

STRUCTURAL SYSTEM RELIABILITY-BASED OPTIMIZATION OF TRUSS STRUCTURES USING GENETIC ALGORITHM

K. Biabani Hamedani and V.R. Kalatjari^{*,†}

Department of Civil Engineering, Shahrood University of Technology, Shahrood, Iran

ABSTRACT

Structural reliability theory allows structural engineers to take the random nature of structural parameters into account in the analysis and design of structures. The aim of this research is to develop a logical framework for system reliability analysis of truss structures and simultaneous size and geometry optimization of truss structures subjected to structural system reliability constraint. The framework is in the form of a computer program called RBO-S>S. The objective of the optimization is to minimize the total weight of the truss structures against the aforementioned constraint. System reliability analysis of truss structures is performed through branch-and-bound method. Also, optimization is carried out by genetic algorithm. The research results show that system reliability analysis of truss structures can be performed with sufficient accuracy using the RBO-S>S program. In addition, it can be used for optimal design of truss structures. Solutions are suggested to reduce the time required for reliability analysis of truss structures and to increase the precision of their reliability analysis.

Keywords: branch-and-bound method; system reliability analysis; size and geometry optimization; truss structures; genetic algorithm

Received: 12 October 2017; Accepted: 10 January 2018

1. INTRODUCTION

Structural optimization can be defined as the creation of a structure of materials, which bear loads optimally [1]. In structural engineering, the optimization aims mainly at designing structures with high efficiency, while their design and construction require minimum cost and materials. In this regard, special attention has been paid to optimization of truss structures, which are among the most common structures in the construction industry. In general, truss structure optimization problems can be classified into three different

*Corresponding author: Associate Professor, Department of Civil Engineering, Shahrood University of Technology, Shahrood, Iran
v_kalatjari@shahroodut.ac.ir (V.R. Kalatjari)

categories of 1) size optimization, 2) geometry optimization, and 3) topology optimization. In most structural optimization problems, either a dual combination of these three areas is used or all three areas are considered.

Although numerous attempts have been made for optimization of truss structures, in most cases the stochastic nature of structural parameters was neglected and optimization was carried out as a deterministic process [2-5]. However, the uncertainties are sometimes so severe that if ignored, the resulting model fails to simulate the actual conditions satisfactorily. Nevertheless, in most common methods of structural design, the stochastic nature of structural parameters is not directly incorporated into the design. In such methods, coefficients known as the safety factor are used to maintain the structure's safety level at a specific level. These factors are used to eliminate the concerns resulted from the alleged certainty of structural parameters and from the incorporation of simplifying approximations and assumptions in structural design.

For optimal design of structures, especially complicated structures, it is necessary to include the stochastic nature of structural parameters directly in their analysis and design. Hence, approaches provide the possibility of assessment of uncertainties in computer models, loads, structure geometry, materials properties, production processes, and operational environment should be considered. The structural reliability theory provides a reasonable approach to consider the aforementioned uncertainties in the analysis and design of structures and introduce the safety and performance requirements quantitatively into their design.

Since the 1930s, researchers have studied the probabilistic methods of structural safety analysis. The first formulations of structural safety problems can be ascribed to Wierzbicki [6] and Streletzki [7]. These researchers realized the stochastic nature of load and resistance parameters. However, practical application of the structural reliability theory was not possible before the pioneering studies by Cornell [8] and Hasofer and Lind [9] in the late 1970s and early 1980s as well as the researches by Rackwitz and Fiessler [10] in the late 1980s. These researchers showed how to assess the reliability of individual structural members in a logical process. However, subsequent studies revealed that to provide an acceptable reliability analysis of structures, it is necessary to employ methods assessing structural reliability at the structural system level rather than on the basis of reliability analysis of individual members [11]. It is because these methods allow the inclusion of the interaction between structural members into the analysis and design and also make it possible to design structural members regarding their status in the structural system. In general, failure of the first member of a statically indeterminate truss structure does not necessarily lead to the failure of its structural system. Methods for reliability analysis of these structures can be classified into three main categories of 1) numerical integration methods, 2) simulation techniques, and 3) failure-path-based methods [12]. In the numerical integration method, to assess the structure's failure probability, it is usually necessary to calculate complicated multidimensional integrals and sometimes insolvable integrals. Hence, application of numerical integration method is limited to two-dimensional spaces and simple structures. Thus, a concept known as the "reliability index" is used to quantify the structural reliability [13].

The main concept of simulation techniques is to simulate a probabilistic phenomenon numerically and then observe the frequency of a certain event in that phenomenon [13].

Some of the most important simulation techniques include the following: 1) Monte Carlo simulation technique, 2) Rosenblueth's point-estimate method, and 3) Latin hypercube sampling method. These techniques can be used easily, but in the case of small failure probabilities, which normally occur in real structures, the number of required simulations is so high that makes these techniques practically useless.

Usually one of the failure-path-based methods is used to analyze the reliability of structures at the structural system level. In such methods, the structure's reliability analysis is done based on its dominant failure modes. The dominant failure modes of a structure are failure modes that are probabilistically dominant and probabilities of their occurrence are larger than its other failure modes. In these methods, it is basically assumed that it is possible to estimate the failure probability of structures satisfactorily using their dominant failure modes. The most important failure-path-based methods are as follows: 1) Branch-and-bound method, 2) β -unzipping method, and 3) Incremental loading method [14]. In these methods, the structure's reliability analysis is usually done in two general steps: 1) Identification of the structure's dominant failure modes, and 2) Calculation of failure probabilities of the identified dominant failure modes and estimation of the failure probability of the total structural system [12].

In most studies using structural reliability theory to optimize truss structures, optimization is carried out based on constraints on the reliability of individual structural members rather than the structural system reliability constraint. Moreover, researchers who have optimized truss structures with structural system reliability constraint, have often avoided actual system reliability analysis and have instead employed simpler methods and assumptions to estimate the failure probabilities.

Some of the most important studies are further reviewed. Murotsu et al. [15] optimized size and topology of truss structures using constraints on the failure probabilities of individual members. Stocki et al. [16] imposed restrictions on the values of component reliability indices corresponding to the allowable displacements of some specified nodal points, allowable stress or local buckling of the structural members as well as a global loss of stability to optimize size and geometry of truss structures. Kaveh and Kalatjari [17-19] employed the force method and genetic algorithm to optimize truss structures. Park et al. [12] proposed a new method for reliability assessment of structural systems. Togan and Daloglu [3] considered the failure probability of a truss structure to be equal to the sum of failure probabilities of its members and optimized cross-sectional areas of truss members. Kalatjari et al. [20] used the algebraic force method and artificial intelligence to assess the reliability of statically indeterminate trusses. Kalatjari and Mansoorian [21] used the branch-and-bound and the competitive distributed genetic algorithm methods to optimize size of truss structures. Kaveh et al. [22] proposed some strategies to improve the accuracy of reliability analysis of truss structures and to increase the speed of optimization of truss structures against system reliability constraint. Kim et al. [23] used the selective search method to identify the dominant failure modes of structural systems. Kaveh et al. [24] utilized charged system search (CSS) algorithm as an optimization tool to achieve minimum reliability index under limit state function. Kaveh and Ilchi Ghazaan [25] utilized some recent metaheuristic algorithms for structural reliability assessment.

Despite many attempts to optimize truss structures subject to structural reliability constraints, no study has been focused on simultaneous size and geometry optimization of

truss structures subject to system reliability constraint. The present study is an attempt to improve the results of system reliability analysis of truss structures and to develop a logical framework for simultaneous size and geometry optimization of truss structures subject to system reliability constraint. The goal of optimization is to minimize the truss structure's total mass against the aforementioned constraint. Optimization is done through genetic algorithm. Reliability analysis of truss structures is performed through a modified branch-and-bound method. Through the modified branch-and-bound method, system reliability analysis of truss structures will be done more accurately in a shorter period of time. Visual Basic 6.0 programming software is employed to write the codes.

2. MATERIALS AND METHODS

2.1 Failure of truss structures

There are two common definitions of failure of truss structures. In the first definition, the truss is considered to be failed once one of its members exceeds its critical capacity. In this case, probability of failure of the truss structure is equal to the failure probability of the member with the largest failure probability among all of the truss members. In the second definition, the measure for the failure of a truss structure is formation of a collapse mechanism in its structural system. According to this definition, failure of the first member of a statically indeterminate truss does not necessarily lead to the failure of the total structural system. Rather, after the failure of a structural member, the internal forces are redistributed among the remaining members and the next member exposed to failure is detected. For stress analysis, after the failure of each member of truss, a force equal to the residual resistance of the failed member is applied to the truss structure along the failed member axis and the failed member stiffness matrix is set to zero. The residual resistance of the failed member is determined based on the failure type (tensile failure, compressive failure, or buckling-induced failure) and the type of the material to be used. The process of discarding members and redistributing the internal forces continues until a collapse mechanism forms and the truss structural system fails completely [11]. In a truss structure, the requisite for the emergence of a collapse mechanism is that the determinant of the truss structure stiffness matrix which consists of the remaining members would be equal to zero. For example, assume that a specific number of p_q members (e.g. members r_1 to r_{p_q}) are omitted from an n -member statically indeterminate truss structure. In this case, singularity of truss structure stiffness matrix consisting of $(n-p_q)$ remaining members is the requisite for the formation of a collapse mechanism. The truss structure stiffness matrix consisting of $(n-p_q)$ remaining members is denoted by $\bar{\bar{K}}^{(p_q)}$. Therefore, the collapse mechanism forms when:

$$|\bar{\bar{K}}^{(p_q)}| = 0 \quad (1)$$

In the above equation, $|\bar{\bar{K}}|$ represents the determinant of the stiffness matrix $\bar{\bar{K}}$.

2.2 Reliability analysis of truss structures

The sequence of some members of a truss in which the consecutive failure and omission of members lead to the formation of a collapse mechanism is known as a complete failure path (failure mode). Statically indeterminate trusses usually have a large number of failure modes. Thus identification of all of them is not practically possible. Some of the failure modes have high failure probabilities, whereas others have relatively low failure probabilities. Reliability of a truss structure should be assessed so that its dominant failure modes could be involved in the assessment. Each of the failure modes of a truss structure can be modeled based on a parallel system. Failure of a truss structure occurs in its most probable failure mode. If any of the failure modes of a structure occurs, the structure fails completely. Hence, the relationship between failure modes of a structure can be modeled based on a series system. As a result, the failure event of a truss structure is equivalent to the union of the events corresponding to its failure modes. The probability of failure of series systems is generally estimated through the bounding methods. Some of the bounding methods specifically developed for series systems with correlated failure modes include the following: 1) Cornell's bounds, 2) Ditlevsen's bounds, and 3) Vanmarcke's upper bound. To identify the failure modes of a truss structure, first it is necessary to generate its failure paths. In the next section, generation of failure paths of truss structures will be explained.

2.3 Limit state functions of truss structures

Consider a three-dimensional n -member statically indeterminate truss structure. The data on configuration of the structure and mechanical properties of the material to be used is also available. The i -th member of this truss fails when its internal force exceeds its resistance. Therefore, the safety margin of i -th member (M_i) is:

$$M_i = R_i(C_{yi}, A_i) - S_i(A_1, \dots, A_n; L_1, \dots, L_{3u}; l_1, \dots, l_n; E_1, \dots, E_n) \quad (2)$$

In the above equation, n and u denote the number of truss members and the number of nodal points, respectively. L_j is the external load exerted on the truss at j -th degree of freedom ($j = 1, 2, \dots, 3u$). R_i , A_i , E_i , l_i , C_{yi} , and S_i stand for the resistance, cross-sectional area, elasticity modulus, length, stress corresponding to load-bearing capacity, and internal force of i -th member, respectively.

If the internal force and resistance of i -th member (S_i , R_i) are statistically uncorrelated random variables with normal probability distributions, it can be proved that the following relationship exists between the reliability index of i -th member (β_i) and its probability of failure ($(P_f)_i$):

$$(P_f)_i = \Phi(-\beta_i) \quad (3)$$

In Equation (3), $\Phi(X)$ denotes the univariate standard normal cumulative distribution function X .

The reliability index of i -th member (β_i) is:

$$\beta_i = \frac{\mu_{R_i} - \mu_{S_i}}{\sqrt{\sigma_{R_i}^2 + \sigma_{S_i}^2}} \quad (4)$$

where, μ_{R_i} and μ_{S_i} show the mean values of resistance and internal force of i -th member, respectively. In addition, σ_{R_i} and σ_{S_i} refer to the standard deviations of resistance and internal force of i -th member, respectively.

The internal force of i -th member (i.e. a structural member that remains sound after failure of r_1 to r_{p-1} members) at p -th stage of failure (after failure of r_1 to r_{p-1} members) is:

$$S_{i(r_1, r_2, \dots, r_{p-1})}^{(p)} = \sum_{j=1}^{3u} b_{ij}^{(p)} L_j^{(p)} = \sum_{j=1}^{3u} b_{ij}^{(p)} L_j - a_{ir_1}^{(p)} R_{r_1} - \dots - a_{ir_{p-1}}^{(p)} R_{r_{p-1}} \quad (5)$$

In Equation (5), p is the failure path length and $(r_1, r_2, \dots, r_{p-1})$ represents the failed members and their sequence of failure. Moreover, $a_{ij}^{(p)}$ is the internal force of i -th member at p -th stage of failure, which results from application of a unit load along j -th failed member. $b_{ij}^{(p)}$ is the internal force of i -th member at p -th stage of failure, which results from application of a unit load along j -th degree of freedom. $L_j^{(p)}$ is the external load applied to j -th degree of freedom of the structure at p -th stage of failure. $S_{i(r_1, r_2, \dots, r_{p-1})}^{(p)}$ is the internal force of i -th member at p -th stage of failure.

Hence, the safety margin of i -th member at p -th stage of failure ($M_{i(r_1, r_2, \dots, r_{p-1})}^{(p)}$) is:

$$M_{i(r_1, r_2, \dots, r_{p-1})}^{(p)} = C_{yi} A_i - \left| S_{i(r_1, r_2, \dots, r_{p-1})}^{(p)} \right| \quad (6)$$

Probability of the failure path $r_1 \rightarrow \dots \rightarrow r_{p-1} \rightarrow r_p$ is calculated through the following equation:

$$P_{fp(r_1 r_2 \dots r_{p-1} r_p)} = 1 - P\left[\bigcup_{i=1}^p M_{r_i(r_1, r_2, \dots, r_{i-1})}^{(i)} > 0\right] \quad (7)$$

In Equation (7), $P_{fp(r_1 r_2 \dots r_{p-1} r_p)}$ refers to the probability of the failure path $r_1 \rightarrow \dots \rightarrow r_{p-1} \rightarrow r_p$.

When the failure path length exceeds 3 ($p > 3$), precise calculation of the failure path probability using Equation (7) becomes complicated and time-consuming. Therefore, the upper and lower bounds of the failure path probability are usually estimated. One of the equations proposed for the upper and lower bounds of the failure path probability is [11]:

$$P_{fp(r_1 r_2 \dots r_{p-1} r_p)(L)} = \max \left[0, P[(M_{r_1}^{(1)} \leq 0)] P[(M_{r_1}^{(1)} \leq 0) \cap (M_{r_2(r_1)}^{(2)} > 0)] \right. \\ \left. - \sum_{j=3}^p \min \left\{ P_{fp(r_1 r_2 \dots r_{j-1})(U)}, P[(M_{r_1}^{(1)} \leq 0) \cap (M_{r_j(r_1, r_2, \dots, r_{j-1})}^{(j)} > 0)] \right\} \right] \quad (8)$$

$$P_{fp(r_1 r_2 \dots r_{p-1} r_p)(U)} = \min_{j \in \{2, \dots, p\}} P[(M_{r_1}^{(1)} \leq 0) \cap (M_{r_j(r_1, r_2, \dots, r_{j-1})}^{(j)} \leq 0)] \quad (9)$$

where, $P_{fp(r_1 r_2 \dots r_{p-1} r_p)(U)}$ and $P_{fp(r_1 r_2 \dots r_{p-1} r_p)(L)}$ denote the upper and lower bounds of the probability of the failure path $r_1 \rightarrow \dots \rightarrow r_{p-1} \rightarrow r_p$, respectively.

2.4 Calculation of the joint failure probability of members

If the loads applied to the truss structure and its members' resistance are uncorrelated random variables with normal probability distributions, the safety margins corresponding to j-th stage of failure (the truss lacking r_1 to r_{j-1} bars) and k-th stage of failure (the truss lacking r_1 to r_{k-1} bars) can be expressed as linear combinations of a number of uncorrelated random variables with standard normal probability distributions. For instance,

$$M_{r_k(r_1, r_2, \dots, r_{k-1})}^{(k)} = a_o + \sum_{i=1}^n a_i z_i \quad (10)$$

$$M_{r_j(r_1, r_2, \dots, r_{j-1})}^{(j)} = b_o + \sum_{i=1}^n b_i z_i \quad (11)$$

In the above equations, the z_i variables are uncorrelated random variables with standard normal probability distributions. In addition, a_i and b_i are the constants of the z_i variables in the safety margins corresponding to k-th and j-th stages of failure ($i = 1, 2, \dots, n$).

It should be noted that the mean of a standard normal random variable is equal to zero, while its standard deviation equals one. Equations (10) and (11) indicate that the safety margins corresponding to j-th and k-th stages of failure will also have standard normal probability distributions. In this case, the statistical correlation coefficient between the safety margins corresponding to j-th and k-th stages of failure (ρ_{jk}) is as follows:

$$\rho_{jk} = \sum_{i=1}^n a_i b_i \quad (12)$$

For an arbitrary pair of structural elements, the bivariate normal cumulative distribution function with zero mean values ($\Phi_2(X_1, X_2; \rho)$) is:

$$\Phi_2(X_1, X_2; \rho) = \Phi(X_1)\Phi(X_2) + \int_0^\rho \frac{\exp\left(-\frac{1}{2(1-z^2)}(X_1^2 + X_2^2 - 2zX_1X_2)\right)}{2\pi\sqrt{1-z^2}} dz \quad (13)$$

In Equation (13), X_1 and X_2 are the random variables corresponding to a desired pair of structural elements and ρ shows the statistical correlation between X_1 and X_2 random variables.

If in Equation (13), $-\beta_j$ and $-\beta_k$ are assumed to be the values of X_1 and X_2 variables and ρ_{jk} is regarded as the value of the ρ variable, then the calculated value of $\Phi_2(-\beta_j, -\beta_k; \rho_{jk})$ is equal to the joint failure probability of j -th and k -th structural members.

2.5 The branch-and-bound method

In the branch-and-bound method, matrix methods are initially used to generate failure paths of the structure. Then, the dominant failure paths of the structure and the failure paths to be discarded are identified through branching and bounding operations, respectively. The branch-and-bound method generally involves three main operations of 1) partitioning operation, 2) branching operation, and 3) bounding operation.

Partitioning operation: During the partitioning operation, a new failure stage is added to the previous failure stages of the incomplete failure path under consideration. In other words, all the structural members potentially prone to failure are added to the incomplete failure path under consideration. "Potentially prone to fail members" refer to members which can fail during the present failure stage. The failure paths generated during this stage are called "partitioned failure paths". In each step of generating failure paths, all newly partitioned failure paths are entered as input to the "set of candidate failure paths for branching operation". Equations (8) and (9) are used to calculate the upper and lower bounds of the failure paths' probability. Close examination of these equations reveals that, in these bounds, only the safety margins of the failure path at the first failure stage and the present one are used. Therefore, only the safety margin of the failure path at the first failure stage needs to be saved. Consequently, this would lead to less computer memory space occupation and reduction of program execution time. This issue is useful, especially about structures with a high degree of indeterminacy where the number of failure paths required for acceptable evaluation of the structural system reliability cannot be predetermined [11]. In addition, these equations take into account the statistical correlation between safety margins of different stages of failure in each failure mode.

Branching operation: Branching operation can be defined as the operations performed to select the members in order to achieve stochastically dominant failure paths [11]. In each step of generating failure paths, selected member must be the member with the most probable branching failure path compared to other newly partitioned failure paths. In other words, selected failure path must be the failure path which has taken the maximum value of $P_{fp(r_1 r_2 \dots r_{p-1} r_p)}(U)$ at the present step of generating failure paths. The member selection process is continued until the formation of a collapse mechanism.

Bounding operation: Bounding operation is for eliminating unnecessary failure paths from among the candidate failure paths for branching operation [11]. Unnecessary failure

paths refer to failure paths which do not play a determinative role in evaluating the probability of failure of the structural system. Suppose that a new member is added to one of the incomplete failure paths. The upper bound of the failure path probability is a non-increasing function of failure path length (p). Therefore, the probability of consecutive failure of 1 to p members of a failure path is smaller than or equal to the probability of consecutive failure of 1 to $p - 1$ members of the same failure path. Therefore, there is no need to continue the branching operation on the failure paths with smaller upper bounds of probabilities in comparison to a certain value. It is because even in this stage of the branching operation, it is possible to recognize that these failure paths are not the probable and dominant failure paths. Hence, these failure paths are ignored and removed from the set of candidate failure paths for branching operation. The bounding operation should be repeated after identifying each new failure mode. After identifying each new failure mode, to find the unnecessary failure paths, the upper bounds of probabilities of all of the failure paths in the set of candidate failure paths for branching operation are compared to the value of the "bounding reference" variable. All of the failure paths with smaller upper bounds of probabilities in comparison to the current value of the bounding reference are omitted from the branching operation. After identification of n -th failure mode of the truss, the bounding reference is a multiple of the largest lower bound of the probability of failure of these n failure modes. The bounding reference (Br) is calculated through the following equation:

$$Br = Re \times 10^{-\delta} \quad (14)$$

where, δ is the branch-and-bound constant, Re is the branch-and-bound variable.

After identification of n -th failure mode of the truss, Re variable's value is equal to the largest lower bound of failure probability of these n failure modes. δ is a constant determined in relation to the precision desired by the user and type of the given structure. As the value of δ constant gets larger, the number of discarded failure paths declines and, consequently, the precision of computations escalates. On the other hand, as the value of δ constant gets larger, the time allocated to the structural reliability analysis expands. It is, therefore, concluded that the upper bounds of probabilities of all of the failure paths omitted from the branching operation are smaller than the final value of Br variable [11].

The branch-and-bound operation continues until the set of candidate failure paths for branching operation becomes null. Reaching this status indicates that the branch-and-bound operation is over. Following the branch-and-bound operation and identification of the dominant failure modes of the structure under consideration, a specific bounding method is applied to estimate the structure's failure probability. In this research, Cornell's method is used to estimate the structure's failure probability. Suppose that a specific number of n_k failure modes are identified as the dominant failure modes of the structure. Based on the Cornell's method, the upper and lower bounds of the structure's failure probability are respectively [26]:

$$P_{up-s} = 1 - \prod_{i=1}^{n_k} \left[1 - P_{fp(r_1 r_2 \dots r_{p-1} r_p)(U)(i)} \right] \quad (15)$$

$$P_{low-s} = \max_{i \in \{1, \dots, n_k\}} \left[P_{fp(r_1 r_2 \dots r_{p-1} r_p)(U)(i)} \right] \quad (16)$$

In the above equations, P_{up-s} and P_{low-s} are the upper and lower bounds of the structure's failure probability, respectively. $P_{fp(r_1 r_2 \dots r_{p-1} r_p)(U)(i)}$ and $P_{fp(r_1 r_2 \dots r_{p-1} r_p)(L)(i)}$ are the upper and lower bounds of failure probability of i -th identified dominant failure mode of the structure.

2.6 Structural system reliability-based optimization of truss structures

In this research, optimization of a truss structures is conducted by minimizing its total mass subject to system reliability constraint. To formulate the problem of minimizing mass of a truss structure subject to system reliability constraint we have:

Minimize:

$$W(X_1, X_2, \dots, X_{NDV}) = \sum_{i=1}^{NE} A_i l_i \rho_i \quad (17)$$

So that:

- 1) The truss structure would be geometrically stable;
- 2) The following conditions would not be violated.

$$\beta_S(X_1, X_2, \dots, X_{NDV}) \geq \beta_{min}^{System} \quad (18)$$

$$X_i^L \leq X_i \leq X_i^U \quad (i = 1, 2, \dots, NDV) \quad (19)$$

In the above equations, W is the total mass of the truss structure, NDV shows the number of design variables, X_i is i -th design variable, NE presents the number of truss members, β_S is the system reliability index of the truss, β_{min}^{System} refers to the minimum allowable value of the system reliability index of the truss, and P_{fpmax}^{System} is the maximum allowable value of the structure's failure probability. Moreover, X_i^L and X_i^U are the lower and upper bounds of i -th design variable, respectively. ρ_i is the density of i -th member of truss.

2.7 Genetic algorithm

In this study, optimization is conducted using the genetic algorithm as one of the most important subcategories of evolutionary algorithms. To optimize through genetic algorithm, first a specific number of random designs (solutions) are generated, then the relative fitness of all of them are assessed against predetermined criteria. The solutions are encoded as bit strings composed of 0's and 1's (binary encoding). The resulting strings are called chromosomes. Next, chromosomes of higher relative fitness are selected and are put in a set called the "mating pool". Afterwards, by imposing a number of operations such as crossover, mutation, etc. on the selected chromosomes, the next generation is created. It is tried to generate the chromosomes of new generation so that their mean fitness would increase compared to the mean fitness values of previous generations. This process is

repeated until the optimal solution of the problem is obtained based on predetermined convergence criterion (or criteria). In order to select the fit chromosomes, a combination of the tournament selection method [27] and the method presented in reference [28] is used. In this method, the mating pool contains a collection of chromosomes including some of the fittest chromosomes of the current generation and some other manipulated chromosomes. Manipulated chromosomes are those created by making slight changes to the two bits on the right side of the substrings of the best chromosome of the current generation. Changes in the substrings of this chromosome are made separately. The elitism strategy is also used, which makes the genetic algorithm introduce some of the fittest chromosomes of the current generation directly to the next generation [29]. Experience has shown that the elitism strategy improves the performance of GA considerably [30]. Through this strategy, it is possible to ensure that the fittest chromosome of each generation is not less fit than the fittest chromosome of former generations. The elitism strategy is always employed along with other selection methods. For the crossover operations, the two-point crossover method is used. Moreover, for the mutation operation, a decreasing mutation rate is used. The highest value of the decreasing mutation rate belongs to the first generation of GA. With an increase in the number of generations, the decreasing mutation rate declines linearly. As a result of the decreasing mutation rate, mutation occurs with a higher probability in the first generations and, consequently, the search space expands. In the next generations, the mutation probability declines and the search focuses on more fit solutions.

2.8 Features of the code written for optimization of truss structures

The code written for optimization of truss structures is called RBO-S>S, which stands for "Reliability-Based Optimization of Size and Geometry of Truss Structures." RBO-S>S program can be divided into three main subprograms of 1) analysis subprogram, 2) genetic algorithm subprogram, and 3) reliability analysis subprogram. As follows, solutions are proposed to reduce the execution time of the optimization process and to increase the precision of reliability analysis results.

2.8.1 The genetic algorithm subprogram solution

In the genetic algorithm subprogram, a simple solution is proposed to reduce the execution time of the subprogram. In this solution, if a duplicate chromosome is produced during the optimization operation, the program does not reassess the design corresponding to the duplicate chromosome. Instead, it directly uses the results saved for the duplicate chromosome. To apply the solution, non-duplicate chromosomes of all generations are stored in a set namely "set of non-duplicate chromosomes" ($\{Set_{Ch}\}$). A non-duplicate chromosome is one that is not generated during previous generations. For each chromosome, the values of penalty function and objective function are stored. From the second generation onward, if a chromosome similar to one of the chromosomes of the $\{Set_{Ch}\}$ set is generated, it is not sent to the assessment phase. Rather, the results saved for the duplicate chromosome are used. Using the string comparison functions of Visual Basic 6.0 software, it is possible to compare the chromosome strings and identify the duplicates.

2.8.2 Reliability analysis subprogram solution (modified branch-and-bound method)

The truss structures reliability analysis subprogram is written based on a modified branch-and-bound method. To increase the accuracy of reliability analysis of truss structures and to reduce the time needed for reliability analysis, the modified branch-and-bound method is proposed as follows:

Consider that at p -th stage of formation of failure paths ($p \geq 2$) a total number of m new failure paths are branched out of a specific failure path such as $r_1 \rightarrow r_2 \rightarrow \dots \rightarrow r_{p-1}$ (Fig. 1). The failure path branched out into the m new failure paths is called “main-branch failure path”, whereas the branched-out failure paths are called “subbranch failure paths”. The lengths of main-branch failure path and subbranch failure paths are $p-1$ and p , respectively.

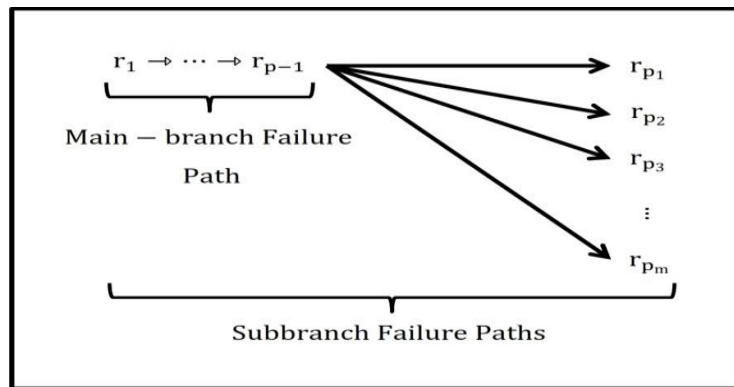


Figure 1. Scheme of the main-branch failure path and its subbranch failure paths

As mentioned earlier, the bound presented in Equation (9) is a non-ascending function of the failure path length. Therefore, each subbranch failure path is a subset of the main-branch failure path. In other words,

$$P_{fp(r_1 \dots r_{p-1} r_{p_i})}(U) \leq P_{fp(r_1 \dots r_{p-1})}(U) \quad (i = 1, 2, \dots, m) \quad (20)$$

Consider an arbitrary probabilistic phenomenon. Assume that a specific number of distinctive events of the probabilistic phenomenon under consideration (e.g. k events) are subsets of another event called common event. In this state, it could be concluded that the union of these k events is definitely a subset of the common event. Similarly, it could be concluded that the union of the failure paths branched out of the main-branch failure path (i.e. the union of subbranch failure paths) is a subset of the main-branch failure path. In other words, the probability of union of subbranch failure paths should not ever exceed the probability of their main-branch failure path. That is to say,

$$P_{fp} \left[\bigcup_{i=1}^m M_{r_{p_i}(r_1 r_2 \dots r_{p-1})}^{(p_i)} \leq 0 \right] \leq P_{fp(r_1 r_2 \dots r_{p-1})}(U) \quad (21)$$

If Cornell's method is used, the probability of union of subbranch failure paths is equal to

the sum of probabilities of subbranch failure paths. In other words, in Cornell's bounds, the real statistical correlation between subbranch failure paths is not considered. Rather, the lower and upper Cornell's bounds correspond to the complete statistical independence of subbranch failure paths and complete statistical correlation of subbranch failure paths, respectively. On the other hand, subbranch failure paths generally have a high level of statistical correlation. Therefore, it cannot be said with certainty that the sum of probabilities of subbranch failure paths is smaller than the main-branch failure path probability. As a result, during the implementation of the truss structures reliability analysis subprogram (especially for truss structures with high degrees of static indeterminacy), the sum of upper bounds of probabilities of subbranch failure paths may exceed the upper bound of the main-branch failure path probability. If no solution is developed for these cases and the program continues in the same way as before, the estimate of the upper bound of failure probability of the total structure will be wrong and conservative.

If the aforementioned state occur (i.e. where the sum of upper bounds of probabilities of subbranch failure paths exceeds the upper bound of the main-branch failure path probability), the modified branch-and-bound method causes RBO-S>S program to omit all subbranch failure paths from the set of candidate failure paths for branching operation and consider the main-branch failure path as one of the failure modes of the structure. In our reliability analysis, the failure probability of this failure mode is considered equal to the sum of probabilities of the discarded subbranch failure paths. As follows, RBO-S>S program goes to the next main-branch failure path and the branch-and-bound operation continues.

3. RESULTS AND DISCUSSION

3.1 Verification of the modified branch-and-bound method

To investigate the efficiency and accuracy of the modified branch-and-bound method, results of reliability analysis of a statically indeterminate truss structure are examined and compared with the findings of other references. In addition, the data resulted from modification of the branch-and-bound method is examined and compared with non-modified condition. Consider the truss shown in Fig. 2 ($L_1 = 121.9$ cm, $L_2 = 91.44$ cm). Data relevant to dimensions of members are summarized in Table 1. The loads applied to the truss and the yield stresses of its members are assumed to be uncorrelated random variables with normal probability distributions (with mean values equal to 44.45 kN and 276 MPa, respectively). Elasticity modulus of members is 206 GPa. Members' behavior under tension and compression are assumed to be identical. Failure due to buckling of members is considered. Buckling stress is assumed to be a normal random variable with the following probability distribution parameters [31]:

$$\mu_{c_c} = \frac{1}{2} (C_y + C_E + \frac{W_0}{R_g} C_E) \left\{ 1 - \sqrt{1 - (4C_y C_E) / (C_y + C_E + \frac{W_0}{R_g} C_E)^2} \right\} \quad (22)$$

$$CV_{C_c} = \frac{\sqrt{(CV_{C_y} C_y)^2 + (C_E/R_g)^2 (CV_{W_0} W_0)^2}}{(C_y + C_E + \frac{W_0}{R_g} C_E) \left\{ 1 - \sqrt{1 - (4C_y C_E)/(C_y + C_E + \frac{W_0}{R_g} C_E)^2} \right\}} \quad (23)$$

where, $C_E = \frac{\pi^2 E}{(l/R_g)^2}$ is the Euler buckling stress. Moreover, W_0 , μ_{C_c} , C_y , R_g , E , and l stand for the initial deflection, mean value of buckling stress, yield stress, radius of gyration, elasticity modulus, and length of member, respectively. CV_{C_c} , CV_{C_y} , and CV_{W_0} denote the coefficients of variation of buckling stress, yield stress, and initial deflection, respectively ($\frac{W_0}{R_g} = CV_{W_0} = 0.1$).

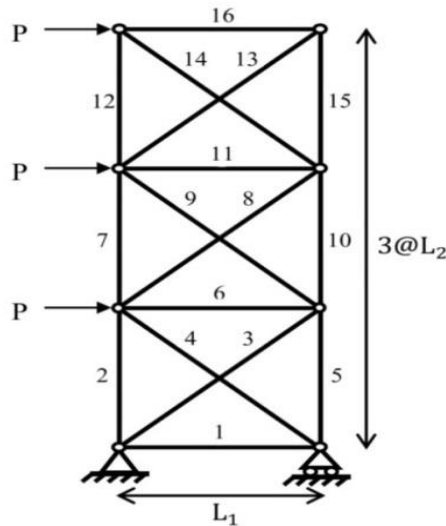


Figure 2. Statically indeterminate 16-bar truss

Table 1: Data of dimensions of members of the 16-bar truss

Member	Cross-sectional area (cm ²)	Radius of gyration (cm)
1	3.35	2.43
2, 5	8.64	4.45
3, 4, 14	5.76	3.03
6	2.29	1.70
7, 8, 10	4.03	2.43
9	7.35	3.82
11, 12, 15	1.58	1.36
13, 16	2.29	2.14

Reliability analysis results of the 16-bar truss are summarized in Table 2. Reliability analysis is carried out for two different conditions, A and B. Conditions A and B are related to the states in which the branch-and-bound method is modified and not modified, respectively. The data shown in Table 2 are obtained through RBO-S>S program. Table

3 shows the results reported by Murotsu et al. [32]. Comparing the results of conditions A and B shown in Table 2 reveals that the modified branch-and-bound method significantly decreases the time allocated to the reliability analyses. In addition, the modified branch-and-bound method reduces the number of failure modes identified as dominant failure modes of the truss structure in all cases. A closer examination of table 2 shows that modification of the branch-and-bound method reduces the upper bound of failure probability of the structure in all cases. Comparing the results presented in Tables 2 and 3 reveals that, almost in all cases, the modified branch-and-bound method makes the upper bound of failure probability of the structure closer to the results reported by Murotsu et al. [32]. These comparisons confirm the accuracy and precision of RBO-S>S program and efficacy of the modified branch-and-bound method. It can be said that our reliability analysis results are more accurate than the results reported by Murotsu et al. [32]. Because our modified branch-and-bound method incorporates statistical correlation of subbranch failure paths more accurately in reliability analysis calculations.

Table 2: Results of the reliability analysis of the 16-bar truss ($\delta = 3$)

Coefficient of variation of loads		0.1	0.1	0.1	0.2	0.2
Coefficient of variation of yield stress of members		0.02	0.05	0.1	0.05	0.1
When the truss is composed of ductile members.						
P_{up-s}^*	Condition A	4.17×10^{-8}	1.22×10^{-7}	8.65×10^{-4}	4.69×10^{-3}	1.03×10^{-2}
	Condition B	4.19×10^{-8}	1.71×10^{-7}	2.72×10^{-3}	6.50×10^{-3}	4.02×10^{-2}
N^{**}	Condition A	2	7	12	10	20
	Condition B	3	27	323	77	423
Time (sec)	Condition A	0.0313	0.0469	0.0625	0.0625	0.125
	Condition B	0.0313	0.0781	0.6094	0.1875	0.8398
When the truss is composed of brittle members.						
P_{up-s}	Condition A	8.65×10^{-8}	2.17×10^{-5}	4.99×10^{-3}	8.29×10^{-3}	2.62×10^{-2}
	Condition B	4.44×10^{-6}	1.32×10^{-3}	2.37×10^{-1}	3.79×10^{-1}	7.91×10^{-1}
N	Condition A	2	2	5	5	6
	Condition B	175	179	305	322	486
Time (sec)	Condition A	0.0352	0.0313	0.0469	0.0313	0.0469
	Condition B	0.3086	0.2773	0.4609	0.5195	0.75

* The upper bound of the structure's failure probability

** Number of dominant failure modes

Table 3: Results of the reliability analysis of the 16-bar truss ($\delta = 3$) (Murotsu et al. [32])

Coefficient of variation of loads		0.1	0.1	0.1	0.2	0.2
Coefficient of variation of yield stress of members		0.02	0.05	0.1	0.05	0.1
When the truss is composed of ductile members.						
P_{up-s}		6.47×10^{-8}	3.02×10^{-7}	8.88×10^{-4}	4.59×10^{-3}	6.87×10^{-3}
N		11	6	58	11	155
When the truss is composed of brittle members.						
P_{up-s}		6.51×10^{-8}	3.87×10^{-6}	3.90×10^{-3}	6.34×10^{-3}	2.18×10^{-2}
N		2	3	12	5	18

3.2 Numerical optimization examples

To describe our proposed reliability-based optimization approach, we use RBO-S>S program to optimize two truss structures. Design variables are discrete and structures are composed of ductile members. The loads applied to structures and the yield stresses of their members are assumed to be uncorrelated random variables with normal probability distributions. The members' behavior under compression and tension are assumed to be the same. In both examples, failure due to buckling of members is not included. The upper bound of failure probability of structural system is considered as problem constraint and the total mass of truss structure as objective function. In each example, convergence history of the structure's total mass is shown.

3.2.1 Example 1: A 25-bar statically indeterminate truss

In the first example, size optimization of a statically indeterminate 25-bar truss shown in Fig. 3 is performed. Cross-sectional areas of members are optimized to achieve minimum mass truss structure subject to system reliability constraint. The modulus of elasticity is 210 GPa, and the material density is 2700 kg/m³. The mean and coefficient of variation of the yield stress of members are 27.6 kN/cm² and 0.05, respectively. Coordinates of the truss nodes, loading condition of the truss, and parameters of probability distribution of the loads applied to the truss are summarized in Tables 4, 5 and 6, respectively. The total number of size design variables is 13. The set of available cross-sections for size design variables ($\{S\}$) is: $S = \{(1 + 0.08 i); i = 0, 1, \dots, 127\} \text{ cm}^2$

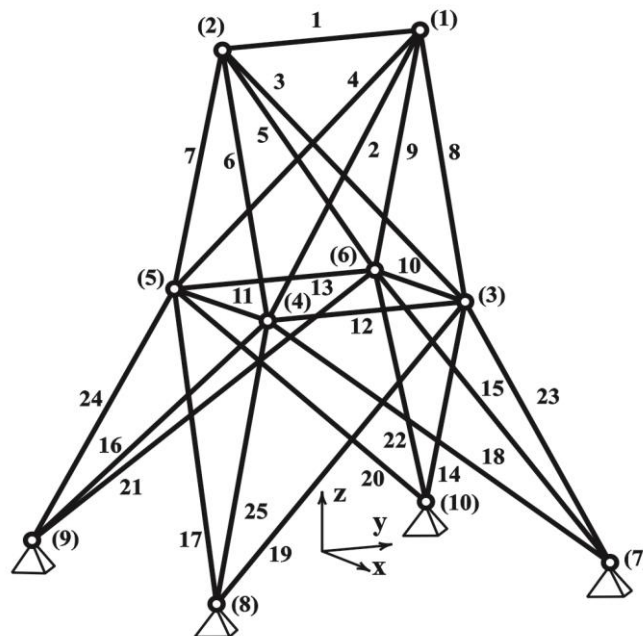


Figure 3. Statically indeterminate 25-bar truss

Table 4: Coordinates of the truss nodes (25-bar truss)

Number of node	x (cm)	y (cm)	z (cm)
1	0	95.25	508
2	0	-95.25	508
3	95.25	95.25	254
4	95.25	-95.25	254
5	-95.25	-95.25	254
6	-95.25	95.25	254
7	254	-254	0
8	-254	-254	0
9	254	254	0
10	-254	254	0

Table 5: Data of the loading condition (25-bar truss)

Number of node	X	Y	Z
1	L_1	L_1	$-L_2$
2	$-L_1$	$-L_1$	$-L_2$
3	L_1	L_1	0
5	$-L_1$	$-L_1$	0

Table 6: Parameters of the probability distributions of loads (25-bar truss)

Load	Mean value (kN)	Coefficient of variation
L_1	88.9	0.2
L_2	22.6	0.2

Table 7 shows the optimization results of 25-bar truss along with the results achieved by other researchers. Although Kalatjari and Mansoorian [21] also optimized the truss structure shown in Fig. 3 against system reliability constraint, but they used a loading condition different from our study. Therefore, we can not compare our optimization results with theirs. According to Table 7, RBO-S>S program (present research) offered better results compared to other researches. The convergence history of optimization of 25-bar truss is shown in Fig. 4. A brief examination of Fig. 4 reveals that the convergence history has a non-ascending course. The non-ascending course of convergence history could be attributed to the use of the elitism strategy in genetic algorithm.

Table 8: Comparison of the optimization results of the 25-bar truss

Design variable	Present research	Thoft-Christensen and Murotsu [11] (Identical failure probability for members)	Togan and Daloglu [33] (Continuous design variables)	Togan and Daloglu [33] (Discrete design variables)
A_1 (cm ²)	4.68	4.36	4.387	6.90
$A_2 = A_5$ (cm ²)	5	4.56	4.588	5.15
$A_3 = A_4$ (cm ²)	7	7.47	7.450	6.90
$A_6 = A_9$ (cm ²)	4.6	2.39	4.376	5.68

$A_7 = A_8$ (cm ²)	7.24	7.52	7.496	9.54
$A_{10} = A_{11}$ (cm ²)	1.72	1.51	2.204	6.90
$A_{12} = A_{13}$ (cm ²)	1.88	1.77	1.778	3.18
$A_{14} = A_{17}$ (cm ²)	4.6	4.88	4.600	3.18
$A_{15} = A_{16}$ (cm ²)	2.12	1.89	2.179	4.31
$A_{18} = A_{21}$ (cm ²)	1.72	1.78	1.810	2.79
$A_{19} = A_{20}$ (cm ²)	2.84	2.63	2.595	2.06
$A_{22} = A_{25}$ (cm ²)	4.76	4.89	4.933	6.90
$A_{23} = A_{24}$ (cm ²)	7.72	7.66	7.483	9.54
Total mass (kg)	95.806	97.8	95.81	118.7
P_{up-s}	9.93×10^{-6}	10^{-5}	10^{-5}	10^{-5}
P_{fpmax}^{System}	10^{-5}	10^{-5}	10^{-5}	10^{-5}

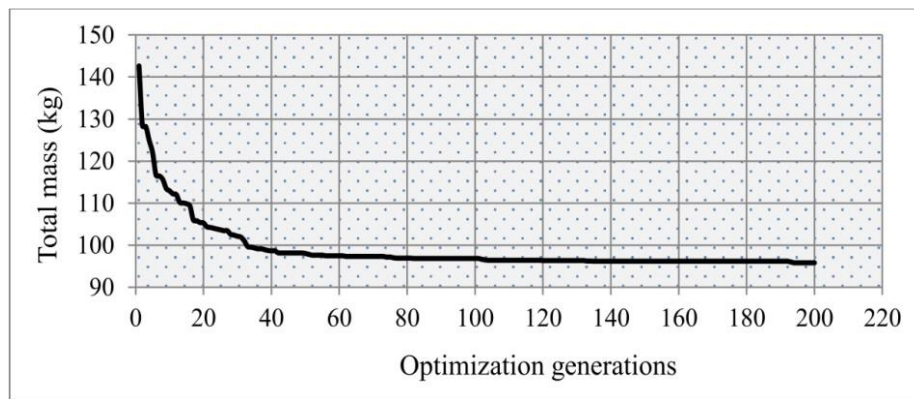


Figure 4. Convergence history of the 25-bar truss

3.2.2 Example 2: A 18-bar statically determinate truss

In the second example, simultaneous size and geometry optimization of a statically determinate 18-bar truss shown in Fig. 5 is carried out. The material density is 2768 kg/m^3 , and the modulus of elasticity is 68.9 GPa . The mean and coefficient of variation of the yield stress of structural members are equal to 13.789 kN/cm^2 and 0.05 . Also, the mean and coefficient of variation of loads applied to the truss are 88.9 kN and 0.1 , respectively. The set of available cross-sections for size design variables ($\{S\}$) is: $S = \{(2 + 0.32 i) \times 2.54^2; i = 0, 1, \dots, 127\} \text{ cm}^2$

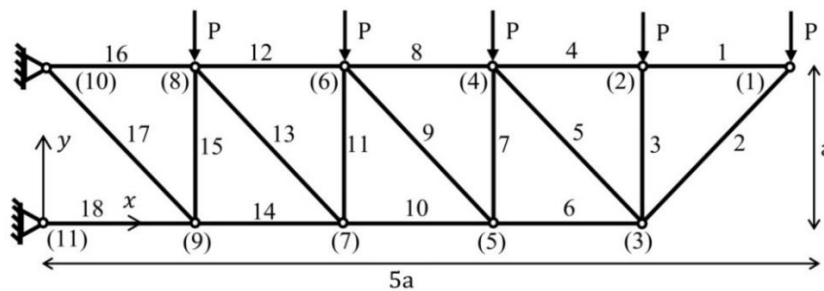


Figure 5. Statically determinate 18-bar truss

The upper and lower bounds of geometry design variables are presented in Table 8. The optimization results of 18-bar truss structure are presented in Table 9. Moreover, Fig. 6 shows the convergence history of 18-bar truss. According to Fig. 6, RBO-S>S program reduces the structure's total mass from 5250 kg to 2300.4 kg in a non-ascending course through two hundred generations. The convergence history shows that the convergence rate of genetic algorithm is very high in the first optimization generations (up to the 20th generation); consequently, the structure's total mass decreases rapidly in the above-mentioned generations. From the 100th generation onward, no change occurs in the optimal design of the structure. Fig. 7 shows the geometry of the structure's final design.

Table 8: Upper and lower bounds of geometry design variables (18-bar truss)

Design variable	Lower bound (cm)	Upper bound (cm)
x_3	1968.5	3110.9
x_5	1333.5	2475.9
x_7	698.5	1840.9
x_9	63.5	1205.9
y_3, y_5, y_7, y_9	-571.5	621.9

Table 9: Optimization results of the 18-bar truss

Geometry design variable	(cm)	Size design variable	(cm ²)
x_3	2258.68	$A_1 = A_4 = A_8 = A_{12} = A_{16}$	91.35
x_5	1619.88	$A_2 = A_6 = A_{10} = A_{14} = A_{18}$	122.32
x_7	1019.93		
x_9	644.75		
y_3	472.14	$A_3 = A_7 = A_{11} = A_{15}$	21.16
y_5	355.14	$A_5 = A_9 = A_{13} = A_{17}$	45.94
y_7	181.98		
y_9	60.3		
Total mass (kg)		2300.4	
P_{up-s}		9.99×10^{-6}	
P_{System}		10^{-5}	
f_{pmax}			

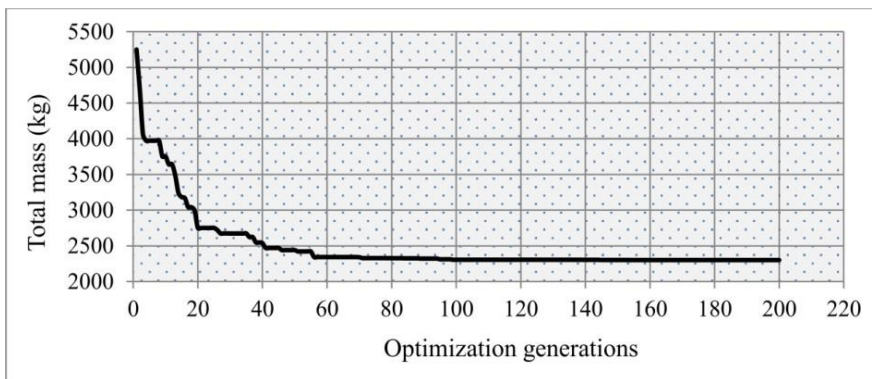


Figure 6. Convergence history of the 18-bar truss

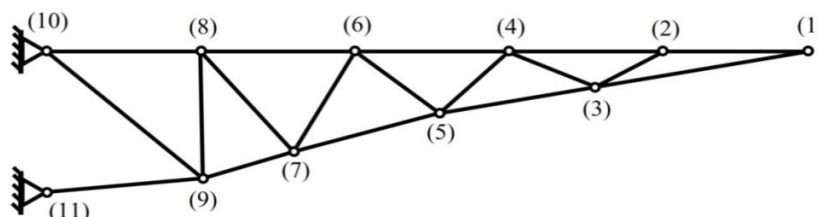


Figure 7. Geometry of the optimum design of the 18-bar truss

4. CONCLUSION

In this study, a computer program, RBO-S>S, is developed for reliability-based optimization of size and geometry of truss structures. A logical framework is introduced to solve simultaneous size and geometry optimization problems of truss structures subjected to system reliability constraint. Results of the optimized examples indicate that efficiency of the developed program. In addition, a modified branch-and-bound method is proposed to perform the structural system reliability analysis of the truss structures with sufficient accuracy. Reliability analysis results indicate that the modified branch-and-bound method makes it possible to obtain a satisfactory system reliability analysis of the truss structures. The modified branch-and-bound method has the following advantages:

- It reduces the time required for reliability analysis of truss structures significantly, thus makes it suitable for optimization.
- It decreases the number of failure modes identified as dominant failure modes of truss structures.
- It makes the upper bound of failure probability of truss structures closer to the exact values, so increase the accuracy of reliability analysis estimations and prevents from overestimating failure probability of truss structures.

Although it is tried to perform structural system reliability analysis of truss structures as accurate as possible, occasionally its time-consuming complicated stochastic nature forced us to use simplifying approximate methods. Therefore, two suggestions can be suggested for further research: (a) More failure stages should be used to estimate the failure probabilities of failure modes, because using only two failure stages may in some cases lead to very conservative estimate of failure probabilities. (b) To employ methods taking into account the real statistical correlation between failure modes, because correlation between failure modes is neglected in Cornell's method.

REFERENCES

1. Christensen PW, Klabring A. *An Introduction to Structural Optimization*, Springer, 1st edition, Netherlands, 2009.
2. Wu SJ, Chow PT. Steady-state genetic algorithms for discrete optimization of trusses, *Comput Struct* 1995; **56**(6): 979-991.
3. Togan V, Daloglu AT. Optimization of 3d trusses with adaptive approach in genetic algorithm, *Eng Struct* 2006; **28**(7): 1019-1027.

4. Dede T, Bekiroglu S, Ayvaz Y. Weight minimization of trusses with genetic algorithm, *Appl Soft Comput* 2011; **11**(2): 2565-2575.
5. Cazacu R, Grama L. Steel truss optimization using genetic algorithms and FEA, *The 7th International Conference Interdisciplinarity in Engineering*, Petru Maior University of Tirgu Mures, Romania, 2013, pp. 339-346.
6. Wierzbicki W. Safety of Structures as a Probabilistic Problem, Technical Report, Warsaw, Poland, 1936.
7. Streletzki NS. *Statistical Basis for the Evaluation of the Structural Safety Factor*, Stroizdat: State Publishing House for Buildings, Moscow, 1947.
8. Cornell CA. A probability-based structural code, *ACI J* 1969; **66**: 974-985.
9. Hasofer AM, Lind NC. An exact and invariant first-order reliability format, *J Eng Mech Div, ASCE* 1974; **100**: 111-121.
10. Rackwitz R, Fiessler B. Structural reliability under combined random load sequences, *Comput Struct* 1978; **9**(5): 489-494.
11. Thoft-Christensen P, Murotsu Y. *Application of Structural Systems Reliability Theory*, Springer, 1st edition, Berlin, 1986.
12. Park S, Choi S, Sikorski C, Stubbs N. Efficient method for calculation of system reliability of a complex structure, *Int J Solids Struct* 2004; **41**(18-19): 5035-5050.
13. Nowak AS, Collins KR. *Reliability of Structures*, McGraw-Hill Higher Education, Singapore, 2000.
14. Mansoorian P. Element and Structural System Reliability-based Optimization of Steel Trusses Using Developed Genetic Algorithm, Ph.D. Thesis, Shahrood University of Technology, Department of Civil Engineering, 2011.
15. Murotsu Y, Kishi M, Yonezawa M. On the shape optimization of truss structure based on reliability concept, *Proceedings of the IUTAM Symposium on Structural Optimization*, Melbourne, Australia, 1988, pp. 193-200.
16. Stocki R, Kolanek K, Jendo S, Kleiber M. Study on discrete optimization techniques in reliability-based optimization of truss structures, *Comput Struct* 2001; **79**(22-25): 2235-2247.
17. Kaveh A, Kalatjari VR. Genetic algorithm for discrete-sizing optimal design of trusses using the force method, *Int J Numer Meth Eng* 2002; **55**(1): 55-72.
18. Kaveh A, Kalatjari VR. Topology optimization of trusses using genetic algorithm, force method and graph theory, *Int J Numer Meth Eng* 2003; **58**(5): 771-791.
19. Kaveh A, Kalatjari VR. Size/geometry optimization of trusses by the force method and genetic algorithm, *Z Angew Math Mech* 2004; **84**(5): 347-357.
20. Kalatjari VR, Kaveh A, Mansoorian P. System reliability assessment of redundant trusses using improved algebraic force method and artificial intelligence, *Asian J Civil Eng (Building and Housing)* 2011; **12**(4): 523-550.
21. Kalatjari VR, Mansoorian P. Bi-level reliability-based optimization of trusses via competitive distributed genetic algorithm, *6th National Congress on Civil Engineering*, Semnan University, Semnan, Iran, 2011.
22. Kaveh A, Kalatjari VR, Mansoorian P. Fast reliability-based optimization using parallel genetic algorithm, developed force method and artificial intelligence, *J Civil Eng Ferdowsi* 2013; **24**(2): 1-26.
23. Kim DS, Ok SY, Song J, Koh HM. System reliability analysis using dominant failure modes identified by selective search technique, *Reliab Eng Syst Safe* 2013; **119**: 316-331.

24. Kaveh A, Massoudi MS, Ghafooni Bagha M. Structural reliability analysis using charged system searched algorithm, *Iran J Sci Technol* 2014; **38**(C2): 439-448.
25. Kaveh A, Ilchi Ghazaan M. Structural reliability assessment utilizing four metaheuristic algorithms, *Int J Optim Civil Eng* 2015; **5**(2): 205-225.
26. Cornell CA. Bounds on the reliability of structural systems, *J Struct Div, ASCE* 1967; **93**(1): 171-200.
27. Goldberg DE, Korb B, Deb K. Messy genetic algorithms: motivation, analysis, and first results, *Complex Syst* 1989; **3**(5): 493-530.
28. Kalatjari VR. Optimization of Trusses Using Genetic Algorithm and the Algebraic Force Method, Ph.D. Thesis, Iran University of Science and Technology, School of Civil Engineering, 2002.
29. Bagchi TP. An Analysis of the Behavior of a Class of Genetic Adaptive Systems, Ph.D. Thesis, University of Michigan, Department of Computer and Communication Sciences, 1975.
30. Bagchi TP. *Multiobjective Scheduling by Genetic Algorithms*, Kluwer Academic Publishers, Massachusetts, 1999.
31. Murotsu Y, Okada M, Niwa K, Miwa S. Reliability analysis of truss structures by using matrix method, *J Mech Des* 1980; **102**(4): 749-756.
32. Murotsu Y, Okada M, Yonezawa M, Taguchi K. Reliability assessment of redundant structures, *Proceeding of Icosar '81, the 3rd International Conference on Structural Safety and Reliability*, the Norwegian Institute of Technology, Trondheim, Norway, 1981, pp. 315-329.
33. Togan V, Daloglu AT. Optimum design of a truss system under the constraint of failure probability, *ARI Bulletin Istanbul Tech University* 2007; **54**(5): 8-19.