STUDY ON THE FUNDAMENTAL FREQUENCIES OF RECTANGULAR CONCRETE TANKS USING FEM AND ANALYTICAL CODES

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ABSTRACT

Impulsive and convective frequencies are one of the most important subjects for evaluation of the seismic behavior of tanks. These two frequencies are defined by Housner and used for obtaining Rayleigh damping in time history analysis. ACI 350 and NZSEE standards have suggested some analytical solutions for finding convective and impulsive frequencies. These frequencies can also extract from modal analysis by finite element (FE) software. In current study, these frequencies are extracted by using FE software and performing modal analysis. Also these modes are compared with analytical methods from ACI and NZSEE standards. Based on the results, convective frequencies obtained from FE and ACI and NZSEE methods are so close together, with just two percent variation between FE and analytical codes, while there are significant differences among these methods for impulsive frequencies. Furthermore, this study shows that the wall thickness has no effect on the convective frequencies, while it is completely opposite for impulsive frequency. When the wall thickness rises by 1.5 times, impulsive frequencies increase by 1.75, 1.55 and 1.48 times for finite element, NZSEE and ACI methods, respectively. In addition, based on the observations, when the liquid height is low, NZSEE method presents high values of impulsive frequency.

Keywords: tanks; convective frequencies; impulsive frequencies; modal.

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1. INTRODUCTION

Liquid storage tanks used widely in petroleum industries, urban water storage facilities, and nuclear water places [1]. Experience of past earthquakes in different countries showed that
these types of structures are so vulnerable under severe earthquakes. The performance of these structures is different from other structures like bridges and buildings. These differences are due to the effect of hydrodynamic pressure on the tank wall. Hence, the stability of the liquid storage tanks under earthquake conditions must be investigated cautiously [2].

Many damages of tanks are observed around the world. For example, many tanks damaged in 1964 Niigata earthquake and it caused a widespread uncontrolled fire eruption in petroleum tanks [3]. Along the Kocaeli earthquake that happened in Northwest Turkey many of tanks were destroyed. Most damages of tanks were observed at 19 kilometer from the earthquake epicenter [4].

The initial and most commonly study on the seismic behavior of the tanks is carried out by Housner [5]. He proposed a new method for rigid tanks using analytical technique in his method, hydrodynamic pressure is divided into convective and impulsive part. The Housner’s method has been selected for seismic analysis in all of past codes and regulations and this method has been considered for basis of many standards. An expanded application of Housner’s method, in the terms of an advanced design method, was presented by Epstein [6]. Haroun and Tayel [7] proposed a FE method for investigating the seismic response of tanks using Housner model. More research on the behavior of tanks under dynamic loads can be carefully observed by [8], [9], [10] and [11].

The seismic behavior of the storage tanks is mainly dependent on their dynamic characteristics such as natural frequencies, mode shapes and damping ratios [12]. Gupta and Hutchinson [13] studied the effects of wall flexibility on the dynamic response of liquid storage tanks and indicated that for both shallow and deep tanks frequencies decrease with reduction of wall thickness. Tedesco et al. [14] investigated free vibration of steel cylindrical tanks incorporating fluid - structure interaction. They proposed an analytical procedure which accurately predicts the fundamental frequencies of tanks. Their method was applicable to tanks both completely full and partially full with liquid. Firouz-Abadi et al [15] used a three dimension boundary element method to identify the natural frequencies and mode shapes of liquid sloshing in cylindrical baffled tanks. Amiri and Yazdi [16] used both experimental tests and finite element modeling of cylindrical tanks. They found good agreement between the numerical and experimental values of dynamic parameters. Yazdanian et al. [17, 18] showed that the natural frequencies obtained from modal analysis are a starting point for time history analysis and it is vital to obtain the frequencies before any further analysis.

In investigation of the seismic behavior of tanks, one of the most crucial issues is selection of convective and impulsive frequencies in storage tanks. As it previously mentioned, these two frequencies are defined by Housner. ACI 350.3 [19] and NZSEE [20] standards have suggested some analytical solutions for finding convective and impulsive frequencies and other parameters. These frequencies can also extract from modal analysis using FE software. In current study, these frequencies are extracted using FE software [21] and performing modal analysis. Then, these frequencies are compared with analytical methods from ACI and NZSEE codes.

Considering the previous studies carried out on this topic, it is clear that very limited research has been done on these two frequencies and most researches has just studied the behavior of tanks under some ground motions, however, there are several studies on
vibration analysis of tanks but they had a special emphasize on the cylindrical tanks. Furthermore, none of the previous studies has investigated the impact of different parameters on vibration analysis of tanks.

This study comprehensively investigates the dynamic behavior of concrete rectangular tanks using the FE method in three-dimensional space in which the coupled fluid–structure equations are solved and two different reliable analytical codes. For this purpose, a parametric study has been conducted and different parameters are investigated. To account for the effect of liquid density and thickness three different liquid height and thickness are taken into account. In addition, three tanks with different liquid height are modeled to consider the impact of this parameter. Finally, five tanks which have a variation in their dimensions are considered to investigate the effect of length to wide. It should be noted that the main objective of this study is to investigate the effect of these two frequencies. As a result, this study is mainly focused towards this part and a brief summary is given on other dynamic parameters.

2. MATERIALS AND METHODS

2.1 Governing equation of dynamic forces

Time history analysis can take all the nonlinear factors into the analysis, so it is used to estimate the exact amount of structural response. The discretized for a multi degree of freedom system subjected to external dynamic forces can be defined as [22]:

\[
[M][\ddot{u}] + [C][\dot{u}] + [K][u] = [f]
\]  

(1)

In which, \([M]\) is the mass matrix of the system; \([C]\) is the damping matrix of the system; \([K]\) is stiffness matrix of the system; \([u]\) is displacement vector; \([\dot{u}]\) is velocity vector; \([\ddot{u}]\) is acceleration vector; \([f]\) is the applied load vector.

In this method, the step by step integration is applied directly to achieve the solution for the original equations of motion of the system. For solving this equation, it is needed to \([C]\), damping matrix resulted. Damping matrix of the system can be resulted from sum of the stiffness matrix and mass matrix of the system. In many research, similar classical damping system said as Rayleigh damping (\(\alpha\) and \(\beta\)) is applied for both structural and fluid domains leading to a classical damping for the whole scheme. Alpha damping and Beta damping are applied to indicate Rayleigh damping constants \(\alpha\) and \(\beta\). The damping matrix within the fluid domain \([C]\) contains two parts which are because of impulsive and convective components of the stored fluid [25].

\[
[C] = \alpha[M] + \beta[K] + \sum_{i=1}^{m}|C_{fi}|
\]  

(2)

\(\alpha\) is defined based on the natural frequency of the primary sloshing mode and calculates for the damping because of sloshing on the free surface of the tank liquid. \(\beta\) is defined based on the primary frequency of the tank and simulates the damping because of the impulsive part.
\[
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = \frac{2 \omega_i \omega_j}{\omega_i^2 + \omega_j^2} \begin{bmatrix}
\frac{\omega_j}{1} & -\frac{\omega_i}{\omega_j} \\
1 & \frac{\omega_i}{\omega_j}
\end{bmatrix} \begin{bmatrix}
\xi_i \\
\xi_j
\end{bmatrix}
\] 
(3)

As suggested by ACI 350.3-06, damping ratios of 5% and 0.5% are specified for the impulsive and convective parts, respectively. Two frequencies of \(\omega_i, \omega_j\) are the impulsive and convective frequencies which are achieved from modal analysis in FE software according to maximum effective mass or they can be obtained by ACI and NZSEE standards. The convective frequency is lower than impulsive frequency. In follow figure, convective and impulsive masses are shown. It is noteworthy that impulsive mass is in lower height than convective mass. According to above equations, in time history analysis one of the most important issues is obtaining of impulsive and convective frequencies. Because for solving of multi degree of freedom system subjected to external dynamic forces, it is needed to damping matrix obtained.

![Figure 1. Convective and impulsive mass](image)

2.2 Finite element method

In the current research, FE software as general purpose computer code is utilized to perform modal analyses. In Fig. 2 the finite element model of one of the tanks is illustrated. Properties of the concrete and fluid materials are as shown in Table 1. The materials of the tanks are assumed isotropic, linearly elastic. For modeling the fluid and solid part of the tanks fluid 80 and shell 63 elements are used respectively. Shell 63 has six degrees of freedom at each node which is suitable for analyzing thin to moderately-thick shell structures and also, for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses. The fluid 80 element is used to model fluids contained within vessels having no net flow rate. The fluid element is particularly well suited for calculating hydrostatic pressures and liquid-solid interactions. In FE modelling of the tanks, the fluid and the structure are coupled at a mesh interface along normal direction without a value. Also, fluid nodes located at the interface with the tank floor are coupled with the base nodes. According to fluid 80 restrictions, fluid element at a boundary should not be attached directly to structural elements but should have separate, coincident nodes that are coupled only in the direction normal to the interface. For interfaces where structure is not present such as fluid free surface, no fluid-structure interaction flags need to be assigned. In addition,
According to fluid80 element inputs, Young’s modulus which is interpreted as the "fluid elastic modulus" should be the bulk modulus of the fluid. In addition, the viscosity property is used to compute a damping matrix for dynamic analyses and as a result, the values of viscosity have no effect on the frequencies. This is due to the fact that in the modal analysis, damping values have no effect on the values of frequencies [21].

Table 1: Properties of liquid and tank for concrete rectangular tanks

<table>
<thead>
<tr>
<th>Materials Properties</th>
<th>Density (kg/m$^3$)</th>
<th>Poisson’s Ratio</th>
<th>Young’s modulus (GPa)</th>
<th>Bulk’s modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>2500</td>
<td>0.3</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>Liquid (water)</td>
<td>1000</td>
<td>-</td>
<td>-</td>
<td>2.07</td>
</tr>
<tr>
<td>Liquid (oil)</td>
<td>800</td>
<td>-</td>
<td>-</td>
<td>1.62</td>
</tr>
<tr>
<td>Liquid (Gasoline)</td>
<td>680</td>
<td>-</td>
<td>-</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Figure 2. Schematic of tank parameter; (a) Finite element modeling of tank; (b) cross-section of a tank [19]

Modal analysis finds the various periods of a structure according to naturally resonate by using the overall mass and stiffness. These periods of vibration can be used in assessing the dynamic behavior of the structure. Natural frequencies and mode shapes of a tank are important parameters in the analysis of tank. Determining these parameters in the first step can be very useful in interpreting the behavior of the tank. Convective and impulsive modes are most important modes which have the maximum effective mass to account for the dynamic analysis. According to the maximum effective mass in modal analysis two modes of impulsive and convective are extracted and these modes amount would be set in Eq (3). In addition, this analysis can be a starting point for other analysis, such as response spectrum analysis or time history analysis. Two vibration mode shapes of tanks are illustrated in Fig. 3. It should be noted that the natural frequency of a system is dependent only on the stiffness of the structure and the mass which participates with the structure. Furthermore, Stiffness depends upon material properties and geometry. The stiffness of a structural element of a given material is the product of the material’s Young’s modulus and the element’s second moment of area.
Figure 3. Vibration mode shape of tanks

2.3 ACI method

According to ACI standard, time period of convective mode of vibration, in seconds, is given as follow:

$$T_c = \frac{2\pi}{\lambda} \sqrt{L}$$  \hspace{1cm} (4)

In which, L is inside length of a storage tank with a rectangular shape, L must be parallel to the direction of the earthquake force and is defined as follow:

$$\lambda = \sqrt{(3.16 \times g \times \tanh(3.16(\frac{H_L}{L}))}$$  \hspace{1cm} (5)

where, $H_L$ is liquid height and $g$ is gravity. Also according to ACI impulsive period of the rectangular tank is as follow:

$$T_i = 2\pi \sqrt{\frac{m}{K}}$$  \hspace{1cm} (6)

where, $m$ is accounted for by adding impulsive mass of contained liquid per unit width of a rectangular tank wall ($m_i$) and mass per unit width of a rectangular tank wall ($m_w$):

$$m = m_i + m_w$$  \hspace{1cm} (7)

Impulsive mass of contained liquid per unit width of a rectangular tank is given as:

$$m_i = (\frac{W}{W_L}) \times (\frac{L}{2}) \times H_L \times \rho_L$$  \hspace{1cm} (8)

In which, $W_i$ is equivalent mass of the impulsive component of the stored liquid; $W_L$ is total mass of the stored liquid; $H_L$ is design depth of stored liquid and $\rho_L$ is mass density of
the contained liquid. Also equivalent masses of accelerating liquid, \( \frac{W_i}{W_L}, \frac{W_c}{W_L} \) are given as:

\[
\frac{W_i}{W_L} = \frac{\tanh(0.866(\frac{L}{H_L}))}{0.866(\frac{L}{H_L})} \tag{9}
\]

\[
\frac{W_c}{W_L} = 0.264(\frac{L}{H_L})\tanh(3.16(\frac{H_L}{L})) \tag{10}
\]

In above equation, \( W_i \) is equivalent mass of the convective component of the stored liquid.

\[
m_w = (H_w) \times \left(\frac{t_w}{10}\right) \times \rho_c \tag{11}
\]

In which, \( H_w \) is height of tank wall; \( t_w \) is average wall thickness and \( \rho_c \) is mass density of concrete. For fixed-base, open-top tanks, flexural stiffness \( k \) may be obtained as follow:

\[
k = \frac{E_c}{4 \times 10^6} \times (\frac{t_c}{h})^3 \tag{12}
\]

\[
h = \frac{(h_u m_w + h_i m_i)}{(m_u + m_i)} \tag{13}
\]

where, \( h_u \) is height from the base of the wall to the center and \( h_i \) is height above the base of the wall to the center of gravity of the impulsive lateral force of gravity of the tank shell.

\[
h_u = 0.5 \times H_u \tag{14}
\]

Also according as follow equations \( h_i \) and \( h_c \) can be computed. In follow equations, \( h_i \) and \( h_c \) are the height above the base of the wall to the center of gravity of the impulsive and convective lateral force.

\[
For \ \tan ks \ with \ \frac{L}{H_L} > 1.33 \ \Rightarrow \ \frac{h_i}{H_L} = 0.375 \tag{15}
\]

\[
\frac{h_i}{H_L} = 1 - \frac{\cosh(3.16(\frac{H_L}{L}) - 1)}{3.16(\frac{H_L}{L}) \sinh(3.16(\frac{H_L}{L}))} \tag{16}
\]

2.4 NZSEE method

In NZSEE method, \( T_i \) is determined according to ACI method. Also \( T_i \) can be defined according as follow:
In which, \( d \) is deflection of the tank wall on the vertical center-line at a height of \( \bar{h} \), when loaded by uniformly distributed pressure of intensity \( q \). Where \( q \) is as follow:

\[
q = \frac{(\frac{W_c}{2} + \bar{W}_w)g}{BH_1}
\]

(18)

\[
d = \frac{p(\bar{h})^3}{3EI_w}
\]

(19)

\[
\bar{h} = \frac{\frac{W_c}{2} + \bar{W}_w + H_L}{2}
\]

(20)

\[
I_w = \frac{1.0 \times 1^3}{12}
\]

(21)

\[
p = q \times H_L \times 1
\]

(22)

**Table 2: Characteristics of the selected tanks**

<table>
<thead>
<tr>
<th>Tank No</th>
<th>Fluid type</th>
<th>L (m)</th>
<th>B (m)</th>
<th>H_w (m)</th>
<th>H_l (m)</th>
<th>t_w (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>water</td>
<td>18</td>
<td>12</td>
<td>5</td>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>water</td>
<td>18</td>
<td>12</td>
<td>5</td>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>water</td>
<td>18</td>
<td>12</td>
<td>5</td>
<td>3</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>oil</td>
<td>18</td>
<td>12</td>
<td>5</td>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>gasoline</td>
<td>18</td>
<td>12</td>
<td>5</td>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>water</td>
<td>9.9</td>
<td>9.9</td>
<td>3</td>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>7</td>
<td>water</td>
<td>11</td>
<td>8.9</td>
<td>3</td>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>8</td>
<td>water</td>
<td>12</td>
<td>8.17</td>
<td>3</td>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>9</td>
<td>water</td>
<td>13</td>
<td>7.54</td>
<td>3</td>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>10</td>
<td>water</td>
<td>14</td>
<td>7</td>
<td>3</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>11</td>
<td>water</td>
<td>14</td>
<td>7</td>
<td>3</td>
<td>2</td>
<td>0.6</td>
</tr>
<tr>
<td>12</td>
<td>water</td>
<td>14</td>
<td>7</td>
<td>3</td>
<td>2</td>
<td>0.6</td>
</tr>
</tbody>
</table>

The geometric characteristics of the studied tanks are shown in Table 2. These tanks are selected for a parametric study on the tanks. Tanks No.1, 2 and 3 are selected for investigating the effect of liquid filling in tanks. Tanks No.1, 4 and 5 are selected for investigating the effect of viscosity type. Tanks No. 6, 7, 8, 9, and 10 are selected for investigating the effect of length to wide. These five tanks cover the length to wide ratio between one and two. It should be mentioned that all these 5 tanks have the same volume of water. Tanks No. 10, 11 and 12 are selected for investigating the effect of thickness on convective and impulsive frequencies.
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3. VERIFICATION OF NUMERICAL MODEL

To verify the FE method used in the present study an analytical model that was proposed in Kianoush and Ghaemmaghami [11] was adopted. The tank used in the validation process had a length of 30 m, width of 60 m, height of 6 m, liquid height of 5.5 m, and wall thickness of 0.6 m. As can be seen in Table 3 the convective and impulsive frequencies calculated using the FE model were in close agreement with those obtained based on the model by Kianoush and Ghaemmaghami.

Table 3: Comparison of impulsive and convective frequencies of current research and Kianoush and Ghaemmaghami

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Current research</th>
<th>Kianoush and Ghaemmaghami</th>
</tr>
</thead>
<tbody>
<tr>
<td>convective</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>impulsive</td>
<td>6.78</td>
<td>6.66</td>
</tr>
</tbody>
</table>

4. RESULT AND DISCUSSION

In Table 4, convective and impulsive frequencies are shown. It can be seen from table that convective frequencies are so close together in both FE and analytically method. Also, it is clearly seen from Table 4 that impulsive frequencies in FE, NZSEE and ACI methods are not as close as convective frequencies.

Table 4: Comparison of impulsive and convective frequencies

<table>
<thead>
<tr>
<th>Tank No</th>
<th>Convective frequencies</th>
<th>Impulsive frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ACI &amp; NZSEE</td>
<td>FEM</td>
</tr>
<tr>
<td>1</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>2</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>3</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>4</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>5</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>6</td>
<td>0.21</td>
<td>0.20</td>
</tr>
<tr>
<td>7</td>
<td>0.19</td>
<td>0.18</td>
</tr>
<tr>
<td>8</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>9</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>10</td>
<td>0.15</td>
<td>0.14</td>
</tr>
<tr>
<td>11</td>
<td>0.15</td>
<td>0.14</td>
</tr>
<tr>
<td>12</td>
<td>0.15</td>
<td>0.14</td>
</tr>
</tbody>
</table>

In Table 5 other impulsive and convective parameters are shown. These parameters are included height ($h_c$ and $h_i$) and mass ratio ($W_C/W_L$ and $W_I/W_L$) which convective and impulsive forces are exerted respectively. As it can be observed, the height which convective force is exerted is higher than impulsive height. This was also shown by Housner (1963). By comparing the height which convective and impulsive force are exerted in tank
No. 1, 2 and 3 it is found that the convective and impulsive height in tank No. 1 is more than two other tanks which have lower level of water. Also it can be observed that the values of convective masses are more than impulsive masses in all tanks.

Comparison of the values obtained from tanks No. 1, 2 and 3 with each other show that by increasing in liquid height the value of impulsive mass is increased and the value of convective mass is decreased.

By comparing the values of \( h_c \) and \( h_i \) in tanks No. 6 to 10 which are selected for investigating effect of length variation, it can be observed that an increase in the length which is parallel to direction of earthquake (L) leads to a decrease in the values of \( h_c \) and \( h_i \) and also, an increase in the length leads to an increase and decrease in the values of convective and impulsive masses. In addition, it is seen from Table 4 that a rise in the wall thickness has no effect on convective and impulsive height and mass.

<table>
<thead>
<tr>
<th>Tank No</th>
<th>Convective</th>
<th>Impulsive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Height</td>
<td>Height</td>
</tr>
<tr>
<td></td>
<td>( h_c ) (m)</td>
<td>( h_i ) (m)</td>
</tr>
<tr>
<td>1</td>
<td>2.649</td>
<td>0.670</td>
</tr>
<tr>
<td>2</td>
<td>2.078</td>
<td>0.719</td>
</tr>
<tr>
<td>3</td>
<td>1.534</td>
<td>0.765</td>
</tr>
<tr>
<td>4</td>
<td>2.649</td>
<td>0.670</td>
</tr>
<tr>
<td>5</td>
<td>2.649</td>
<td>0.670</td>
</tr>
<tr>
<td>6</td>
<td>1.033</td>
<td>0.736</td>
</tr>
<tr>
<td>7</td>
<td>1.027</td>
<td>0.753</td>
</tr>
<tr>
<td>8</td>
<td>1.022</td>
<td>0.765</td>
</tr>
<tr>
<td>9</td>
<td>1.019</td>
<td>0.774</td>
</tr>
<tr>
<td>10</td>
<td>1.017</td>
<td>0.782</td>
</tr>
<tr>
<td>11</td>
<td>1.017</td>
<td>0.782</td>
</tr>
<tr>
<td>12</td>
<td>1.017</td>
<td>0.782</td>
</tr>
</tbody>
</table>

4.1 Effect of liquid height to tank height

In Fig. 4, the convective and impulsive frequencies of tank No. 1, 2 and 3 are shown. It is seen from Fig. 4(a) that the convective frequencies are so near together according to ACI, NZSEE and FE method. Also, it can be observed by increasing the level of liquid, convective frequencies are increased, while Fig. 4 (b) shows that an increase in the liquid level leads to a decrease in the impulsive frequencies. The values of impulsive frequencies are close to each other in tank No. 1 which has five meter level of liquid. When the level of liquid is decreased the values of impulsive frequencies are differed. In the tank No. 3 the level of water is three meter and NZSEE code shows a very high value of impulsive frequency. It should be noted that the average differences between impulsive frequencies of NZSEE and FE method is 22 percent, while this figure for ACI and FE is just 6 percent. By decreasing the level of liquid, the NZSEE code presents high values of impulsive frequencies. A more detailed look at the graphs reveals that when a liquid level of a tank increase by 1.66 times, the figure for impulsive frequencies of NZSEE code falls over 60
percent. On the other hand, figures for convective frequencies grow by above 20 percent in all three methods. The relative proportions of the convective and impulsive liquid are highly dependent on the ability of the tank to confine the liquid. Greater the confinement, greater is the impulsive liquid and smaller the convective liquid. As a result, when a tank has a lower liquid level, it presents higher impulsive and smaller convective frequencies.

![Image](https://via.placeholder.com/150)

**Figure 4.** Comparison of convective and impulsive frequencies. (a): convective (b): impulsive

### 4.2 Effect of liquid density

In Fig. 5, the convective and impulsive frequencies of tanks No.1, 4 and 5 is shown. These tanks have the same material and dimension and the liquid of them are different. It is clearly seen from Fig. 5 (a) that liquid density value has no effect on convective frequencies because of all the three convective response of tanks are the same, in other words, this figure clearly shows that for the viscosity values considered in this study the convective frequencies remain unaffected. It can be clearly seen from figure that ACI, NZSEE and FE methods present the same values. It is seen from Fig. 5 (b) that an increase in the viscosity leads to a decrease in the figures for impulsive frequencies. The case study show that 47 percent rise in density pose an eight percent decrease in values of impulsive frequencies. The impulsive frequency is highly dependent on mass matrix, while there is no close relation between mass and convective frequency.

![Image](https://via.placeholder.com/150)

**Figure 5.** Comparison of convective and impulsive frequencies. (a): convective (b): impulsive
4.3 Effect of dimension variation

Fig. 6 compares the effect of length to wide (L/B) ratio on both convective and impulsive frequencies. It is seen from Fig. 6 (a) that in all three methods, a rise in (L/B) ratio leads to a decrease in the values of convective frequencies. As previously observed, the values of convective frequencies are the same in all three methods. It can be observed from Fig. 6 (b) that an increase in the (L/B) ratio leads to a rise in the values of impulsive frequencies in FE method, while the values of impulsive frequencies are the same in all tanks in ACI and NZSEE methods. The values of impulsive frequencies are more dependent on the volume of liquid in ACI and NZSEE standard and in this case, all these tanks have the same volume, so the impulsive frequencies values for these standards remain unchanged for all five tanks and there are differences among the impulsive frequencies of three different methods. However, the values of impulsive frequencies in ACI and FE method are near together. Overall, it should be noted that the differences in convective frequencies are more significant than discrepancy for impulsive frequencies. As it was mentioned before, the values of impulsive frequencies are highly dependent on the boundary condition of tank walls and variation in (L/B) has not effect on boundary condition of tank walls. In contrast, the values of convective frequencies are deeply affected by changes in (L/B).

4.4 Effect of thickness

Fig. 7 compares the convective and impulsive frequencies of tanks No.10, 11 and 12. These tanks are selected for investigating the effect of wall thickness on the convective and impulsive frequencies. It can be seen from Fig. 7 (a), the convective frequencies remain unchanged by variation of wall thickness. Also it is seen from this figure that convective frequencies are so close in ACI & NZSEE and FE methods, while according to Fig. 7 (b) there are some differences in values of impulsive frequencies. Furthermore, the values of impulsive frequencies in ACI and FE methods are close to each other and the values of NZSEE method is more than FE and ACI methods. The average discrepancy between impulsive frequencies obtained from ACI and FE are about eight percent, while this difference for NZSEE and FE method are highlighted, with 47 percent. It is seen from Fig. 7 (b) that an increase the in wall thickness leads to an increase in the impulsive frequencies. This is due to the fact that increases in the thickness bring about rise in the stiffness of the
tank wall and the stiffness of the tanks wall has a direct relation with frequency. A more
detailed look at the graphs reveals that when the thickness of the wall become 1.5 times, the
figures for impulsive frequencies increased by 1.75, 1.55 and 1.48 times for FE, NZSEE and
ACI method, respectively.

![Graph](image)

Figure 7. Comparison of convective and impulsive frequencies. (a): convective (b): impulsive

5. CONCLUSIONS

In this paper, two fundamental frequencies are obtained using FE, ACI and NZSEE
methods. Also, these frequencies are compared with each other. For this purpose a
comprehensive study is done and many parameters such as Effect of liquid height to tank
height, Effect of liquid density, Effect of dimension variation and Effect of thickness are
investigated. Based on the results and discussions reported in this paper, the following
conclusions can be drawn:

1) The results show that an increase in the level of water led to an increase in the value of
convective and impulsive height. In addition, it can be resulted that an increase in the
liquid height leads to an increase in value of impulsive masses while this increase results
in decrease in the convective masses. It was also, concluded that an increase in the (L) to
wide (B) ratio leads to an increase in the values of \( h_c \) and \( h_i \) and also an increase in the
(L/B) ratio leads to a corresponding increase in the values of convective masses and
decrease in values of impulsive masses. In addition, variation of the wall thickness has no
effect on convective and impulsive mass and height.

2) This study showed that an increase in the level of liquid led to an increase and decrease in
the convective frequencies and impulsive frequencies, respectively. It was shown that by
decreasing the level of liquid the NZSEE standard presented considerably higher
impulsive frequencies than ACI and FE methods, when a liquid level increased by 1.66
times, the values of impulsive frequencies of NZSEE code experienced a fall of 60
percent, while this was less marked for FE and ACI, with 25 and 35 percent declines
respectively. On the other hand, figures for convective frequencies grow by above 20
percent in all three methods. It was also, resulted that in high level of liquid, codes and
FE methods show the same results.
3) Based on the results, by changing in the values of density, convective frequencies remain constant, while it was found that a 47 percent rise in density posed an eight percent decline in values of impulsive frequencies. The impulsive frequency is highly dependent on mass matrix, while there is no close relation between mass and convective frequency.

4) Results of this study showed that variation in dimension of a tank has a higher impact on the convective frequencies than impulsive frequencies. A rise by two times in the (L/B) ratio led to drop by 30 percent in the figures for convective frequencies, while the values of impulsive frequencies in FE method grow by 14 percent. Furthermore, based on the observations, the values of impulsive frequencies remained steady in ACI and NZSEE methods. The values of impulsive frequencies are more dependent on the volume of fluid in ACI and NZSEE standards and this subject showed defections of ACI and NZSEE methods. However, the values of impulsive frequencies are highly dependent on the condition of tank walls and mass matrix and changing the (L/B) does not affect significantly on these.

5) This study showed that the wall thickness has no effect on the convective frequencies, while when the thickness of the wall become 1.5 times, impulsive frequencies rose by 1.75, 1.55 and 1.48 times for FE, NZSEE and ACI method, respectively. In fact, the thickness led to rise in the stiffness of the tank wall and the stiffness of the tanks wall has a direct relation with frequency. It is noteworthy that the convective frequencies are independently of the tank wall and the impulsive liquid moves with the tank wall. In addition, the average difference between impulsive frequencies obtained from ACI and FE are about eight percent, while this is about 47 percent for NZSEE and FE method.

6) Overall, Based on the obtained results, convective frequencies are so close together in all three methods, with just two percent variation between FE and analytical codes, while these discrepancies are more highlighted for impulsive frequencies. The average discrepancies of all the impulsive frequencies between ACI and FE is about nine percent, while this is more highlighted for NZSEE and FE method, with 45 percent. In addition, there are less marked 37 percent differences between ACI and NZSEE method. When the liquid height is low and also volume of liquid is great, NZSEE method presents a high impulsive frequency.

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