



## OPTIMIZATION OF STEEL MOMENT FRAME BY A PROPOSED EVOLUTIONARY ALGORITHM

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### ABSTRACT

This paper presents an improved multi-objective evolutionary algorithm (IMOEa) for the design of planar steel frames. By considering constraints as a new objective function, single objective optimization problems turned to multi objective optimization problems. To increase efficiency of IMOEa different Crossover and Mutation are employed. Also to avoid local optima dynamic interference of mutation and crossover are considered. Feasible particles called elites which are very helpful for better mutation and crossover considered as a tool to increase efficiency of proposed algorithm. The proposed evolutionary algorithm (IMOEa) is utilized to solve three well-known classical weight minimization problems of steel moment frames. In order to verify the suitability of the present method, the results of optimum design for planar steel frames are obtained by present study compared to other researches. Results indicate that, as far as the convergence, speed of the optimization process and quality of optimum design are concerned behavior, IMOEa is significantly superior to other meta-heuristic optimization algorithms with an acceptable global answer.

**Keywords:** frame design; metaheuristic optimization algorithms; evolutionary algorithm; constraint handling; Structural optimization.

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### 1. INTRODUCTION

Generally, the optimization techniques in structural design can be categorized into two main fields: classical and heuristic search methods [1]. Classical optimization methods such as linear programming, nonlinear programming and optimality criteria often require basic gradient information to achieve best answer. In these methods the final results depend on the initial points and the number of computational operations increases as due to the increase the size of the structure members. The solution in these methods does not respond to the global

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optimum necessarily. Complexity of many engineering design problems are twisted enough that cannot be handled with mathematical programming methods [2-4]. In comparison, heuristic search methods do not require the initial data as in the mathematical programming and have better global search abilities than the classical optimization algorithms. Therefore, in the last decades considerable attention has been paid to heuristic search methods [5-15].

The genetic algorithm (GA) is one of popular heuristic methods; it has been used to solve structural optimization problems by some researches such as Camp et al. [16], Pezeshk et al. [17], Erbatur et al. [18], Kaveh and et al. [19]. Other heuristic algorithm such as ant colony optimization (ACO) is used to solve structural optimization problems too [20-22]. Also NSGA-II is widely used for optimization such problems [22-24]. Major variety of well-perform algorithms like genetic algorithm and NSGA-II are categorized as evolutionary algorithms (EA). The main advantages of EA is their simplicity to implementation to different problem especially structural problems. EA is randomized search technique heavily depended on its operators: mutation and crossover. This operators collect information from the previous cycles and produced new points in each iteration [25, 26].

Despite all the advantages of EA, the optimization time for solving structural problems is too high. In order to make EA less time-consuming, this paper utilizes a special attitude to structural design constraint and transform frame single-objective optimization problem to multi-objective optimization problem. This strategy is capable of decreasing the number of analyses notably without any reduction in the possibility of finding the optimal solution; as consequent, it is capable of reducing the optimization time[27].

The proposed algorithm is tested on several well-known frames, and numerical results are compared to other similar studies. The fast convergence of proposed algorithm is demonstrate the effectiveness of the proposed method to find optimal solution.

## 2. FRAME OPTIMIZATION PROBLEMS

The purpose of the optimum design of steel frames is to find a design with minimum weight in the most studies. The optimal design of frame structures is formulated as:

$$f(X) = \sum_{i=1}^n \gamma_i A_i L_i \quad (1)$$

where  $f(X)$  is objective function (total weight of structure);  $\gamma$  is density of material;  $A$  is the cross sectional of member;  $L$  is the length member;  $i$  is the current member of frame and  $n$  is the number of frame members. According to AISC-LRFD [28], structures are subjected to several design constraints. These constraints are:

For stress constraints of each element:

$$v_i^\sigma = \left| \frac{\sigma_i}{\sigma_i^\alpha} \right| - 1 \leq 0 \quad i = 1, 2, \dots, n \quad (2)$$

For maximum lateral displacement:

$$v^{\Delta} = \frac{\Delta_T}{H} - R \leq 0 \quad (3)$$

For inter-story displacements constraints (drift):

$$v_j^d = R_I - \frac{d_j}{h_j} \leq 0 \quad j = 1, 2, \dots, n_s \quad (4)$$

where  $\sigma_i$  and  $\sigma_i^{\alpha}$  are the current stress and allowable stress of each member, respectively;  $R$  is the maximum allowable drift;  $\Delta_T$  is the maximum lateral displacement;  $H$  is the total height of the structure;  $d_j$  is the inter-story drift;  $h_j$  is the story height of the  $j$ th story;  $n_s$  is the total number of stories;  $R_I$  is the inter-story allowable drift;  $i$  is the current member of frame and  $n$  is the total frame members. According to the AISC, the allowed drift for inter-story is given as  $1/300$ , and the LRFD interaction formula constraints are define as:

$$v_i^l = 1 - \frac{P_u}{2\phi_c P_n} - \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \geq 0 \quad \text{For } \frac{P_u}{\phi_c P_n} < 0.2 \quad (5)$$

$$v_i^l = 1 - \frac{P_u}{\phi_c P_n} - \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \geq 0 \quad \text{For } \frac{P_u}{\phi_c P_n} \geq 0.2 \quad (6)$$

where  $P_u$  is the required strength and  $P_n$  is the nominal axial strength;  $\phi_c$  is the resistance factor (for tension  $\phi_c = 0.9$ , for compression  $\phi_c = 0.85$ );  $M_{ux}$  and  $M_{uy}$  are the required flexural strengths in the  $x$  and  $y$  directions, respectively;  $M_{nx}$  and  $M_{ny}$  are the nominal flexural strengths in the  $x$  and  $y$  directions (for 2d frames,  $M_{uy} = 0$ );  $\phi_b$  is the flexural resistance reduction factor ( $\phi_b = 0.90$ ). Furthermore,  $P_n$  and  $M_n$  are calculated with effective length factors and unbraced length factor according to AISC-LRFD. In this paper, the following approximate effective length formula is used based on Dumonteil [29], which is accurate within about  $-1.0\%$  and  $+2.0\%$  of the exact results:

$$K = \sqrt{\frac{1.6G_A G_B + 4G_A G_B + 7.5}{G_A + G_B + 7.5}} \geq 1 \quad (7)$$

where  $G_A$  and  $G_B$  refer to the stiffness ratio or the relative stiffness of a column at its two ends.

### 3. THE IMPROVED MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM (IMOEA) IMPLEMENTATION

A simple EA proceeds by randomly generating an initial population. The next generation is evolved from this initial population by crossover and mutation operators. By this consideration, the weak designs are removed and the strong ones are transfer to the next

generation. After several generations, the best individual of the population is considered as the final solution of the algorithm. The stochastic nature of the method and considering a population of design points in each generation usually leads to the global optimum. The full details of the method can be found in the literature [27, 30]. Up to now standard EA and its improved versions have been extensively employed by researchers to efficiently tackle the different problems in the area of structural engineering [25, 31, 32].

### 3.1 Basic definitions for transforming single-objective problem to multi objective problem

Before explaining the method introduced in this paper, several basic concepts that have been used as below. The general form of the single-objective optimization problem with equality and inequality constraints is shown in (8):

$$\begin{aligned} & \text{Minimize } f(X) \\ \text{Subject to: } & \begin{cases} h_i(X) = 0, & i = 1, 2, \dots, m_1 \\ g_j(X) \leq 0, & j = 1, 2, \dots, m_2 \\ L_k \leq x_k \leq U_k, & k = 1, 2, \dots, m_3 \end{cases} \end{aligned} \quad (8)$$

where  $X$  is n-dimensional vector of design variables and  $f(X)$  is the objective function which in this particular case minimization of  $f(X)$  is the objective of optimization;  $g_j(X)$  and  $h_i(X)$  are constraints of optimization problem and also known as inequality and equality constraints, respectively;  $L_k$  and  $U_k$  are lower band upper band of each variables, respectively;  $m_1$  and  $m_2$  are the number of equality and inequality constraints[26].

To reduce the complexity for solving single-objective problems, due to increase the accuracy and speed of solving such problems simultaneously, following equation for transforming constraints to objectives are considered.

For equal constraints:

$$v_{1i}(X) = \max(h_i(X) - \sigma, 0) \quad i = 1, 2, 3 \dots, m_1 \quad (9)$$

And for unequal constraints:

$$v_{2i}(X) = \max(g_i(X), 0) \quad i = 1, 2, 3 \dots, m_2 \quad (10)$$

Finally, the objective function derived from the constraints is the sum of above-mentioned objectives:

$$v(X) = \sum_1^{m_1} v_{1i}(X) + \sum_1^{m_2} v_{2i}(X) \quad (11)$$

where  $\sigma$  is a small positive value used for equal constraints in order to convert them to unequal constraints and  $v(X)$  is the violation function (second objective function). Now, any constrained single-objective problem can easily turned into an unconstrained bi-objective optimization problem [26].

$$F(X) = (f(X), v(X)) \quad (12)$$

Minimum value of  $f(X)$  versus  $v(X)$  can be depicted as Fig. 1 and the curve called Pareto front. By explained definitions, the best answer will be in the possible minimum of violations (without any violation ( $v(x) = 0$ )), which is the global answer of optimization problem. While the other solutions whose value of their second objective function (constraint) is not equal to zero (definitely a positive number due to above definition ( $v(x) > 0$ )), has violated the initial constraints of the main problem and are not in feasible space and also are not qualified to be an acceptable answer to the problem. Thus, the speed of this algorithm is guaranteed by limiting the search space, due to removing many mislead particles in the search space and also local optimums which can mislead the optimization process to reach the global optimum.

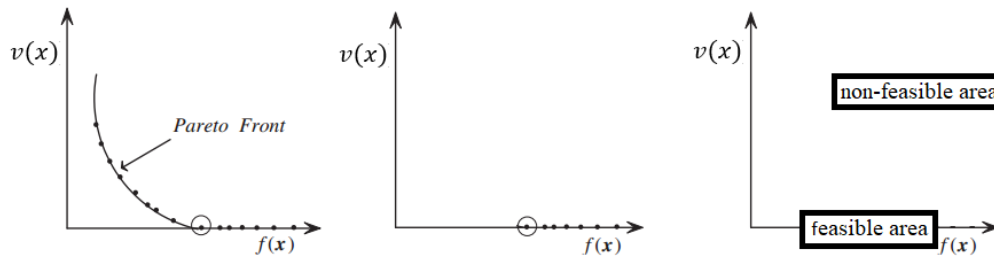


Figure 1. Global optimum position in this method

### 3.2 Mutation operator

The mutation operator has always been one of the most influential operators in evolutionary algorithms [33, 34]. In the mutation operation, certain digits of the chromosomes are altered randomly. For continuous problems, the use of more complicated mutations is more prevalent. The general form of the mutation operator in continuous problems is in the form of the Equation 13 given:

$$x'_i = x_i + (u_i - l_i)\bar{\delta}_i \quad (13)$$

where  $x'_i$  is the mutated gene (child);  $x_i$  is the primary gene (parent);  $u_i$  is the upper limit;  $l_i$  is lower limit and  $\bar{\delta}_i$  is the distribution function. In this study, normal mutation[35] which distributes a set of random numbers normally is selected as mutation of algorithm.

### 3.3 Crossover operator

Another influential operator in the evolutionary algorithms is the crossover operator. In the crossover operation, two members of the population are randomly selected, as parents, and two new offsprings are produced by exchanging a chromosome of parents' string [36]. Mask crossover is chosen crossover for this study, produce a completely randomized gene as the size of parents' genes. In case of having a value of 0 in the randomized gene, the

corresponding strand should be found in the first parent and poured into a new gene. In case of having the value of 1 in a randomized gene, corresponding value of the second gene will assign to the new gene. The schematic of mask crossover is shown in Fig. 2.

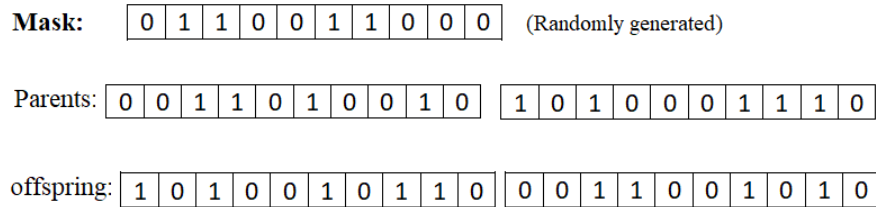


Figure 2. Schematic of mask crossover

### 3.4. Interference of operators

As mentioned in other studies, evolutionary algorithms are used to find the optimal solution by mutation and crossover operator functions [37]. Now, the impact percentage of these two operators is very important for the production of new generations for a reasonable moving toward the global optimum. Generally, in the initial cycles, the crossover operator makes a better convergence of the algorithm, and in the final cycles, the Mutation Operator prevents particles from being captured at local optima. Therefore, in order to consider this feature in the proposed algorithm and also to avoid additional calculations, the impact coefficients of these two operators are considered dynamically and it is related to momentary cycle of the procedure. Crossover operator linearly reduces and mutation operator is linearly increases as new generations being produced.

### 3.5 Importance of feasible particles

Crossover and mutation operators use particles of each cycle to search the solution space to achieve optimal solution in next generation. If they operate among particles with violations, there would be very little chance of finding a particle which is in the feasible area. So due to the specific performance of the proposed algorithm and also that many points can be in the infeasible area, a certain percentage of the solution in feasible area (elites) of each cycle are allocated. Now crossover and mutation can also select elites which are definitely in the feasible area and combine them with other particles (whole search space), will bring a convenient probability of finding global answer. There is also a special way to prioritize these particles. Since none of these particles has constraints violation ( $v(x) = 0$ ), these particles are arranged according to the value of the main objective function of the problem ( $f(x)$ ) in an ascending order, which is clear that the smallest value of  $f(x)$  of these particles is the optimal answer of the problem, and then the rest of the particles with higher  $f(x)$  are arranged. It should be noted that if the number of existed particles without a violation is less than a selected specified percentage set at the beginning of optimization, the algorithm fills the population of itself with other particles without violation, and if the present particles with no violation is greater than is needed, the algorithm chooses the particle as much as the elite number, and does not consider the rest.

#### 4. DESIGN EXAMPLES

In this section, three benchmark frame structures are optimized using the proposed method and results were compared with previous studies to demonstrate the validity of the proposed algorithm. Population for each problem is different but for all problems, maximum and minimum of operators (crossover and mutation) percentage are 0.9 and 0.3, respectively. Also in all problems, 0.3 of best feasible point (elites) reserved for operators calculation of next generation.

##### 4.1 Two-bay, three-story planar frame

The first benchmark problem is a two-bay, three-story frame subject to a single-load case as shown in Fig. 3. This frame was optimized according to the AISC–LRFD specification and the values of the uniform and the point loads in Fig. 3. The modulus of elasticity of steel is taken as  $E = 200 \text{ GPa}$  (29,000 ksi), the yield stress is  $F_y = 248.2 \text{ MPa}$  (36 ksi) and the material unit weight of  $\gamma = 7861 \text{ kg/m}^3$  ( $0.284 \text{ lb/in}^3$ ) were used. The unbraced length

factor for each beam member was specified to be 0.167. The beam group section should be chosen from the entire W-shapes of AISC standard list; however, the column group section is limited to W10 sections [17, 20]. Population for each cycle is 40 individuals.

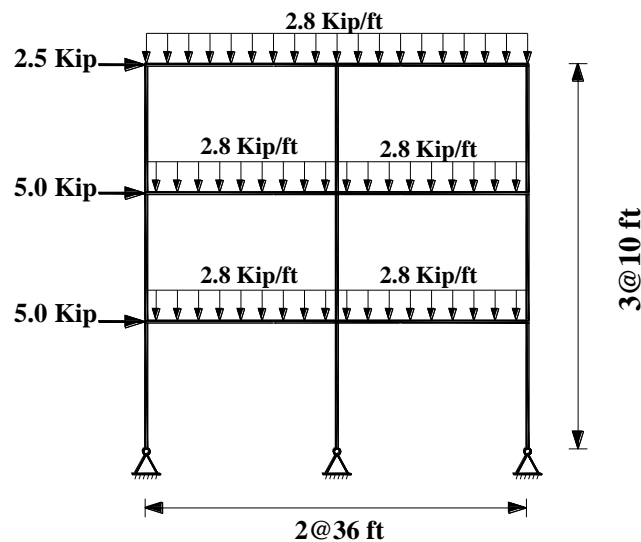


Figure 3. Two-bay, three-story planar frame

Fig. 4 shows the convergence history for the IMOEA. The optimum design of the frame is obtained after 250 analyses and reached the global optimum of this problem.

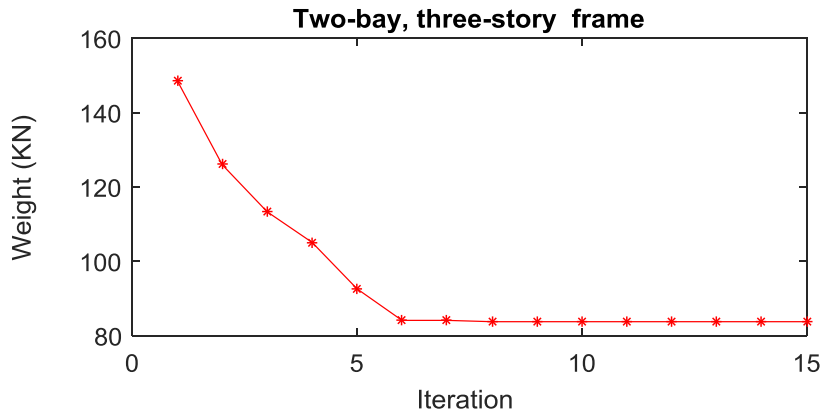


Figure 4. Convergence history for two-bay, three-story planar frame

Optimization results are compared with the literature as listed in Table 1. IMOEA acquired the answer with lowest analyses among other algorithms.

Table 1: Statistical results of two-bay, three-story planar frame

Element Group no.	AISC W-shapes				
	Element kind	GA[17]	ACO[20]	DDHS[38]	IMOEA
1	Beams	W24X62	W24X62	W24X62	W24X62
2	Columns	W10X60	W10X60	W10X60	W10X60
Weight(kN)		83.7759	83.7759	83.7759	83.7759
Number of Analyses		1800	3000	270	250

In summary, the proposed IMOEA algorithm seems to be the most efficient optimizer overall in terms of structural weight and computational cost of the optimization process in this specific problem.

#### 4.2 One-bay ten-story frame design

Fig. 5 shows the configuration and applied loads of 1-bay 10-story frame structure consisting of 30 members. Many researchers tested this example [17, 21, 39]. The beam element groups are chosen from all 267 W-shaped sections of the AISC standard list, while the column element groups are limited to W12 and W14 sections (66 W-shapes). The frame was designed according to the AISC-LRFD specifications and uses inter-story drift constraints: inter story drift < story height/300. The modulus of elasticity of the material  $E$  is 200 GPa and the yield stress  $f_y$  is 248.2 MPa. Same beam section to be used for three consecutive stories, beginning at the foundation, and that the same column section is used every two consecutive stories. The element grouping resulted in four beam sections and five column sections for a total of nine design variables and is shown in Fig. 5. 100 individuals are selected as Population of each cycle.

Fig. 6 shows the convergence history for optimization of one-bay ten-story planar frame problem by IMOEA. The optimum design of the frame is obtained 277.9 KN after 2200 analyses. Only IACO found better solution than IMOEA but there is no information about needed analyses.





Table 2: Statistical results of one-bay ten-story planar frame

Element Group no.	Element kind	AISC W-shapes			
		GA[17]	ACO[20]	IACO[4]	IMOEA
1	Beam 1-3S	W33X118	W30X108	W33X118	W30X124
2	Beam 4-6S	W30X90	W30X90	W30X90	W24X103
3	Beam 7-9S	W27X84	W27X84	W24X76	W21X93
4	Beam 10S	W24X55	W21X44	W14X30	W18X46
5	Column 1-2S	W14X233	W14X233	W14X233	W30X211
6	Column 3-4S	W14X176	W14X176	W14X176	W36X160
7	Column 5-6S	W14X159	W14X145	W14X145	W21X122
8	Column 7-8S	W14X99	W14X99	W14X90	W30X90
9	Column 9-10S	W12X79	W12X65	W12X65	W21X62
Weight(kN)		289.72	278.48	274.99	277.9401
Number of Analyses		3000	8300	n/a	2200

#### 4.3 Three-bay fifteen-story frame design

The topology and the service loading conditions for a three-bay fifteen-story frame consisting of 105 members are shown in Fig. 7. Displacement and AISC combined strength constraints were included as optimization constraints. Elasticity of the material ( $E$ ) and yield stress ( $f_y$ ) is similar to the previous problem. The beam and column element groups are chosen from all 267 W-shaped sections of the AISC standard list. The element grouping resulted in ten column sections and all beam are in same grouping. Details of grouping is shown in Fig. 7. 150 individuals are selected as Population of each cycle.

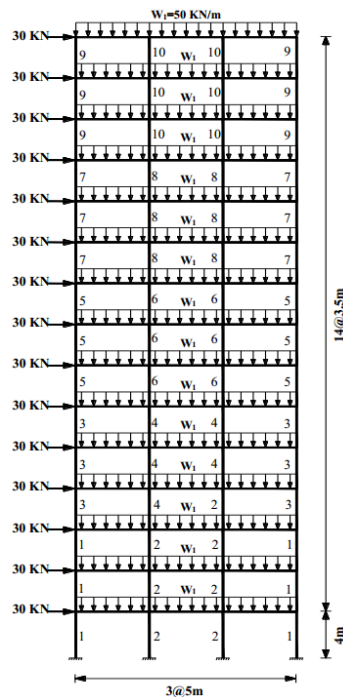


Figure 7. Convergence history for three-bay fifteen-story planar frame

Fig. 8 shows the convergence history for the IMOEA. The optimum design of the frame is obtained after 6500 analyses and reached the global optimum of 425.7 KN for this problem. Only CBO found better solution than IMOEA but there is no information about needed analyses.

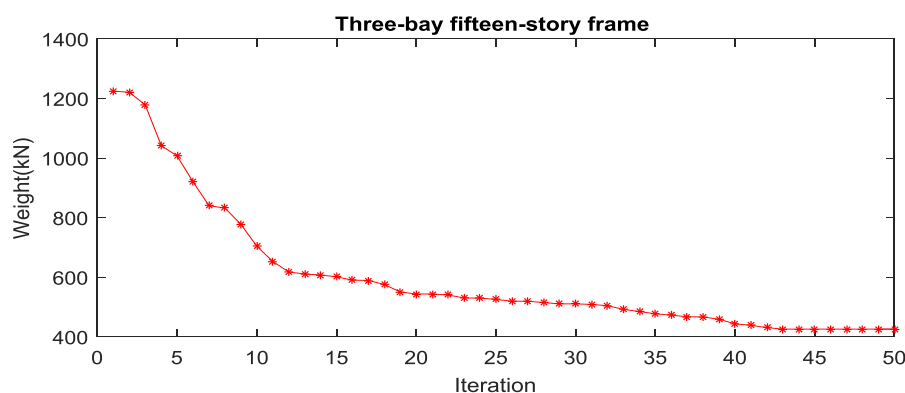


Figure 7. Convergence history for one-bay ten-story planar frame

Optimization results are compared with the literature as listed in Table 3. IMOEA acquired the convenient answer with lowest analyses among other algorithms.

Table 3: Statistical results of three-bay fifteen-story planar frame

Element Group no.	AISC W-shapes				
	Element kind	PSO[40]	CBO[41]	HPSACO[40]	IMOEA
1	Ex.Column 1-3S	W33X118	W24X104	W21X111	W33X118
2	In.Column 1-3S	W33X263	W40X167	W18X158	W36X160
3	Ex.Column 4-6S	W24X76	W27X84	W10X88	W18X86
4	In.Column 4-6S	W36X256	W27X114	W30X116	W14X120
5	Ex.Column 7-9S	W21X73	W21X68	W21X83	W21X68
6	In.Column 7-9S	W18X86	W30X90	W24X103	W30X90
7	Ex.Column 10-12S	W18X65	W8X48	W21X55	W10X60
8	In.Column 10-12S	W21X68	W21X68	W27X114	W24X68
9	Ex.Column 13-15S	W18X60	W14X34	W10X33	W10X33
10	In.Column 13-15S	W18X65	W8X35	W18X46	W12X40
11	Beams	W21X44	W21X50	W21X44	W21X50
Weight(kN)		496.39	416.5	426.5	425.7233
Number of Analyses		n/a	n/a	6800	6500

## 5. CONCLUSION

Evolutionary algorithms are generic population-based metaheuristic optimization algorithm. It is stochastic search technique uses mechanisms inspired by biological evolution, such as mutation and crossover, which is easy to implementation to optimization problems.

Despite all these advantages, optimization process for solving frame structures still very time-consuming.

In this paper, in order to improve the convergence speed and searching ability of the EAs, the implementation of an efficient converting constraint to objective function based on specific definition is introduced. Also In this method, mutation and crossover operators are chosen so that, the IMOEA shows better performance. Dynamization of crossover and mutation operators' percentage impact in population of every cycle and considering elite particles on this operators improved algorithm capability to find better solutions.

The proposed algorithm was tested on 3 benchmark frame problem. In all presented problems, IMOEA shows better performance against single-objective optimization algorithm in convergence and speed of providing optimal solution. IMOEA also present convenient global optimum, but there is a little weakness against powerful optimization algorithm. This algorithm by narrow down search space could be recommended in some of the problems where there is no limit in the search space. Obviously, implementation of the main idea of algorithm may leads to better algorithm while the result of IMOEA shows a great performance and could be tested on many other optimization challenges.

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