OPTIMUM COST DESIGN OF REINFORCED CONCRETE SLABS USING CUCKOO SEARCH OPTIMIZATION ALGORITHM

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ABSTRACT

This paper presents a Cuckoo Optimization Algorithm (COA) model for the cost optimization of the one-way and two-way reinforced concrete (RC) slabs according to ACI code. The objective function is the total cost of the slabs including the cost of the concrete and that of the reinforcing steel. In this paper, One-way and two-way slabs with various end conditions are formulated as ACI code. The two-way slabs are modelled and analyzed using direct design method. The problems are formulated as mixed-discrete variables such as: thickness of slab, steel bar diameter, and bar spacing. The presented model can be applied in design offices to reduce the cost of the projects. It is also the first application of the Cuckoo Optimization Algorithm to the optimization of RC slabs. In order to demonstrate the superiority of the presented method in convergence and leading to better solutions, the results of the proposed model are compared with the other optimization algorithms.

1. INTRODUCTION

RC slab is a structural element that its thickness is smaller than the two other dimensions. In general, slabs are classified as being one-way or two-way. If the loads in RC slabs are distributed in one direction, they are referred to as one-way slabs. Two-way slabs distribute loads in two perpendicular directions. Two-way slabs can be strengthened by the addition of beams between the columns, by thickening the slabs around the columns (*drop panels*), and by

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flaring the columns under the slabs (*column capitals*). *Flat plates* (Fig. 1) are solid RC concrete slabs of uniform depths that transfer loads directly to the supporting columns, as shown in the figure, without the aid of beams or capitals or drop panels. Flat plates can be constructed quickly because of their simple formwork and reinforcing bar arrangements. Today, flat plates systems are popular for use in the slab systems for hotels, motels, apartment houses, hospitals, and dormitories [1].
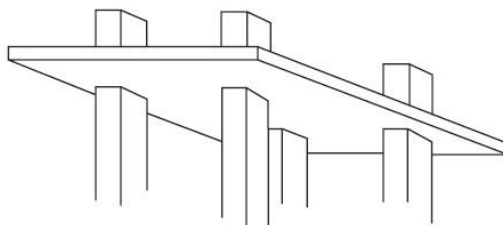
Figure 1. RC Flat plate system

Safety and cost are the most important part of the design of structures. Thus, structural optimization algorithms must be used for cost optimization and applied to realistic structures subjected to the actual constraints of commonly used design codes such as the American Concrete Institute Code. The major articles on cost optimization of reinforced concrete structures were reviewed by Sarma and Adeli [2]. The early works on optimization of RC slabs were based on many simplifying assumptions. These works were considered one-way slabs. Traum [3] discussed the optimum cost design of one-way concrete slabs according to the 1956 ACI code subjected to pure moment only. His cost function included cost of the concrete and steel. The problem was formulated to find the reinforcing ratio and an explicit formula for this variable by distinguishing the cost function. Brown [4] presented a single-variable optimization to find the optimum thickness of one-way slabs for uniformly loaded simply supported slabs considering flexural deformations. His cost function was similar to Traum's work. Brondum-Nielsen [5] introduced a method for minimizing the cost of reinforcement in reinforced concrete shells, folded plates, walls, and slabs by minimizing the summation of the forces in the steel reinforcement in two perpendicular directions. Brondum-Nielsen solved an academic example without any code of practice in defining the constraints. Hanna and Senouci [6] described a design cost optimization method for all-wood concrete-slab forms. Four different variables (sheathing, joist, stringer, and wood shore) were considered in this paper. They concluded that cost savings as high as 9.9% were achieved by using the design optimization method compared with the traditional methods. Tabatabai and Mosalam [7] presented a system for optimum reinforcement design and non-linear analysis of reinforced structures. They solved two examples of deep beam with duct and one-way slabs with ends fully fixed on one side and simply supported on the other side. Ahmadkhanlou and Adeli [8] applied a neural dynamics model to the cost optimization of RC one-way slabs according to ACI, 1999 code provisions. They formulated the problem as a mixed-discrete optimization problem with three design variables: thickness of slab, steel bar diameter, and bar spacing. Sahab et all. [9,10] presented a hybrid genetic algorithm for the optimal cost design of reinforced concrete flat slab buildings based on British Code of

Practice (BS8110). Ahmadi-Nedushan and Varaee [11] used particle swarm optimization with varying acceleration time coefficients to the cost optimization of reinforced concrete one-way slabs according to ACI, 2008 code provisions. A. Kaveh et al. [12,13] optimized the cost of reinforced concrete one-way ribbed slabs with harmony Search algorithm, PSO, CBO, and democratic PSO.

In the last decade, many new metaheuristics algorithms have been developed and used in structural optimization. Sahab et all. [10], Atabay [14] and Augusto et all. [15] used hybrid and classic genetic algorithms for concrete structures, Varaee et al [11] and Kaveh et al [16] developed particle swarm optimization for reinforced and prestressed concrete slabs, Camp and Huq [17], Kaveh and Sabzi [18] used big bang-big crunch algorithm for design of reinforced concrete frames, and harmony search algorithm have been applied by Akin and Saka [19], Kaveh et al [20,21,22] for structural optimization.

Cuckoo Search (CS) is a new metaheuristics algorithm. It has been developed by Yang and Deb [23]. Gandomi et all. [24,25], Kaveh and Bakhshpoori [26] showed that it has outperformed other optimization algorithms. Standard CS algorithm is usually quick at the exploitation of the solution though its exploration ability is relatively poor. Therefore, Wang et all. [27], Babukartik et al. [28] showed CS can perform the local search. Rajabioun [29] investigated more details about the life style of cuckoos and developed Cuckoo Optimization Algorithm (COA). The COA can perform the local and global search efficiently. This paper presents cost optimization of one-way RC slabs and RC flat slabs by COA. Since one-way RC slabs with four different support condition were optimized previously with neural dynamics model by Ahmadkhanlou and Adeli [8], the results of neural dynamics and COA methods are compared. In order to compare the results, the same designing code must be considered. For this reason, one-way RC slabs and RC flat slabs were designed by ACI 1999 and ACI 318-14 code provision, respectively [30,31].

The rest of this paper is arranged as follows. Section 2 describes the detailed optimum design problem. In this section, objective function, design constraints, and design variables are described for one-way RC slabs and RC flat slabs separately. In section 3, the Cuckoo Optimization Algorithm is introduced and mixed integer-discrete optimization of one-way and flat slabs using COA is presented. In section 4, examples are provided and results are presented. Finally, in section 5, the concluding remarks are given.

2. MODELS FORMULATION

2.1 One-way reinforced concrete slabs

2.1.1 Objective function

A total cost function can be written as follows:

$$C_t = C_c + C_r + C_f \tag{1}$$

where C_c, C_r, and C_f are cost of concrete, reinforcement bars and formwork, and finishing materials, respectively. The formwork cost does not vary significantly for any given locality

and consequently can be dropped from formulation (Ahmadkhanlou and Adeli [8]). They are defined as follows

$$C_c = LbhC_c^1 \tag{2}$$
$$C_r = w_s LA_s C_r^1 \tag{3}$$

where L, b, h, C_c^1, w_s, A_s, and C_r^1 are the span length, the span width, thickness of slab (Fig. 2), cost of concrete per unit volume, unit weight (specific weight per unit volume) of steel, cross section area of reinforcement bars, and cost of reinforcement bars per unit weight, respectively. The quantity A_s is calculated by

$$A_s = \frac{\pi d_b^2}{4} \left(\frac{b}{s}\right) \tag{4}$$

where d_b and s are the diameter and the spacing of the reinforcement bars, respectively.
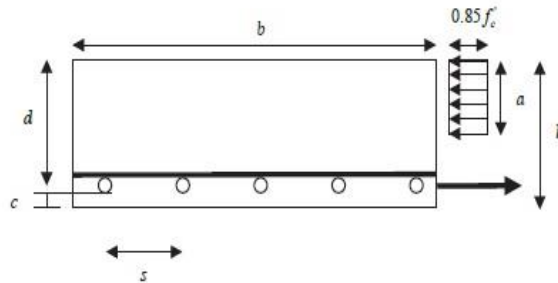

Figure 2. Typical cross-section of RC slab

2.1.2 Design constraints

As previously mentioned, the optimization of cost function is based on the constraints defined by ACI 1999 code [30]. The constraints included flexural constraint, shear constraint, serviceability constraint, and deflection constraint. They are defined and expressed in a normalized form as given below.

2.1.2.1. Flexural constraint

Nominal flexural strength, ϕM_n, should be greater than the ultimate design moment, M_u;

$$g_1(x) = \frac{M_u}{\phi M_n} - 1 \leq 0, \qquad \phi = 0.9, \tag{5}$$

In Eq. (5), M_u is calculated as follows:

$$M_u = kwl_n^2 \tag{6}$$

where l_n and k are, respectively, the clear span length and the moment coefficient for

continuous slab that depends on the type of slab supports. The values of k are given in Table 1. In Eq. (6), the maximum value of moment coefficient for four different support conditions (simply-supported, continuous at one end and simply-supported at the other, continuous at both ends, and cantilever) is used which is given in Table 2.

In Eq. (6), w is the factored uniformly distributed load. In the article of Ahmadkhanlou and Adeli [8] loading cases were considered as suggested by ACI 1999 code [30]:

$$w = 1.4 \times (DL \times b + DL_s) + 1.7 \times LL \times b, \tag{7}$$

in which DL, LL, and DL_s are the dead load of floor excluding the self-weight of slab, live load, and self-weight of slab. DL_s is calculated as follows:

$$DL_s = (bh - A_s)w_c + A_s w_s, \tag{8}$$

where w_c is the weight of the concrete per unit volume.

Table 1: Moment coefficient for continuous slabs

Exterior span			Interior span		
Support	Middle	Support	Support	Middle	Support
-1/24	+1/14	-1/10	-1/11	+1/16	-1/11

Table 2: Maximum moment coefficient, k, used for design of RC slabs

Simply Supported	One end continuous	Both ends continuous	Cantilever
1/8	1/10	1/11	½

The nominal bending moment, M_n, is calculated as follows:

$$M_n = A_s f_y (d - \frac{a}{2}) \tag{9}$$

where f_y is the specified yield strength of the reinforcement bars and a is the equivalent depth of the concrete compressive stress block that is calculated from (Fig. 2).

$$a = \frac{A_s f_y}{0.85 f_c' b} \tag{10}$$

where f_c' is the specified compressive strength of concrete.

2.1.2.2 Shear constraint

The nominal shear strength of concrete, ϕV_n, should be greater than the ultimate factored shear force, V_u;

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$$g_2(x) = \frac{V_u}{\emptyset V_n} - 1 \le 0, \qquad \emptyset = 0.85, \tag{11}$$

The ultimate factored shear force is defined as follows:

$$V_u = k_v \frac{w l_n}{2} \tag{12}$$

where k_v is the shear coefficient for continuous slab that depends on the type of slab supports. The values of k_v are given in Table 3. The nominal shear strength of concrete is defined as follows:

$$V_c = 2\sqrt{f_c'}bd \tag{13}$$

Table 3: Shear coefficient for continuous slabs

Simply Supported	One end continuous	Both ends continuous	Cantilever
1	1.15	1	2

2.1.2.3. Serviceability constraints

The percentage of the longitudinal reinforcement steel, ρ, and the bar spacing, s, in one-way RC slabs should be between minimum and maximum limits permitted by the design specification.

$$g_3(x) = \frac{\rho}{\rho_{max}} - 1 \le 0 \tag{14}$$

$$g_4(x) = \frac{\rho_{min}}{\rho} - 1 \le 0 \tag{15}$$

$$g_5(x) = \frac{s_{min}}{s} - 1 \le 0 \tag{16}$$

$$g_6(x) = \frac{s}{s_{max}} - 1 \le 0 \tag{17}$$

where the ρ_{max} is given by:

$$\rho_{max} = 0.75\rho_b \tag{18}$$

ρ_b is defined as follows:

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y} \left(\frac{87000}{87000 + f_y}\right) \tag{19}$$

and β_1 is calculated from

$$\text{for } f_c' \le 4000 \text{ psi}, \beta_1 = 0.85,$$

$$\text{for } f_c' > 4000 \text{ psi}, \beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) \geq 0.65 \tag{20}$$

The minimum area of flexural(longitudinal) reinforcement is chosen as follows:

$$A_{s\,min} = \begin{cases} 0.0020bh, & \text{for Steel Grade 40 and 50} \\ 0.0018bh, & \text{for Steel Grades 60} \end{cases} \tag{21}$$

and the minimum and maximum bar spacing are defined as follows:

$$s_{min} = \max(1", d_b) \tag{22}$$
$$s_{max} = \min(18", 3h) \tag{23}$$

2.1.2.4 Deflection constraints

Slab thickness, h, shall not be less than the minimum slab thickness, h_{min};

$$g_7(x) = \frac{h_{min}}{h} - 1 \leq 0 \tag{24}$$

where h_{min} is given in Table 4, with an absolute minimum thickness of 1.5 in (38.1 mm). The values of Table 4 are applicable for normal weight concrete and $f_y = 60,000$ psi. For f_y other than 60,000 psi, the values shall be multiplied by α_1 (which is given in Eq. (25)). For lightweight concrete having w_c in the range of 90 to 115 lb/ft³, the values shall be multiplied by α_2 (which is given in Eq. (26)).

$$\alpha_1 = 0.4 + \frac{f_y}{100,000} \tag{25}$$
$$\alpha_2 = \max(1.65 - 0.005w_c, 1.09) \tag{26}$$

Table 4: Minimum thickness for solid one-way slab according to ACI code

Simply Supported	One end continuous	Both ends continuous	Cantilever
L/20	L/24	L/28	L/10

2.1.3. Design variables

Design variables for the one-way RC concrete slab consist of three variables: thickness of slab (h), the diameter of reinforcement bars (d_b), and the spacing of reinforcement bars (s). Thickness of slab and spacing of reinforcement can be considered as integer variables, for example, centimeters in the SI system, or a multiple of 1/8" or 1/4" in the US customary system. Since the diameter of the reinforcement bars has to be assigned from limited numbers, it has to be considered as a discrete variable. ACI supplies eleven different bar sizes starting with the bar size #3 with the diameter of 0.375" (0.953 cm) to the bar size #18 with the diameter of 2.257" (5.733 cm).

2.2 Reinforced concrete flat slabs

2.2.1 Design constraints

This paper analyzes the concrete flat slabs by Direct Design Method based on ACI 318-M14 [31]. For using direct design method, ACI presents six limitations;

1. There must be at least three continuous spans in each direction.
2. Successive span lengths (center-to-center of supports) in each direction must not differ by more than one-third of the longer span.
3. Panels must be rectangular, with the ratio of longer to shorter panel dimensions, measured center-to-center of supports, and not exceed 2.
4. Column offset shall not exceed 10 percent of the span in the direction of the offset from either axes between centerlines of successive columns.
5. All loads shall only be due to gravity and uniformly distributed over an entire panel.
6. Unfactored live load must not exceed two times the unfactored dead load.

In this section, a general formulation is presented for cost optimization of single- and multiple- span RC flat plates with various end conditions (interior span, exterior edge unrestrained, exterior edge fully restrained, slab without beam between interior support and edge beam). These spans conditions are shown in Fig. 3.
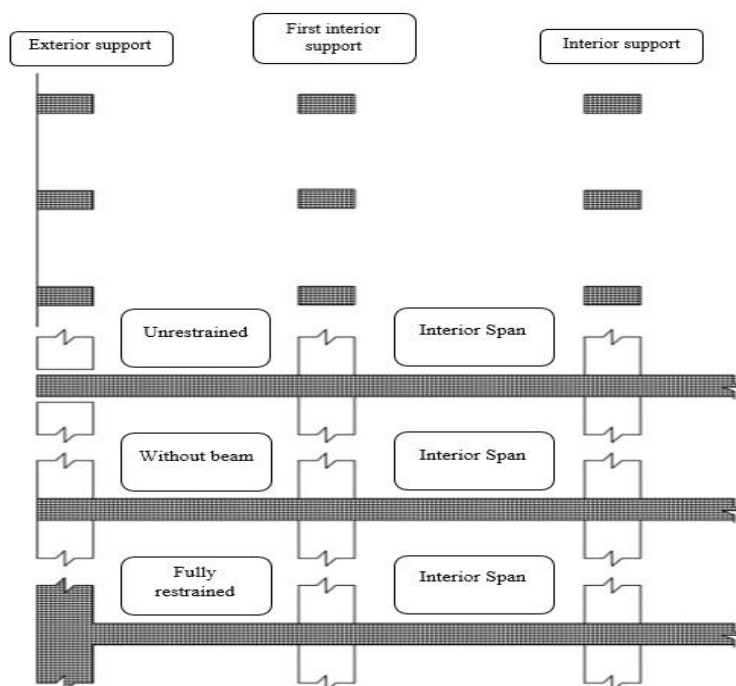

Figure 3. Flat plates with various end conditions

The constraints include flexural constraint, one-way shear constraint, two-way shear constraint, serviceability constraint, and deflection constraint. The constraints are explained and expressed in a normalized form below.

2.2.1.1 Flexural constraint

In direct design method, the design moments are distributed across each panel. The panels are divided into a column and middle strips, as shown in Fig. 4, and positive and negative moments are obtained in each strip. The column strip is a slab with a width on each side of the column centerline that is defined as follows:

$$b_c = \min(l_1/4, l_2/4) \tag{27}$$
$$b_m = l_2 - 2 \times b_c \tag{28}$$

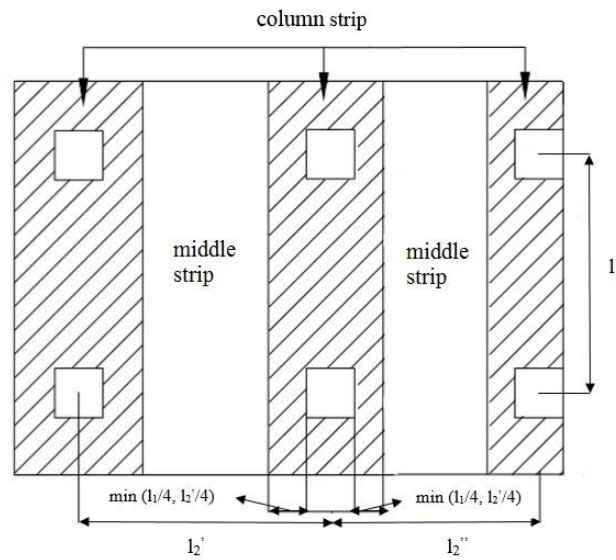where b_c and b_m are the width of half column strip and middle strip, respectively.

Figure 4. Middle and column strips

Positive nominal flexural strength at the middle, and the negative nominal flexural strength at the two ends of the column strip and middle strip, ϕM_n, should be greater than the ultimate design moment in column strips (col) and middle strips (mid) at these parts, M_u;

$$g_{1\,col}(x) = \frac{M_{ul}^-}{\emptyset M_n^-} - 1 \le 0 \tag{29}$$

$$g_{2\,col}(x) = \frac{M_u^+}{\emptyset M_n^+} - 1 \le 0 \tag{30}$$

$$g_{3\,col}(x) = \frac{M_{ur}^-}{\emptyset M_n^-} - 1 \le 0 \tag{31}$$

$$g_{4\,mid}(x) = \frac{M_{ul}^-}{\emptyset M_n^-} - 1 \le 0 \tag{32}$$

$$g_{5\,mid}(x) = \frac{M_u^+}{\emptyset M_n^-} - 1 \le 0 \tag{33}$$

$$g_{6\,mid}(x) = \frac{M_{ur}^-}{\emptyset M_n^-} - 1 \le 0 \tag{34}$$

In Eqs. (29-34), $\emptyset = 0.9$, and M_u is calculated as follows:

$$M_{u\,cs} = k_\propto k_m M_o \tag{35}$$
$$M_{u\,ms} = (1 - k_\propto)k_m M_o \tag{36}$$

where, k_m is the distribution of the total span moment coefficient for four different span conditions (interior span, exterior edge unrestrained, exterior edge fully restrained, slab without beam between interior support and edge beam) and $k_\propto$ is the portion of the interior negative moment, the exterior negative moment, and the positive moment resisted by the column strip. The values of k_m and $k_\propto$ are expressed in Table 5 and Table 6, respectively. M_o is the total factored static moment for a span that is defined as:

$$M_o = \frac{w_u l_2 l_n^2}{8} \tag{37}$$

where l_n is the clear span of supports in the direction of moments which are considered, measured face-to-face of the supports, and are not less than $0.65l_1$, and w_u is the factored uniformly distributed load that is defined as:

$$w_u = 1.2 \times (DL + w_{Rc}h) + 1.6 \times LL \tag{38}$$

where w_{Rc} is the density of the reinforced concrete and h is the thickness of the slab.

Table 5: Distribution of total span moment, k_m

	Exterior edge unrestrained	Slab without beam between interior support and edge beam	Exterior edge fully restrained	Interior span
Interior negative factored moment	0.75	0.70	0.65	0.65
Positive factored moment	0.63	0.52	0.35	0.35
Exterior negative factored moment	0	0.26	0.65	0.65

Table 6: Portion of interior negative moment, exterior negative moment, and positive moment resisted by column strip, $k_\propto$

	l_2/l_1		
	0.5	1	2
Interior negative moment	0.75	0.75	0.75
Exterior negative moment	1	1	1
Positive moment	0.6	0.6	0.6

The nominal bending moment, M_n, is defined as follows:

$$M_{n\,col} = A_{s\,cs}f_y(d - \frac{a}{2}) \tag{39}$$

$$M_{n\,mid} = A_{s\,ms}f_y(d - \frac{a}{2}) \tag{40}$$

where a is the equivalent depth of the concrete compressive stress block that is calculated as follows:

$$a_{col} = \frac{A_{s\,col}f_y}{0.85f_c' \times 2b_c} \tag{41}$$

$$a_{mid} = \frac{A_{s\,mid}f_y}{0.85f_c'b_m} \tag{42}$$

the quantity of A_s is calculated by;

$$A_{s\,col} = \frac{\pi d_b^2}{4}(\frac{2b_c}{s} + 1) \tag{43}$$

$$A_{s\,mid} = \frac{\pi d_b^2}{4}(\frac{b_m}{s} + 1) \tag{44}$$

In effective slab width, a fraction of factored slab moment resisted by the column, $\gamma_f M_{sc}$ should be less than the nominal flexural strength, ϕM_n;

$$g_7(x) = \frac{\gamma_f M_{sc}}{\phi M_n} - 1 \le 0 \quad \emptyset = 0.9, \tag{45}$$

The effective slab width shall be the width of column plus 1.5h of slab. M_{sc} in interior column (int col) and edge column (edge col) and γ_f are defined as follows:

$$M_{sc\,int\,col} = 0.07[(q_{Du} + 0.5q_{lu})l_2l_n^2 - q_{Du}'l_2'(l_n')^2] \tag{46}$$

where q_{Du}', l_2', and l_n' refer to the shorter span

$$M_{sc\,edge\,col} = 0.3M_o \tag{47}$$

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} \tag{48}$$

The maximum values for γ_f is provided in Table 7.

Table 7: Maximum modified values of γ_f

Column location	Span direction	v_{ug}	ε_t (within b_{slab})	Maximum modified γ_f
Corner column	Either direction	$\leq 0.5\emptyset v_c$	≥ 0.004	1.0
Edge column	Perpendicular to the edge	$\leq 0.75\emptyset v\_c$	≥ 0.004	1.0
	Parallel to the edge	$\leq 0.4\emptyset v_c$	≥ 0.010	$\dfrac{1.25}{1 + (\frac{2}{3})\sqrt{\frac{b_1}{b_2}}} \leq 1.0$
Interior column	Either direction	$\leq 0.4\emptyset v_c$	≥ 0.010	$\dfrac{1.25}{1 + (\frac{2}{3})\sqrt{\frac{b_1}{b_2}}} \leq 1.0$

where b_1 is the length of the shear perimeter, which is perpendicular to the axis of bending, and b_2 is the length of the shear perimeter parallel to the axis of bending. Also, c_1 is the width of column perpendicular to the axis of bending, while c_2 is the column width parallel to the axis of bending. These perimeters are calculated from Fig. 5.
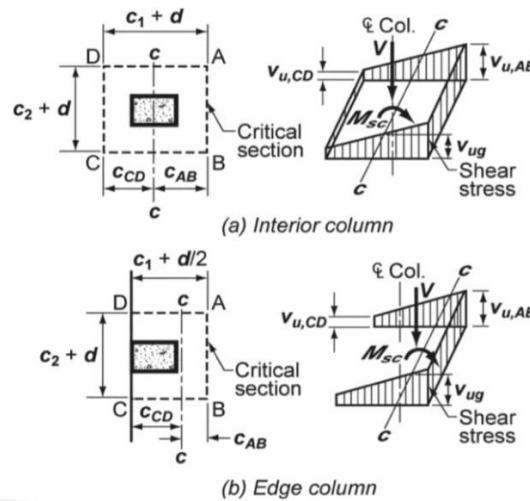

Figure 5. Assumed distribution of shear

2.2.1.2 One-way shear constraint

The nominal shear strength of concrete, $\emptyset V_n$, should be greater than the ultimate factored shear force, V_u;

$$g_8(x) = \frac{V_u}{\emptyset V_n} - 1 \leq 0, \qquad \emptyset = 0.75, \tag{49}$$

The ultimate factored shear force in interior spans (int) and edge spans (edge) are calculated as follows:

$$V_{u\ int} = w_u l_2 (\frac{l_n}{2} - d) \tag{50}$$

$$V_{u\ edge} = w_u l_2 (\frac{1.15 l_n}{2} - d) \tag{51}$$

The nominal shear strength of concrete is defined as follows:

$$V_n = V_c = 2\sqrt{f_c'} l_2 d \tag{52}$$

2.2.1.3 Two-way shear constraint

The shear stress strength, ϕv_n, should be greater than the ultimate factored shear stress, v_u;

$$g_9(x) = \frac{v_u}{\phi v_n} - 1 \le 0, \qquad \emptyset = 0.75, \tag{53}$$

In Eq. (53), v_u is defined as follows:

$$v_u = \frac{V_{ug}}{A_c} + \frac{\gamma_v M_{sc} c}{J_c} \tag{54}$$

where V_{ug} = the ultimate factored shear that is calculated by Eq. (55), A_c = the area of the concrete along the assumed critical section that is calculated by Eqs. (56-57), J_c = the property of assumed critical section analogous to the polar moment of inertia that is calculated by Eqs. (58-59) c = the distance between central axis and outlines in the critical section that is calculated by Eqs. (60-61), and γ_v is given in Eq. (62):

$$V_{ug} = w_u (l_1 l_2 - b_1 b_2) \tag{55}$$

$$A_{c\ int\ col} = 2(b_1 + b_2) \times d \tag{56}$$

$$A_{c\ edge\ col} = (2b_1 + b_2) \times d \tag{57}$$

$$J_{c\ int\ col} = d \left(\frac{b_1^3}{6} + \frac{b_2 b_1^2}{2} \right) + \frac{b_1 d^3}{6} \tag{58}$$

$$J_{c\ edge\ col} = d \left(\frac{2b_1^3}{3} - (2b_1 + b_2) \times c^2 \right) + \frac{b_1 d^3}{6} \tag{59}$$

$$c_{int\ col} = \frac{b_1}{2} \tag{60}$$

$$c_{edge\ col} = \frac{b_1^2}{2b_1 + b_2} \tag{61}$$

$$\gamma_v = 1 - \gamma_f \tag{62}$$

where c_1 is the width of the column perpendicular to the axis of bending and c_2 is the width of the column parallel to the axis of bending.

The shear stress strength for two-way members without shear reinforcement is calculated by:

$$v_n = v_c = \min(4\sqrt{f_c'}, (\frac{\alpha_s d}{b_0} + 2)\sqrt{f_c'})$$

(63)

In Eq. (63), the value of α_s is 40 for interior columns and 30 for edge columns.

2.2.1.4 Serviceability constraints

The area of reinforcement bars, A_s, should be greater than the minimum area of reinforcement, $A_{s\ min}$, and the bar spacing, s, in reinforced one-way slabs should be between minimum and maximum limits permitted by the design specification.

$$g_{10}(x) = \frac{A_{s\ min}}{A_s} - 1 \leq 0$$

(64)

$$g_{11}(x) = \frac{s_{min}}{s} - 1 \leq 0$$

(65)

$$g_{12}(x) = \frac{s}{s_{max}} - 1 \leq 0$$

(66)

The minimum area of flexural reinforcement is presented in Table 8:

Table 8: The minimum area of flexural reinforcement, $A_{s\ min}$

Reinforcement type	f_y, psi		$A_{s\ min}$
Deformed bars	< 60000		$0.0020A_g$
Deformed bars or welded wire reinforcement	≥ 60000	Greater of:	$\frac{0.0018 \times 60000}{f_y}A_g$ $0.0014A_g$

and the minimum and maximum bar spacing are defined as follows:

$$s_{min} = \max(1\ in, d_b, 4/3\ d_{agg})$$

(67)

where d_{agg} is the diameter of aggregate.

$$s_{max} = \begin{cases} \min\ (18'', 2h) & at\ critical\ sections \\ \min(18'', 3h) & at\ other\ sections \end{cases}$$

(68)

in this paper the formula for critical sections is assumed for all sections of the RC flat slabs.

2.2.1.5 Deflection constraints

Slab thickness, h, shall not be less than the minimum slab thickness, h_{min};

$$g_{13}(x) = \frac{h_{min}}{h} - 1 \leq 0$$

(69)

where h_{min} is presented in Table 9.

Table 9: Minimum thickness of slabs without interior beams, h_{min}

| f_y, psi | Without drop panels | |
| | Exterior panels | Interior panels |
	Without edge beams	
40000	$l_n/33$	$l_n/36$
60000	$l_n/30$	$l_n/33$
75000	$l_n/28$	$l_n/31$

where l_n is the clear span in the long direction, measured face-to-face of supports.

2.2.2. Design variables

The compressive strength of the concrete (f_c'), the thickness of the slab (h), the diameter of the reinforcement bars (d_b), and the spacing of the reinforcement (s) were included as the design variables. The numbers of the diameters of the reinforcement bars (d_b) are different for four end spans because the ultimate design moment is different in them. The thickness of the slab (h) and the spacing of the reinforcement (s) can be considered as integer variables (multiple of 1/4 "), while the compressive strength of the concrete (f_c') and the diameter of the reinforcement bars (d_b) have to be assigned discrete variables. A number of possible values for d_b and f_c' are listed in Table 10.

Table 10: List of possible values for f_c' and d_b

Concrete strength (f_c') psi	Diameter of reinforcement bars (d_b) in
3000 (21 Mpa)	0.375 (9.525 mm)
4000 (28 Mpa)	0.500 (12.7 mm)
5000 (35 Mpa)	0.625 (15.875 mm)
6000 (40 Mpa)	0.750 (19.05 mm)

2.2.3. Objective function

A total cost function for the reinforced concrete flat slab can be defined as follows:

$$C_t = C_r^1(C_c + C_r + C_f) \tag{70}$$

where C_c is the cost of the concrete. C_c can be calculated as:

$$C_c = r_1 l_1 l_2 h \tag{71}$$

where l_1 is the length of span, center to center of supports in the direction in which moments are being considered, l_2 is the length of span, center to center of supports in the direction transverse to l_1, and r_1 is the cost ratio of the cost of a unit volume of concrete to a unit volume of concrete ($\frac{C_c^1}{C_r^1}$).

C_r and C_f are the cost of negative and positive reinforcement bars in interior and exterior

supports, formwork, and finishing materials, respectively. As mentioned before, the formwork cost can be dropped from the formulation. For a slab with interior span, C_r and C_f are defined as:

$$C_r = w_s \sum (2 \times 0.5 \times A^-_{s\,col,int\,sup}(l_{col,top1} + l_{col,top2}) + l_{col,bot3} A^+_{s\,col} \\ + 2 \times l_{mid,top4} A^-_{s\,mid,int\,sup} + 0.5 \times A^+_{s\,mid}(l_{mid,bot5} + l_{mid,bot6}))$$ (72)

For a slab with exterior edge unrestrained, C_r is defined as:

$$C_r = w_s \sum (0.5 \times A^-_{s\,col,int\,sup}(l_{col,top1} + l_{col,top2}) + l_{col,bot7} A^+_{s\,col} \\ + l_{mid,top4} A^-_{s\,mid,int\,sup} + 0.5 \times A^+_{s\,mid}(l_{mid,bot8} + l_{mid,bot6}))$$ (73)

For a slab with exterior edge fully restrained and slab without beam between interior support and edge beam, C_r is defined as:

$$C_r = w_s \sum (0.5 \times A^-_{s\,col,int\,sup}(l_{col,top1} + l_{col,top2}) + 0.5 \\ \times A^-_{s\,col,ext\,sup}(l_{col,top9} + l_{col,top10}) + l_{col,bot7} A^+_{s\,col} \\ + l_{mid,top4} A^-_{s\,mid,int\,sup} + 0.5 \times A^+_{s\,mid}(l_{mid,bot8} + l_{mid,bot6}))$$ (74)

$l_{col,top}$, $l_{col,bot}$, $l_{mid,top}$, and $l_{mid,bot}$ are the required reinforcement lengths which are defined as in Fig. 6.
where

$l_{col,top1} = 0.3l_n + \frac{C_1}{2}$, $l_{col,top2} = 0.2l_n + \frac{C_1}{2}$, $l_{col,bot3} = l_n + C_1$,

$l_{mid,top4} = 0.22l_n + \frac{C_1}{2}$, $l_{mid,bot5} = l_n - \left(2 * \left(0.15l_n - \frac{C_1}{2}\right)\right)$, $l_{mid,bot6} = l_n + 12''$,

$l_{col,bot7} = l_n + \frac{C_1}{2} + 6''$, $l_{mid,bot8} = l_n - \left(0.15l_n - \frac{C_1}{2}\right) + 6''$,

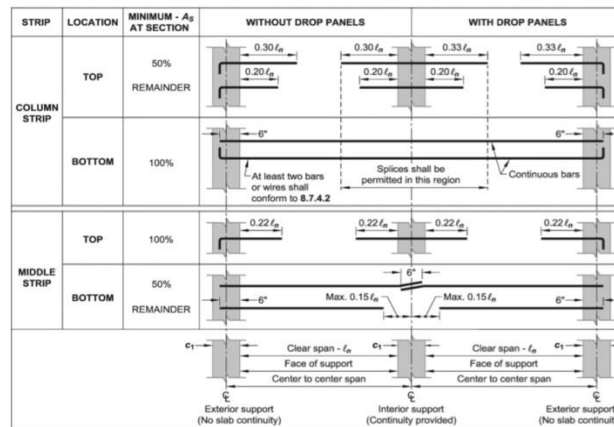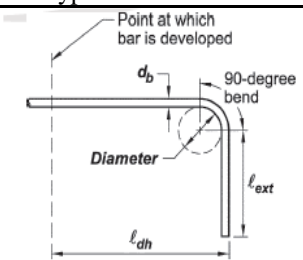$l_{col,top9} = 0.3l_n + 6d_b + 12d_b$, $l_{col,top10} = 0.2l_n + 6d_b + 12d_b$

Figure 6. Minimum extensions for deformed reinforcement in two-way slabs without beams

For the exterior column, the standard hook is expressed as in Table. 11.

Table 11: Standard hook geometry

Minimum inside bend diameter, in.	Straight extension (l_{ext}), in	Type of standard hook
$6d_b$	$12d_b$	

3.1 Introducing cuckoo optimization algorithm

The Cuckoo Optimization Algorithm (COA) was proposed by Rajabioun [29]. This optimization algorithm is inspired by the lifestyle and characteristics in egg laying and breeding of a bird family, called Cuckoo. The cuckoo belongs to the "brood parasites family", birds which never build their own nests and instead lay their eggs in the nest of other species. The cuckoo mother removes the egg laid by the host mother, lays her own egg whose color and pattern is similar to host's eggs, and flies off with the host egg in her bill. Some bird species throw out the strange egg or abandon the nests to prevent the cuckoo from laying eggs. The struggle between host and parasite is akin to an arms race, each trying to out-service the other [29].

Fig. 7 shows a flowchart of the proposed algorithm. Like other evolutionary methods, COA starts with an initial population. These initial cuckoos have some eggs to lay in some host birds' nests. Some of the eggs which have less similarity to the host bird's eggs will be detected and killed. Other eggs will have the opportunity to grow up and become a mature cuckoo. This area is where COA is going to optimize. Cuckoos search for the most suitable area to lay eggs in order to maximize their eggs survival rate. After the remaining eggs grow and turn into a mature cuckoo, they make some societies. Each society has its habitat region to live in. The best habitat of all societies will be the destination for all the cuckoos in other societies. Then they immigrate toward this best habitat. They will inhabit somewhere near the best habitat. Considering the number of eggs each cuckoo has and also the cuckoo's distance to the target point (best habitat), some egg laying radii are dedicated to it. Then, cuckoos start to lay eggs in some random nests inside their egg laying radius. This process continues until the best position with maximum profit value is obtained and most of the cuckoo population is gathered around the same position [29].

In order to solve an optimization problem, the values of problem variables should be formed as an array. In COA the array is called "*habitat*". In a N_{var}-dimensional optimization problem, a habitat is an array of $1 \times N_{var}$, representing current living position of cuckoo. This array is described as:

$$habitat = [x_1, x_2, \ldots, x_{N_{var}}] \tag{75}$$

Each of the variable values $(x_1, x_2, ..., x_{N_{var}})$ is floating point number. The profit of a habitat is obtained by evaluating the profit function, f_p, at a habitat of $(x_1, x_2, ..., x_{N_{var}})$. So

$$\text{Profit} = f_p(\text{habitat}) = f_p(x_1, x_2, ..., x_{N_{var}}) \tag{76}$$

As can be seen, COA is an algorithm that maximizes a profit function. To use COA in cost minimization problems, one can easily maximize the following profit function:

$$\text{Profit} = -\text{Cost (habitat)} = -f_c(x_1, x_2, ..., x_{N_{var}}) \tag{77}$$

To start the optimization algorithm, a candidate habitat matrix of size $N_{pop} \times N_{var}$ is generated. Then some randomly produced number of eggs is supposed for each of these initial cuckoo habitats. In nature, each cuckoo lays from 5 to 20 eggs. These values are used as the upper and lower limits of egg dedication to each cuckoo at different iterations. Another habit of real cuckoos is that they lay eggs within a maximum range which is called "*Egg Laying Radius (ELR)*". In an optimization problem with upper limit of var_{hi} and lower limit of var_{low} for variables, each cuckoo has an egg laying radius (ELR) which is proportional to the total number of eggs, number of current cuckoo's eggs and also variable limits of var_{hi} and var_{low}. So ELR is defined as:

$$ELR = \alpha \times \frac{Number\ of\ current\ cuckoo's\ eggs}{Total\ number\ of\ eggs} \times (var_{hi} - var_{low}) \tag{78}$$

where α is an integer, supposed to handle the maximum value of ELR.

So, after the egg laying process. $\rho\%$ of all eggs (usually 10%) with less profit values will be destroyed. The rest of the eggs will power up and grow in host nests [29].

The young cuckoos grow up in their own zone, but when the laying time comes, they immigrate to new and better habitats where the eggs have more chance to survive. When groups of cuckoos are formed in different zones, the group with best profit value will be targeted and other cuckoos will immigrate there. When the grown cuckoos live in all over the environment, it is hard to recognize which cuckoo belongs to which group. To solve this problem, the cuckoos will be grouped by the k-means clustering method (a k between 3 and 5 seems to be acceptable). When the cuckoos immigrate to the target point, they do not fly the direct way to the destination habitat. They only fly a part of the way and also have a deviation as it is shown in Fig. 8. They just travel $\lambda\%$ of all distance toward the target point and also have a deviation of φ radians. These two parameters, λ and φ, help cuckoos search much more position in all environment. For each cuckoo, λ is a random number between 0 and 1, and φ is a number between $-\frac{\pi}{6}$ and $\frac{\pi}{6}$ [29].
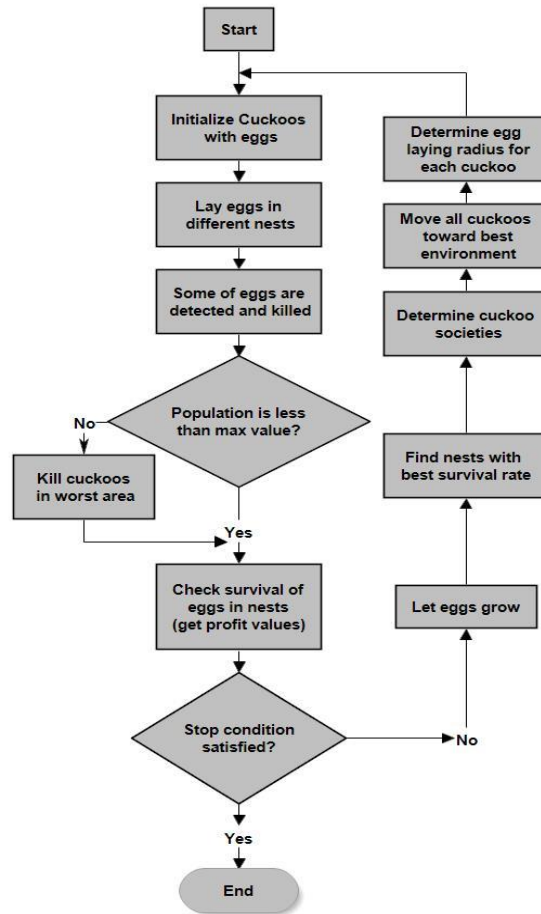
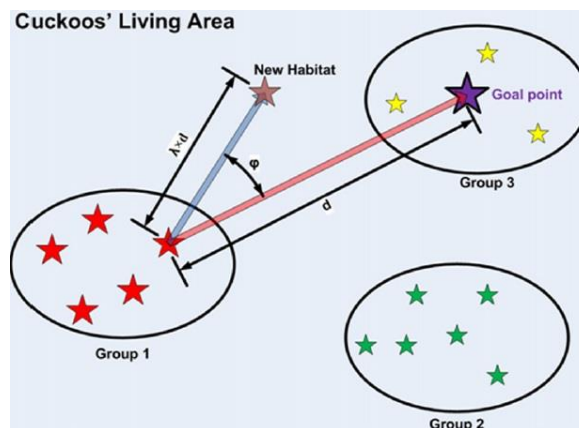Figure 7. Flowchart of cuckoo optimization algorithm

Figure 8. Immigration of a sample cuckoo toward goal habitat

3.2 Optimum design process using COA

The mixed integer-discrete cost optimization problem with constraints can be expressed as:

Minimize $f(x)$

subjected to the following constraints:

$$g_r(x) \leq 0, \quad r = 1, 2, \ldots, m \tag{79}$$

$x_i \in D_i \qquad D_i(d_{i1}, d_{i2}, \ldots, d_{ini}), \quad i = 1, 2, \ldots, N$

where x, $f(x)$, $g_r(x)$, m, N, D_i, and ni are the real vector of design variables, the cost function, the rth inequality constraint, the total number of inequality, the number of integer and discrete design variables, the set of feasible integer and discrete values for the ith variable, and the number of feasible integer/discrete values for the ith variable, respectively. Furthermore, d_{ik} is the kth integer/discrete value for the ith variable. Then, the continuous optimum design values are mapped to the nearest integer/discrete values. All the constraints are handled by using penalty function that is calculated by:

$$C = \sum_{q=1}^{Q} \max[0, g_q(x)] \tag{80}$$

where Q is the number of constraints. Now, the modified objective function, $fM(x)$, is defined as:

$$fM(x) = f(x) + \lambda' C \tag{81}$$

where λ' is the penalty coefficient which is used to tune the intensity of penalization.

The steps of mixed integer-discrete cost optimization are described as follows:

(1) Initial design variables and a value of algorithm parameters are selected.
(2) The values of the continuous design variables are mapped to the nearest integer and discrete variables.
(3) Design variables are updated.
(4) The algorithm calculates the cost function if the constraints are satisfied.
(5) If the new cost function is better than the worst habitat vector, the new habitat replaces the existing worst habitat.
(6) If the termination criterion is satisfied, the procedure stops; otherwise, it goes to step 3.

4. NUMERICAL EXAMPLES

Two RC slabs (one-way RC slab and RC flat slab) are designed by the COA in order to show its efficiency. Slabs are solved several times using different sets of algorithm parameters. The performance of COA depends on the initial COA parameter. These parameters are given in Table 12. A MATLAB computer programing is used to design RC slabs. The optimization software was run on a core i5 laptop with 1.8 GHz of processor speed and 4GB of memory under the Microsoft Windows 8 operating systems. The COA algorithm ran 1000 iterations for each example. All the examples were performed at least 30 times to assure the optimality of result.

Table 12: The values of COA parameters

Number of Cuckoos	50
Minimum number of eggs	2
Maximum number of eggs	7
Maximum iteration	1000
Number of clusters	10
Motion coefficient (λ)	2
Maximum number of Cuckoos	10

4.1 One-way RC slab

One-way RC slabs were previously optimized with neural dynamics model by Ahmadkhanlou and Adeli [8] and PSO by Varaee and Ahmadi-Nedushan [11]. In this paper one-way RC slab is optimized with COA. The results of the examples are compared to PSO and neural dynamics model. Four examples with different support conditions are presented in this section. The input data of the slab is given in Table 13 and Fig. 9. The cost of reinforcement steel and concrete are \$1300/ton (\$1.43/kg) and \$76/cyd (\$ $9.272/m^3$) [8]. For variables h and s, practical values are assumed to be a multiple of $\frac{1}{4}$ " and $\frac{1}{2}$ ", respectively. Example 1 is simply supported at both ends. Example 2 is simply supported at one end and continuous at the other. Example 3 is continuous at both ends (it is part of a multi-space RC slabs). Example 4 is a one-way cantilever RC slab. The obtained optimum values for design variables are summarized in Table 14. Comparison between the COA and neural dynamics model design history for examples 1-4 are shown in Figs. 10-13. According to Table 14 and Fig. 10-13, COA has acceptable performance and speed of convergence to optimize the RC slab. It can be seen that example 4 (cantilever slab) has the maximum cost (\$59.96) and example 3 (continuous at both ends) has the minimum cost (\$20.50) among all examples. Between common and practical slab configurations (examples 1-3), the simply supported slab has the maximum cost. The comparison in Table 14 shows that increasing the amount of steel and decreasing the amount of concrete is more economical than increasing the thickness of slab and decreasing the amount of steel.
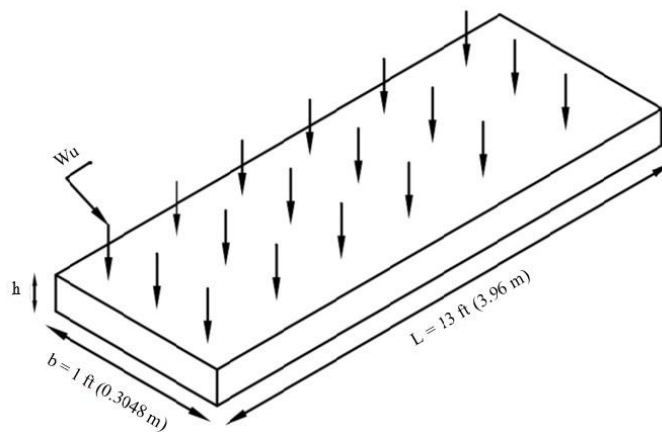

Figure 9. Details of one-way slab

Table 13: Input data for one-way RC slabs

f_y	40 Ksi (275.8 Mpa)	B	1 ft (0.3048 m)
w_s	490lb/ft³(77 KN/m³)	L	13 ft (3.96 m)
f_c'	3 Ksi (20.68 Mpa)	DL	10lb/ft²(0.48 KN/m²)
w_{Rc}	150lb/ft³ (23.6 KN/m³)	LL	40 lb/ft² (2.39 KN/m²)
Cover	0.75 in (19.05 mm)	C_r^1	1300 \$/ton (1.43 \$/kg)
C_c^1	76 \$/cyd (\$9.272/ m³)		

Table 14: Cost optimization results for examples 1-4

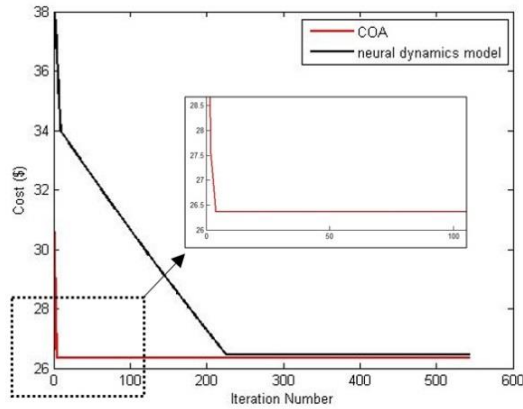|	Ahmadkhanlou [8]				Ahmadi-Nedushan [11]				This paper			
	h (in)	d_b (in)	S (in)	Total cost (\$)	h (in)	d_b (in)	S (in)	Total cost (\$)	h (in)	d_b (in)	s (in)	Total cost (\$)
Example 1	6.75	0.375	6.5	26.45	6.25	0.5	9	26.57	6.25	0.625	14.5	26.36
Example 2	5.57	0.375	7	22.98	5.25	0.375	5.5	22.76	5.25	0.5	10	22.78
Example 3	4.75	0.375	7	19.93	4.5	0.375	5.5	20.64	4.5	0.5	10	20.5
Example 4	13.5	0.375	2	60.22	12.5	0.625	12.5	59.31	12.5	0.875	9.5	59.96

Figure 10. Comparison of the convergence rates between the COA and neural dynamics model for example 1
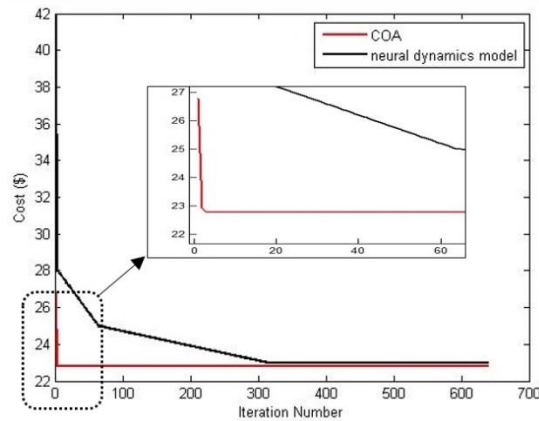

Figure 11. Comparison of the convergence rates between the COA and neural dynamics model for example 2
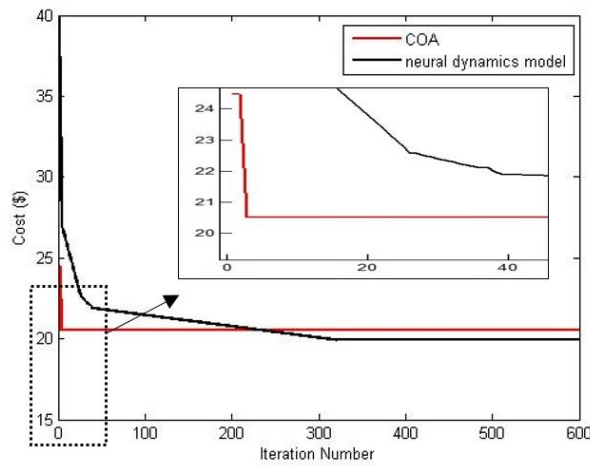
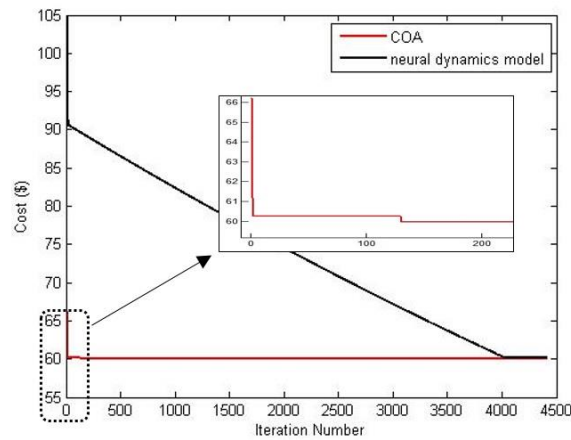Figure 12. Comparison of the convergence rates between the COA and neural dynamics model for example 3

Figure 13. Comparison of the convergence rates between the COA and neural dynamics model for example 4

4.1 RC flat slab

After evaluating the efficiency of the proposed algorithm by comparing it to other algorithms, RC flat slab is designed by COA. Four examples of RC flat slab with different end span conditions are designed in this section. The common data for the examples are given in Table. 15-16. For variables h and s, practical values are assumed to be a multiple of $\frac{1}{4}$". As mentioned before, the number of d_b in column and middle strips are different in various end spans.

Examples 5-8 are the RC flat slabs with interior span, exterior edge unrestrained, slab without beam between interior support and edge beam, and exterior edge fully restrained, respectively. For exterior edge span, the span direction is perpendicular to the edge and the edge column and interior column have been controlled. The obtained optimum values for design variables are given in Table 17. The convergence results for examples 5-8 are shown

in Fig. 14. Table 17 indicates that the slab without beam between interior support and edge beam, and exterior edge fully restrained has the maximum total cost among all examples. The slab with interior span has minimum total cost and from among the slabs with exterior spans, exterior edge unrestrained span is more economical than the other exterior spans.

Table 15: Input data for reinforced flat slabs

f_y	60 ksi (420 MPa)	$l_{col,top2}$	53.4 in (1.36 m)
w_s	490 lb/ft³ (77 KN/m³)	$l_{col,bot3}$	240 in (6.1 m)
w_{RC}	155 lb/ft³ (24.35 KN/m³)	$l_{mid,top4}$	57.84 in (1.47 m)
Cover	0.75 in (19.05 mm)	$l_{mid,bot5}$	173.4 in (4.4 m)
$C_{x,y\,col}$	18 in (45.72 cm)	$l_{mid,bot6}$	234 in (5.94 m)
LL	42 lb/ft² (2.011 KN/m²)	$l_{col,top7}$	237 in (6.01 m)
DL	42 lb/ft² (2.011 KN/m²)	$l_{col,top8}$	203.7 in (5.17 m)
$l_1 = l_2$	20 ft (6.096 m)	$l_{col,bot9}$	$66.6 + 18d_b$ in
$l_{col,top1}$	75.6 in (1.92 m)	$l_{mid,top10}$	$44.4 + 18d_b$ in

Table 16: Cost ratio of the cost of a unit volume of concrete to a unit volume of concrete

f_c'	3000 psi (21 MPa)	4000 psi (28 MPa)	5000 psi (35 MPa)	6000 psi (40 MPa)
r_1 (lb/in³)	0.0025 lb/in³ (69.2 kg/m³)	0.0027 lb/in³ (74.73 kg/m³)	0.0028 lb/in³ (77.5 kg/m³)	0.0032 lb/in³ (88.6 kg/m³)

Table 17: Cost optimization for reinforced flat slabs

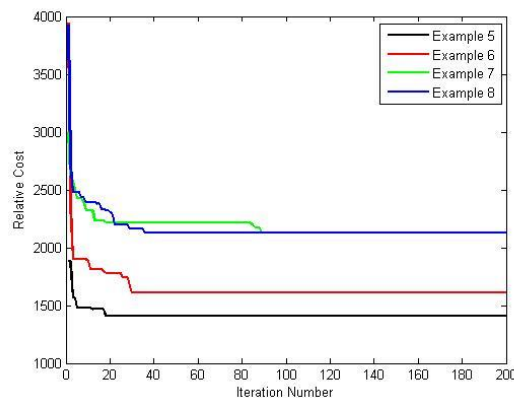|	f_c' (psi)	h (in)	$d_{b\,col,int\,sup}^-$ (in)	$d_{b\,col,ext\,sup}^-$ (in)	$d_{b\,col}^+$ (in)	$d_{b\,mid,int\,sup}^-$ (in)	$d_{b\,mid}^+$ (in)	S (in)	Total cost/C_r^1 (lb)
Example 5	3000	7.00	0.625	-	0.375	0.375	0.375	9.25	1409.61
Example 6	5000	7.50	0.75	-	0.625	0.50	0.50	15.00	1606.53
Example 7	5000	10.25	0.50	0.50	0.375	0.375	0.375	6.25	2123.18
Example 8	5000	10.25	0.50	0.50	0.375	0.375	0.375	6.25	2123.18

Figure 14. Convergence result for examples 5-8 (for first 200 iteration)

5. CONCLUSIONS

Cost optimization of RC one-way slabs and RC flat slabs with various end conditions using the COA was presented in this study. The design of the slabs was based on ACI code and the procedure included finding the optimum thickness of slab, diameter of reinforcement bars, and spacing of reinforcement. The constraints were handled using penalty function. As mentioned before, the main goal of this paper was to demonstrate that natural evolutionary algorithm can design and optimize the real life structures efficiently. The design of flat slabs like a practical application do not only considers the design code requirements but also determine reinforcement detailing constraints. Furthermore, according to the results, the COA that was used for the first time to optimize the concrete slab proved to have acceptable speed of convergence to optimize the concrete structures. In addition, for the one-way slabs the comparison of the optimization result of COA with neural dynamics model and PSO demonstrated the superiority of the COA to achieve better results than the other two algorithms.

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