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A GUIDED TABU SEARCH FOR PROFILE OPTIMIZATION OF FINITE ELEMENT MODELS

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ABSTRACT

In this paper a Guided Tabu Search (GTS) is utilized for optimal nodal ordering of finite element models (FEMs) leading to small profile for the stiffness matrices of the models. The search strategy is accelerated and a graph-theoretical approach is used as guidance. The method is evaluated by minimization of graph matrices pattern equivalent to stiffness matrices of finite element models. Comparison of the results with those of some powerful methods, confirms the robustness of the algorithm.

Keywords: Guided Tabu Search; optimal nodal ordering; finite element models; profile reduction; graph-theoretical; stiffness matrices.

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1. INTRODUCTION

Nowadays even though the computational tools have been improved, on the other side, growing new intricate models and methods are the major obstacles to resolving the computational cost problems. For instance for the finite element model (FEMs), as a popular numerical method for solving engineering problems, a great deal of computational effort and memory are required for the analysis of large-scale structures. The solution of a large number of equations, often requires high computational resources or usage of parallel computers. However, there are some efficient and swift methods to analyze an engineering system to construct sparse, well-conditioned and well-structured matrices for such model. The study of these techniques is the main objective of optimal analysis developed by Kaveh [1].

In the case of a linear static structural analysis, the assembled set of equations is of the form $Kd = r$, where K is the total stiffness matrix, d is the nodal displacement vector,

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and r is the applied nodal load vector. This equation involves a positive definite and symmetric matrix coefficient K. Some suitable patterns for the coefficients of the corresponding equations have been provided, like profile form, banded form, and partitioned form. These patterns are often achieved by nodal ordering of the corresponding models which reduces the computational effort and dedicated memory. The nodal numbering is an NP-complete problem [2] and therefore only approximate approaches are proposed. These approaches can be divided into two main categories, namely graph based methods and meta-heuristic based approaches.

Graph based methods: A procedure was presented for the automatic renumbering of network equations by King [3], an algorithm for reducing the bandwidth and profile of a sparse matrices was described by Gibbis et al. [4]. A direct method for nodal numbering corresponding to narrow bandwidth was presented by Cuthill and McKee [5] and a 4-step systematic algorithm based on graph concepts was presented by Kaveh [6].

Metaheuristic based methods: An ordering for bandwidth and profile minimization problems via charged system search and ant system algorithm was presented by Kaveh and Sharafi [7,8], three recently developed meta-heuristic algorithms are used for optimum nodal ordering to reduce bandwidth, profile and wavefront of sparse matrices by Kaveh and Bijari [9]. In the above mentioned papers, meta-heuristics are applied to tune the parameters of Sloan's method, and the results are obtained by the tuned versions of Sloan are reported. Envelope reduction of sparse matrices using a genetic programming system can be found in the work of Koohestani and Poli [10].

Metaheuristic based methods usually find feasible solutions very slowly, but because of their search capability, these algorithms find more near optimal solutions than graph-theoretical approaches [9]. This feature has encouraged the authors to present a new hybrid techniques. In this method, first King’s algorithm is utilized to find an optimal profile solution, then a modified tabu search algorithm is applied. This hybridization naturally leads to better solutions than King’s algorithm.

The rest of this paper is organized as follows: Section 2 provides the basic definitions of the problem. In section 3 the necessary preliminaries of graph theory and King’s method are presented. In section 4 tabu search and the applied modification are introduced. Numerical examples are provided in section 5 and the final section concludes the study.

2. PROBLEM DEFINITION

Let $Kd = r$ be a system of linear equation, which is generated by the finite element displacement method in structural analysis. K is sparse, symmetric and positive definite matrix. To expedite an automated solution of the system, a data structure is desired which avoids the storage of a significant number of the zero entries in K and which allows trivial arithmetic operations to be circumvented by program logic. If K is an $n \times n$ matrix, p_j of row j $(1 \leq j \leq n)$ equals $j - i + 1$, where i is the biggest integer such that $a_{ij} \neq 0$. Profile of the matrix will be equal to $\sum_{j=1}^{n} p_j$. Profile of a finite element model can be related to the ordering of its nodes. The main aim of this work is to find an ordering, which minimize the profile. This will lead to an efficient finite element analysis. The objective function can be defined as:

$$To\ minimize\ Profile = \sum_{j=1}^{n} p_j$$

This optimization problem can be transferred to a well-known problem in graph theory, so called travelling salesman problem (TSP). The only difference between these is their objective functions, which of profile should be replaced that of TSP. Therefore, any solution of the TSP by search approaches like meta-heuristics, can be converted to that of ordering for profile reduction. The necessary description of the TSP, is provided in the next section.

3. PRELIMINARIES ON GRAPH THEORY

3.1 Basic definitions

Graph theory as a branch of discrete mathematics has many applications in engineering such as electrical, civil, computer and mechanical engineering. Here, some of the basic definitions are explained:

A graph S consists of a non-empty set $N(S)$ of elements called nodes (vertices or points) and a set $M(S)$ of elements called members (edges or arcs), together with a relation of incidence which associates each member with a pair of nodes, called its ends. A graph may be visualized as a set of points connected by line segments in Euclidean space; the nodes of a graph are identified with points, and its members are identified as line segments without their end points. Such a configuration is known as a topological graph. Two nodes of a graph are called adjacent if these nodes are the end nodes of a member, the nodes adjacent to x can be represented by $Adj(x)$. A member is called incident with a node if this node is an end node of the member. Two members are called incident if they have a common end node. The degree of a node n_i of a graph, denoted by $deg(n_i)$, is the number of members incident with that node. A walk w of S, is a finite sequence $w = \{n_0, m_1, n_1, \ldots, m_k, n_k\}$ whose terms are alternately nodes nj and members m_j of S for $1 \leq j \leq k$, and $n_j - 1$ and n_j are the two ends of m_j. A path P in S, is a trail in which neither nodes nor members appear more than once. A complete graph is a graph in which every two distinct nodes are connected by exactly one member. Topological properties of finite element models can be transformed into graph models using clique graphs [11]. This graph has the same nodes as those of the corresponding finite element model, and the nodes of each element are cliqued, avoiding the multiple edges for the entire graph.

3.2 King's algorithm

An important and simple algorithm for profile reduction is that of King [3], which operates as follows:

Take a node of minimum degree and number it "1". The set of nodes is now divided into three subsets, A, B and C. The subset A consists of nodes already numbered. The subset B is defined as $B = Adj\ (A)$; i.e. it consists of all nodes adjacent to any node of A. C contains the remaining nodes. Then, at each step number the node of subset B which causes the smallest number of nodes of subset C to be transferred to subset B, and redefine A, B and C, accordingly.

3.3 Travelling salesman problem (TSP)

Given a set of cities along with the cost of travel between each pair of them, the traveling salesman problem, or TSP for short, is to find the cheapest way of visiting all the cities and returning to the starting point. The "way of visiting all the cities" is simply the order in which the cities are visited; the ordering is called a tour or circuit through the cities [12]. It is an NP-hard problem in combinatorial optimization. Various heuristics and approximation algorithms, which quickly yield good solutions have been devised [13]. Modern methods can find solutions for extremely large problems within a reasonable time which are with a high probability just 2–3% away from the optimal solution [14].

Slightly modified, it appears as a sub-problem in many areas. As mentioned each route of TSP can be considered as an ordering of cities. The difference between TSP and optimum ordering for profile reduction problem, is their objective functions. It means that some of the TSP solutions that are not intensive to cost function can be used for profile reduction.

Usually meta-heuristics are not sensitive to the structure of cost functions and they use only the evaluation results. This feature makes them convenient tool for TSP based solution of the ordering problem.

4. SEARCH ALGORITHM

Tabu search is a strategy for solving combinatorial optimization problem whose applications range from graph theory to mixed integer programming problems [15]. The steps of the utilized tabu search are explained in the next paragraphs.

4.1 Initialization

The initialization usually is done by producing a random solution. Because of the verified large-size examples and available simple solutions, King's algorithm is utilized to produce primary solution. As seen in Ref. [9], usually the solutions obtained by King's algorithm have close or better than that of meta-heuristics. This point encouraged the authors to utilize tabu search as a means for improving rather than finding a competitor for the King's Algorithm.

4.2 Main loop

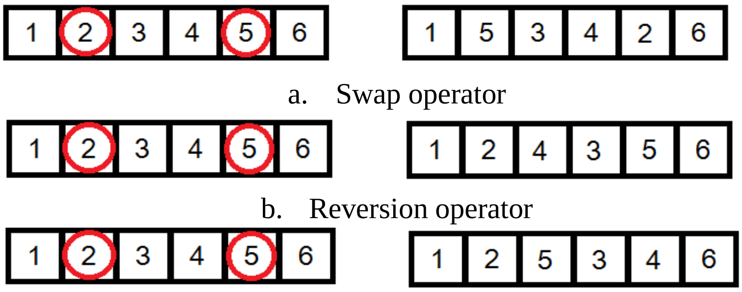
Tabu search repeats a cycle of actions until the termination conditions is satisfied. The cycle contains two main parts which are mentioned in two subsections. Once all neighbors of the current solution are evaluated, the best solution between them is replaced. Because of the significant time needed to do this, a modification is imposed and it is changed as follows:

The neighbors are evaluated until the last evaluated solution has lower cost than that of the current solution. This modification accelerates the operation of the algorithm.

4.2.1 Operations

A permutation of integer numbers from $1\ to\ n$ is vectored in a vector L with the dimension of $n \times 1$. In this, the node should be considered as the ith node, to be inserted in the ith array. Three operators are designed to create the neighbor solution as follows:

- **Swap operator**
  This operator selects two arrays and swap their positions as illustrated in Fig. 1.a.
- **Reversion operator**
  This selects two arrays and reverse the arrays between them as shown in Fig 1.b.
- **Insertion operator**
  This selects two arrays and inserts one of them in the front of another as illustrated in Fig 1.c.



a. Swap operator

b. Reversion operator

c. Insertion operator

Figure 1. Operations

A list of all possible operations producible by operators is made and denoted by O. Different operations sometimes produce the same solutions, these are unified to one operation in O. In every production of neighbor solutions, one of the allowable operations is selected randomly and evaluated. This is repeated until the operations lead to $Profile(solution) \leq Profile(best\ solution\ sofar)$. By finding new best solution the solution is replaced, tabu memory is updated and the number of iterations is increased by one.

4.2.2 Update tabu memory

Tabu search saves a sequence of the last operations whose solutions are replaced as the best. The memory is abbreviated as TM and its contents are limited to Ts. The saved operations are forbidden to apply. In this paper, Ts is set to10, so the contents of the memory are updated to 10 past solutions as explained above and they are forbidden in the next operations.

4.3 Termination condition

The procedure is terminated when the number of iterations reaches a maximum number. The predetermined maximum number of iterations is set to 500, except the example 4 where is set to 250. The pseudo code of the applied search algorithm is summarized as follows in Table 1:

Table 1: The pseudo code of the guided and accelerated tabu search

Begin
Generate a primary solution using King's algorithm;[Guidance]
Evaluate the profile of the solution;
Define Ts;

Define the maximum number of iterations;
while (t<Max number of iterations)
while Profile(solution) > Profile(best solution sofar)
Generate random neighbor using operations of (O − TM);
Evaluate the neighbor solution;
If Profile(solution) ≤ Profile(best operation sofar); [Acceleration]
Update best solution so far and its profile;
Update TM;
t → t+1;
End if
end while
end while
End

5. NUMERICAL EXAMPLES

In this section five finite element models (FEMs) are considered and the results are compared to those of the other algorithms, Table 1. The topological properties of the finite element models are transferred to the connectivity properties of graphs, by the clique graphs. This graph has the same nodes as those of the corresponding finite element model, and the nodes of each element are cliqued, avoiding the multiple edges for the entire graph.

Example 1: First example is a Z-shaped finite element model of a shear wall as shown in Fig. 2. The clique graph has 550 nodes and 2007 edges.

Example 2: In the second example a rectangular FEM is considered with four equal openings.

The clique graph of this model has 760 nodes and 2692 edges. The graph model is illustrated in Fig. 3.

Example 3: A fan with eight 1D beam is considered in the third example. As shown in Fig. 4, the clique graph of this model has 1575 nodes and 2925 edges.

Example 4: Consider an H-shaped finite element model of a shear wall with 4949 nodes and 9688 elements. Fig. 5 shows the clique graph of this.

Example 5: As illustrated in Fig. 6 a hexagon finite element model is designed and the algorithm is tested on this model. The graph model has 144 nodes and 390 edges.

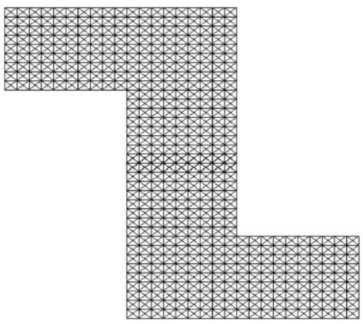


Figure 2. The FEM of a shear wall with 550 nodes

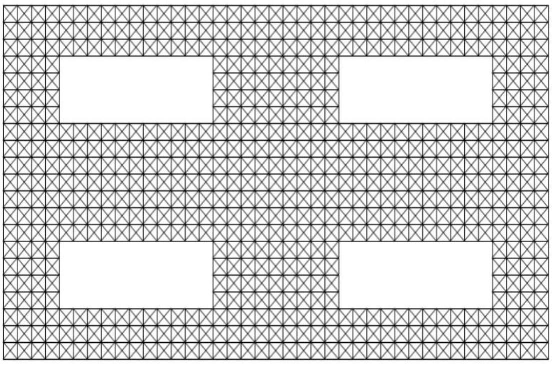


Figure 3. The element clique graph of a rectangular FEM with 760 nodes

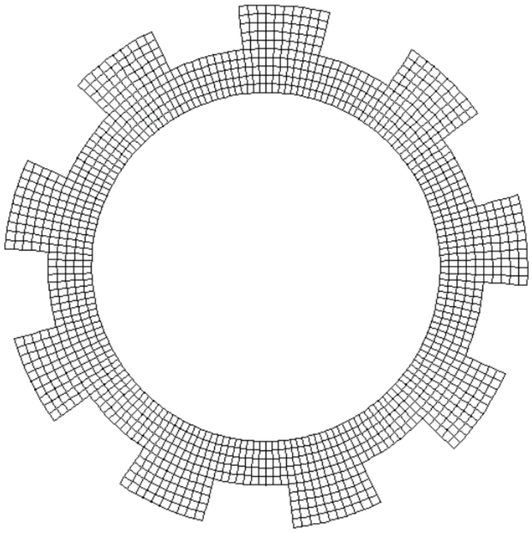


Figure 4. The graph model of a fan containing 1575 nodes

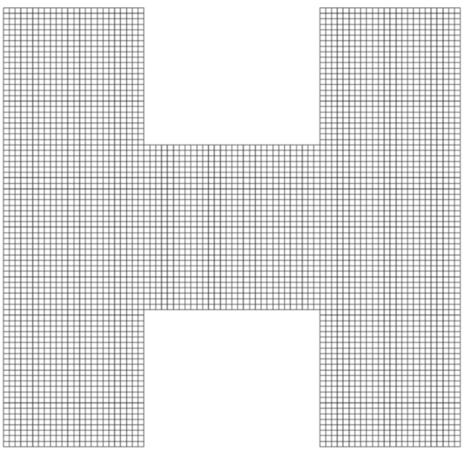


Figure 5. The clique graph of an H-shaped shear wall with 4949 nodes

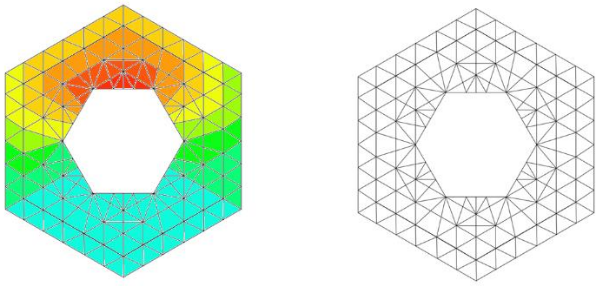
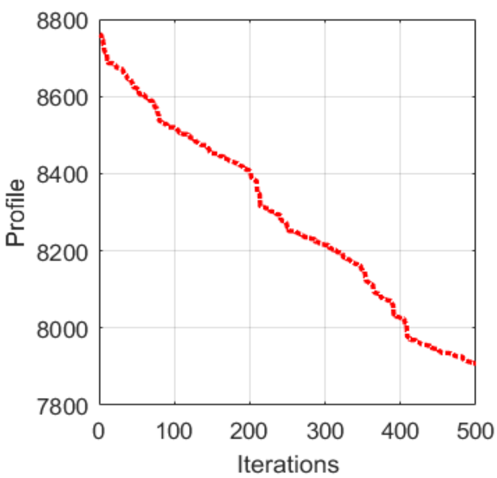


Figure 6. The clique graph of an FEM with 144 nodes

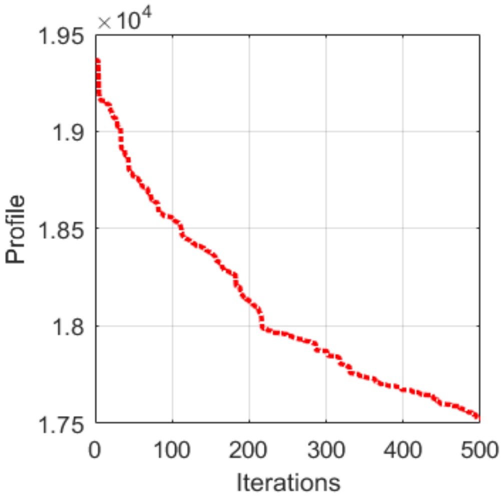
The results of the proposed method are compared with King's method and Sloan's improved versions, simple and tuned by different meta-heuristics. The graph theoretical methods are King's and Sloan's algorithms [3, 16]. The compared meta-heuristics contain PSO [17], ACO [18], CBO [19] , ECBO [20] , TWO [21] and CSS [22] algorithms . The results of different methods are presented in Table 2 and the convergence diagrams of present work are shown in Fig. 7. The statistical results of ten independent runs for example 5 are presented in Table 3.The best results are written in bold font.

Table 2: Comparison of the results of different algorithms for profile reduction

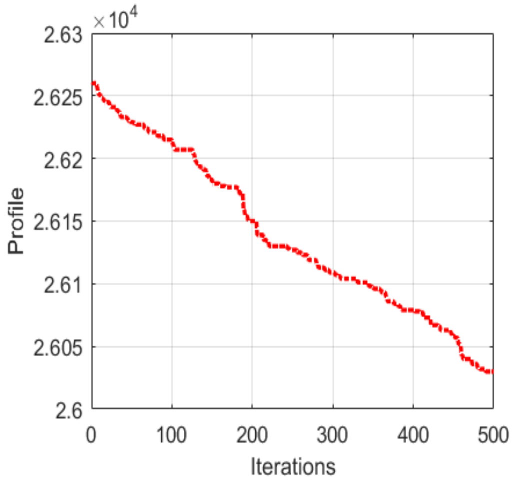
Methods	Example 1	Example 2	Example 3	Example 4
King	10974	18839	28853	211731
Sloan	10530	18719	28703	210845
PSO [9]	10501	18690	28629	**157095**
CBO [9]	10501	18689	28608	**157095**
ECBO [9]	10501	18581	28587	**157095**
TWO [9]	10501	18581	28579	**157095**
CSS [7]	-	19232	28770	206649
Present work	**7908**	**17525**	**26030**	185131



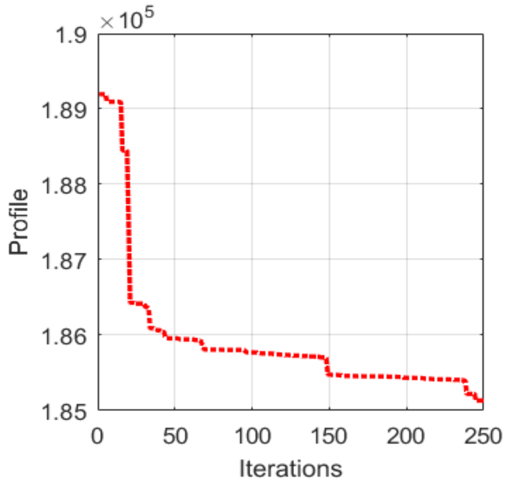
a. The convergence history of Example 1



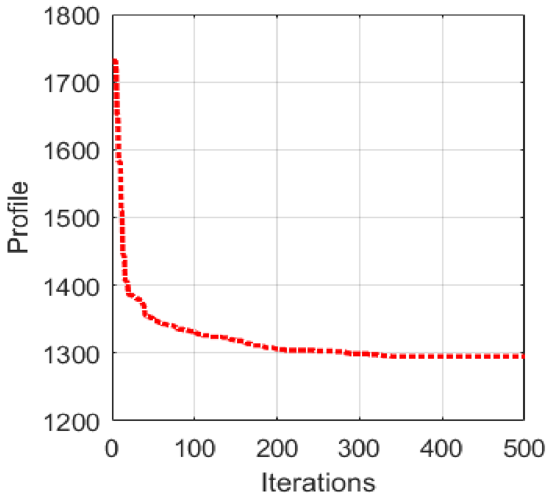
b. The convergence history of Example 2



c. The convergence history of Example 3



d. The convergence history of Example 4



e. The convergence history of Example 5

Figure 7. Convergence histories of the presented method for the examples

Table 3: Statistical results for Example 5

Best result	Worst result	Mean result	Standard deviation
1295	1305	1302.6	3.5653

6. CONCLUSIONS

The main objective of this paper has been to propose a new hybrid method containing graph-theoretical method which guides a well-known search strategy so called tabu search. Because of examined large-size problems, tabu search has been accelerated. The ordering problem is simulated to TSP and the applicability of this technique has been demonstrated by reduction of profile of five finite element models.

The results of the presented method are compared with those of other powerful methods. From Table 2 it can be seen that the proposed algorithm has obtained the best results in some of the examples. The differences between the results, shows the good performance of this method in term of global search.

Utilizing King's algorithm as an initial solution to increase the chance of finding better solutions. However, this guided initial solution may lead to trapping the solution process in a local optima. Trying to escape from local optima sometimes is time consuming. Especially in large-scale problems (like Example 4) this problem was more cumbersome, and further research is needed to make the metaheuristic algorithms as a competitive approach to the graph theoretical methods.

This method also can be employed in other type of structures and problems. In FE model updating problems, this may lead to good results. Particularly in aeronautics that the computing resources are more limited, the optimal ordering should be more valuable.

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