



## OPTIMAL DESIGN OF ARCH DAMS BY COMBINING PARTICLE SWARM OPTIMIZATION AND GROUP METHOD OF DATA HANDLING

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### ABSTRACT

Optimization techniques can be efficiently utilized to achieve an optimal shape for arch dams. This optimal design can consider the conditions of the economy and safety simultaneously. The main aim is to present an applicable and practical model and suggest an algorithm for optimization of concrete arch dams to enhance their seismic performance. To achieve this purpose, a preliminary optimization is accomplished using PSO procedure in the first stage. Capabilities of Ansys Parametric Design Language (APDL) are applied for modeling the Dam-Foundation-Reservoir system. In the second stage with training the neural network, Group Method of Data Handling (GMDH) and replacement of Ansys analyst, optimal results have been achieved with the lowest error and less number of iteration respectively. Then a real world double-arch dam is presented to demonstrate the effectiveness and practicality of the PSO-GMDH. The numerical results reveal that the proposed method called PSO-GMDH provides faster rate and high searching accuracy to achieve the optimal shape of arch concrete dams and the modification and optimization of shape have a quite important role in increasing the safety against dynamic design loads.

**Keywords:** optimization; concrete arch dams; particle swarm optimization; group method of data handling; neural network.

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### 1. INTRODUCTION

Dams are one of the most important structures in engineering because of the economical and social utilization, largeness of structural scale and the intensity of sensitivity to the damages. Moreover, proper shape designing and modelling of double-arch dams has been considered

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them as an important problem in dam engineering.

In double-arch dams, geometry of the structure has a great influence on safety and economy of design and also is an important factor on stability of the dam. During the last years, various studies related to optimized design of arch dams were reported in Refs. [28, 9]. In these studies, the effect of dam-water-foundation rock interaction was neglected. In recent years, the optimal shape design of arch dams considering dam-water-foundation rock interaction subjected to earthquake loading has been developed by few researchers [1, 10, 14].

Due to accelerating the convergence for exploring and finding promising regions in the search space, meta-heuristic optimization methods are quite suitable for global or near-global searches. The particle swarm optimization (PSO) is one of the most popular meta-heuristic algorithms which is inspired by the social behavior of flock population. Despite high speed of convergence in finding optimum design in the search space for global searches, facing a great number of structural analyses for finding solution in complex problems, slow searching speed especially in last iterations and trapping into local optimums are the most important deficiencies of PSO [15].

In this study, the concrete weight and the geometric parameters of the arch dam have been considered as the objective function and design variables, also principle stresses, sliding, overturning and eccentricity constrains have been defined as the constraints of the optimization process. For this purpose arch dam-water-foundation rock system has been simulated using the finite element method. In order to assess the validity of finite element method, the result of model have been compared with those of valid models provided in the literature and its performance has been verified [2].

A preliminary optimization is accomplished using PSO procedure in the first stage of PSO-GMDH. With the first optimization results a neural network has been built. Then it will be replaced instead of Ansys analysts in the next stages of the optimization. The results demonstrated the computational efficiency of particle swarm optimization (PSO). In addition, using approximation method could significantly reduce the total time of double arch dam optimization while having a high accuracy.

## 2. GEOMETRICAL MODEL OF DOUBLE-ARCH DAM

The most important stage of shape optimization of double arch dam is selecting an appropriate geometrical model. In this study to define the upstream curve of dam central section, as shown in Fig. 1, a polynomial of 3rd order is considered as following [2]:

$$y(z) = b(z) = -S_1z + \frac{S_1 - \beta^2(S_1 + S_2)}{2\beta h(1 - \beta)} - \frac{S_1 - \beta(S_1 + S_2)}{3\beta h^2(1 - \beta)}z^3 \quad (1)$$

where  $h$ ,  $S_1$  and  $S_2$  are the height of dam, the slope at crest and the foundation of dam, also the point where the slope of upstream curve equals to zero is a position  $Z = \beta h$  in which  $0 < \beta \leq 1$  is constant.

By dividing the height of dam into  $n$  segment containing  $n + 1$  levels, the thickness of the central vertical section of dam is expressed as following:

$$t_c(z) = \sum_{i=1}^{n+1} L_i(Z) t_{ci} \quad (2)$$

where  $t_{ci}$  indicates the thickness of the central vertical section at  $i$ th level. Also, in the above mentioned equation,  $L_i(Z)$  is the Lagrange interpolation function corresponding to  $i$ th level shown as:

$$L_i(z) = \frac{\prod_{k=1, k \neq i}^{n+1} (z - z_k)}{\prod_{i=1}^{n+1} (z_i - z_k)}, \quad k \neq i \quad (3)$$

where  $z_i$  is the  $z$  coordinate of  $i$ th level in the vertical section of dam.

For the radii of curvature corresponding to upstream and downstream levels of dam ( $r_u, r_d$ ), as shown in Fig.2, two functions of  $n$ th order with respect to  $z$  can be utilized as:

$$r_u(z) = \sum_{i=1}^{n+1} L_i(Z) r_{ui} \quad (4)$$

$$r_d(z) = \sum_{i=1}^{n+1} L_i(Z) r_{di} \quad (5)$$

In this stage, the shape of a double-arch dam can be defined using the two parabolic surfaces as [23]:

$$y_u(x, u) = \frac{1}{2r_u(z)} x^2 + b(z) \quad (6)$$

$$y_d(x, u) = \frac{1}{2r_d(z)} x^2 + b(z) + t_c(z) \quad (7)$$

In which  $y_u$  and  $y_d$  are the levels of upstream and downstream.

According to the models shown in Fig. 1 and Fig. 2, a double arch dam can be created by a vector called  $X$  that has  $(3n + 7)$  components including shape parameters of concrete double - arch dam as:

$$X^T = \{S_1, S_2, \beta, x_p, t_{c1}, t_{c2}, \dots, t_{cn+1}, r_{u1}, r_{u2}, \dots, r_{un+1}, r_{d1}, r_{d2}, \dots, r_{dn+1}\} \quad (8)$$

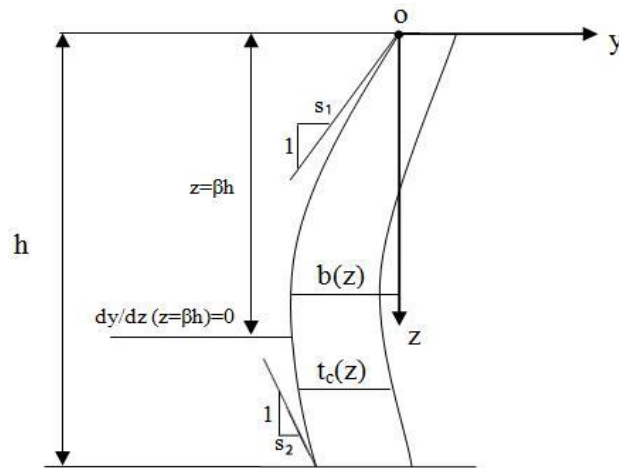


Figure 1. Central vertical section of double- arch dam [2]

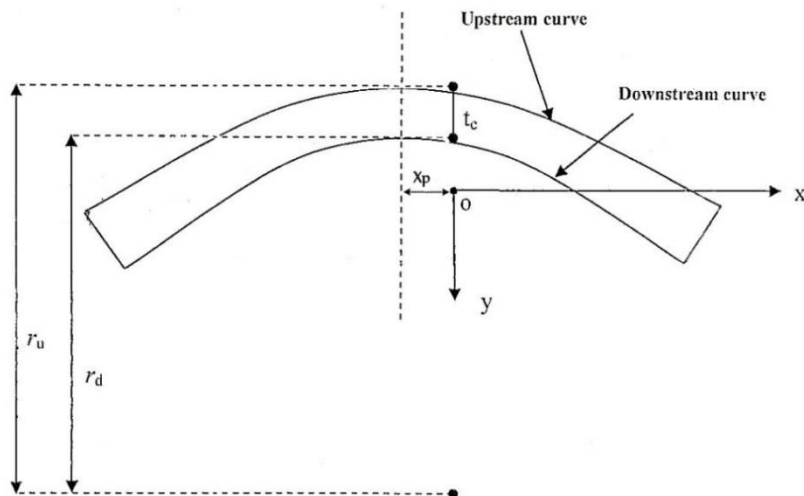


Figure 2. Section of double – arch dam [2]

### 3. FINITE ELEMENT MODEL OF DAM-WATER-FOUNDATION ROCK SYSTEM

In solving the fluid-structure interaction problem using finite element method (FEM), the discretized dynamic equations of the fluid and structure need to be considered simultaneously to obtain the coupled fluid-structure equation.

#### 3.1 Structural responses

The solid dam is discretized using finite elements and its equations of seismic motion including the effects of the reservoir and the foundation are expressed as [24].

$$M_s \ddot{u}_e + C_s \dot{u}_e + K_s u_e = -M_s \ddot{u}_g + Q p_e \quad (9)$$

where  $M_s$  is the structural mass matrix,  $C_s$  is the structural damping matrix,  $K_s$  is the structural stiffness matrix,  $u_e$  is the vector of the nodal displacements relative to the ground,  $\ddot{u}_g$  is the vector of the ground acceleration, and  $Q p_e$  represents the nodal force vector associated with the hydrodynamic pressures produced by the reservoir.

The structural damping in the system is usually included by a Rayleigh type of damping matrix given by [24]:

$$C_s = \bar{\alpha} M_s + \bar{\beta} K_s \quad (10)$$

where  $\bar{\alpha}$  and  $\bar{\beta}$  are constants adjusted to obtain a desirable damping in the system, usually on the basis of given modal damping ratios.

### 3.2 Reservoir responses

For a compressible and inviscid fluid, the hydrodynamic pressure  $p$  resulting from the ground motion of the rigid dam satisfies the wave equation in the form [25, 26].

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad (11)$$

where  $c$  is the velocity of sound in water and  $\nabla^2$  is the Laplacian operator.

The following boundary conditions are defined by assuming that the effects of surface waves and the viscosity of the fluid are neglected. As shown in Fig. 3, some boundary conditions may be imposed on the fluid domain as follows:

- I. At the fluid-solid interface,

$$\frac{\partial p}{\partial n} = -\rho_w a_n \quad (12)$$

where  $n$  is the unit normal vector,  $a_n$  is the normal acceleration on the interface and  $\rho$  is the mass density of the fluid.

- II. At the bottom of the fluid domain

$$\frac{\partial p}{\partial n} = -\rho a_n - \bar{q} \frac{\partial p}{\partial t} \quad (13)$$

where  $\bar{q}$  is a damping coefficient which is the fundamental parameter characterizing the effects of the reservoir bottom materials. relation between damping coefficient and ratio of reflected hydrodynamic pressure wave,  $\alpha$ , is [27]:

$$\alpha = \frac{1 - \bar{q}c}{1 + \bar{q}c} \quad (14)$$

- III. At the far end; a summerfield –type radiation boundary condition can be implemented, namely:

$$\frac{\partial p}{\partial n} = -\frac{\dot{p}}{c} \quad (15)$$

- IV. At the free surface:

$$p = 0 \quad (16)$$

Eqs. (11) and (15) can be discretized to get the matrix form of the wave equations as [25, 26].

$$M_f \ddot{p}_e + C_f \dot{p}_e + K_f p_e + \rho_w Q^T (\ddot{u}_e + \ddot{u}_g) = 0 \quad (17)$$

where  $M_f$ ,  $C_f$  and  $K_f$  are the fluid mass, damping and stiffness matrices, and  $p_e$  and  $\ddot{u}_e$  are the nodal pressure and relative nodal acceleration vectors, respectively. The term  $\rho_w Q^T$  is also often referred to as coupling matrix.

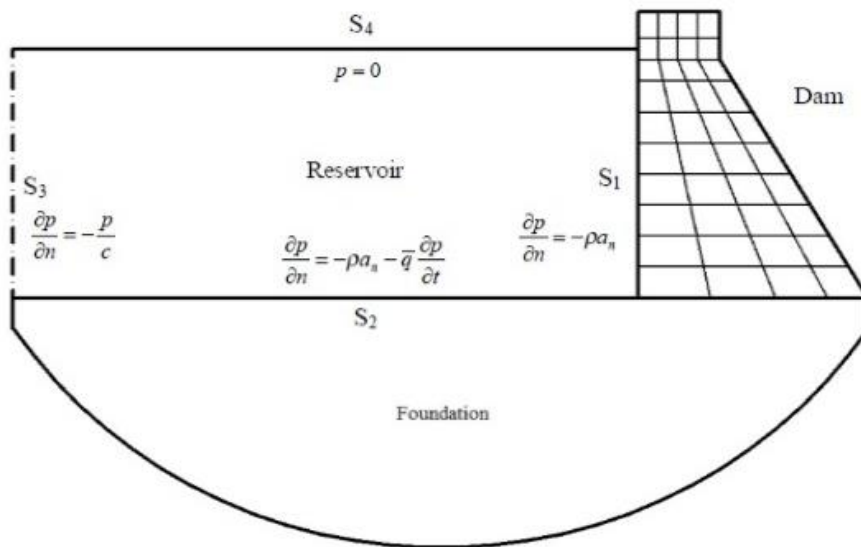


Figure 3. The boundary conditions of the fluid domain

### 3.3 The coupled fluid- structure equation

The complete finite element discretized equations for the dam-water-foundation rock interaction problem are described by Eqs. (9) and (17) and they can be written in an assembled form as [25, 26]:

$$\begin{bmatrix} M_s & 0 \\ M_{fs} & M_f \end{bmatrix} \begin{Bmatrix} \ddot{u}_e \\ \ddot{p}_e \end{Bmatrix} + \begin{bmatrix} C_s & 0 \\ 0 & C_f \end{bmatrix} \begin{Bmatrix} \dot{u}_e \\ \dot{p}_e \end{Bmatrix} + \begin{bmatrix} K_s & K_{sf} \\ 0 & K_f \end{bmatrix} \begin{Bmatrix} u_e \\ p_e \end{Bmatrix} = \begin{Bmatrix} -M_s \ddot{u}_g \\ -M_{fs} \ddot{u}_g \end{Bmatrix} \quad (18)$$

where  $M_{fs} = \rho_w Q^T$  and  $K_{sf} = -Q$ . Eq. (18) expresses a second order linear differential equation having unsymmetrical matrices and may be solved by means of direct integration methods.

In the present study, FEM of double curvature arch dam including the interaction effects of dam-water-foundation rock system is based on the above mentioned theory and assumption. The arch dam-water-foundation rock system as a three dimensional linear structure is subjected to earthquake loading. The foundation rock without mass is considered flexibly and the inertia and damping effects of the foundation rock are neglected. [25, 26]

The foundation rock is extended to three times of dam height in upstream, downstream and downward directions. In the analysis phase, first, a static analysis of the arch dam-water-foundation rock system is implemented under a gravity load and a hydrostatic pressure, and then the dynamic analysis of the system is performed using the Newmark time integration method [24].

Therefore the nodal relative displacement vector of arch dam is used to evaluate the principle stresses at the center of dam elements via conventional finite-element procedure.

#### 4. OPTIMIZATION PROBLEM OF DOUBLE-ARCH DAM

The general shape of an optimization problem is defined as follows:

$$\text{Find } X = \{x^1, x^2, \dots, x^d\}^T$$

$$\text{To minimize } f(X) \quad (19)$$

$$\text{Subjected to } g_i(X, t) \leq 0; \quad i = 1, 2, \dots, m; \quad t = 0, \dots, Te$$

$$X^L \leq X \leq X^U$$

where  $f$ ,  $g_j$  and  $t$  are the objective function,  $i$ th constraint from  $m$  inequality constraints and the time.  $X^L$  and  $X^U$  are the lower bound and the upper bound of the design variables and  $Te$  is the earthquake duration.

##### 4.1 Objective function

The most effective design variables of double-arch dam are its geometrical parameters. The design variables are expressed as:

$$X = \{S_1, S_2, \beta, t_{c1}, t_{c2}, \dots, t_{cn+1}, r_{u1}, r_{u2}, \dots, r_{un+1}, r_{d1}, r_{d2}, \dots, r_{dn+1}\}^T \quad (20)$$

In the present study, six different levels in the height of arch dam are considered  $n = 5$ , hence the vector of the design variables has 22 elements. Because of the symmetrical shape of double-arch dam, the vector of variables consists of  $2l$  variables. The concrete volume of the arch dam body is considered as the objective function, which can be obtained from integrating the arch dam surfaces as:

$$f(X) = \iint_A |y_d(x, z) - y_u(x, z)| dx dz \quad (21)$$

where  $y_d$  and  $y_u$  are the upstream and downstream surfaces of the arch dam, and  $A$  is an area produced by projecting the dam body on  $xz$  plane.

#### 4.2 Design constraints

In this study, the design constraints include behavior, geometric and stability constraints. The principle stresses in the center of each element are considered as the behavior constrain that can be defined as:

$$\sigma_C^t \leq k_1 f_c \quad (22)$$

$$\sigma_T^t \leq k_2 f_t \quad (23)$$

where  $\sigma_C^t$  and  $\sigma_T^t$  are the principal compression and tension stresses in time  $t$ , respectively  $f_c$  and  $f_t$  are the uniaxial compressive strength and the uniaxial tension strength for concrete.  $k_1$  and  $k_2$  are the incremental coefficients which are considered as 1.3 and 1.5 in this study. Therefore these constraints are defined as:

$$g_{C,e}(X, t) = \frac{\sigma_C^t}{k_1 f_c} - 1, \quad t = 0, \dots, T, \quad e = 1, 2, \dots, ne \quad (24)$$

$$g_{T,e}(X, t) = \frac{\sigma_T^t}{k_2 f_t} - 1, \quad t = 0, \dots, T, \quad e = 1, 2, \dots, ne \quad (25)$$

To control the stability of arch dams, the central angle of dam is limited in various levels of height. This constraint is expressed as follows:

$$\varphi^L \leq \varphi_i \leq \varphi^U \Rightarrow \left\{ \begin{array}{l} g_{S,i}^U = \frac{\varphi_i}{\varphi^U} - 1 \\ g_{S,i}^L = 1 - \frac{\varphi_i}{\varphi^L} \end{array} \right\} \quad i = 1, 2, \dots, n + 1 \quad (26)$$

where  $\varphi^L$  and  $\varphi^U$  are the central angle of dam for downstream and upstream levels in  $i$ th level which is equal to  $90^\circ$  and  $130^\circ$ .

The most important geometric constraint is to prevent from intersecting of downstream and upstream faces optimization process as:

$$r_{di} \leq r_{ui} \Rightarrow g_{G,i} = \frac{r_{di}}{r_{ui}} - 1 \quad i = 1, 2, \dots, n + 1 \quad (27)$$

where  $r_{di}$  and  $r_{ui}$  are the radii of curvature at the down and upstream faces of the dam in  $i$ th position in  $z$  direction.

The slope of curve in the central section of crest and foundation is considered as



geometric constraint, which is defined as:

$$S_1 \leq S_{all} \Rightarrow g_{S1} = \frac{S_1}{S_{all}} - 1 \quad (28)$$

$$S_2 \leq S_{all} \Rightarrow g_{S2} = \frac{S_2}{S_{all}} - 1 \quad (29)$$

where  $S_1$  and  $S_2$  are the slope of curve in the central section of crest and foundation. In this study the thickness of the central vertical section in the height of dam as geometric constraint is limited as follows:

$$t^L \leq t_{c,i} \leq t^U \Rightarrow \begin{cases} g_{t,L} = \frac{t_{c,i}}{t^L} - 1 \\ g_{t,U} = 1 - \frac{t_{c,i}}{t^U} \end{cases} \quad i = 1, 2, \dots, n + 1 \quad (30)$$

where  $t^L$  and  $t^U$  are the upper and lower value of the thickness of the central vertical section dam.

Also, to achieve the acceptable shape of arch dam in various levels, the following constraint is considered as a geometric constraint:

$$t_{c,i} \leq t_{c,i+1} \Rightarrow g_{tC} = \frac{t_{c,i}}{t_{c,i+1}} - 1 \quad i = 1, 2, \dots, n \quad (31)$$

#### 4.3 Penalty function

In the present study, the external penalty function method is employed to transform constrained optimization problem into unconstrained one as follows [29, 31]:

$$\check{f}(X) = f(X)(1 + r_p Pf) \quad (32)$$

where  $\check{f}(X)$ ,  $Pf$  and  $r_p$  are the modified function, the penalty function and an adjusting coefficient. The penalty function based on the violation of normalized constraints [29] is expressed as the sum of all active constraints violation as follows:

$$P_f = \sum_{t=0}^{Te} \max \left[ \sum_i \max(g_i(X, t), 0.0)^2, 0.0 \right] \quad (33)$$

This formulation allows solutions with violated constraints and the objective function is always greater than non-violated one. Therefore, the penalty function ( $P_f$ ) in the optimization problem of arch dam is defined as the sum of all active constraint violations that expressed in Eqs. (24-31).

## 5. OPTIMIZATION METHOD

In this study, the optimization method based on a combination of PSO and GMDH is presented for finding the optimal shapes of double-arch dams. In this section PSO and GMDH methods are introduced at first, then the proposed PSO-GMDH is described.

### 5.1 Particle swarm optimization (PSO)

PSO was introduced by Kennedy and Eberhart (15) in the mid 1990s. this method of optimization is inspired by social behaviour of animals such as fish, insects and birds. PSO involves a number of particles that are initialized randomly in the search space of an objective function. Each particle of the swarm represents a potential solution of the optimization problem. The  $i$ th particle in the  $t$ th iteration is associated with a position vector  $X_i^t$ , and a velocity vector  $V_i^t$ , that are shown in the following, where  $d$  is the dimension of the solution space.

$$X_i^t = \{x_{i1}^t, x_{i2}^t, \dots, x_{id}^t\} \quad (34)$$

$$V_i^t = \{v_{i1}^t, v_{i2}^t, \dots, v_{id}^t\} \quad (35)$$

The particle fly through the solution space and its position is updated based on its velocity, the best position particle (p best) and the global best position (g best) that swarm has visited since the first iteration as:

$$V_i^{t+1} = w^t V_i^t + c_1 r_1 (pbest_i^t - X_i^t) + c_2 r_2 (gbest^t - X_i^t) \quad (36)$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \quad (37)$$

where  $r_1$  and  $r_2$  are two uniform random sequences generated from interval  $[0,1]$ ,  $c_1$  and  $c_2$  are the cognitive and social scaling parameters and  $w^t$  is the inertia weight that controls the influence of the previous velocity. The performance of PSO is very sensitive to the inertia weight ( $w$ ) parameter which may decrease with the number of iteration as follows [33]:

$$w = w_{max} - \frac{w_{max} - w_{min}}{t_{max}} t \quad (38)$$

where  $w_{max}$  and  $w_{min}$  are the maximum and minimum values of  $w$ , and  $t_{max}$  is the limit numbers of optimization iteration.

### 5.2 Group method of data handling (GMDH)

The GMDH is a multilayer self organizing procedure able to build a polynomial approximation of the relationship between a dependent variable  $y$  (output) and a number of independent variables  $x_1, \dots, x_n$  (inputs). The original idea was proposed by Ivakhnenko in 1970. By means of the GMDH algorithm, a model can be represented as a set of neurons in which different pairs in each layer are connected through a quadratic polynomial and thus produce new neurons in the next layer. The formal definition is to find an approximate function  $\hat{f}$  so that it can be used instead of the actual one  $f$ , in order to predict output  $\hat{y}$  for a

given input vector  $X = (x_1, x_2, x_3, \dots, x_n)$  as close as possible to the actual  $y$ . [5, 8]

General connection between inputs and outputs variables can be expressed by a complicated discrete form of

$$y = a_0 + \sum_{i=1}^n a_i x_i + \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n a_{ijk} x_i x_j x_k + \dots \quad (39)$$

which is known as the Kolmogorov-Gabor polynomial. [12] This full mathematical description can be represented by a system of partial quadratic polynomials consisting of only two variables (neurons) in the form of:

$$\hat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^2 + a_5 x_j^2 \quad (40)$$

To predict the output  $y$ , a typical feed forward GMDH – type network is shown in Fig. 4. The coefficient  $a_i$  in Eq. (40) is calculated using regression techniques [11, 13] so that the difference between actual output,  $y$  and the calculated one  $\hat{y}$ , for each pair of  $x_1, x_j$  as input variables is minimized.

Indeed, it can be seen that a tree of polynomials is constructed using the quadratic form given in Eq. (40) whose coefficients are obtained in a least-square sense. In this way, the coefficients of each quadratic function  $G_i$  are obtained to optimally fit the output in the whole set of input - output data pair, that is

$$r^2 = \frac{\sum_{i=1}^M (y_i - \hat{y})^2}{\sum_{i=1}^M (y_i)^2} \Rightarrow \min \quad (41)$$

In the basic form of the GMDH algorithm, all the possibilities of two independent variables out of a total of  $n$  input variables are taken in order to construct the regression polynomial in the form of Eq. (40) that best fits the dependent observation  $y_i$  ( $i=1,2,\dots,M$ ) in a least square sense. Consequently,

$$\binom{n}{2} = \frac{n(n-1)}{2}$$

Neurons will be built up in the second layer of the feed forward network from the observations. [12] In other words, it is now possible to construct  $M$  data triples  $\{(y_i, x_{ip}, x_{iq}); (1,2, \dots, M)\}$  from observation using such  $p, q \in \{1,2, \dots, n\}$  in the form of

$$\begin{bmatrix} x_{1p} & x_{1q} & y_1 \\ x_{2p} & x_{2q} & y_2 \\ \vdots & \vdots & \vdots \\ x_{Mp} & x_{Mq} & y_M \end{bmatrix}$$

Using the quadratic sub\_ expression in the form of Eq. (40) for each row of  $M$  data triples, the following matrix equation can be obtained as:

$$Aa = Yt \quad (42)$$

where  $a$  is the vector of unknown coefficients of the quadratic polynomial in Eq. (40),

$a = \{a_0, a_1, a_2, a_3, a_4, a_5\}$  and  $Y = \{y_1, y_2, y_3, \dots, y_M\}^T$  is the vector of output values from observations. It can be readily seen that the least – square technique from the multiple-regression analysis leads to the solution of the normal equations in the form of

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x_{Mp} & x_{Mq} & x_{Mp}x_{Mq} & x_{Mp}^2 & x_{Mq}^2 \end{bmatrix} \quad (43)$$

$$a = (A^T.A)^{-1}A^TY \quad (44)$$

which determines the vector of the best coefficients of the quadratic Eq. (40) for the whole set of  $M$  data triples. It should be noted that this procedure is repeated for each neuron of the next hidden layer according to the connectivity topology of the network. However, such a solution directly from normal equations is rather susceptible to round off errors and more importantly to the singularity of these equations [5].

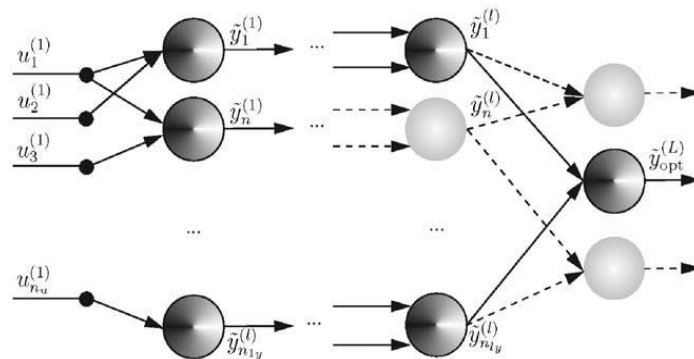


Figure 4. A feed forward GMDH-type network [5]

### 5.3 The proposed PSO-GMDH

In the optimization problem of large scale structures such as double arch dams, the structural analysis is time-consuming procedure. Furthermore, the optimization problem of double arch dam on shape has many design variables. Due to the aforementioned problems and the slow searching speed of PSO in the iterations, it can easily trap in local optimum, and therefore the exploitation capability of PSO is decreased.

In the first stage of PSO-GMDH, a preliminary optimization is accomplished using PSO with Ansys analysis. In the second stage, with a number of particles randomly selected from the search space and the outputs that have been obtained from precise analysis, the GMDH network has been built. In the GMDH network the available data is separated into training and testing sets. At each layer, all possible input pairs are generated and the output of each layer is fitted to the training set. In fact, only  $n$  out of  $m$  outputs of a stage are chosen as intermediate variables and passed as inputs to the next stage.

The procedure can be stopped according to one of the following two criteria. One is based on the rate of decrease of the approximation error, so that the algorithm can be stopped when this rate goes below a fixed threshold. The other criterion takes into account

the maximum admissible degree of the polynomial model, so that the algorithm is stopped when the degree of the approximating polynomial reaches that value.

The PSO-GMDH method is executed by the following steps:

Step 1: Utilize the PSO algorithm and find the optimum design with Ansys analyst.

Step 2: Create neural network, GMDH with initial swarm of PSO and outputs of step1

Step 3: Utilize the PSO with random selection of the swarm and replace GMDH network instead of Ansys.

The algorithm flow of PSO – GMDH strategy is shown in Fig. 5.

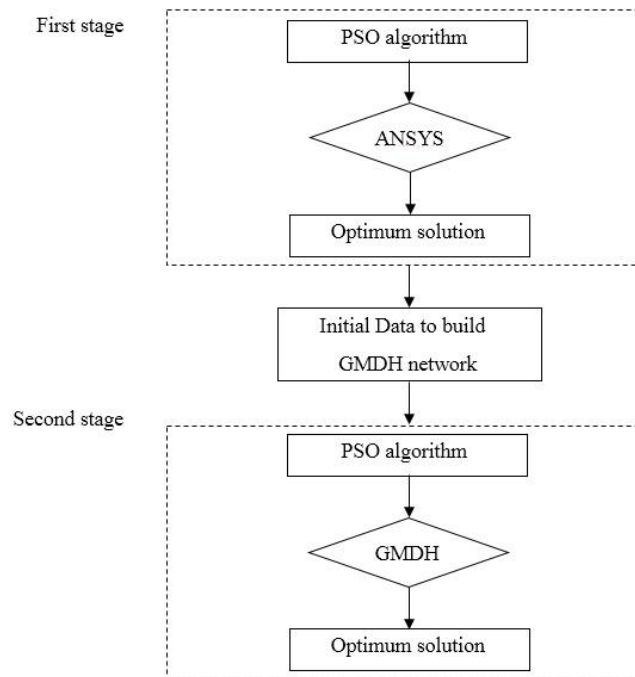


Figure 5. The flow chart of the proposed PSO – GMDH

#### 5.4 Verification of the proposed PSO\_GMDH

The performance of the proposed PSO\_GMDH depends on some parameters. The appropriate selection of these parameters can lead to better solution. In this proposed optimization method, a great number of neurons are evaluated in each layer in order to achieve the optimal solution. However, there is no possibility of developing all neurons for each layer, where some of them have to be used for next layer. Thus a specified number of neurons will be chosen regarding the difference between approximate output error and precise error in each layer. As it follows, the parameter  $e_c$  will be defined as:

$$e_c = \alpha e_{min} + (1 - \alpha)e_{max} \quad (45)$$

where  $e_{min}$  and  $e_{max}$  are the minimum and maximum error of neurons and  $\alpha$  is the selected pressure parameter in which  $0 < \alpha < 1$ .

The output of each layer are defined as the inputs of next layer in this method, so to control the elements of network, the number of layers and neurons will be defined as well. In this study, to achieve the optimal design of an arch dam-water-foundation rock system subjected to earthquake loading, the evaluation of dam requires a dynamic analysis of FEM. Hence, the dynamic analysis of arch dam has high computational effort. Also, the stochastic nature of evolutionary search technique (FEA) makes the convergence of the process slow and the slow searching speed of PSO in the iteration can easily trap it in to local optimum but the proposed PSO\_GMDH method can efficiently accelerate the optimization process and reduce the computational cost.

## 6. TEST EXAMPLE AND RESULTS

In order to investigate the computational efficiency of the proposed meta-heuristic optimization method for the shape optimization of double-arch dams, Morrow Point arch dam, located 263 km southwest of Denver, Colorado, is considered as a real world structure. The dam structure is 143 m high with a crest length of 221 m and its construction consumed about  $273600 \text{ m}^3$  concrete. The detailed properties of the arch dam-water system have been provided in Ref [32].

### 6.1 The shape optimization of double arch dam

In the present study, twenty one variables are considered for creating the double arch dam geometry. The lower and upper bounds of design variables required for the optimization process can be determined using some preliminary design methods. These bounds are shown in Table 1. Also, for finding the optimum shape of the arch dam, the properties of concrete water and foundation as shown in Table 2 are considered.

The ground motion recorded at Taft Lincoln School Tunnel during Kern country, California, earthquake of July 21, 1952, is selected as the excitation for analyses of Morrow Point arch dam. The ground motion acting transverse to the axis of the dam is defined by the S69E component of the recorded motion. This component of the recorded ground motion is shown in Fig. 6.

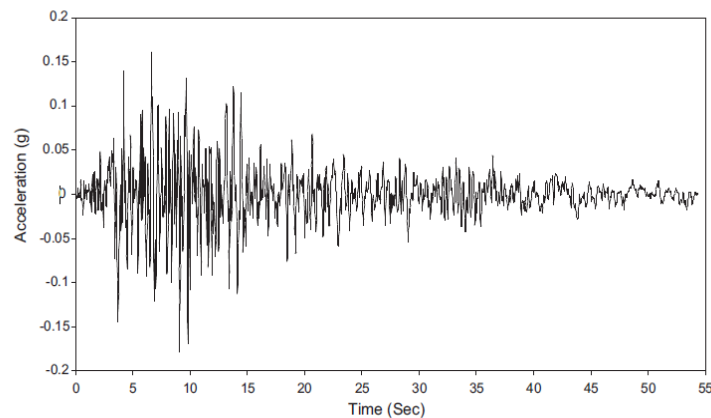


Figure 6. Ground motion at Taft Lincoln Tunnel, Kern country, California, 1952

Table 1: The lower and upper bounds of design variables Morrow Point arch dam

Design variable	Lower bound (m)	Upper bound (m)
$t_{c1}$	5	10
$t_{c2}$	8	15
$t_{c3}$	12.5	20
$t_{c4}$	15	25
$t_{c5}$	17.5	30
$t_{c6}$	20	35
$r_{u1}$	100	135
$r_{u2}$	90	125
$r_{u3}$	80	105
$r_{u4}$	65	90
$r_{u5}$	50	75
$r_{u6}$	30	45
$r_{d1}$	95	125
$r_{d2}$	80	115
$r_{d3}$	70	95
$r_{d4}$	55	75
$r_{d5}$	45	65
$r_{d6}$	25	45
$\beta$	0.5	0.9
$S_1$	0	0.36
$S_2$	0	0.36

Table 2: The properties of materials

Material	Property	Value	Unit
Dam body	Elasticity modulus of concrete	270.58	MPa
	Poisons ratio of concrete	0.2	
	Mass density of concrete	2483	Kg/m <sup>3</sup>
	Uniaxial compressive strength of concrete	30	MPa
	Uniaxial tensile strength of concrete	1.5	MPa
	Biaxial compressive strength of concrete	53.6	MPa
Water	Mass density of water	1000	Kg/m <sup>3</sup>
	Speed of pressure wave	1440	m/s
	Wave reflection coefficient	0.9	-
Foundation rock	Elasticity modulus of foundation rock	27.580	MPa
	Poisson's ratio of foundation rock	0.2	
	Mass density of foundation rock	0	-

The parameters of PSO and PSO\_GMDH used in optimization process are given in Table 3 and 4, respectively.

Table 3: The parameter of PSO method

Swarm size	30
Cognitive parameter (C1)	2
Social parameter (C2)	2
Wmin	0.4
Wmax	0.9
Number of interaction	100

Table 4: The parameter of PSO\_GMDH method

Swarm size	15
Selection pressure parameter	0.6
Maximum number of Layers	20
Maximum neurons of each Layers	40
Training Parameter	0.7
Number of interaction	140

### 6.2 Finite element model of Morrow point arch dam

An idealized FE model of Morrow Point arch dam- water- foundation rock system is simulated using FEM as shown in Fig. 7(a). Also, the FEM of dam body is depicted in Fig. 7(b). The geometric properties of the dam can be found in Ref. [32].

In order to validate FEM with the employed assumptions expressed, the first natural frequency of the symmetric mode of the arch dam for nine cases are determined from the frequency response function for the crest displacement. The results are compared with those reported by Hall and Chopra [32] as given in Table 5.

Table 5: A comparisons of the natural frequencies from the literature with FEM

case	water	Wave reflection coefficient	Foundation rock		Natural frequency (HZ)		
			$\frac{E_f}{E_d}$	condition	Hall and Chopra [32]	Present work	Error %
1	Empty	-	*	Rigid	4.2735	4.27	0.08
2	Empty	-	2	interaction	4.0816	4.06	0.53
3	Empty	-	1	interaction	3.9216	3.88	1.06
4	Empty	-	1/2	interaction	3.6101	3.60	0.28
5	Empty	-	1/4	interaction	3.1746	3.26	2.7
6	full	1	*	Rigid	2.8169	2.74	2.73
7	full	1.5	*	Rigid	3.2258	3.13	2.97
8	Empty	-	1/2	interaction	3.6101	3.10	0.28
9	Empty	-	1/4	interaction	3.1746	3.26	2.7



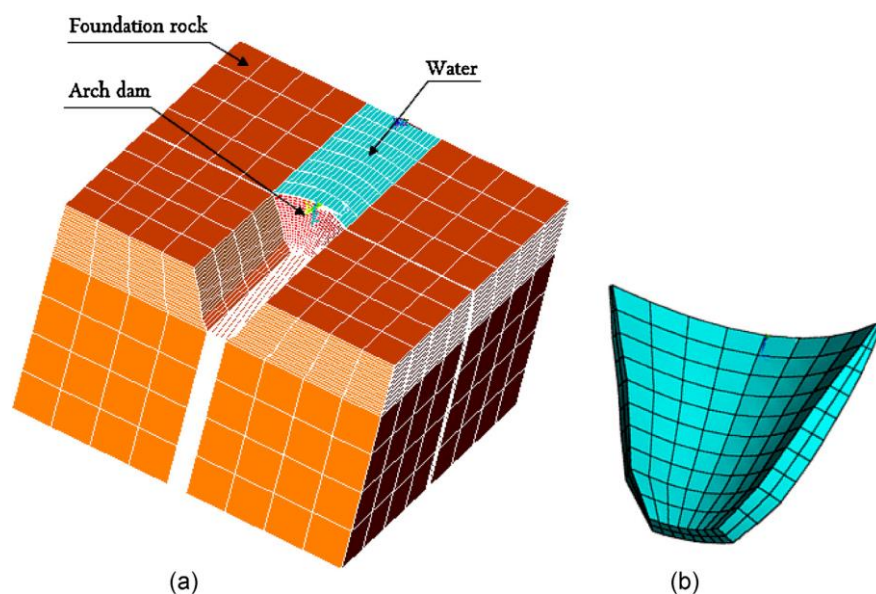


Figure 7. (a) A finite element of Morrow point dam\_water\_foundation system, (b) The finite element of arch dam body

### 6.3 Result of optimization

In order to consider the stochastic nature of the optimization process, 10 independent optimization runs are performed for each method and the three best solutions are reported. The optimum designs of the double arch dam for using PSO and PSO\_GMDH are given in Table 6 and 7, respectively.

As revealed from the results of Fig. 8 and 9, the proposed PSO\_GMDH performs acceptable solution with low error compared with PSO in less time.

Table 6: Optimum designs of the dam obtained by PSO for the three best cases

Variable (m)	Optimum design		
	Case 1	Case 2	Case 3
$t_{c1}$	7.20	5	7.63
$t_{c2}$	10	11.12	10
$t_{c3}$	12.50	12.5	12
$t_{c4}$	15	15	15
$t_{c5}$	17.50	17.50	17.50
$t_{c6}$	20	33.74	20
$r_{u1}$	117.5	100	135
$r_{u2}$	95	95	102.57
$r_{u3}$	80	85	89.89
$r_{u4}$	69.40	70.74	65
$r_{u5}$	58.89	59.92	60
$r_{u6}$	45	37.50	37.50
$r_{d1}$	95	95	130

$r_{d2}$	90	88.37	97.57
$r_{d3}$	75	78.37	83.56
$r_{d4}$	62.20	65.74	57.39
$r_{d5}$	50.28	45	55
$r_{d6}$	35	37.50	36.25
$\beta$	0.72	0.93	0.85
$S_1$	0.03	0.026	0.026
$S_2$	0.36	0.102	0.073
<i>Conc Vol (m<sup>3</sup>)</i>	266938	274874	257898
<i>Approximation Error</i>	2.43%	0.46%	5.73%
<i>Optimization time(min)</i>	220	210	230
<i>Average conc Vol (m<sup>3</sup>)</i>		266570	

Table 7: Optimum designs of the dam obtained by PSO\_GMDH for the three best cases

Variable (m)	Optimum design		
	Case 1	Case 2	Case 3
$t_{c1}$	5.16	5.00	10
$t_{c2}$	10	10	10.57
$t_{c3}$	12.54	13.13	12.5
$t_{c4}$	15	15	15
$t_{c5}$	17.5	17.5	17.5
$t_{c6}$	35	20	20
$r_{u1}$	100	135	117.5
$r_{u2}$	95	95	95
$r_{u3}$	90	85	80
$r_{u4}$	75	80	70
$r_{u5}$	52.70	60	50
$r_{u6}$	37.5	40.59	36.92
$r_{d1}$	95	95	95
$r_{d2}$	85	90	90
$r_{d3}$	72.50	70	75
$r_{d4}$	70	70	65
$r_{d5}$	47.7	52.07	45
$r_{d6}$	37.5	32.79	25
$\beta$	0.5	0.64	0.5
$S_1$	0.082	0.30	0.30
$S_2$	0.30	0.20	0.20
<i>Conc Vol (m<sup>3</sup>)</i>	264375	256500	25988 0
<i>Approximation Error</i>	3.37%	6.2%	5.01%
<i>Optimization time(min)</i>	40	50	42
<i>Average conc Vol (m<sup>3</sup>)</i>		261933.33	

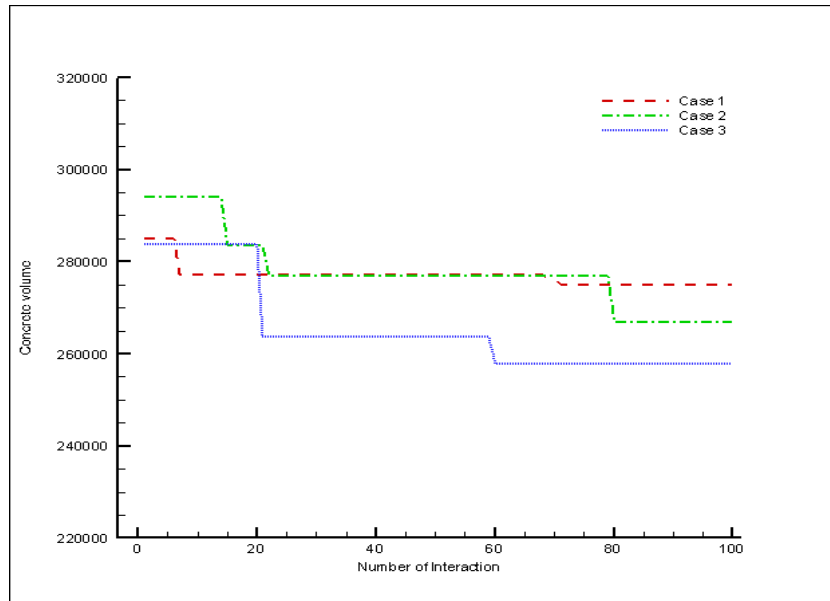


Figure 8. Convergence histories of the three best solutions of PSO for case 1 to 3

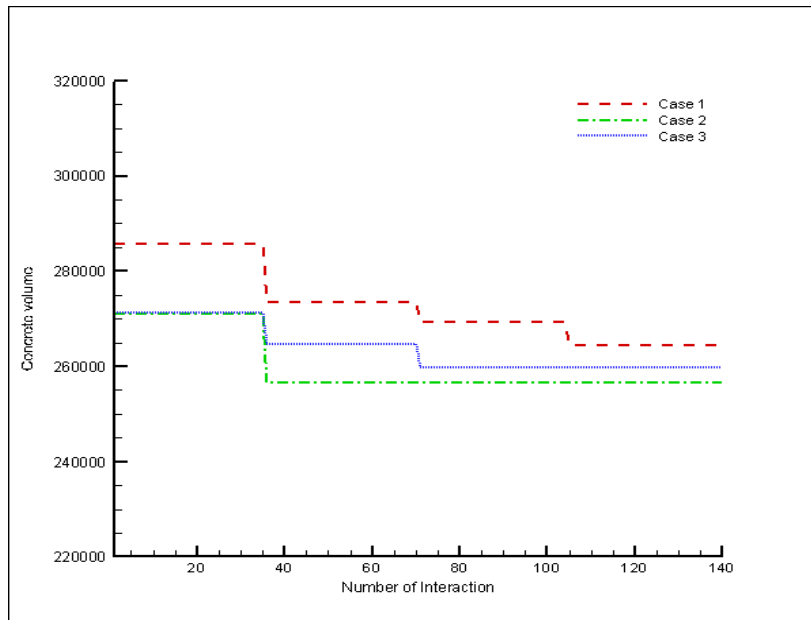


Figure 9. Convergence histories of the three best solutions of PSO-GMDH for case 1 to 3

## 7. CONCLUSION

A two stage meta-heuristic optimization method is introduced to find the optimal shapes of double arch concrete dams including dam-water-foundation rock interaction subjected to earthquake loading. The proposed optimization method is based on the particle swarm optimization (PSO) and group method of data handling, which is called PSO\_GMDH.

The main idea of this proposed method is to combine the advantages of PSO and GMDH method and improve the global search ability of PSO. Hence an approximation system can effectively accelerate the convergence of PSO. The randomly selected particles which are achieved from PSO will be the initial data of neural network and the optimization process is continued with training this network. In this study, a real world double arch dam is optimized. The optimum designs obtained by the proposed PSO\_GMDH are also compared with those produced by PSO.

PSO\_GMDH shows the improvement in terms of computational efficiency, speed of convergence, capability of training network, optimum solution and low error of results in the optimization process.

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