



## CRACK DETECTION IN CONCRETE BEAM USING OPTIMIZATION METHOD

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### ABSTRACT

Structural damage detection is a field that has attracted a great interest in the scientific community in recent years. Most of these studies use dynamic analysis data of the beams as a diagnostic tool for damage. In this paper, a massless rotational spring was used to represent the cracked sections of beams and the natural frequencies and mode shape were obtained. For calculation of rotational spring stiffness equivalent of uncracked and cracked sections, finite element models and experimental test were used. The damage identification problem was addressed with two optimization techniques of different philosophers: ECBO, PSO and SQP methods. The objective functions used in the optimization process are based on the dynamic analysis data such as natural frequencies and mode shapes. This data was obtained by developing a software that performs the dynamic analysis of structures using the Finite Element Method (FEM). Comparison between the detected cracks using optimization method and real beam shows an acceptable agreement.

**Keywords:** crack; rotational spring; objective function; optimization; ECBO; bending rigidity.

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### 1. INTRODUCTION

The occurrence of cracks in reinforced concrete elements under service loads is expected due to the low tensile strength of concrete, weathering, creep and aging effects. Cracking in reinforced concrete structures has an effect on structural performance including stiffness, energy absorption, capacity and ductility. Reduction in the strength and stiffness properties of a structure can be dangerous and may lead to catastrophic structural failures. The effect of

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location and depth of cracks on the static and dynamic behavior of concrete structural element has been the subject of several studies and investigations.

One of the first studies on crack detection is the one by Adams et al. [1] that used both sensitivity and finite element method to determine crack location and depth. Gudmundson [2] in their investigation, used perturbation as well as transfer matrix method to study the influence of small cracks on the Eigen frequencies and Eigen modes of slender structures.

The modal assurance criterion (MAC) was used by West [3] to determine the level of correlation between modes from the test of an undamaged Space Shuttle Orbiter body flap and the modes from the test of the flap after it was exposed to an acoustic loading.

The coordinate modal assurance criterion (COMAC) is a mode shape based damage indicator proposed by Lieven and Ewins [4]. Qian et al. [5] used a finite element model to determine the natural frequencies of a cantilevered beam, taking into account, the effect of crack closure. Also, a method based on the relationship between the crack and modal parameters, to determine the crack position from known natural frequencies was developed. Chondros et al. [6] developed a continuous cracked beam theory for free vibration analysis; their basic assumption was that the crack caused a continuous change in flexibility in its neighborhood which they modelled by incorporating a consistent displacement field with singularity. A different but related approach in which a crack in rotational shaft is replaced by a mass-less spring-link located at the crack position became popular due to much effort by Dimarogonas and Papadopolous [7]. Kheyroddin [8] proposed new models for estimating the flexural rigidity and deflection of R.C. beams to account for the influencing parameters. Ismail et al. [9] determined the location of damage due to single cracks and honeycombs in R.C. beams using mode shape derivatives from modal testing. Experimental modal analysis was performed on the beams with cracks prior to and after each load cycle, on a control beam and beams with honeycombs. In studies of Labib et al. [10, 11], rotational spring model was used and natural frequencies of beams and frames with multiple single-edge cracks were obtained. Aktas and Sumer [12] modelled pre-damaged RC beams in finite element program and indicated that inclusion of pre-damage levels by means of cracks into the cross sections have significant effect on beams moment capacity. Gerist et al. [13] presented a new method to detect the structural damages. In this method, the sensitivity matrices of structural responses with respect to elemental damages were evaluated by finite difference method with various finite difference increments at first. Then, various systems of equations were formed for the structure and solved by BP. Shakti et al. [14] investigated the influence of parameters like crack depth and crack inclination angles, on the dynamic behaviour of deteriorated structures excited by time-varying mass. Analysis of the structure was carried out at constant transit mass and speed. Sarvi et al. [15] used Levenberg-Marquardt algorithm to update the truss structural finite element model. Kaveh and Mahdavi [16] in their study, considered an elemental stiffness reduction factor, to introduce the damage in the element of steel trusses. The reduction factor ( $\alpha_i$ ) indicates the damage severity at the  $i$ th element in the finite element model whose values are between 1 for an element without damage and 0 for a ruptured element. They used the new optimization algorithms called Colliding Bodies Optimization (CBO) and Enhanced Colliding Bodies Optimization (ECBO) (see Kaveh [17] for further explanation of recently developed metaheuristics). An efficient method for structural damage localization based on the concepts of flexibility matrix and strain energy of a structure was suggested by Nobahari and Seyedpoor [18]. An efficient indicator for structural damage localization using the change of strain energy based on static noisy data

(SSEBI) was proposed by Seyedpoor and Yazdanpanah [19]. Eroglu and Tufekci [20] introduced a finite element formulation for straight beams with an edge crack, including the effects of shear deformation and rotatory inertia. The main purpose of these studies is to present a more accurate formulation to improve the beam models used in crack detection problems. Shayanfar et al. [21] used time-domain responses for damage detection of a bridge structure. The method proposed by them include, measuring acceleration responses of the time-domain and also creating a finite element model of the structure, based on the equations of motion of the bridge under a moving load. Afterwards, an objective function for solving the inverse problem of damage detection was defined; and by using ECBO algorithm, the problem was solved.

In the study of Sabuncu et al. [22], the effects of number of stories, static and dynamic load parameters, crack depth and crack location, on the in-plane static and dynamic stability of cracked multi-storey frame structures subjected to periodic loading, were investigated numerically using the Finite Element Method.

In this study, the natural frequencies of concrete beams with multiple single-edge cracks were obtained. The rotational spring model was used to represent the crack sections. For crack detection in concrete beams, the following steps were taken:

- Use of finite element analysis and Matlab code for calculation of the stiffness of rotational spring equivalent of intact rectangular concrete section.
- Experimental test using finite element analysis for determination of the effect of crack's depth on stiffness of rotational spring and a new equation for reduction of stiffness due to crack's depth proposed.
- The objective functions was obtained and used for updating the stiffness matrix of model in the optimization process.
- Stiffness matrix of beams was updated with optimization process and their crack location and depth were detected.

## 2. MODELS OF CRACKS

A rotational spring model has been used in many studies to identify cracks in beams [10, 23]. In this study, the cracked beams were modelled by elements and components connected by hinge and less-mass rotational spring to determine the effects of cracking on bending behavior of beams. As shown in Fig. 1, the Bernoulli–Euler beam was divided into two halves at the crack location. The beam sections were then pinned together and a rotational spring was used to model the increased flexibility due to the crack. It is assumed that the axial stiffness of the beam at the crack location remained intact.

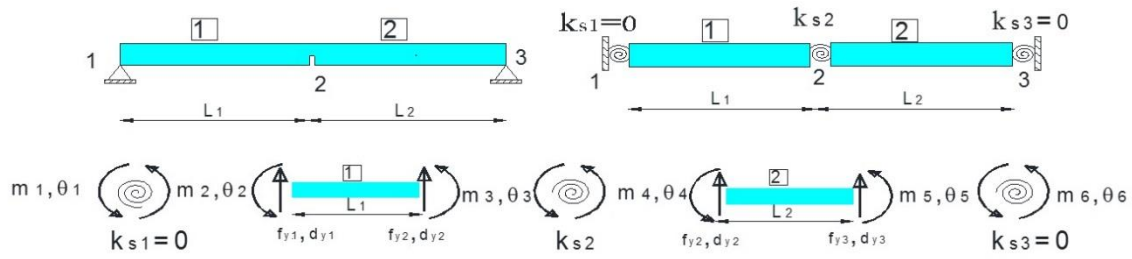


Figure 1. Simple supported cracked beam modelling by torsional springs

Because crack in concrete beams develops in early stages of loading, the behaviour of material is still linear and the reserved strain energy in beam equals the work done by the force. In Eq. (1), strain energy for cracked beam was calculated.

$$W = U = U_b + U_s; \quad U_b = \int_0^L \frac{M^2}{2EI} dx, \quad U_{si} = \frac{1}{2} M_{si} \theta_i = \frac{M_{si}^2}{2k_{si}} \quad (1)$$

$$U_s = \sum_{i=1}^{N_s} \frac{M_{si}^2}{2k_{si}}, \quad U = \int_0^L \frac{M^2}{2EI} dx + \sum_{i=1}^{N_s} \frac{M_{si}^2}{2k_{si}}$$

where  $k_{si}$  is stiffness of rotational spring equivalent of cracked section  $i$ ,  $M_{si}$  is bending moment in crack location and  $N_s$  is number of cracked section.

### 3. STIFFNESS OF SPRING EQUIVALENT OF CRACKED SECTION

To study the effect of cracking on concrete beam stiffness, experimental samples with rectangular section were built and cracks with different depths were made in them. These samples were loaded in 3 points as shown in Fig. 2, and their load-displacement diagrams were plotted. Changes in crack depths result in inclination change as shown in the diagram. Plates with 0.1 mm thickness were used in two layers to develop cracks in the samples. Therefore, the width of the crack was 0.2 mm. The applied concrete properties are shown in Table 1.

Table 1: Concrete properties of the test beams

$f'_c (MPa)$	$E_c (MPa)$	$w_c \left(\frac{kg}{m^3}\right)$	$\nu$
48	33200	2320	0.21

The height and width of the samples are  $d = 200 \text{ mm}$  and  $b = 150 \text{ mm}$ , respectively, and their length is  $L = 1000 \text{ mm}$ . As shown in Fig. 2, these samples were tested after processing on the 28th day.

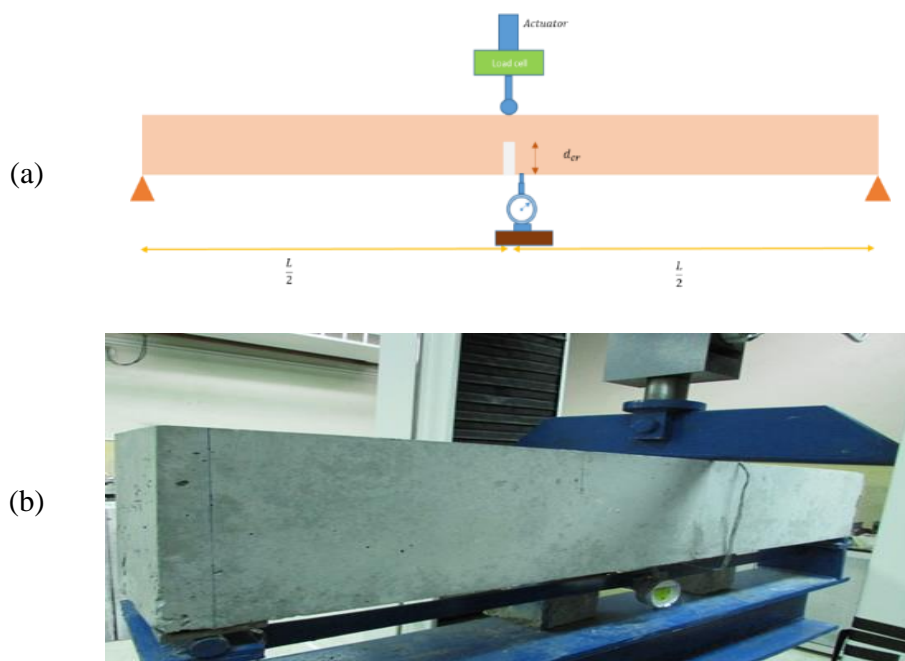


Figure 2. Experimental test. a: Test setup b: Test sample

After loading, displacement of middle section of the beam was measured and the equivalent stiffness of the beam ( $K_{eb}$ ) was calculated. In Eq. (2), strain energy for test sample was calculated.

$$\begin{aligned}
 M(x) &= \frac{px}{2} ; M_s = \frac{p\xi}{2} ; W = \frac{p\delta}{2} \\
 U &= 2 \int_0^{\frac{L}{2}} \frac{\left(\frac{px}{2}\right)^2}{2EI} dx + \frac{\left(\frac{p\xi}{2}\right)^2}{2k_s} = \frac{p^2 L^3}{96EI} + \frac{p^2 \xi^2}{8k_s} \\
 W = U &\rightarrow \delta = p \left[ \frac{L^3}{48EI} + \frac{\xi^2}{4k_s} \right] = p \left[ \frac{1}{k_{eb}} \right] \rightarrow \frac{1}{k_{eb}} = \left[ \frac{1}{k_b} + \frac{\xi^2}{4k_s} \right] \\
 \xi = \frac{L}{2} &\rightarrow \frac{1}{k_{eb}} = \left[ \frac{1}{k_b} + \frac{L^2}{16k_s} \right]
 \end{aligned} \tag{2}$$

$k_b$  is stiffness of intact beam and that for test sample is:

$$k_b = \frac{48EI}{L^3} = 159360 \text{ N/mm} \tag{3}$$

Stiffness of rotational spring equivalent of cracked section in the middle of beams obtained using Eqs. (2) and (3) are presented in Table 2.

Table 2: Stiffness of rotational spring equivalent of cracked section in the experimental samples

No	Beam	$h$ (mm)	$d_{cr}$ (mm)	$\frac{d_{cr}}{h}$	Force of failure (N)	Displacemen t of failure (mm)	$K_{eb}$ (N/mm)	$\frac{K_{eb}}{K_b}$	$\frac{K_s}{\left(\frac{N \cdot mm}{rad}\right)}$
1	CB0	200	0	0	16596	0.105	158061	0.99	1.2e12
2	CB10	200	20	0.1	16412	0.11	149200	0.94	1.46e11
3	CB20	200	40	0.2	15548	0.115	135200	0.85	5.57e10
4	CB30	200	60	0.3	12965	0.120	108041	0.67	2.096e10
5	CB50	200	100	0.5	9045	0.096	94200	0.59	1.44e10

In order to complete the required information and control the obtained results, the tested beam was modelled in Abaqus software, and its behavior under 3 point-loading was investigated. The analysis of cracked and uncracked beams by Abaqus finite element software also shows that with increase in crack depth, the beam stiffness decreases. The quality of variation of rotational spring stiffness equivalent to section, according to the crack depth, is demonstrated in Fig. 3.

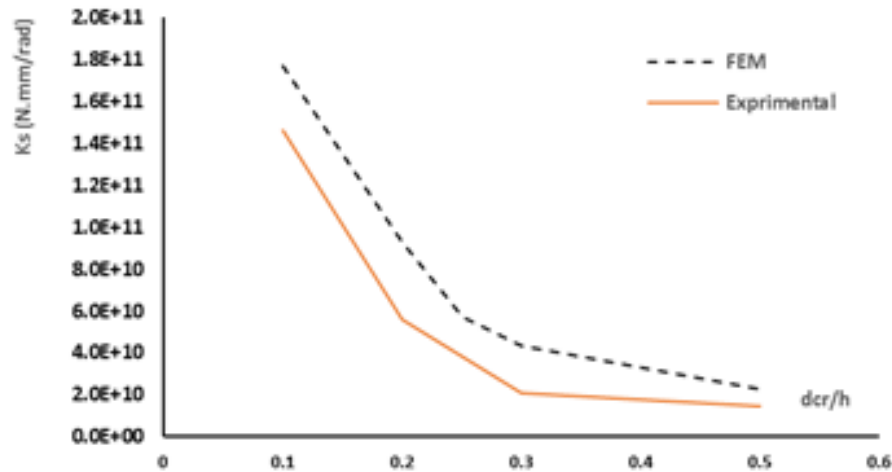


Figure 3. Stiffness of rotational spring stiffness in experimental and FE models

According to this diagram, the decrease in spring stiffness versus the crack depth is not linear; on the other hand, the relation is valid in border conditions. If the crack depth is zero, the spring stiffness will tend to infinity and if the crack depth equals the beam height, then its value will be zero. Accordingly, the rotational spring stiffness equivalent to cracked section is suggested as Eq.(4).

$$k_s = 0.70 \left[ \left( \frac{h}{d_{cr}} \right)^{1.2} - 1 \right] \frac{EI}{h} \quad (4)$$

where,  $h$  is height of beam section,  $d_{cr}$  is depth of crack and  $EI$  is bending rigidity of beam section. Change in the concrete properties during the construction of the samples, due to human error, caused differences in the graphs as shown in Fig. 4.

4. STIFFNESS MATRIX OF MODELS

There are three degrees of freedom  $\delta = \{\theta_L, d_y, \theta_R\}$  in cracked section of the 2D beams as shown in Fig.1.  $\theta_L$  and  $\theta_R$  are rotations at near end of beam elements at the left and right sides of rotational spring, respectively, and  $d_y$  is displacement of node. The stiffness matrix of the cracked beam can be written as Eq.(5). In this matrix, the array corresponding to the displacement of the common point of the beams equals the sum of the stiffness of two beam elements, and the array corresponding to the rotation of the beam elements end adjacent to the middle node equals the sum of the rotational stiffness of the beam and the rotational spring.

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{12}^t & K_{22} & K_{23} \\ K_{13}^t & K_{23}^t & K_{33} \end{bmatrix}; K_{11} = \begin{bmatrix} k_{s1} & 0 & -k_{s1} \\ 0 & \frac{12EI}{L_1^3} & \frac{6EI}{L_1^2} \\ -k_{s1} & \frac{6EI}{L_1^2} & \frac{4EI}{L_1} + k_{s1} \end{bmatrix}; K_{12} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{6EI}{L_1^2} & -\frac{12EI}{L_1^3} & 0 \\ \frac{2EI}{L_1} & -\frac{6EI}{L_1^2} & 0 \end{bmatrix}; K_{13} = \text{zeros}(3)$$

$$K_{22} = \begin{bmatrix} \frac{4EI}{L_1} + k_{s2} & -\frac{6EI}{L_1^2} & -k_{s2} \\ -\frac{6EI}{L_1^2} & \frac{12EI}{L_1^3} + \frac{12EI}{L_2^3} & \frac{6EI}{L_2^2} \\ -k_{s2} & \frac{6EI}{L_2^2} & \frac{4EI}{L_2} + k_{s2} \end{bmatrix}; K_{23} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{6EI}{L_2^2} & -\frac{12EI}{L_2^3} & 0 \\ \frac{2EI}{L_2} & -\frac{6EI}{L_2^2} & 0 \end{bmatrix}; K_{33} = \begin{bmatrix} \frac{4EI}{L_2} + k_{s3} & -\frac{6EI}{L_2^2} & -k_{s3} \\ -\frac{6EI}{L_2^2} & \frac{12EI}{L_2^3} & 0 \\ -k_{s3} & 0 & k_{s3} \end{bmatrix} \tag{5}$$

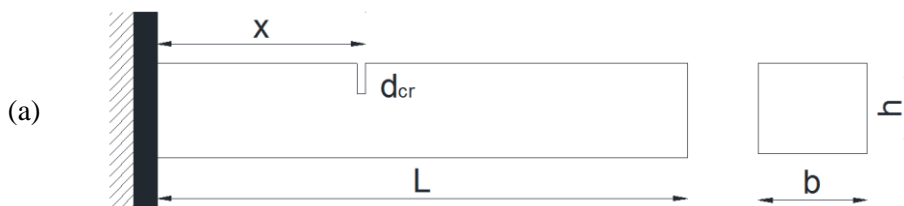
where  $k_{sj}$  is stiffness of rotational spring equivalent of cracked section  $j$ ,  $EI$  is bending rigidity,  $L_i$  is length of element  $i$  and  $[K]$  is proposed global stiffness matrix.

Global stiffness matrix  $[K]$  can be used to obtain the natural frequency and mode shape of undamped beam in free vibration analysis. It is generally known that the Eigen value equation of an undamped structure is as follows:

$$[K - \omega^2 M]\{\phi\} = 0 \tag{6}$$

where  $M$  is mass matrix,  $K$  is global stiffness matrix,  $\omega$  is natural frequency and  $\phi$  is mode shape of beam. Matlab code was used to obtain natural frequency and mode shape of beam.

In order to test the accuracy and convergence of the present method, the effects of edge-cracks on the natural frequencies of a cantilever beam that has been studied extensively by Wentland [24] experimentally and by Gudmundson [2], Kisa et al. [25] and Labib et al. [10] numerically, were examined. The cantilever beam was modeled by rotational spring in cracked section as shown in Fig. 4.



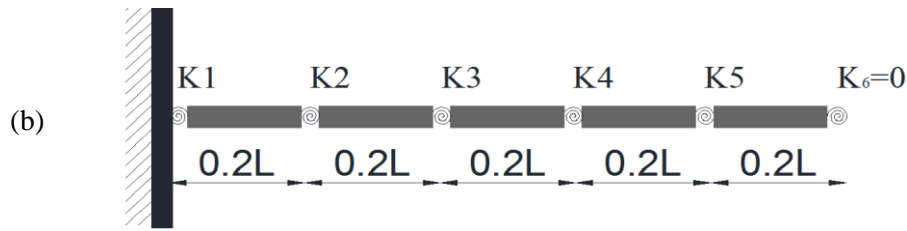


Figure 4. Cantilever beam with an edge-crack. a: Geometry b: proposed model

In the cracked section, the stiffness of equivalent rotational spring was calculated using Eq.(4), and in uncracked section, it was  $\infty$  theoretically. For numerical analysis, it is assumed that  $\frac{d_{cr}}{h} = 0.01$ . Therefore, the stiffness of rotational spring equivalent of intact section ( $k_{s0}$ ) can be obtained using Eq. (7).

$$k_{s0} = 0.70[(100)^{1.2} - 1] \frac{EI}{h} = 175 \frac{EI}{h} \quad (7)$$

The first three natural frequencies of a cantilever beam (Fig. 4) with length of 0.20 m, depth  $h = 0.0078$  m, mass per unit length  $\rho = 1.5308$  kg/m and bending rigidity  $EI = 231.548$  N.m<sup>2</sup> [10], having a single open crack, are shown in Table 4. Different variations of the crack to depth ratio  $\frac{d_{cr}}{h}$  and crack locations were employed. Eqs. (4) and (5) were used to obtain the equivalent spring stiffness, and global stiffness matrix and natural frequency of beam, respectively. The predicted Eigen frequency changes calculated from the proposed Equation were compared with the numerical data obtained by Kisa et al. [25] and Labib et al. [10] for the first three Eigen frequencies.(Table 3).

Table 3: Natural frequencies of a cantilever beam with a single crack located at a distance  $\xi = 0.2.L$

$\frac{d_{cr}}{h}$	$K_s \left( \frac{N \cdot mm}{rad} \right)$		$\omega_1 \left( \frac{rad}{s} \right)$			$\omega_2 \left( \frac{rad}{s} \right)$			$\omega_3 \left( \frac{rad}{s} \right)$		
	Present	[10]	Present	[25]	[10]	Present	[25]	[10]	Present	[25]	[10]
0	4.8e6	-	1038.29	1037.01	1038.2	6510.28	6458.34	6506.3	18459.94	17960.56	18218
0.2	113045	130000	1028.3427	1020.13	-	6509.69	6457.39	-	18403.70	17872	-
0.4	38380	28800	1009.40	966.9	-	6508.60	6454.48	-	18297.65	17596.57	-
0.6	16200	8400	973.1	842.2	-	6506.55	6448.175	-	18098.13	16944	-

## 5. THE PROPOSED DAMAGE DETECTION METHOD

The choice of objective function and constraints is critical for all constrained optimization problems. The objective is a function of the system parameters that will be adjusted by the algorithm. A carefully chosen objective function is needed to provide the algorithm with a means of determining the remaining design variables of the structure. In the present study, the discussion on cracks was limited to beams. The un-damped free vibration equation of this system is known as:



$$K\ddot{U} + MU = 0 \tag{8}$$

where  $M$  and  $K$  are the mass matrix and the stiffness matrix of the structure, respectively, and  $U$  is the displacement vector. The solution of this equation is  $N$  values of  $\omega^2$ , the undamped system's natural frequencies. The Eigen value problem for the undamped system can be written as:

$$[K - \omega^2 M]\{\phi\} = 0 \rightarrow [M^{-\frac{1}{2}}KM^{-\frac{1}{2}}\{\phi\} - \omega^2 I\{\phi\}] = 0 \tag{9}$$

$$\rightarrow M^{-\frac{1}{2}}KM^{-\frac{1}{2}}\{\phi\} = \omega^2 I\{\phi\}$$

This problem intends to find the natural frequencies  $\omega$  and the mode shapes  $\{\phi\}$  of the structure, given its stiffness and mass matrix. Assuming no volume or mass changes due to the cracks or other geometrical changes, the natural frequencies and the mode shapes will change if the stiffness matrix changes as shown in Eq.(10).

$$KM^{-1}\{\phi\} = \omega^2 I\{\phi\} \rightarrow K = \{\phi\}^{-1}\omega^2 I\{\phi\}M$$

$$K_d M^{-1}\{\phi_d\} = \omega_d^2 I\{\phi_d\} \rightarrow K_d = \{\phi_d\}^{-1}\omega_d^2 I\{\phi_d\}M \tag{10}$$

$$K_d = K + \Delta K \rightarrow \Delta K = K - K_d = (\{\phi\}^{-1}\Omega^2\{\phi\} - \{\phi_d\}^{-1}\Omega_d^2\{\phi_d\})M$$

where  $\Omega$  is the diagonal matrix and their arrays are square of Eigen frequency.  $K, \omega, \phi$  represents the stiffness matrix, natural frequency and mode shape of the intact beam,  $K_d, \omega_d, \phi_d$  represents the stiffness matrix, natural frequency and mode shape of the cracked beam.  $\Delta K$  is the stiffness reduction on the intact beam stiffness matrix due to the crack.  $\omega_d$  and  $\phi_d$  are measured in the existing beam by sensors. For detection of the crack location and depth, it is necessary to minimize arrays of  $\Delta K$ , for this reason, the proposed objective function was introduced as shown in Eq. (11)

$$\min F = \Delta K = \sum_{i=1}^N \sum_{j=1}^M (\{\phi\}_{ij}^{-1}\Omega_i^2\{\phi\}_{ij} - \{\phi_d\}_{ij}^{-1}\Omega_{di}^2\{\phi_d\}_{ij})^2 \tag{11}$$

where,  $N$  and  $M$  are the number of modes shape considered and number of degree of freedom measured by sensors, respectively. For that,  $\{\phi\}$  is an invertible matrix, it is necessary for  $N = M$ .

It should be noted that, as the damage locations are unknown for the damaged structure regarding real data applications, for this case, the element stiffness matrix of the healthy structure is used to estimate the parameters  $\{\phi\}$  and  $\Omega$  for minimizing the objective function.

The crack occurrence in a beam can reduce the stiffness of rotational spring equivalent of cracked section. The stiffness matrix of the cracked beam,  $K$ , is then obtained through the assemblage of the intact element stiffness matrices and damage spring stiffness matrix. As a result, in this study, to construct a damage indicator, health index ( $HI$ ) for spring stiffness was used according to Eq.(12).

$$[K] = [k_b] + HI_n[k_{s0}] \quad (12)$$

In order to form the optimization problem, damage will be quantified using a scalar variable or index  $HI$  whose values are between 0 and 1. The value 1, refers to no crack ( $d_{cr} = 0$ ) at all, whereas values close to zero ( $d_{cr} \cong h$ ) imply devastating damage in the corresponding section of the beam. The depth of cracked is obtained in cracked sections by Eq.(13).

$$HI_n = \frac{k_{sn}}{k_{s0}} = 0.004 \left[ \left( \frac{h}{d_{cr}} \right)^{1.2} - 1 \right] \rightarrow \left( \frac{d_{cr}}{h} \right) = [250.HI + 1]^{-\frac{1}{1.2}} \quad (13)$$

## 6. OPTIMIZATION METHOD

In the present study, the optimization problems were solved using the following three methods:

- Enhanced Colliding Bodies Optimization (ECBO)
- Particle Swarm Optimization (PSO)
- Sequential Quadratic Programming (SQP).

Colliding bodies optimization (CBO) is a new meta-heuristic search algorithm developed by Kaveh and Mahdavi [17]. In this technique, one object collides with another object and they move towards a minimum energy level. The CBO is simple in concept and does not depend on any internal parameter. The CBO is a multi-agent algorithm inspired by a collision between two objects in one dimension. Each agent is modeled as a body with a specified mass and velocity. A collision occurs between pairs of objects and the new positions of the colliding bodies are updated based on the collision laws. The ECBO uses memory to save some historically best solutions to improve the CBO performance without increasing the computational cost. In this method, some components of agents are also changed to jump out from local minimum. A Matlab code which was prepared by Kaveh and Ilchi Ghazaan was used for this purpose [26].

PSO is a population-based optimization method built on the premise that social sharing of information among the individuals can provide an evolutionary advantage. The mathematical optimizer used in this study is a SQP method. SQP methods are the standard general purpose mathematical programming algorithms for solving nonlinear programming (NLP) optimization problems, Matlab programming language was used for this optimization.

In this paper, damage detection of a prismatic beam with a specified length was studied. First, the beam was divided into a number of finite elements (beams and spring). Then, mode shapes of the healthy beam in measurement points were evaluated using the finite element method. A Matlab (R2013a) code was prepared here for this purpose.

An example is the simply supported beam of Fig. 5, where there is crack in some of the sections of the beam; the cracked section and stiffness of rotational spring equivalent of this section is shown in Table 4.

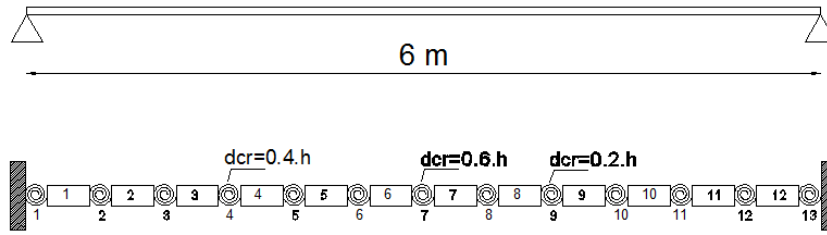


Figure 5. Simple supported beam divided into 12 beam elements and 13 spring elements (sample1)  
 (Beam properties:  $E = 33.2 \text{ Gpa}$ ,  $A = 1.2e5 \text{ mm}^2$ ,  $I = 1.6e9 \text{ mm}^4$ ,  $\rho = 285 \text{ kg/m}$ )

Table 4: Crack's depth of various sections of beam and stiffness of equivalent spring

Node	1	2	3	4	5	6	7	8	9	10	11	12	13
$\frac{d_{cr}}{h}$	1	0	0	0.4	0	0	0.6	0	0.2	0	0	0	1
$K_s$	0	$2.32e13$	$2.32e13$	$1.86e11$	$2.32e13$	$2.32e13$	$7.86e10$	$2.32e13$	$5.48e11$	$2.32e13$	$2.32e13$	$2.32e13$	0

In Table 4, stiffness of spring equivalent of cracked and uncracked sections were calculated using Eqs.(4) and (7), respectively. The ratio of the proposed objective functions were calculated by optimization algorithms, ECBO and PSO algorithms and mathematical optimization; the ratio for simple support beams is shown in Fig. 6.

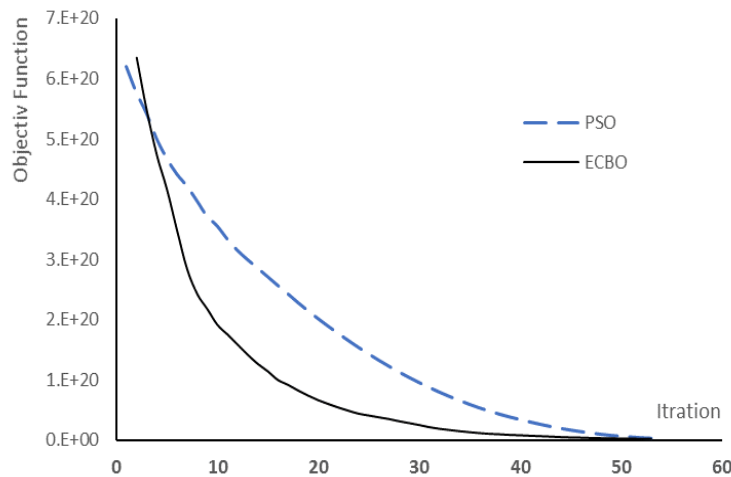


Figure 6. Ratio of objective functions in ECBO and PSO algorithm

HI index were obtained in optimization process and  $\frac{d_{cr}}{h}$  ratio can be obtained by Eq.(13). The result for simple support beam cases is shown in Table 5 and Fig.7.

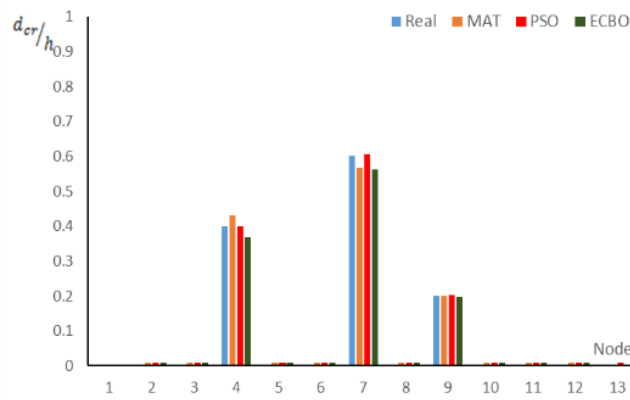


Figure 7. Cracked section results for simple support beam

In this example, the proposed objective function provides some good results for the position and the extent of crack in the beam sections, although there are some errors between the real and the calculated damage.

Table 5: Optimization results for simple support cracked beam

Node		1	2	3	4	5	6	7	8	9	10	11	12	13
MAT	HI	0.002	0.999	0.999	0.007	1	1	0.0039	1	0.023	0.99	0.98	0.99	0
	$\frac{d_{cr}}{h}$	0.922	0.01	0.01	0.42	0.01	0.01	0.57	0.01	0.21	0.01	0.01	0.01	1
PSO	HI	1	1	1	0.008	1	1	0.0033	1	0.023	1	1	1	1
	$\frac{d_{cr}}{h}$	0	0.01	0.01	0.4	0.01	0.01	0.606	0.01	0.204	0.01	0.01	0.01	0.01
ECBO	HI	0	1	1	0.008	1	1	0.004	1	0.023	1	1	1	0
	$\frac{d_{cr}}{h}$	1	0.01	0.01	0.39	0.01	0.01	0.599	0.01	0.201	0.01	0.01	0.01	1

In another example, two bays beam with several cracked section as shown in Fig. 8 were examined. The cracked section and stiffness of rotational spring equivalent of this section are shown in Table 6. Beam properties are same with that of sample.

The result of crack detection for two bays beam cases is shown in Table 7 and Fig. 9.

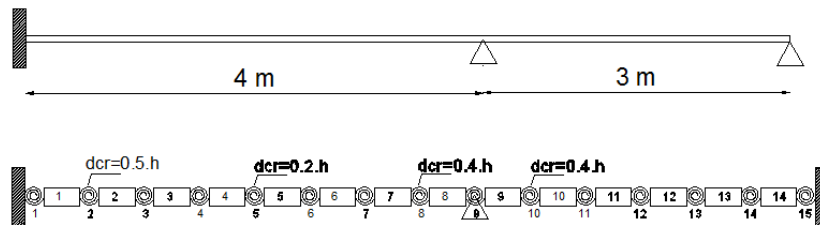


Figure 8. Two bays beam divided in 14 beam elements and 15 spring elements (sample 2)

Table 6: Crack's depth of various sections of two bays beam and stiffness of equivalent spring

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\frac{d_{cr}}{h}$	0	0.5	0	0	0.2	0	0	0.4	0	0.4	0	0	0	0	1
$K_s$	2.32e13	1.21e11	2.32e13	2.32e13	5.48e11	2.32e13	2.32e13	1.86e11	2.32e13	1.86e11	2.32e13	2.32e13	2.32e13	2.32e13	0

Table 7: Optimization results for tow bays cracked beam

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
MAT	HI	1	0.0059	1	1	0.0239	1	1	0.0081	1	0.0082	1	1	1	1
	$\frac{d_{cr}}{h}$	0.01	0.47	0.01	0.01	0.198	0.01	0.01	0.398	0.01	0.395	0.01	0.01	0.01	0
ECBO	HI	1	0.0055	1	1	0.0219	1	1	0.0065	1	0.0073	1	1	1	0
	$\frac{d_{cr}}{h}$	0.01	0.486	0.01	0.01	0.211	0.01	0.01	0.447	0.01	0.421	0.01	0.01	0.01	0
PSO	HI	0.921	0.005	1	1	0.023	1	1	0.008	1	0.008	1	1	1	0
	$\frac{d_{cr}}{h}$	0.011	0.47	0.01	0.01	0.2	0.01	0.01	0.4	0.01	0.40	0.01	0.01	0.01	0

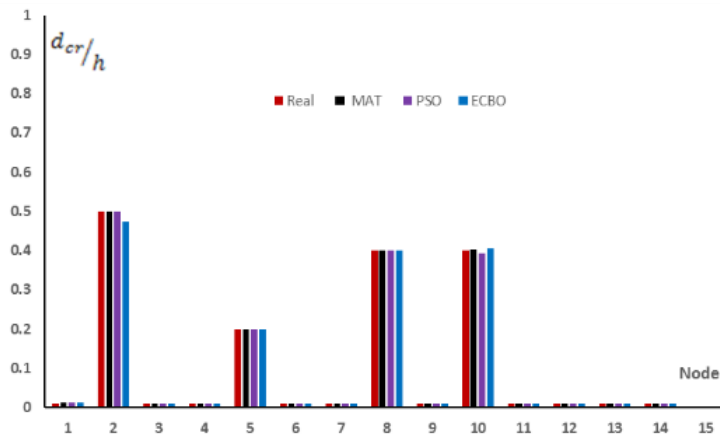


Figure 9. Cracked section results for tow bays beam

In the next example, it is assumed that uniform damage in two bays beam (sample 2) occurred. Module of elasticity of concrete in this sample is  $E_c = 26560 \text{ MPa}$ . The stiffness of

rotational spring equivalent of beam sections are shown in Table 8.

The result of crack detection for two bays beam cases is shown in Fig. 10.

Table 8. Stiffness of spring equivalent of beam sections

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$K_s$	$1.86e13$	$1.86e13$	$1.86e13$	$1.86e13$	$1.86e13$	$1.86e13$	$1.86e13$	$1.86e13$	$1.86e13$	$1.86e13$	$1.86e13$	$1.86e13$	$1.86e13$	$1.86e13$	0

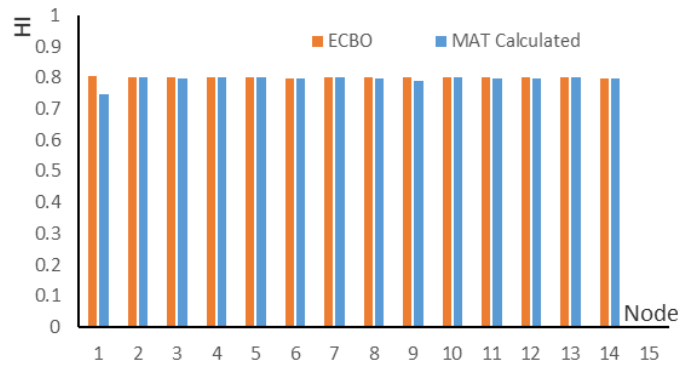


Figure10. Cracked section results for two bays beam with uniform damage

In the uniform damage case, the proposed objective function criteria led to an acceptable match between the real damage and the calculated crack in the beam sections.

The one story concrete frame shown in Fig. 11 has several cracked sections in the beam and columns. The beam and columns have the same cross-sectional dimensions:  $300 \times 300 \text{ mm}$ . Therefore, the stiffness of rotational spring equivalent of uncracked section is:

$$K_{s0} = 175 \frac{EI}{h} = 1.3e13 \text{ N.mm/rad}$$

The crack to depth ratio for the cracked sections of beam and columns are shown in Table 9.

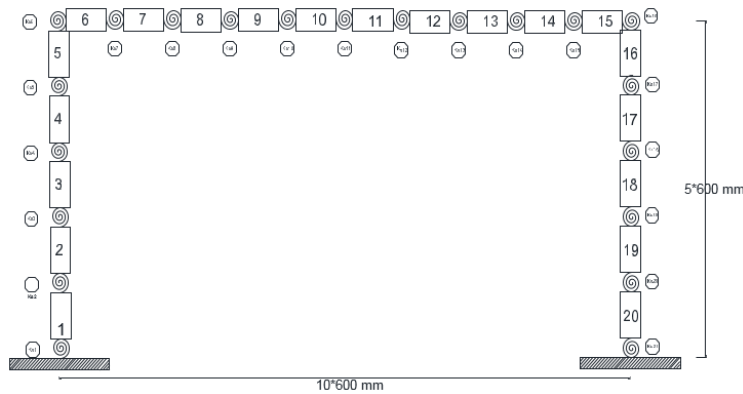


Figure 11. Model of one by concrete frame

Table 9: Cracked section in one storey frame

Cracked Node	2	4	8	13	19
$\frac{d_{cr}}{h}$	0.4	0.3	0.6	0.3	0.2

To detect the cracks, the proposed objective function and optimization algorithms were used. Stiffness of rotational spring equivalent of cracked section was obtained from optimization program shown in Table 9. The crack to depth ratio obtained by Eq. (13) is shown in Table 10 and Fig.12. The corresponding results of estimated damage extent show that the exact value can be obtained using the proposed objective function with used optimization algorithms.

Table 10: The crack to depth ratio for the cracked sections of frame

Cracked Node	MAT(SQP)		PSO		ECBO	
	$\frac{K_s}{K_{s0}}$	$\frac{d_{cr}}{h}$	$\frac{K_s}{K_{s0}}$	$\frac{d_{cr}}{h}$	$\frac{K_s}{K_{s0}}$	$\frac{d_{cr}}{h}$
	2	0.009	0.374	0.0082	0.395	0.0088
4	0.0145	0.279	0.0136	0.291	0.0138	0.288
8	0.0037	0.579	0.0033	0.606	0.0034	0.599
13	0.0138	0.288	0.0141	0.284	0.013	0.299
19	0.026	0.187	0.024	0.198	0.023	0.204

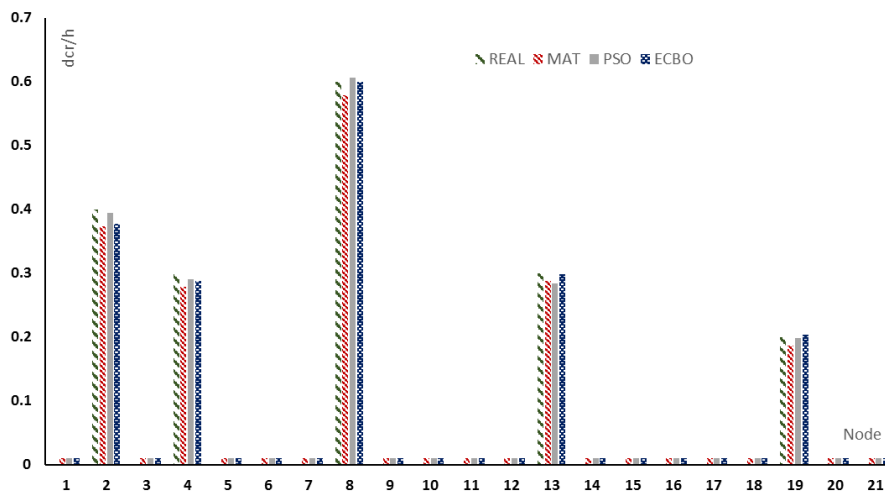


Figure 12. Cracked section results for frame

To accurately calculate the depth and location of the cracks, it is necessary to place the node of the finite element model on the cracked section. If the sections of the finite element model do not match the crack location, the crack depth obtained will not be accurate; therefore, increase in the number of nodes will lead to increase in the accuracy of crack detection.

## 7. CONCLUSIONS

The main purpose of the current study was to identify crack in concrete beams, both in position and in depth, by using some distinct and easily measured dynamic characteristics of the beams. To solve this damage identification problem, the cracked beam was essentially idealized by two sub-beams connected through a massless elastic rotational spring at the crack location and the following three optimization algorithms were used:

- I. Enhanced Colliding Bodies Optimization (ECBO), which is a meta-heuristic search algorithm.
  - II. Particle Swarm Optimization (PSO), which is a population based optimization method.
  - III. Sequential Quadratic Programming (SQP), which is a mathematical optimization method.
- Stiffness of rotational spring equivalent of uncracked section was proposed as Eq. (4) by finite element studies.

Stiffness of rotational spring equivalent of cracked section was proposed as Eq. (14) by experimental studies.

Objective function based on natural frequency and mode shape was proposed as Eq. (12) and optimization result showed that the proposed objective function can indicate the acceptable accuracy, position and depth of crack in beams.

Uniform damage cannot be distinguished with objective functions based on mode shape alone.

Increase in the number of nodes leads to increase in the accuracy of crack detection.

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