902

#### **Population and Generation**

After finding the best combination of coefficients values F and CR, algorithms for solving the independent populations were examined. For this purpose, the population of 4, 25, 50, 100, 500 and 1000 members were studied in two generations (G=50 and 100). Fig. 6 shows these results.

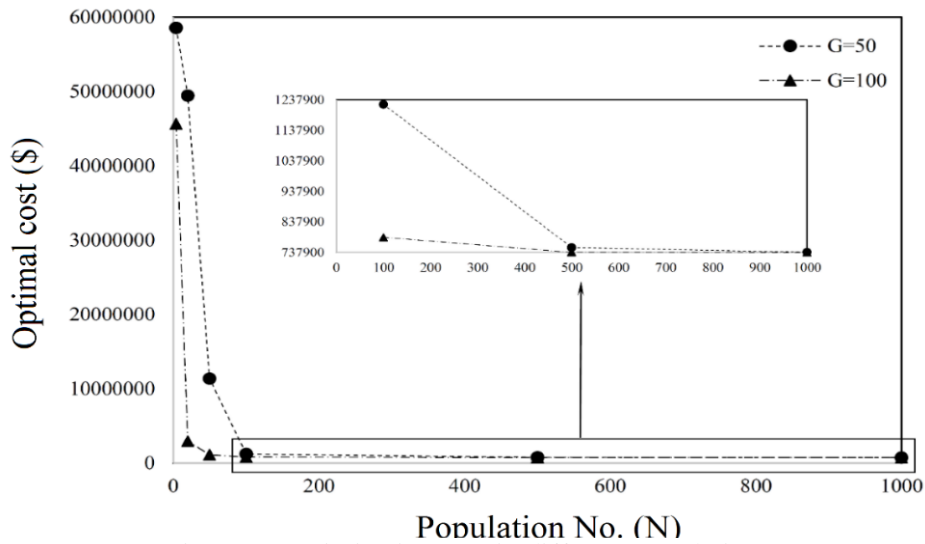


Figure 6. Optimization cost in different populations

Based on the DE algorithm, the initial population is very important to select the initial three members, when the population gets more, the selection of four initial members has more variety, which causes the algorithm to reach convergence.

According to Fig. 6, it is clear that by increasing population, the optimal cost will be lower. In addition, it is proved that the increasing population will extend the domain of the search; and more members are used for optimization.

Finally, the best combination of coefficients and population were used to examine the effect of generations' number, so ten generations (30, 40, 50, 100, 200, 300, 500, 1000, 2000, and 3000) were studied. The results are shown in Table 7.

Table 7 indicates that the generation number 200 is suitable for optimizing water distribution networks. This results show that DE algorithm for optimizing water distribution networks in the generation of 200 gives acceptable results.

Table 7: The effect of generation on optimization cost

No. Generation	Optimal cost (\$)	Runtime (s)
30	55293902	629
40	2065347	780
50	1222823	1005
100	786416	1950
200	737920	4024
300	737931	6164
500	737920	9324
1000	737924	20163
2000	737920	39826
3000	737920	53911

Table 7 indicates that the generation number 200 is suitable for optimizing water distribution networks. This results show that DE algorithm for optimizing water distribution networks in the generation of 200 gives acceptable results.

The increase in time per the number of population has almost a linear trend, which indicates the effect of population in the runtime algorithm. Hence specifying suitable population to obtain an optimal results is very important.

The runtime algorithm for 100 members of population and 50 generations is 935s and 100 generation is 1950s. According to the numbers, the running time of the algorithm to reach new member in each generation takes an average of 0.19s (Fig. 7).

Results of Fig. 8 indicates a fairly linear relationship between runtime and number of generations. In general it can be said that the population and number of generations to run the algorithm, in order to optimize water distribution network is 100 and 200, respectively that requires nearly an hour to reach the optimal answer.

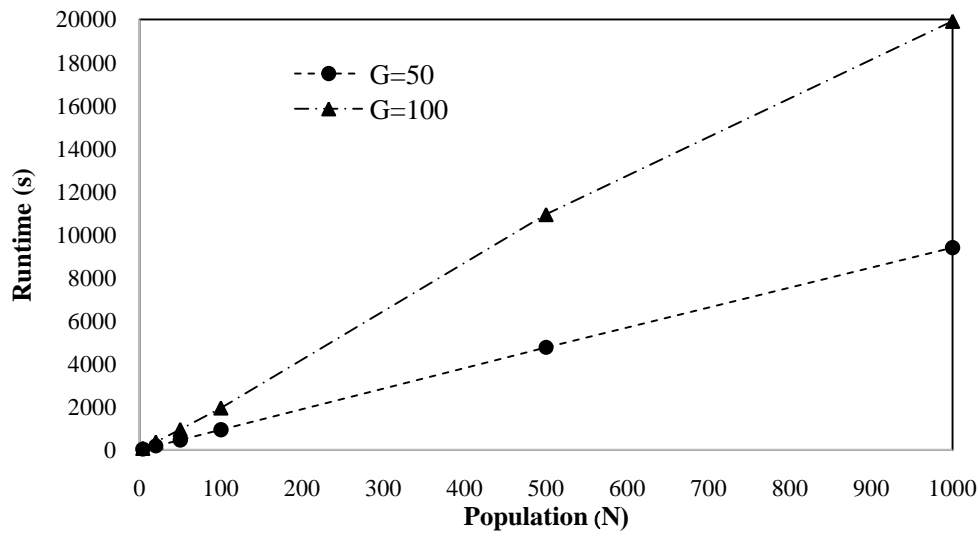


Figure 7. Runtime in different population

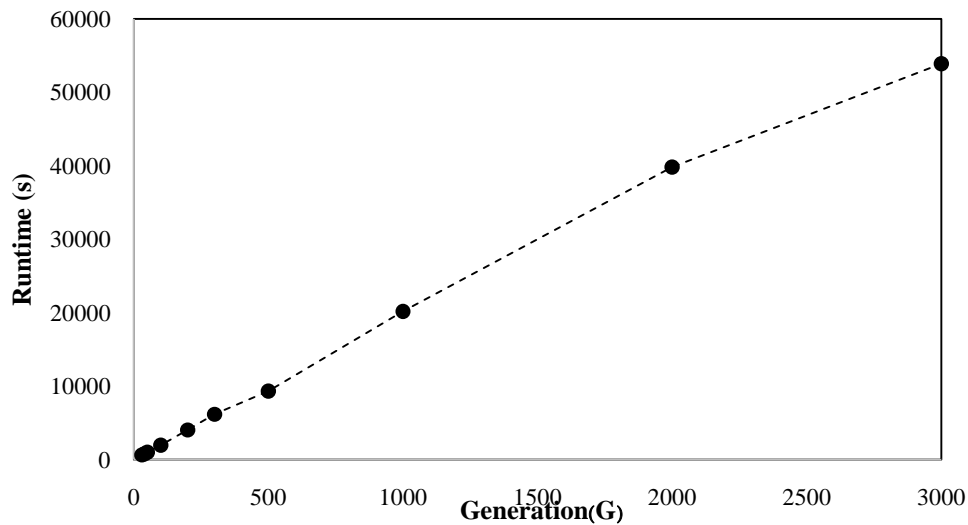


Figure 8. Runtime in different generation

So it can be revealed that one of the advantages of this algorithm is the high speed runtime. Another advantage is rapid convergence of the algorithm, that takes 16 minutes ( $G=50$  and  $N=100$ ) to reach convergence.

### Differential Evolution Algorithm Optimization

The network has been optimized with conditions  $Cr=0.5$ ,  $F=0.6$ , 100 members of population and 200 generations in the differential evolution algorithm. The algorithm makes relationship between Epanet and MATLAB software to optimize the water distribution network. The combination of optimum pipe diameter is shown in Table 8.

Table 8: Optimum pipe diameter in Differential Evolution Algorithm

Pipe	No. Pipe	Optimum Diameter (inch)	Internal Optimum Diameter (mm)	Outer Optimum Diameter (mm)
Res.-J1	P1	10.575	268.6	315
J2-J3	P2	31.496	800	800
J3-J4	P3	31.496	800	800
J4-J5	P4	23.622	600	600
J4-J6	P5	7.551	191.8	225
J4-J7	P6	11.921	302.8	355
J4-J8	P7	16.787	426.4	500
J8-J9	P8	15.110	383.8	450
J9-J10	P9	11.921	302.8	355
J10-J11	P10	8.394	213.2	250
J11-J12	P11	4.701	119.4	140
J5-J13	P12	15.110	383.8	450
J13-J14	P13	11.921	302.8	355
J5-J15	P14	10.575	268.6	315
J15-J16	P15	6.039	153.4	180
J5-J17	P16	11.921	302.8	355
J17-J18	P17	7.551	191.8	225
J5-J19	P18	7.551	191.8	225
<b>Optimal cost (\$)</b>			<b>737920</b>	
<b>Runtime</b>			<b>1:07:00</b>	

### 3.4 Comparison of CFO, DE, MILP and classical methods

Shahinezhad [2010] optimize this network by using mixed integer linear programming method. In this paper the network is optimized by Central Force Optimization (CFO) and Differential Evolution Algorithm (DE), the results are compared with absolute optimum that is obtained from Mixed Integer Linear Programming (MILP) by shahinezhad. Table 9 shows the results of optimizing from CFO, DE, MILP and classic method.

In all optimization methods, the factor of time is important. MILP method to find absolute optimum needs more time than CFO, that it's one of the disadvantages of this method. Although MILP Method achieves the absolute optimum, this method is not recommended in the engineering works that the time is important. The biggest problem in this method is that this method cannot be used in the loop network. So you cannot use this method to networks that combine the loop and branched network (complex network).

In this study, we compared the CFO with this method, Therefore, According to great

potential of CFO, the algorithm can be used in the loop, branch and complex network. In Table 10 optimal cost obtained by each method can be seen.

Table 9: Optimum diameter from CFO, DE, MILP method and classic method

Pipe	No. Pipe	Optimum Diameter (mm)			
		CFO Algorithm	DE Algorithm	Classic Method	MILP Method
Res.-J1	P1	-	-	-	-
J2-J3	P2	800	800	900	800
J3-J4	P3	800	800	900	800
J4-J5	P4	600	600	700	600
J4-J6	P5	250	225	250	225
J4-J7	P6	355	355	355	355
J4-J8	P7	500	500	500	500
J8-J9	P8	450	450	500	450
J9-J10	P9	355	355	400	355
J10-J11	P10	250	250	250	250
J11-J12	P11	140	140	160	140
J5-J13	P12	450	450	400	400
J13-J14	P13	355	355	315	355
J5-J15	P14	315	315	315	315
J15-J16	P15	180	180	200	180
J5-J17	P16	355	355	400	355
J17-J18	P17	225	225	250	225
J5-J19	P18	225	225	160	225

Table 10: Inlet pressure head and network cost by CFO, DE, Classic method and MILP Method

Methods	Inlet Pressure Head (m)	Cost (\$)
MILP Method	139.66	726463
CFO Algorithm	134.69	737924
DE Algorithm	135.37	737920
Classic Method	140	825935

According to Table 10, it can be said that algorithm presents very good results for optimizing water distribution network. So that Central Force Optimization estimates cost, %1.61 more than the lowest cost (MILP Method) and Differential Evolution algorithm estimates cost, % 1.57 more than that.

#### 4. CONCLUSION

In this study, to optimize water distribution network used CFO and DE algorithm. Since probes in the proposed CFO method share global and individual information, this method has been accounted as one of the intelligent swarm algorithms and having mutation operator enhance increasing the global exploration ability and convergence velocity.

Conclusions show CFO runtime is less than the MILP method that provides absolute optimum. While optimization of CFO (737,924 \$) is %1.61 more than the absolute optimum

that determined by the MILP method. Also, CFO estimates cost %10.61 less than classic method.

In DE algorithm, the best scale and probability coefficients (F and Cr) are 0.6 and 0.5, respectively. About the initial population and the number of generations investigation revealed that the initial population of 100 members and generations 200 are the best, in terms of time and efficiency.

While optimization of differential evolution algorithm (737,920 \$) is %1.57 more than the absolute optimum that determined by the MILP method. Also, DE algorithm estimates cost %10.66 less than classic method.

Conclusions show CFO algorithm runtime is less than DE algorithm and DE algorithm runtime is less than MILP method that provides absolute optimum.

Another advantage of CFO and DE in comparison with MILP method is that CFO and DE can be used in the loop network and complex network. Whereas MILP Method is unable to solve loop and complex network (loop and branch).

About major networks with many pipes, using CFO and DE is recommended compared with MILP method and other evolutionary algorithms, because of high-speed runtime and convergence to reach the optimum.

## REFERENCES

1. Swamee PK, Sharma AK. *Design of Water Supply Pipe Networks*, John Wiley, Sons Inc Hoboken NJ., 2008
2. Schaake J, Lai D. *Linear programming and Dynamic Programming –Application of Water Distribution Network Design*, Report 116 MIT Press: Cambridge MA, 1969
3. Haghghi A, Ramos HM. Detection of leakage freshwater and friction factor calibration in drinking networks using Central Force Optimization, *Water Res Manag* 2012; **26**(8): 2347-63.
4. Kaveh A, Ahmadi B, Shokohi F, Bohlooli N. Simultaneous analysis, design and optimization of water distribution systems using supervised charged system search, *Int J Optim Civil Eng* 2013; **3**(1): 37-55.
5. Tahershamsia A, Kaveh A, Sheikholeslamia R, Talatahari S. Big bang - big crunch algorithm for least-cost design of water distribution systems, *Int J Optim Civil Eng* 2012; **2**(1): 71-80.
6. Vairavamoorthy K, Ali M. Optimal design of water distribution systems using genetic algorithms, *Comput-Aided Civil Infrastruct Eng* 2000; **15**(2): 374-82.
7. Montesinos P, Guzman AG, Ayuso JL. Water distribution network optimization using a modified genetic algorithm, *Water Res Res* 1999; **35**(11): 3467-73.
8. Kaveh A, Shokouhi F, Ahmadi B. Analysis and design of water distribution systems via colliding bodies optimization, *Int J Optim Civil Eng* 2014; **2**(4): 165-85.
9. Neelakantan TR, Suribabu CR. Optimal design of water distribution networks by a modified genetic algorithm, *J Civil Environ Eng* 2005; **1**(1): 20-34.
10. Kaveh A, Shokohi F, AhmadiKadu B. Optimal analysis and design of water distribution systems using tug of war optimization algorithm, *Int J Optim Civil Eng* 2017; **7**(2): 193-210.

11. Cunha M, Sousa J. Water distribution network design optimization: simulated annealing approach, *J Water Res Plan Manag* 1999; **125**(4): 215-21.
12. Eusuff MM, Lansey KE. Optimization of water distribution network design using the shuffled frog leaping algorithm, *J Water Res Plan Manage* 2003; **129**(3): 210-25.
13. Madadgar S, Afshar A. Forced water main design mixed ant colony optimization, *Int J Optim Civil Eng* 2011; **1**(1): 47-71.
14. Sheikholeslami R, Kaveh A, Tahershamsi A, Talatahari S. Application of charged system search algorithm to water distribution networks optimization, *Int J Optim Civil Eng* 2014; **1**(4): 41-58.
15. Keedwell E, Khu ST. Novel cellular automata approach to optimal water distribution network design, *J Comput Civil Eng* 2006; **20**(1): 49-56.
16. Suribabu CR, Neelakantan TR. Design of water distribution networks using particle swarm optimization, *Urban Water J* 2006; **3**(2): 111-20.
17. Suribabu CR. Differential evolution algorithm for optimal design of water distribution networks, *J Hydroinform* 2010; **12**(1): 66-82.
18. Sheikholeslami R, Kaveh A. Vulnerability assessment of water distribution networks: graph theory method, *Int J Optim Civil Eng* 2015; **3**(5): 283-99.
19. Formato RA. Central force optimization: a new metaheuristic with applications in applied electromagnetic, *Prog Electromagnetics Res* 2007; **77**(1): 425-91.
20. Formato RA. Improved CFO algorithm for antenna optimization, *Prog Electromagnetics Res B* 2010a; **19**: 405-25.
21. Formato RA. Central force optimization applied to the PBM suite of antenna benchmarks, Computing Research Repository abs/1003.0221, 2010b.
22. Ding D, Luo X, Chen J. A convergence proof and parameter analysis of central force optimization algorithm, *J Conver Inform Technol* 2011; **6**(10): 16-23.
23. Babu BV, Angira R. Optimization of water pumping system using differential evolution strategies, In: *Proceedings of the Second International Conference on Computational Intelligence, Robotics, and Autonomous Systems (CIRAS- 2003)*, Singapore, 2003, pp. 25–30.
24. Janga Reddy M, Nagesh Kumar D. Multi-objective differential evolution with application to reservoir system optimization, *J Comput Civil Eng* 2007; **21**(2): 136-46.
25. Vasan A, Raju K. Application of differential evolution for irrigation planning: an Indian case study, *Water Res Manag* 2007; **21**(8): 1393-1407.
26. Shahinezhad B. Optimal design of water distribution networks using mixed integer linear programming, Ph.D. thesis, Chamran University, Ahvaz, Iran, 2011
27. Rossman LA. EPANET 2 User's Manual, EPA/600/R-00/057, 2000
28. Storn R, Price K. Differential evolution - a simple and efficient heuristic for global optimization over continuous spaces, *J Global Optim* 1997; **11**: 341-359.