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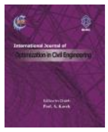

THE CAPABILITY OF OPTIMAL SINGLE AND MULTIPLE TUNED MASS DAMPERS UNDER MULTIPLE EARTHQUAKES

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ABSTRACT

The main focus of this research has been to investigate the effectiveness of optimal single and multiple Tuned Mass Dampers (TMDs) under different ground motions as well as to develop a procedure for designing TMD and MTMDs to be effective under multiple records. To determine the parameters of TMD and MTMDs under multiple records various scenarios have been suggested and their efficiency has been assessed. For numerical simulations, a ten-story linear shear building frame subjected to 12 real earthquakes as well as a filtered white noise record and optimum parameters of TMDs and MTMDs have been determined by solving an optimization problem using genetic algorithm (GA). The results show that when designing optimal TMD and MTMD under a specific ground motion, using the optimization procedure leads to achieve the best performance while the characteristics of the design earthquake strongly affects the performance of TMDs. Furthermore, it has been found that TMDs and MTMDs designed using only one earthquake as the design record have not worked successfully under multiple ground motions. For determining the parameters of TMDs to be effective under multiple records it has been suggested to use the mean of optimal TMDs parameters obtained using each of the design records.

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Key Words: tuned mass damper (TMD); multiple tuned mass damper (MTMD); design records; optimization; multiple records.

1. INTRODUCTION

Over the past decades, extensive research has been conducted to investigate the

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effectiveness and feasibility of using control systems strategies in improving the seismic behavior of structures and mitigating the damage caused by earthquakes. This has encouraged researchers to develop and examine different types of control systems including passive, active, semi-active and hybrid control systems [1]. Among the various proposed control mechanisms, tuned mass damper (TMD) has been highly regarded as one of the simplest and effective passive control mechanisms which can be used in passive, active and semi-active forms, too. Many full-scale applications of TMD systems to buildings and bridges have been accomplished worldwide.

The first theory about application of TMD was proposed by Den Hartog and Ormondroyd [2]. Since then, a great deal of research has been performed on the designing and application of this control mechanism to protect different kinds of structures against earthquake loads [3-6]. Generally, in most of the methods proposed for designing TMD on linear structures under earthquake excitations, such as the methods proposed by Sadek et al. [7] and Villaverde [8], the frequency of optimal TMD is tuned to the frequency of a particular mode to be controlled without paying much attention to design record characteristics. Bernal [9] investigated the effect of ground motion characteristics on the effectiveness of TMD and found that the optimum TMD damping is only dependent on the ratio of duration of excitations to period of structure and is sensitive to the bandwidth of the earthquake. It was also suggested that TMD units may be able to provide notable reductions in spectral response for periods near the dominant period when the excitation is narrow band and of long duration. Murudi and Mane [10] evaluated the effectiveness of TMD in controlling the seismic response of structure by considering the effect of ground motion parameters. For a single degree of freedom (SDOF) structure, their results showed that TMD is effective for lightly damped structures under both actual records and artificially generated earthquakes while its performance depends on the frequency content, bandwidth and duration of the ground motion. However, the seismic effectiveness of TMD is not affected by the intensity of the ground motion.

Kamrani and Rahimian [11] investigated the performance of TMD in reducing the response of 3, 9 and 20 story structures under near-field and far-field earthquakes and concluded that the performance of TMD depends on the input excitation.

Soto-Brito and Ruiz [12] studied the effectiveness of TMD on structures subjected to moderate and high intensity ground motions. By performing analysis on a 22- story four-bay nonlinear reinforced concrete frame subjected to ground motions with different intensities, they found that the effectiveness of TMD is better in systems with high nonlinear behavior produced by high intensity motions.

Mohebbi and Jogataei [13] proposed designing optimal TMDs for nonlinear frames using genetic algorithms (GAs) and investigated the effect of ground motion on the designed TMDs performance. It was found that using TMD decreases the maximum response of structures under different earthquakes while the performance of TMD depends on the characteristics of the input earthquake excitation.

To enhance the accuracy of design and also to overcome some of the shortcomings of using single TMD such as its detuning issues, researchers have examined the application of multiple tuned mass dampers (MTMDs) in controlling more than one mode of the structure [14]. The effect of earthquake characteristics on the effectiveness of MTMDs has also been studied [15-17]. Mohebbi and Ghanbarpour [18] studied the effect of input earthquake on performance of MTMD with different mass ratio, where TMD units were designed for a white noise excitation and then tested under near-field and far-field earthquakes. They found that the most reductions in the structural response were achieved under far-field excitations while under near-field records such as Northridge earthquake in some cases, MTMD has even increased the response of structure. Hence, their results showed that the performance of MTMD depends on the input record's characteristics. Li and Lio [19, 20] examined the effect of dominant frequency of ground motion on the optimum parameters and effectiveness of MTMD, and found that according to the dominant frequency ratio of ground motion and mass ratio values, white noise, Kanai-Tajimi spectrum [21] or Clogh-Penzien spectrum [22] can be used for designing MTMDs. Also the capability of MTMDs in mitigating the damage of nonlinear steel structure subjected to far-field and near-field earthquakes [23] has been studied.

In the area of active control systems, Mohebbi et al. [24] have also investigated the effect of design excitation on performance of active mass damper (AMD) in mitigating the seismic response of nonlinear frames by testing the controllers under a number of scaled and real earthquakes including both near and far-field earthquakes. It was found that to enhance the performance of AMD, an earthquake with proper peak ground acceleration (PGA) and frequency content should be used in the design process. Furthermore, the performance of active control of multi-story frame structures under both near and far field earthquakes has been studied [25].

While the results of previous research show that the performance of mass damper systems depends strongly on characteristics of the design record, in most of the previous studies, only one real earthquake or a white noise excitation has been used in the design procedure of TMDs and/or MTMDs. Consequently in these studies, the effect of different earthquake characteristics on TMDs and MTMDs parameters has not been investigated in detail. In addition, selecting a proper design record for TMDs and MTMDs has still remained as an essential and controversial problem. According to seismic design codes, to take into account the effect of different earthquake characteristics in response of structure and design a structure to be resisstant under different excitations, it is recommended to consider multiple design records in the design procedure where the design records are selected based on site's seismic condition. In extending this recommendation for designing structures equipped with TMD or MTMDs, there is an essential question whether TMD or MTMD has the capability of being effective under multiple records or not. To clarify this issue, in this paper it has been decided to evaluate the performance of TMD and MTMDs designed using different methods under multiple ground motions. Also the possibility of developing a method to determine the parameters of TMD and MTMDs so that it can be efficient under different excitations has been assessed.

2. STRUCTURE-MTMDS EQUATION OF MOTION

The equation of motion for a linear *n* degree-of-freedom shear building frame subjected to ground excitation, $\ddot{x}_g(t)$, and equipped with N_{tmd} at its top floor with parallel configuration, can be written as:

$$[M]\ddot{X}(t)+[C]\dot{X}(t)+[K]X(t)=[M]e\ddot{X}_g \tag{1}$$

where $e^T=[-1 \quad -1 \quad \cdots \quad -1]_{1\times(n+N_{tmd})}$ is the ground acceleration mass transformation vector, [M], [K] and [C] are respectively mass, stiffness and damping matrices and X , $\dot{X}$ and $\ddot{X}$ are displacement, velocity and acceleration vectors relative to the ground, respectively. By considering m_{di}, c_{di} and k_{di} as mass, damping and stiffness of i^{th} TMD, [M], [K] and [C] can be written as:

$$M=\begin{bmatrix} m_1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & m_2 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & m_{n-1} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & m_n & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & m_{d_1} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & m_{d_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & m_{d_{N_{tmd}-1}} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & m_{d_{N_{tmd}}} \end{bmatrix} \tag{2}$$

$$K=\begin{bmatrix} k_1+k_2 & -k_2 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -k_2 & k_2+k_3 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & k_{n-1}+k_n & -k_n & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & -k_n & k_n+\sum_{i=1}^{N_{tmd}} k_{d_i} & -k_{d_1} & -k_{d_2} & \cdots & -k_{d_{N_{tmd}-1}} & -k_{d_{N_{tmd}}} \\ 0 & 0 & \cdots & 0 & -k_{d_1} & k_{d_1} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & -k_{d_2} & 0 & k_{d_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -k_{d_{N_{tmd}-1}} & 0 & 0 & \cdots & k_{d_{N_{tmd}-1}} & 0 \\ 0 & 0 & \cdots & 0 & -k_{d_{N_{tmd}}} & 0 & 0 & \cdots & 0 & k_{d_{N_{tmd}}} \end{bmatrix} \tag{3}$$

$$C=\begin{bmatrix} c_1+c_2 & -c_2 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -c_2 & c_2+c_3 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & c_{n-1}+c_n & -c_n & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & -c_n & c_n+\sum_{i=1}^{N_{tmd}} c_{d_i} & -c_{d_1} & -c_{d_2} & \cdots & -c_{d_{N_{tmd}-1}} & -c_{d_{N_{tmd}}} \\ 0 & 0 & \cdots & 0 & -c_{d_1} & c_{d_1} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & -c_{d_2} & 0 & c_{d_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -c_{d_{N_{tmd}-1}} & 0 & 0 & \cdots & c_{d_{N_{tmd}-1}} & 0 \\ 0 & 0 & \cdots & 0 & -c_{d_{N_{tmd}}} & 0 & 0 & \cdots & 0 & c_{d_{N_{tmd}}} \end{bmatrix} \tag{4}$$

In this study, the Wilson's-Θ numerical method [26] has been used for solving the structure-MTMD equations.

3. DESIGN RECORDS

To evaluate the effect of ground motion characteristics on performance of TMD and MTMD, 12 real earthquakes from California district shown in Table1 have been selected for numerical simulations. As shown in Table1, a number has been assigned to each earthquake for convenience in reporting the results. Furthermore, since the optimal TMDs need to be efficient under different earthquakes, a white noise record has also been considered in designing and testing procedure.

In this paper, the artificial earthquake has been simulated by passing a Guassian white noise process through Kanai-Tajimi filter [21, 27] with power spectral density function given by:

$$S_g = \frac{1 + 4\xi_g^2 \left(\omega/\omega_g\right)^2}{\left(1 - \left(\omega/\omega_g\right)^2\right)^2 + 4\xi_g^2 \left(\omega/\omega_g\right)^2} S_0 \tag{5}$$

where S_0 is constant spectral density and ξ_g and ω_g are the ground damping and frequency, respectively, where $\xi_{g=}0.3$ and ω_g=37.3 (rad/sec) have been used in this paper. The time history of filtered white noise excitation, W(t), with peak ground acceleration (PGA)=0.475g ,used for designing TMD and MTMDs has been shown in Fig. 1.

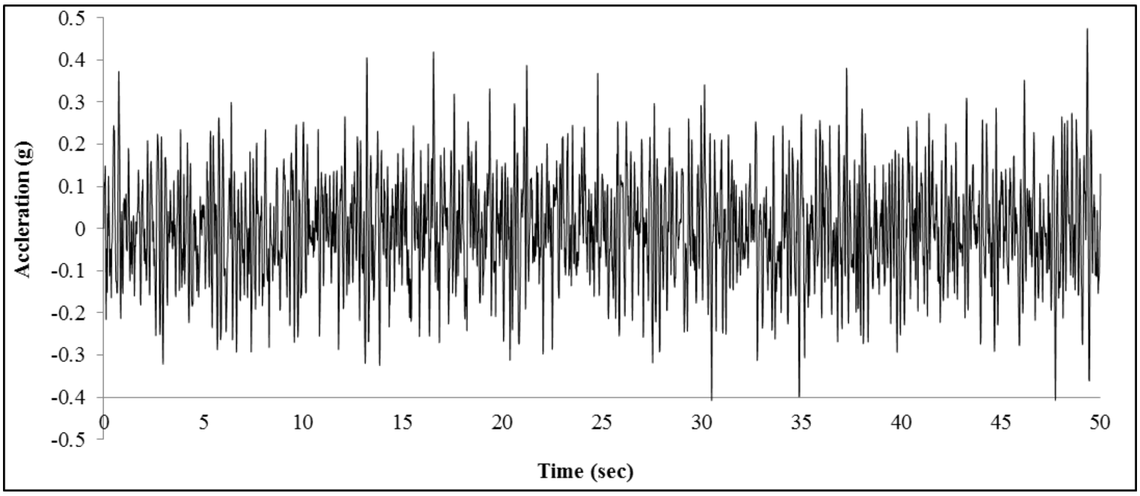


Figure 1. Time history of W(t)

4. NUMERICAL EXAMPLE

4.1 Evaluating the performance of single TMD

To evaluate the effectiveness of TMD under different input records, a 10- storey shear building frame assuming linear behavior and uniform mass, damping and stiffness for all stories has been selected. The properties of each story are as follows:

$m=360$ tons, $k=650$ MN/m , $c=6.2$ MN.s/m

In this research the performance of TMD for two mass ratios μ=2% and 4% has been studied where the TMD is considered on the top floor.

4.1.2 Optimal design of TMD

In this study for optimal design of TMD the optimization-based design method (OBDM) proposed by Mohebbi and Joghataei [13] has been used. In this method for optimal design of TMD, an optimization problem has been defined which considers the minimization of structural response as the objective function and the parameters of TMD as variables to be determined. In this paper, minimization of maximum displacement of structure, X_{max}, has been considered as the objective function. Assuming a constant mass ratio for TMD, the optimal value of TMD parameters are determined by solving the following optimization problem:

$$\text{Find:} \quad k_d,\ c_d \tag{6a}$$

$$\text{Minimize:} \quad X_{max}=max(|x_k\ (i)|\ ,\ k=1,2,....,k_{max}) \quad , \quad i=1,2,...,n \tag{6b}$$

$$0<k_d<k_{d_{\max}} \tag{6c}$$

$$0<c_d<c_{d_{\max}} \tag{6d}$$

where k_{max} is the total number of time steps and $x_k(i)$ is the lateral displacement of i^{th} storey at k^{th} time step. The optimization problem can be solved using gradient-based or powerful evolutionary optimization methods such as charged system search (CSS) algorithm [3, 28, 29] and genetic algorithm (GA) [13]. In this paper GA has been employed to solve the optimization problem and determine optimum TMD parameters. The parameters of GA used in this research are as follows:

Number of individuals =25, Insertion rate=0.9, Mutation rate=0.05, Crossover rate: 1.0, and Maximum number of Generation: 2000

4.1.3 Designing TMD under different earthquakes

The uncontrolled structure has been subjected to different earthquakes given in Table 1 and the maximum displacement for different stories has been reported in Fig. 2. From Fig. 2, it is clear that the maximum lateral displacement of structure has occurred under Imperia Valley record.

Table 1: Characteristics of real earthquakes from California district

Earthquake No.	Earthquake	COMP	PGA(g)	Soil condition
1	Kem Country	S69E	0.179	Rock
2	Imperia Valley	S00E	0.348	Stiff soil
3	Helena Montana	S00E	0.146	Rock
4	Borrego	N57W	0.046	Stiff soil
5	Long Beach	N51W	0.097	Rock
6	Parkfield I	N65W	0.269	Rock
7	Parkfield II	N85E	0.434	Rock
8	Sanfernando I	S74W	1.075	Rock
9	Sanfernando II	N90E	0.165	Stiff soil
10	Sanfernando III	N37E	0.2	Stiff soil
11	Sanfransisco I	S80E	0.105	Rock
12	Sanfransisco II	S09E	0.085	Stiff soil

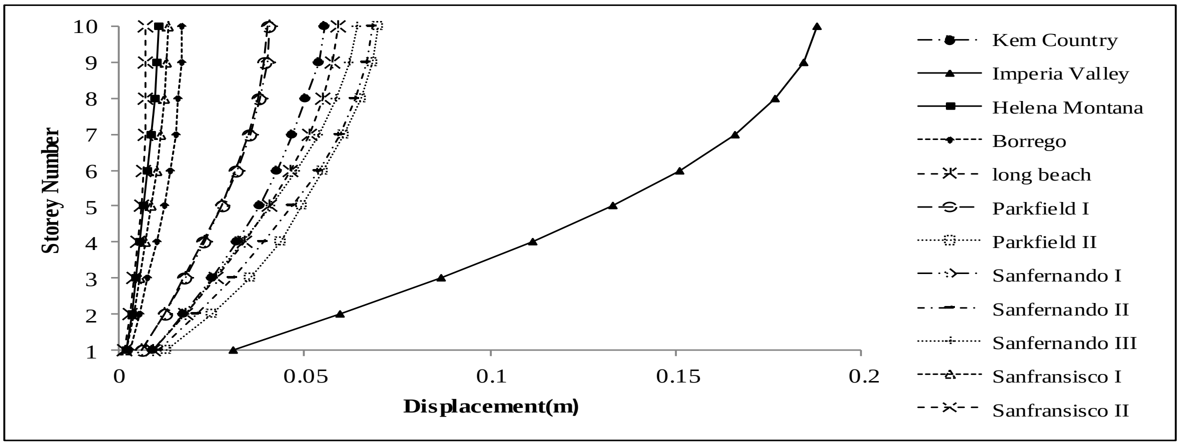


Figure 2. Lateral displacement of uncontrolled structure under different earthquakes

To show the effect of design record on performance of TMD, for each of the considered earthquakes and the W(t) excitation, optimal TMDs have been designed using the optimization procedure explained in previous section. Additionally, to assess the capability of different TMD design methods proposed in literature, the parameters of TMDs have also been determined by using Sadek et al. [7] and Den Hartog[30] methods suggested for linear structures. The structure equipped with TMD has been subjected to each of the design records and the reductions in maximum displacement under all excitations have been determined and reported in Tables 2 and 3 for different design methods and mass ratios. The results show that (1): under each design record, the optimal performance of TMD has been achieved by using OBDM. For example when μ =2%, the reductions in the maximum displacement under different design records have been about 9% to 45%, 0% to 43% and 0% to 42% for OBDM, Sadek et al and Den Hartog methods, respectively, while under Long Beach earthquake(earthquake No.5) using Den Hartog procedure has resulted in increasing the maximum displacement. In addition, the average of reductions under all design records has been respectively about 25%, 14% and 16% for OBDM, Sadek et al. and Den Hartog methods for μ=2%; (2): the effectiveness of TMD in reducing the displacement response varies with the design records. For example by using OBDM procedure, the maximum displacement of the structure has been reduced by about 50% under W(t) record for μ =4% while the corresponding reduction has been about 6% under Long Beach earthquake record. Hence, it can be concluded that application of TMD, even if designed optimally, cannot deliver the desired performance under a number of ground motions. Therefore, before using TMD in practical applications for a given structure and region, an initial assessment of TMD efficiency under design records of that area should be conducted, where design records can be selected based on site condition and seismic design codes.

Table 2: Reductions (%) in the maximum displacement of structure under design records for μ=2%

Earthquake / Design method	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**W (t)**
OBDM	9	33	13	20	14	24	25	16	40	45	26	18	45
Sadek et al.	4	24	2	4	0	10	20	8	10	27	13	14	43
Den Hartog	4	31	1	6	-3	15	21	10	20	31	17	17	42

Table 3: Reductions (%) in the maximum displacement of structure under design records for μ=4%

Earthquake / Design method	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**W(t)**
OBDM	13	39	16	23	6	29	32	26	41	53	39	19	50
Sadek et al.	5	35	4	7	0	12	24	12	19	29	22	17	47
Den Hartog	6	37	2	7	-1	17	27	1	29	37	29	16	49

Comparing the response of controlled structure using different design methods for TMD shows that under some of the earthquakes, there is no significant difference between the results obtained using OBDM and Sadek et al or Den Hartog procedures. To evaluate the performance of TMDs designed for each earthquake under other records which have different characteristics from the design record, the structure equipped with the optimal TMD has been designed for a specific record and then subjected to other earthquakes. The maximum displacement of controlled structure has been determined and divided to its corresponding value of uncontrolled structure. The normalized displacement values have been given in Tables 4 and 5 for all considered records. In addition, the mean of normalized maximum displacement under all earthquakes have been shown in Tables 4 and 5. The results show that TMDs have worked differently under the testing records. It can be seen that under a number of records TMD's application has even resulted in increasing of the maximum displacement of the structure. The maximum reductions in peak displacements under testing records have been 44% and 48% for μ=2% and 4%, respectively, which belongs to the TMD designed using Sanfransisco I record as the design excitation and tested under Sanfernando III earthquake. Also, the maximum reductions in the average of maximum displacements have been about 14% and 21% for μ=2% and μ=4% achieved when TMD has been designed under Imperia Valley as the design record. Also under Sanfernando I (Earthquake No.8), the mean of normalized displacement under different excitations is 1.07 that shows 7% increase in the average of maximum displacement.

According to the response obtained under different earthquakes, it has been found that the TMD designed for a specific earthquake cannot provide an acceptable performance when the structure is excited by other ground motions. Therefore, due to the uncertainty in input records, using only one specific earthquake in designing TMD is not reliable. Hence in the next sections of the paper, various procedures for designing TMD under multiple earthquakes have been proposed.

Table 4: Normalized maximum displacement of controlled structure under different earthquakes for μ=2%

Design Record No. / Test Record No.	1	2	3	4	5	6	7	8	9	10	11	12	W(t)
1	**0.91**	1.00	1.06	1.03	0.91	1.33	1.31	1.35	1.02	1.39	1.39	1.02	.96
2	0.71	**0.67**	1.02	0.98	0.75	0.83	0.81	0.89	0.68	0.89	0.90	0.68	.72
3	0.94	1.03	**0.87**	1.03	0.93	1.01	1.01	1.00	1.03	1.02	1.03	1.00	.97
4	1.69	0.95	1.29	**0.80**	1.57	1.01	1.00	1.11	0.95	1.01	1.01	0.94	1.01
5	0.92	1.09	1.10	1.06	**0.86**	1.16	1.15	1.13	1.10	1.23	1.26	1.08	1.00
6	0.95	0.93	1.11	1.02	0.97	**0.76**	0.83	0.81	0.93	0.90	0.91	0.85	.88
7	1.15	0.83	1.10	0.85	1.14	0.77	**0.75**	0.83	0.85	0.86	0.87	0.77	.79
8	0.88	0.90	0.97	0.94	0.89	0.85	0.85	**0.84**	0.90	0.86	0.86	0.88	.91
9	1.25	0.60	1.22	0.89	1.22	1.02	0.98	1.26	**0.60**	1.15	1.17	0.78	.93
10	0.97	0.60	0.96	0.96	0.97	0.64	0.65	0.80	0.62	**0.55**	0.56	0.64	.75
11	0.83	0.79	0.96	0.89	0.83	0.79	0.79	0.75	0.79	0.74	**0.74**	0.80	.85
12	1.01	0.94	0.93	1.11	0.97	0.97	0.97	1.05	0.95	1.01	1.03	**0.82**	.86
W(t)	0.81	0.84	0.92	1.07	0.71	0.80	0.75	0.91	0.71	0.88	0.91	0.64	**0.55**
Mean	1.00	0.86	1.04	0.97	0.98	0.92	0.91	0.98	0.86	0.96	0.97	0.84	0.86

Table 5: Normalized maximum displacement of controlled structure under different earthquakes for μ=4%

Test Record No. \ Design Record No.	1	2	3	4	5	6	7	8	9	10	11	12	W(t)
1	**0.87**	0.95	0.93	1.06	0.97	1.02	1.08	1.60	0.97	1.27	1.27	0.95	1.02
2	0.8	**0.61**	0.93	0.94	0.67	0.89	0.76	1.15	0.82	0.84	0.88	0.64	0.61
3	0.92	1.00	**0.84**	1.04	0.92	1.06	1.07	1.00	1.05	1.03	1.06	0.98	0.99
4	1.34	0.93	1.48	**0.77**	1.13	0.9	0.99	1.18	0.87	0.98	1	0.93	0.94
5	1.14	1.02	1.02	1.11	**0.94**	1.08	1.10	1.22	1.07	1.05	1.08	1.01	1.01
6	1.06	0.85	1.09	0.9	0.99	**0.71**	1.10	1.08	0.75	0.92	1.06	0.82	0.85
7	1.20	0.72	1.14	0.76	0.92	0.71	**0.68**	1.23	0.71	0.83	0.78	0.75	0.71
8	0.85	0.84	0.93	0.89	0.84	0.86	0.83	**0.74**	0.86	0.8	0.81	0.87	0.83
9	1.49	0.65	1.44	0.79	1.09	0.72	0.92	1.19	**0..59**	0.78	0.74	0.74	0.75
10	1.06	0.54	0.9	0.9	0.91	0.9	0.81	0.78	0.85	**0.47**	0.63	0.64	0.59
11	0.76	0.68	0.92	0.79	0.73	0.71	0.68	0.76	0.69	0.62	**0.61**	0.76	0.67
12	1.05	0.92	0.92	1.16	0.95	1.12	1.19	1.18	1.07	1.05	1.15	**0.81**	0.88
W(t)	0.87	0.52	0.84	0.9	1.00	0.81	0.9	0.74	0.77	0.65	0.89	0.52	**0.5**
Mean	1.03	0.79	1.03	0.92	0.93	0.88	0.93	1.07	0.85	0.87	0.92	0.8	0.80

4.1.4. The effect of earthquake characteristics on parameters of optimal TMD

The optimal parameters of the TMDs determined using OBDM for each earthquake as well as Sadek et al. and Den Hartog methods in previous section have been reported in Table 6 for μ=2% and μ=4%.

Table 6: Optimal parameters of TMDs under different earthquakes for μ=2% and μ=4%

Earthquake	K_{opt}(N/m)		C_{opt}(N-s/m))	
	μ=2%	**μ=4%**	**μ=2%**	**μ=4%**
Kem Country	3673000	7870000	0.142	0.000356
Imperia Valley	2456200	4576202	15611	134500
Helena Montana	5388300	11124000	0.2371	139720
Borrego	1356100	2695800	1024.3	23818
Long Beach	3940600	969780	92484	1.07E-05
Parkfield I	2812100	3346600	7511	31832
Parkfield II	2719600	4011700	9825.3	14780
Sanfernando I	2967500	5764500	0.0012	0.31259
Sanfernando II	2217800	3480500	29163	55942
Sanfernando III	2631600	4807200	1163.3	59240
Sanfransisco I	2613700	4490100	0.0087	24785
Sanfransisco II	2608800	4501000	66343	388940
W(t)	2976400	5129700	128750	189690
Den Hartog	2694960	5018296	103076	276343
Sadek et al.	2727649	5130570	181695	483059

From the results, it is clear that the stiffness and damping coefficients of TMDs have been different under each earthquake and thus depends on the input record characteristics. In previous research for designing TMDs mostly the optimal frequency of TMDs is tuned to the frequency of first vibration mode while in this research the optimal parameters of TMDs using the OBDM are determined such that the maximum displacement of the structure is minimized. To assess the effect of input records on the frequency of optimal TMDs, in Fig. 3 the frequency of optimal TMDs designed for different earthquakes has been compared with the frequency of 1st and 2nd vibration modes of structure which are f_1=1.01 *Hz* and f_2=3.0 *Hz*, respectively. Furthermore, the optimal frequencies of TMDs using Sadek et al.[7] and Den Hartog[30] methods, which are constant under all records, have been shown in Fig. 3 as well.

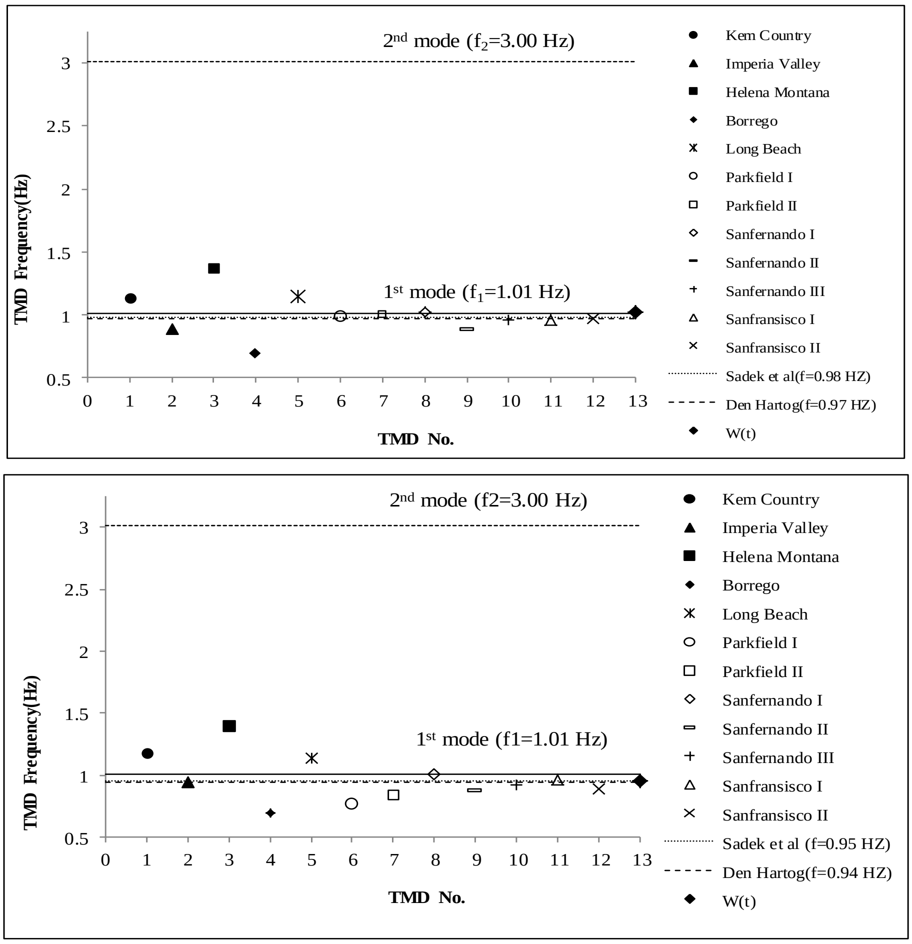


Figure 3. Frequency of optimal TMDs under different earthquakes for (a): μ=2% and (b):μ =4%

It is clear that for most earthquakes, the frequency of optimal TMDs designed using OBDM, Sadek et al. and Den Hartog methods is around the frequency of 1st vibration mode. Hence under different earthquakes, it can be recommended that the frequency of TMD can be selected near the frequency of first vibration mode of structure without sensitivity to the input record. This conclusion confirms the capability of methods suggested in literature for determination of TMD parameters for linear structures based on tuning the frequency of TMD to the frequency of 1st vibration mode of main structure. While the frequency of optimal TMDs have been close to each other under different earthquakes, according to Table 6, it is clear that the damping of TMDs have been different for each earthquake as well as for Sadek et al. and Den Hartog methods, which has resulted in different performance of TMDs. Therefore, it is concluded that for a given mass ratio of TMD, the stiffness of TMD is not affected significantly by earthquake characteristics while the damping value of optimal TMD varies by the input record.

4.1.5 Different scenarios for designing TMDs under multiple earthquakes

According to the results reported in section 4.1.3 regarding the reductions obtained in the average of maximum displacement under all records, it has been found that TMDs designed according to different design procedures by using only one design record, though effective under a number of earthquakes, have not performed well under all records. Therefore, it is necessary to develop methods to design TMDs under multiple earthquakes and evaluate TMDs performance accordingly. In this paper following scenarios have been considered to determine TMDs parameters:

Case A: to take into account the effect of all records in determining TMD parameters, in this case the parameters of the TMD has been considered as the mean of optimal TMD parameters obtained separately for each earthquake.

Case B: in this scenario, the standard deviations of optimal parameters obtained for each earthquake separately have been added to the mean value of TMD parameters.

Case C: in this procedure to consider the effect of each record, the weighted average of optimal parameters under different earthquakes has been used. The weighting parameter of a specific earthquake in this case has been defined as the reduction percentage in displacement of uncontrolled structure.

Case D: in this case the parameters of TMD have been determined using the method proposed by Sadek et al. For a specific TMD mass ratio, the stiffness and damping of TMD are constant for all earthquakes.

Case E: in this case, Den Hartog method has been used to determine the parameters of TMD. In this method, too, the parameters of TMD are independent of input excitation.

By using data given in Table 6, for various scenarios the parameters of TMDs have been determined and reported in Tables 7 and 8 for μ=2% and 4%.

To compare the effectiveness of TMDs designed base on different scenarios under multiple records, the maximum displacement of controlled structure has been determined under all records and the average of normalized maximum displacement has been shown in Fig. 4 for each case. Based on the results shown in Fig. 4, it can be seen that TMDs have not been more effective in mitigating the average of maximum displacement, which the maximum reduction has been about 14% and 18% for μ=2% and 4% when using Den Hartog method. Therefore, it can be said that though TMD can be effective in mitigating the response of structure when subjected to a number of earthquakes separately, its performance under multiple earthquakes is not notable.

Table 7: Parameters of TMD for μ=2%

TMD parameters	Case A	Case B	Case C	Case D	Case E
C_{opt}(N-s/m)	12256	30708	14664	181695.6	103076.06
K_{opt}(N/m)	2923816	3923111	2692733	2727649.8	2694960

Table 8: Parameters of TMD for μ=4%

TMD parameters	Case A	Case B	Case C	Case D	Case E
C_{opt}(N-s/m)	72796.4	183310.3	73984.2	483059.23	276343.8
K_{opt}(N/m)	4803115	7399559	4650418.6	5130570.38	5018296.12

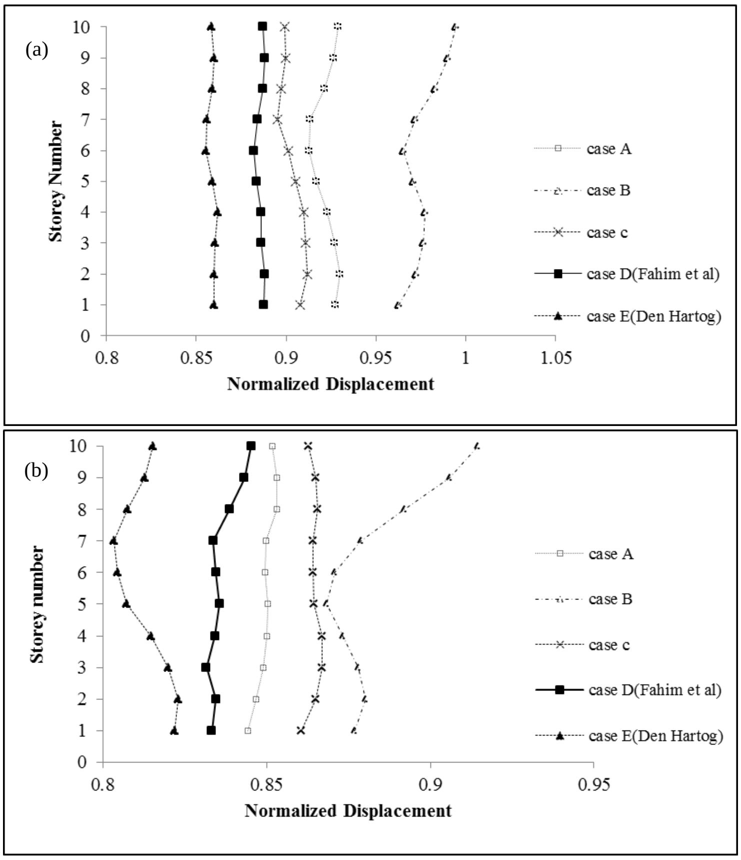


Figure 4. Mean of normalized maximum displacement of controlled structure under testing earthquakes for (a): μ=2% and (b):μ =4%

4.1.6 Assessment of TMDs performance under testing records

In order to evaluate the effectiveness of the proposed scenarios under other real earthquakes which are different from design records in characteristics, the structure equipped with TMDs designed according to cases *A* to *E*, has been subjected to Hachinohe (1968, PGA=.23 g), Northridge (1994, PGA=.84 g), Kobe (1995, PGA=0.83 g), Orovill (1975, PGA=0.198 g) and Lytle Creek (1970, PGA=0.084) records. The normalized maximum displacement and its mean value under testing earthquakes have been shown in Tables 9 and 10 for μ=2% and μ=4%. The results indicate that different TMDs have worked similarly under testing records and also their efficiency has not been more considerable.

Table 9: Normalized maximum displacement of controlled structure for μ=2%

Test Records \ Scenario	*Case A*	*Case B*	*Case C*	*Case D*	*Case E*
Hachinohe	0.89	1.10	0.91	0.87	0.88
Northridge	0.98	1.01	0.98	1.01	1.00
Kobe	0.90	0.89	0.90	0.93	0.92
Orovill	0.88	0.89	0.88	0.92	0.90
Lytle Creek	0.95	0.89	0.97	0.95	0.96
Mean	0.92	0.95	0.93	0.94	0.93

Table 10: Normalized maximum displacement of controlled structure for μ=4%

Test Records \ Scenario	*Case A*	*Case B*	*Case C*	*Case D*	*Case E*
Hachinohe	0.85	0.81	0.85	0.84	0.84
Northridge	0.98	1.05	0.97	1.04	1.01
Kobe	0.82	0.82	0.82	0.87	0.84
Orovill	0.79	0.82	0.80	0.86	0.83
Lytle Creek	0.96	0.84	0.97	0.91	0.93
Mean	0.88	0.87	0.88	0.91	0.89

Based on the results under design and testing records, it can be said that (1): TMD has the capability of suppressing structural response when the structure is subjected to a large number of earthquakes separately, and it is recommended to design an effective TMD under a specific earthquake by using the optimization-based design method (OBDM) suggested by Mohebbi and Joghataie[13]. This suggestion is also useful when it is desired to mitigate the maximum response of structure under multiple earthquakes ,which for this case the earthquake that induces the maximum response is selected as the design earthquake; (2): for reducing the mean of maximum response of the structure under multiple records, it can be said that while the capability of different TMDs has not been significant, to design a TMD to protect the structure under several records, to avoid complexity and to take into account the effect of different earthquakes in TMD design, it is suggested to use *case A* for determining TMD parameters.

4.2 Designing optimal MTMD

To evaluate the performance of MTMD under different earthquakes, in this section MTMDs for two mass ratios i.e. μ=2% and μ=4% has been designed when ten TMDs are considered on the top floor of the structure in parallel configuration. To design MTMDs, the method proposed by Mohebbi et al. [31] has been applied where the parameters of optimal TMDs are determined such that the maximum displacement of structure is minimized.

To assess the effectiveness of MTMD under different earthquakes, similar to previous section, the controlled structure has been subjected to 12 real earthquakes reported in Table 1 as well as the W(t) excitation, and the normalized maximum displacement of structure under each record and its mean value under all records have been determined. The results have been shown in Tables 11 and 12 for μ=2% and μ=4%.

Table 11: Normalized maximum displacement of controlled structure under different excitations for μ=2%

Design Record No. / Test Record No.	1	2	3	4	5	6	7	8	9	10	11	12	W(t)
1	**0.91**	1.11	1.09	0.98	1.05	0.95	1.29	1.30	0.99	1.40	1.39	1.08	0.96
2	0.74	**0.66**	1.02	0.81	0.79	0.71	0.78	0.84	0.72	0.9	0.9	0.69	0.69
3	0.93	1.02	**0.9**	0.99	0.96	0.99	1.01	1.00	1.02	1.02	1.03	1.00	1.00
4	1.67	1.01	1.12	**0.69**	1.15	1.36	1.08	1.16	0.92	1.00	1.01	0.93	0.98
5	0.96	1.11	1.05	1.03	**0.81**	0.99	1.14	1.12	1.09	1.10	1.26	1.14	0.99
6	0.96	0.95	1.10	1.01	0.95	**0.72**	0.82	0.78	0.9	0.9	0.91	0.89	0.81
7	1.16	1.00	1.13	0.8	1.01	0.94	**0.76**	0.82	0.82	0.87	0.87	0.78	0.78
8	0.89	0.88	1.00	0.94	0.92	0.9	0.86	**0.84**	0.91	0.85	0.86	0.9	0.9
9	1.26	0.84	1.18	0.82	1.03	1.17	0.96	1.28	**0.56**	1.16	1.17	0.77	0.93
10	0.96	0.57	1.02	0.84	0.91	0.89	0.66	0.81	0.66	**0.54**	0.56	0.61	0.67
11	0.84	0.76	1.03	0.87	0.87	0.84	0.77	0.78	0.83	0.74	**0.74**	0.92	0.82
12	0.95	0.93	0.98	0.94	0.91	0.91	0.97	1.04	0.93	1.03	1.03	**0.82**	0.86
W(t)	0.93	1.02	1.01	0.69	0.93	0.83	0.81	0.91	0.7	0.89	0.92	0.68	**0.44**
Mean	1.01	0.91	1.05	0.88	0.95	0.94	0.92	0.98	0.85	0.95	0.97	0.86	0.83

Table 12: Normalized maximum displacement of controlled structure under different earthquakes for μ=4%

Design Record No. / Test Record No.	1	2	3	4	5	6	7	8	9	10	11	12	W(t)
1	**0.87**	1.01	0.98	0.94	0.93	0.93	1.31	1.60	0.94	1.29	1.48	1.02	0.94
2	0.75	**0.58**	1.00	0.81	0.64	0.66	0.82	1.15	0.61	0.86	1.00	0.67	0.63
3	0.93	0.98	**0.82**	0.96	0.96	0.99	1.04	1.00	1.00	1.02	1.04	0.99	0.97
4	1.50	0.86	1.54	**0.60**	1.35	1.08	1.13	1.18	0.79	0.96	1.08	0.9	0.96
5	0.99	0.93	1.05	1.00	**0.76**	0.93	1.06	1.22	0.93	1.09	1.06	1.13	0.94
6	1.00	0.95	1.11	0.96	1.11	**0.59**	1.08	1.08	0.9	0.91	1.03	0.95	0.77
7	1.21	1.00	1.21	0.76	1.09	0.85	**0.66**	1.23	0.82	0.81	0.85	1.05	0.74
8	0.85	0.8	0.97	0.89	0.88	0.85	0.79	**0.74**	0.84	0.8	0.77	0.89	0.86
9	1.40	0.77	1.47	0.82	1.09	1.06	1.05	1.19	**0.47**	0.83	0.89	0.71	0.97
10	1.06	0.62	0.98	0.83	0.98	0.83	0.63	0.78	0.71	**0.48**	0.58	0.6	0.67
11	0.77	0.71	0.97	0.81	0.74	0.73	0.64	0.76	0.66	0.62	**0.61**	0.79	0.75
12	1.04	0.98	0.92	0.96	0.84	0.96	1.01	1.18	0.95	0.99	1.08	**0.78**	0.82
W(t)	0.94	1.27	0.9	0.73	0.98	0.72	0.96	0.73	0.72	0.64	0.83	0.76	**0.44**
Mean	1.02	0.88	1.07	0.85	0.95	0.86	0.94	1.06	0.80	0.87	0.95	0.86	0.8

The results show that the reductions in maximum displacement when the structure has been subjected to different earthquakes, has been about 9% to 56% and 13% to 56% for μ=2% and μ=4%, respectively. It is clear from the results that the performance of MTMDs depends on the input earthquake characteristic as well. Here the control system has had its best performance under the W(t) record. In addition, the maximum reductions in the mean of maximum displacement under all earthquakes has been about 17% and 20% for μ=2% and 4% respectively, which has been achieved for MTMDs designed by using *W(t)* as the design record and tested under other ground motions. Moreover, MTMDs designed by using a number of earthquakes such as Kem Country record under other records has led to increases in the maximum displacement. Hence, similar to the result concluded for single TMD, generally MTMDs designed by using a single specific record has not worked successfully under other records and due to the uncertainty in input records, using only a specific earthquake in design of MTMD is not much efficient.

To assess the performance of different MTMDs under multiple records, following the procedure explained for single TMD and using the scenarios defined in *cases A* and *B* in section *4.1.5*, two sets of MTMDs have been designed. The mean of normalized maximum displacements of each story of the controlled structure under all records has been given in Fig. 5. For comparison purposes, the corresponding values for MTMDs designed under Helena Montana and Sanfernando II earthquakes and tested to different earthquakes have also been reported in Fig. 5. The results show that in reducing the mean of maximum displacement under all ground motions, MTMDs designed for Sanfernando II earthquake has worked better in comparison with other MTMDs. However, since there is no significant difference between performance of MTMDs designed using *case A* and MTMDs designed using Sanfernando II earthquake as the design record, to avoid complexity and generalize the design procedure using *case A* can be recommended as a design method for MTMDs under multiple earthquakes.

4.2.1 The effect of input earthquake characteristics on parameters of MTMD

To assess the effect of input records on frequency of optimal MTMDs, the frequency of optimal TMDs under different earthquakes have been compared with the frequency of 1st and 2nd vibration modes of structure for μ=2% and 4% in Fig. 6.

As can be seen optimal frequency of most of the TMDs under different earthquakes has been close to the frequency of 1st vibration mode of structure while a number of TMDs have been tuned to a frequency between 1st and 2nd vibration modes frequency. Therefore, as a design procedure for MTMDs, it can be suggested to distribute the frequency of optimal TMDs around the structure 1st vibration mode frequency. In addition, it has been found that for optimal MTMDs, the damping of TMDs depends strongly on the design record characteristics.

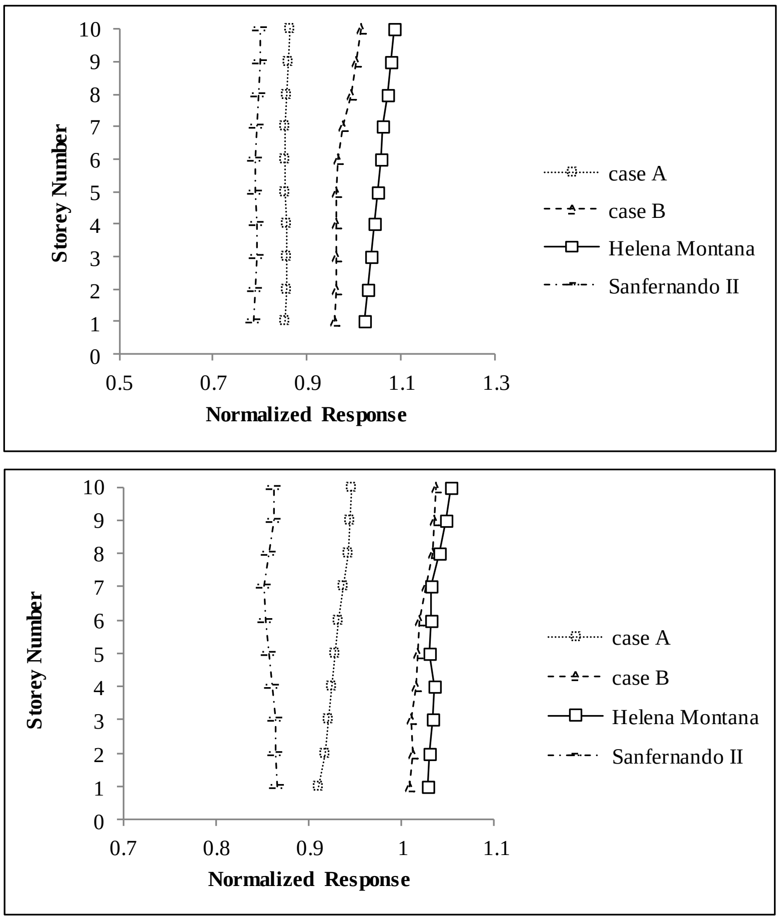
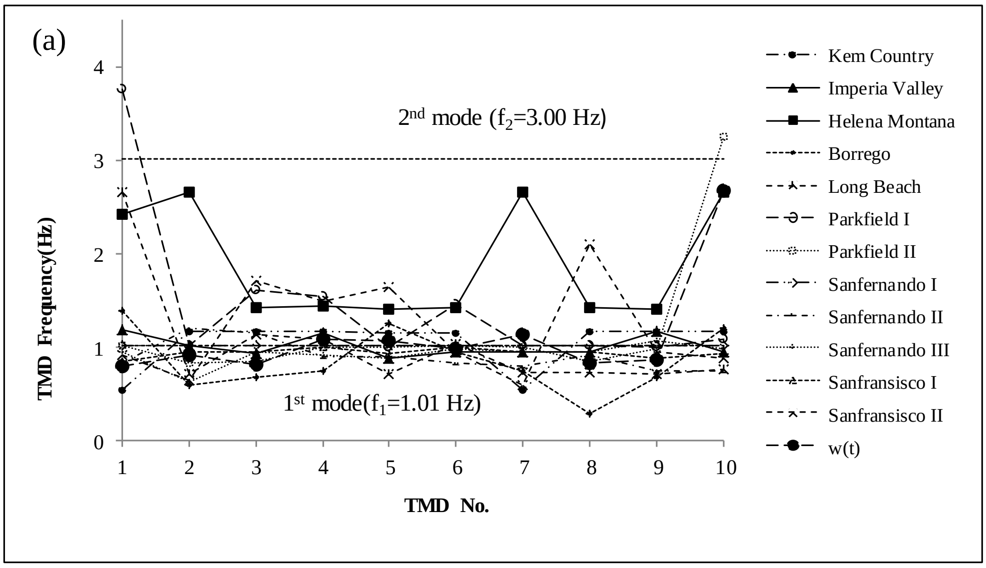


Figure 5. Mean of normalized maximum displacement of controlled structure for (a): μ=2% and (b):μ =4%



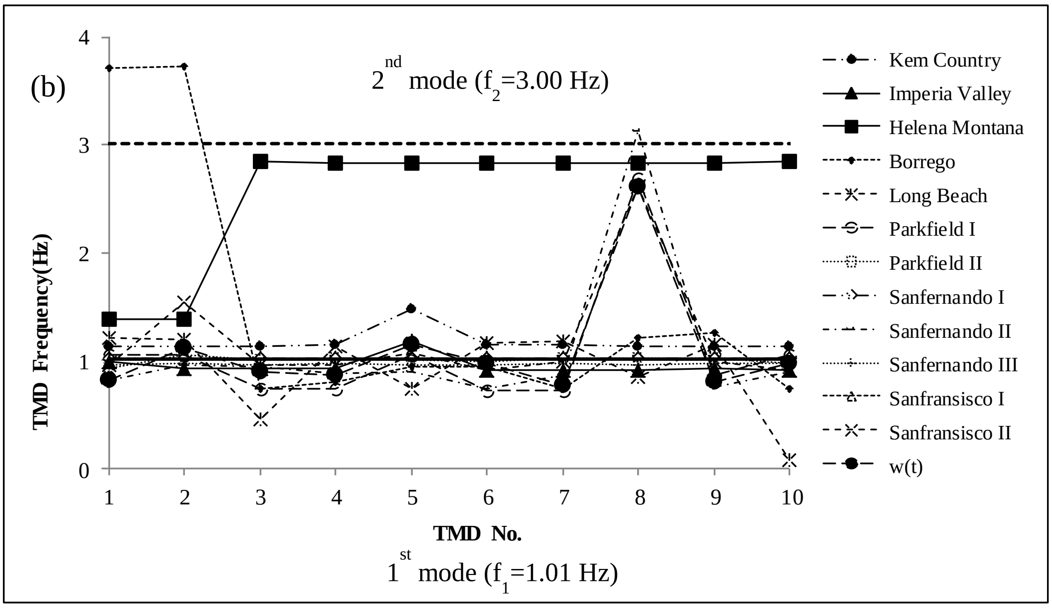


Figure 6. Frequency of optimal TMDs under different earthquakes for (a): μ=2% and (b):μ =4%

5. CONCLUSIONS

In this paper, the effectiveness of TMD and MTMDs in mitigating the response of shear frame subjected to different earthquakes has been studied. Optimum parameters of single and multiple TMD have been determined by solving an optimization problem to minimize the maximum displacement of structure using genetic algorithm (GA). For illustration, a ten-storey shear frame with linear behavior has been considered and subjected to 12 earthquakes of California district as well as a white noise excitation. Optimal single and multiple TMDs have been designed for each ground motion separately and tested under other records. The results have shown that the performance of TMD and MTMD depends strongly on the input earthquake characteristics which for the case study of this research, the maximum reductions in the maximum displacement have been about 6% to 53% and 13% to 56% for single and multiple TMDs , respectively when μ =4%. In addition, evaluating the efficiency of TMD and MTMDs designed for a specific earthquake under other ground motions shows that they have not performed well under other records. In this case study the maximum reductions in the average of maximum displacement under multiple records has been 21% for μ =4% while using a number of TMDs and MTMDs have resulted in increasing the average of maximum displacement. According to the results obtained by using different procedures for designing TMD and MTMDs, it has been concluded that for designing TMD and MTMD under a specific ground motion, it is recommended to use the optimization procedure while for determination of single or multiple TMDs parameters under multiple ground motions, to avoid complexity and take into account the characteristics of input ground motions, using the mean of optimal TMD parameters obtained under each record provides effective results. Furthermore, comparing the frequency of optimal TMD and MTMDs under different earthquakes with that of structure has shown that under a large number of records the frequency of TMDs has been close to the frequency of 1st vibration mode of the structure while the damping of TMDs has been dependent on the design record. Therefore, in designing TMD and MTMDs under different records the frequency of single and multiple TMDs can be selected around the 1st mode and between 1st and 2nd modes frequency, respectively.

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