

## SIZE AND GEOMETRY OPTIMIZATION OF TRUSS STRUCTURES USING THE COMBINATION OF DNA COMPUTING ALGORITHM AND GENERALIZED CONVEX APPROXIMATION METHOD

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### ABSTRACT

In recent years, the optimization of truss structures has been considered due to their several applications and their simple structure and rapid analysis. DNA computing algorithm is a non-gradient-based method derived from numerical modeling of DNA-based computing performance by new computers with DNA memory known as molecular computers. DNA computing algorithm works based on collective intelligence. It works with doing random search in the search space and creating the initial random population by modeling DNA-based computing operators and applies the operators derived from genetic algorithm to achieve the optimum solution of the objective function. Generalized Convex Approximation (GCA) method is a gradient-based method that with approximation of the main function and starting from a point, finds the optimum solution using information about functions and their gradient. In this research, in order to minimize the weight of truss, the cross-section areas of the elements as discrete variables are optimized by DNA computing algorithm, and the coordinates of truss nodes as continuous variables are optimized by Generalized Convex Approximation (GCA) method. Therefore, to simultaneously optimize the size and geometry of truss structures, these two methods are used in combination. The results of numerical examples show the proper functioning of this process.

**Keywords:** Optimization; Truss; DNA computing algorithm; Generalized Convex Approximation (GCA) method.

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## 1. INTRODUCTION

Nowadays, optimization plays an important role in solving many engineering problems such as structural engineering. So, the growth and development of computer science, and also the importance of achieving optimum solution in short time have caused the researchers welcome using meta-heuristic algorithms with random search in solving optimization problems. The meta-heuristic algorithms such as genetic algorithm, ant colony algorithm, particle swarm optimization algorithm and simulated annealing algorithm are the most attractive. These algorithms do not need common computations such as computing gradient of functions. So, they are more rapid than ordinary algorithms in solving optimization problems.

Also, because of random search in the search space during the process of solving optimization problems, the exploration and exploitation performance of these algorithms rise and therefore, the chance of achieving global optimum solution increases.

DNA-based computing is a new research method. In this method, the computation is performed based on DNA (Deoxyribonucleic acid) molecules and using new computers with DNA memory called molecular computers. At first, DNA-based computing was proposed by Adelman in 1994 to solve travelling salesman problem [1]. By successful solving of the travelling salesman problem, the ability to do DNA-based computing in solving NP (Non Polynomial) problems was distinguished. This method was used to solve other problems such as vertex coloring problem [2], the minimum spanning tree problem [3], the knapsack problem [4] and the N-Queens Problem [5].

In DNA-based computing, by taking advantage of high volume data storage feature in molecular computers, DNA strings are produced. Then, by applying operators in parallel on all strings, DNA strings are purified. By evaluating the remained strings, the computational problem will be solved [6].

DNA computing algorithm is a meta-heuristic algorithm and a non-gradient-based method resulted from numerical modeling of DNA-based computing performance by molecular computers that works based on collective intelligence, doing random search in search space, creating initial random population by modeling DNA-based computing operators and applying operators derived from genetic algorithm to achieve the optimum solution for objective function [7].

Generalized Convex Approximation (GCA) method is a step-by-step mathematical programming method that converges to the optimum solution by producing convex approximations of optimization problem called sub-problem and solving these sub-problems [8].

In this research, in order to minimize the weight of truss structures, the cross-section areas of the elements as discrete variables are optimized by DNA computing algorithm (size optimization), and the coordinates of truss nodes as continuous variables are optimized by Generalized Convex Approximation (GCA) method (geometry optimization).

Therefore, to simultaneously optimize the size and geometry of truss structures, these two methods are used in combination and the results will be discussed in the numerical examples.

## 2. THE PROBLEM OF SIZE AND GEOMETRY OPTIMIZATION OF TRUSS STRUCTURES

The general form of an optimization problem is formulated as Equation (1):

$$\begin{aligned}
 & \text{Minimize : } f(X) \\
 & \text{Subject to : } \frac{g_i(X)}{g_i(X)} - 1 \leq 0 \quad i = 1, 2, \dots, m \\
 & X = \{x_1, x_2, \dots, x_j, \dots, x_n\}
 \end{aligned} \tag{1}$$

where,  $f(X)$  is the objective function,  $X$  is the set of design variables,  $m$  is the number of constrains,  $n$  is the number of variables and  $\bar{g}_i(X)$  is the allowable value of  $n$ -th constraint [9].

In size and geometry optimization of truss structures, the goal is minimizing the weight of truss structures under design constraints. In size optimization, the cross-section areas of the elements are selected as discrete variables, and in geometry optimization, the coordinates of truss nodes are selected as continuous variables [10].

The general form of optimization problem of truss structures is formulated as Equation (2) with selecting the truss weight as objective function, and selecting constraints such as axial stresses of the elements, displacements in the degrees of freedom of truss nodes and slenderness ratios of the elements:

$$\begin{aligned}
 & \text{Minimize : } W(X) = \sum_{i=1}^{n_e} \gamma_i A_i L_i \\
 & \text{Subject to : } \frac{|\sigma_i(X)|}{\bar{\sigma}_i} - 1 \leq 0 \quad i = 1, 2, \dots, n_e \\
 & \quad \quad \quad \frac{|\Delta_j(X)|}{\bar{\Delta}_j} - 1 \leq 0 \quad j = 1, 2, \dots, n_j \\
 & \quad \quad \quad \frac{\lambda_i(X)}{\bar{\lambda}_i} - 1 \leq 0 \quad i = 1, 2, \dots, n_e \\
 & \quad \quad \quad x_j^{\min} \leq x_j \leq x_j^{\max} \\
 & \quad \quad \quad A_i \in S
 \end{aligned} \tag{2}$$

where  $X$  is the set of design variables,  $W(X)$  is the truss weight as objective function,  $n_e$  is the number of truss elements,  $n_j$  is the total number of the degrees of freedom of truss nodes,  $\gamma_i$  is the specific weight of  $i$ -th element,  $A_i$  is the cross-section area of  $i$ -th element,  $L_i$  is the length of  $i$ -th element,  $\sigma_i(X)$  is the stress of  $i$ -th element,  $\bar{\sigma}_i$  is the allowable stress of  $i$ -th element,  $\Delta_j(X)$  is the displacement in the  $j$ -th degree of freedom,  $\bar{\Delta}_j$  is the allowable displacement in the  $j$ -th degree of freedom,  $\lambda_i(X)$  is the slenderness ratio of  $i$ -th element,  $\bar{\lambda}_i$  is the allowable slenderness ratio of  $i$ -th element,  $x_j$  is the coordinate of the  $j$ -th degree of freedom,  $x_j^{\min}$  and  $x_j^{\max}$  are respectively the lower limit and upper limit of the  $j$ -th degree of freedom and  $S$  is the cross-section list of elements [11].

### 3. DNA-BASED COMPUTING BY MOLECULAR COMPUTERS

DNA molecule is made up of two protein strings wrapped and called polynucleotide. These strings are made of ingredients known as organic base called nucleotide. Nucleotides are divided into four groups: Adenin, Guanin, Cytosine and Thymine which are respectively abbreviated with the letters A, G, C and T. In each DNA molecule by using the complementarity feature of bases A with T and C with G and hydrogen bond between complementary bases, polynucleotide strings bond together.

DNA-based computing is a new research method to solve computational problems which computation is done based on DNA molecule and using new computers with DNA memory called molecular computers [1]. Molecular computers are the next generation computers. They use DNA molecules as storage memory of information [12]. DNA molecules can store more information than current computers memory. Therefore, molecular computers store more information than silicone computers. Also, DNA molecules provide the energy of molecular computers in such a way that by breaking the bonds between DNA strings, the released energy is used in molecular computers [13].

DNA-based computing was first used by Adelman in 1994 to solve travelling salesman problem. Then this method was used to solve NP problems [14]. In solving a computational problem by DNA-based computing, trillions DNA strings with bases A, G, C and T are produced as the solutions which is due to high capacity of information storage. These strings are placed in test tubes. Then by applying specific and biochemical operators in parallel on all strings, DNA strings are purified and the needed strings remain. After that, by evaluating the remained strings as desired solutions to the problem, the final solution of the computational problem is achieved.

The operators used in DNA-based computing are:

- (1) Synthesis operator that produces DNA strings based on bases A, G, C and T with a specific length. It is possible to assign these strings as sub-string to design variables of problem.
- (2) Ligation operator with salt and water and ligase pastes the sub-strings related to the problem variables together and some strings are produced as the computational problem solutions.
- (3) PCR<sup>1</sup> operator amplifies the desired complementary strings. In this way, according to Fig. 1, if the double-string molecule of XYZ $\uparrow$  is available, the temperature is increased to 95 celsius degree and the two strings of DNA molecule by melting process are converted to single strings of XYZ $\uparrow$  and XYZ $\downarrow$ . Then, the initial signals of the beginning and end of the considered string that are Z $\downarrow$  and X $\uparrow$  are added to combine with molecules and to achieve YZ $\uparrow$ , X $\downarrow$ , Z $\uparrow$  and XY $\downarrow$ . After that, polymerase enzyme can produce two strings of XYZ $\uparrow$ . At this step, two versions of the original string have been produced. By repeating this process it is possible to produce 2<sup>n</sup> versions of complementary string after n steps. Then, by the bonding of DNA strings with complementary strings, DNA strings with desired coding are purified. Therefore, the strings which present the problem desired variables remain.

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<sup>1</sup> Polymerase Chain Reaction

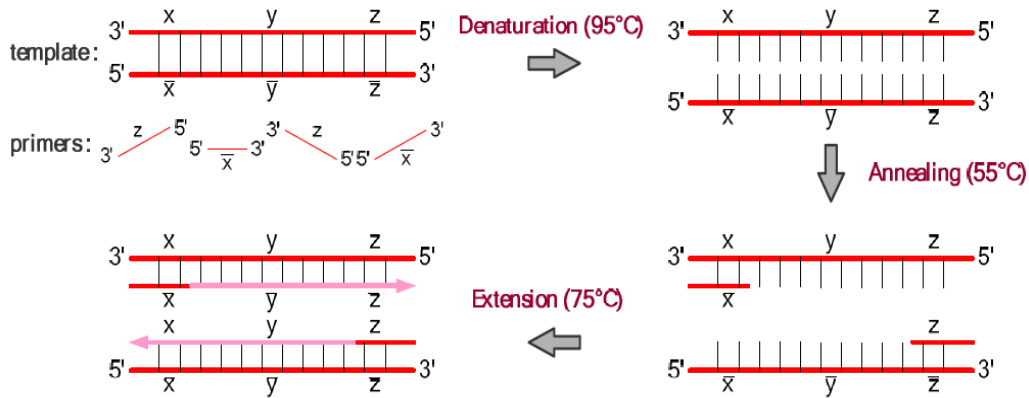


Figure 1. The process of PCR operator [15]

(4) Gel electrophoresis operator in which DNA strings with different lengths are placed on a gel surface. Then, with addition of electrical current to the surface and increasing the temperature, the strings are negatively charged and move towards positively charged electrodes. In this process, due to higher speed of strings with less length, the strings are divided based on length. So, gel electrophoresis operator purifies the strings with desired length [6].

#### 4. DNA COMPUTING ALGORITHM

DNA computing algorithm is a meta-heuristic algorithm and a non-gradient-based method derived from numerical modeling of DNA-based computing performance by molecular computers. DNA computing algorithm works based on collective intelligence. It works with doing random search in the search space and creating the initial random population by modeling DNA-based computing operators and applies the operators derived from genetic algorithm to achieve the optimum solution of the objective function [7]. Below, the details of the stages of DNA computing algorithm are discussed.

##### 4.1 The production of the initial population individuals strings by modeling operators of DNA-based computing

In DNA computing algorithm, an individual is recognized as a point of search space or a solution of optimization problem. In this way, by randomly producing a number of individuals that are called the initial population, the optimization process starts from several different points of search space. The number of population individuals is called population size, which is typically determined at the beginning of the algorithm. In DNA computing algorithm, all population individuals have coded string with given length and based on DNA molecule bases (A, G, C and T), which is formed by pasting the defined sub-strings of design variables.

The strings of population individuals are produced by modeling DNA-based computing operators. So, by modeling of Synthesis operator, the sub-strings of design variables are randomly produced with given length. Then, by modeling of ligation operator, the sub-

strings related to design variables paste together and the general string of every individual in population is formed. The modeling of PCR operator causes the sub-string related to every variable be appointed to the place of that variable.

Also, by modeling of gel electrophoresis operator, the length of general string of every individual in population that is the sum of the lengths of sub-strings related to design variables is controlled. Fig. 2 shows an example of the string of an individual in population with length 12 which contains 3 design variables with sub-strings with length 4.

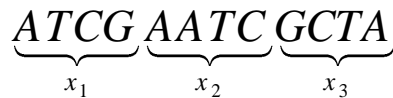


Figure 2. An example of produced string by modeling DNA-based computing operators

#### 4.2 Decoding the design variables

To decode the strings related to discrete variables, it is possible to assign numbers to the DNA strings and using the one-to-one correspondence of the numbers with the numerical values of the variables. In this way, a number is assigned to every sub-string. Then, with one-to-one correspondence of the available numbers with the table of numerical values related to every variable, the numerical value of that variable is specified.

#### 4.3 Evaluating the fitness function

In DNA computing algorithm, the unconstrained function which must be maximized is called the fitness function. Generally, DNA computing algorithm is used to optimize such functions. So, in the unconstrained maximization problems, the fitness function is the objective function. But, in the constrained minimization problems, the fitness function is achieved by some reformations.

In this research, to solve constrained minimization problems, by using the exterior penalty function method, the constrained problem is converted to unconstrained problem [16]. In this way, according to the general form of the constrained minimization problem shown in Equation (1), the constraint violation coefficient is determined with Equation (3):

$$c = \sum_{i=1}^m \max \left\{ \frac{g_i(X)}{\bar{g}_i(X)} - 1, 0 \right\} \quad (3)$$

where,  $X$  is the set of design variables,  $m$  is the number of problem constraints and  $\bar{g}_i(X)$  is the allowable amount of  $i$ -th constraint. Therefore, the objective function of the unconstrained problem is defined as Equation (4):

$$\text{Minimize : } fm = f(1 + kc) \quad (4)$$

where,  $f$  is the objective function,  $fm$  is the reformed objective function,  $c$  is the constraint violation coefficient and  $k$  is the constraint participation coefficient in the optimization problem. Finally, the fitness function is defined by the reverse of reformed objective

function and as Equation (5):

$$Maximize : h = \frac{1}{fm} = \frac{1}{f(1+kc)} \tag{5}$$

Also, in this research, to increase the convergence power of algorithm, the elitist approach has been used. In this way, after producing the population and applying algorithm operators, the most graceful individuals are selected as the next generation [17].

4.4 Selection operator

The selection operator increases the average fitness of population individuals in generations. In this research, the roulette wheel method has been used to perform selection operator. In this method, every population individual is given the probability of being selected and the participation in new generation production. So, every individual with more fitness, has more selection probability. In this way, if a population is  $N$  individuals and fitness function value of  $x_i$  is  $f(x_i)$ , the selection probability of  $x_i$  is determined with Equation (6):

$$p(x_i) = \frac{f(x_i)}{\sum_{i=1}^N f(x_i)} \tag{6}$$

Therefore, according to the cumulative probability of selecting individuals and by randomly selecting them, the individuals are selected for the production of the new generation [18].

4.5 Crossover operator

The crossover operator rises the exploration and exploitation performance during the algorithm process. In crossover operator by using a percentage of the population called the crossover probability ( $\rho_c$ ), the number of produced individuals in this operator in every generation is determined. In this research, the crossover operator has been used as the one-point method.

The one-point crossover operator breaks the DNA strings of two selected individuals as parent at one point, and replaces the broken parts. Therefore, two new individuals called child are produced [18]. Fig. 3 is an example of the one-point crossover operator. The strings break has been happened between third and fourth nucleotides on the left of strings.

|                 |            |              |
|-----------------|------------|--------------|
| <b>Parent 1</b> | <b>ATA</b> | <b>CGGTC</b> |
| <b>Parent 2</b> | <b>GCT</b> | <b>AATTG</b> |
| <b>Child 1</b>  | <b>ATA</b> | <b>AATTG</b> |
| <b>Child 2</b>  | <b>GCT</b> | <b>CGGTC</b> |

Figure 3. An example of one-point crossover operator

#### 4.6 Mutation operator

The mutation operator prevents the getting caught of algorithm in the trap of local optimum points. In mutation operator, by using a percentage of the population called the mutation probability ( $\rho_m$ ), the number of individuals that should be randomly under the mutation operator is determined. In this research, the mutation operator has been used with uniform method. In this way, by determining a string called mask, and containing numbers 0 and 1, the sub-strings of variables corresponding to number 1 are changed and a new sub-string is determined randomly [19].

As an example, if the selected individual for the mutation operator has 3 variables and every variable has a sub-string with length 4, the general string is as Fig. 4:



Figure 4. An example of the string of selected individual for mutation operator

And if the mask string be formed as Fig. 5:

|   |   |   |
|---|---|---|
| 0 | 1 | 0 |
|---|---|---|

Figure 5. An example of mask string

Then, the sub-string related to  $x_2$  should be changed. Therefore, a string like the string in Fig. 6 is created.

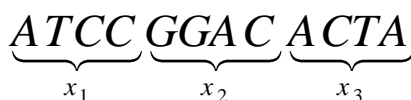


Figure 6. The changed string under mutation operator

## 5. GENERALIZED CONVEX APPROXIMATION (GCA) METHOD

In large scale optimization problems, specifically structures optimization problems, solving the problem is nearly impossible due to various problem constraints and difficulty in achieving derivatives of objective function and problem constraints at different stages. In this condition, the problem can be solved by convex approximation. In this way, the solution of the problem converges to the optimum solution by producing series of simple sub-problems which are approximations of the original problem and solving these sub-problems with different methods [10].

In this research, the Generalized Convex Approximation (GCA) method has been used [8]. The Generalized Convex Approximation (GCA) method is a method to solve



optimization problems as Equation (7):

$$\begin{aligned}
 & \text{Minimize : } f(X) \\
 & \text{Subject to : } g_i(X) \leq 0 \quad i = 1, 2, \dots, m \\
 & X = \{x_1, x_2, \dots, x_j, \dots, x_n\}^T \\
 & x_j^{\min} \leq x_j \leq x_j^{\max} \quad j = 1, 2, \dots, n
 \end{aligned} \tag{7}$$

where,  $n$  is the number of design variables,  $m$  is the number of problem constrains,  $X$  is the vector of design variables,  $f(X)$  is the objective function,  $g_i(X)$  is the  $i$ -th constraint function of the problem and  $x_j^{\min}$  and  $x_j^{\max}$  are respectively the lower limit and upper limit of the  $j$ -th variable of the problem. In this way, every optimization problem by using Generalized Convex Approximation (GCA) method converts to Equation (8):

$$\begin{aligned}
 & \text{Minimize : } \tilde{f}(X) \\
 & \text{Subject to : } \tilde{g}_i(X) \leq 0 \quad i = 1, 2, \dots, m \\
 & X = \{x_1, x_2, \dots, x_j, \dots, x_n\}^T \\
 & L_j^{(k)} \leq x_j \leq U_j^{(k)} \quad j = 1, 2, \dots, n
 \end{aligned} \tag{8}$$

where,  $n$  is the number of design variables,  $m$  is the number of problem constrains,  $X$  is the vector of design variables,  $\tilde{f}(X)$  is the approximation of the objective function,  $\tilde{g}_i(X)$  is the approximation of  $i$ -th constraint function and  $L_j^{(k)}$  and  $U_j^{(k)}$  are called moving limits which are updated at each iteration and are defined as Equations (9) and (10):

$$L_j^{(k)} = \frac{x_j^{\min} + x_j^{(k)}}{2} \tag{9}$$

$$U_j^{(k)} = \frac{x_j^{\max} + x_j^{(k)}}{2} \tag{10}$$

Therefore, at each iteration by solving the  $k$ -th approximation sub-problem with an approximation method and achieving the optimum design point  $X^{(k)}$ , the  $(k+1)$ -th sub-problem is approximated. Generally, these approximations are based on the information of current and previous design point. Then, by solving the  $(k+1)$ -th sub-problem, the optimum design point  $X^{(k+1)}$  is achieved and this process continues until the problem converges to an optimum point.

In Generalized Convex Approximation (GCA) method, the approximation of the objective function and problem constraints are defined as follows:

$$\tilde{f}(X) = \tilde{f}_0 + \sum_{j=1}^n \tilde{f}_j(x_j) \tag{11}$$

where

$$\tilde{f}_0 = f(X^{(k)}) - \sum_{j=1}^n b_j (x_j^{(k)} - c_j)^{r_j} \quad j = 1, 2, \dots, n \quad (12)$$

$$\tilde{f}_j(x_j) = b_j (x_j - c_j)^{r_j} \quad j = 1, 2, \dots, n \quad (13)$$

where,  $f(X^{(k)})$  is the value of original function for design point  $X^{(k)}$ , and  $c_j$  and  $r_j$  and  $b_j$  are approximation parameters which are determined for every design variable. The value of the first and second order derivatives of the original function at every design point are defined as Equations (14) and (15):

$$f_j'(X^{(k)}) = \frac{\partial f(X^{(k)})}{\partial x_j} \quad (14)$$

$$f_j''(X^{(k)}) = \frac{\partial^2 f(X^{(k)})}{\partial x_j^2} \quad (15)$$

Also, due to the accuracy of the approximation,

$$\tilde{f}_j''(X^{(k)}) = f_j''(X^{(k)}).$$

In Generalized Convex Approximation (GCA) method, the approximation parameters are determined as follows:

$$c_j = \frac{-x_j^{(k)} + d^{\frac{1}{(r_j-1)}} x_j^{(k-1)}}{-1 + d^{\frac{1}{(r_j-1)}}} \quad (16)$$

$$r_j = 1 + \frac{x_j^{(k)} - c_j}{e} \quad (17)$$

$$b_j = \frac{f_j'(X^{(k)})}{r_j (x_j^{(k)} - c_j)^{r_j-1}} \quad (18)$$

where

$$d = \frac{f_j'(X^{(k)})}{f_j'(X^{(k-1)})} \quad (19)$$

$$e = \frac{f_j'(X^{(k)})}{f_j''(X^{(k)})} \quad (20)$$

By combination of Equations (16) and (17), the  $c_j$  parameter is defined as Equation (21):

$$c_j = \frac{-x_j^{(k)} + d \frac{e}{(x_j^{(k)} - c_j)} x_j^{(k-1)}}{-1 + d \frac{e}{(x_j^{(k)} - c_j)}} \tag{21}$$

where, by solving the equation  $f(c_j)=0$  the value of  $c_j$  is achieved. Also, to prevent the difficulty in computing the functions value and derivatives, the limitation  $c_j \leq x_j^{min}$  is used. At the first iteration, the information at design point  $X^{(k-1)}$  is not available. So, the approximation parameters are determined as Equation (22):

$$\begin{aligned} c_j &= 0 \\ r_j &= \frac{f_j''(X^{(k)})}{f_j'(X^{(k)})} x_j^{(k)} + 1 \\ b_j &= \frac{f_j'(X^{(k)})}{r_j (x_j^{(k)})^{r_j - 1}} \end{aligned} \tag{22}$$

In this research, the simplified form of Generalized Convex Approximation (GCA) method that uses first order derivative is used. In this method, the approximation process is like second order form. The only difference is in determining the approximation parameters. The approximation parameters  $c_j$  and  $r_j$  and  $b_j$  in the simplified form of Generalized Convex Approximation (GCA) method are determined as follows:

$$(i) \frac{f_j'(X^{(k)})}{f_j'(X^{(k-1)})} > 0: \begin{cases} c_j = 0 \\ r_j = \begin{cases} 1 & \text{if } \frac{x_j^{(k)}}{x_j^{(k-1)}} = 1 \\ 1 + \frac{\log(\frac{f_j'(X^{(k)})}{f_j'(X^{(k-1)})})}{\log(\frac{x_j^{(k)}}{x_j^{(k-1)})}} & \text{else} \end{cases} \\ b_j = \frac{f_j'(X^{(k)})}{r_j (x_j^{(k)})^{r_j - 1}} \end{cases} \tag{23}$$

$$(ii) \frac{f'_j(X^{(k)})}{f'_j(X^{(k-1)})} < 0: \begin{cases} c_j = \frac{x_j^{(k)} - dx_j^{(k-1)}}{1-d} \leq x_j^{\min} \\ r_j = 2 \\ b_j = \frac{f'_j(X^{(k)})}{2(x_j^{(k)} - c_j)} \end{cases} \quad (24)$$

$$(iii) f'_j(X^{(k)}) = 0 \text{ and } f'_j(X^{(k-1)}) = 0: \begin{cases} c_j = 0, r_j = 1 \\ b_j = f'_j(X^{(k)}) \text{ or } f'_j(X^{(k-1)}) = 0 \end{cases} \quad (25)$$

$$(iv) f'_j(X^{(k)}) = 0 \text{ and } f'_j(X^{(k-1)}) \neq 0: \begin{cases} c_j = 0, r_j = 2 \\ b_j = \begin{cases} f'_j(X^{(k-1)}) & \text{if } \frac{x_j^{(k)}}{x_j^{(k-1)}} = 1 \\ \frac{f'_j(X^{(k-1)})}{2(x_j^{(k-1)} - x_j^{(k)})} & \text{else} \end{cases} \end{cases} \quad (26)$$

$$(v) f'_j(X^{(k)}) \neq 0 \text{ and } f'_j(X^{(k-1)}) = 0: \begin{cases} c_j = 0, r_j = 2 \\ b_j = \begin{cases} f'_j(X^{(k)}) & \text{if } \frac{x_j^{(k)}}{x_j^{(k-1)}} = 1 \\ \frac{f'_j(X^{(k)})}{2(x_j^{(k)} - x_j^{(k-1)})} & \text{else} \end{cases} \end{cases} \quad (27)$$

At the first iteration, since there is no information at design point  $X^{(k-1)}$ , the approximation parameters are determined as Equation (28):

$$\begin{aligned} c_j &= 0 \\ r_j &= 1 \\ b_j &= f'_j(X^{(k)}) \end{aligned} \quad (28)$$

The first and second order derivatives of the functions at Generalized Convex Approximation (GCA) method are achieved from Equations (29) and (30):

$$\tilde{f}'_j(X^{(k)}) = b_j r_j x_j^{r_j-1} \quad (29)$$

$$\tilde{f}''_j(X^{(k)}) = b_j r_j (r_j - 1) x_j^{r_j-2} \quad (30)$$

In Generalized Convex Approximation (GCA) method, at each iteration after production of sub-problems that include the approximation of objective function and constraints of the original problem, many methods can be used to solve sub-problems. In this research, the exterior penalty function method has been used to solve the constrained sub-problems [9].

**6. THE COMBINATION OF DNA COMPUTING ALGORITHM AND GENERALIZED CONVEX APPROXIMATION (GCA) METHOD**

In this research, in order to minimize the weight of truss structures, the cross-section areas of the elements as discrete variables are optimized by DNA computing algorithm, and the coordinates of truss nodes as continuous variables are optimized by Generalized Convex Approximation (GCA) method.

In this way, at the first stage by selecting appropriate and similar cross sections for truss elements, the coordinates of truss nodes are optimized by Generalized Convex Approximation (GCA) method. Then, at the second stage with achieved coordinates from the first stage and by using DNA computing algorithm, the cross-section areas of the elements are optimized.

In Fig. 7 the flowchart of the process of size and geometry optimization of truss structures by using the combination of DNA computing algorithm and Generalized Convex Approximation (GCA) method is shown.

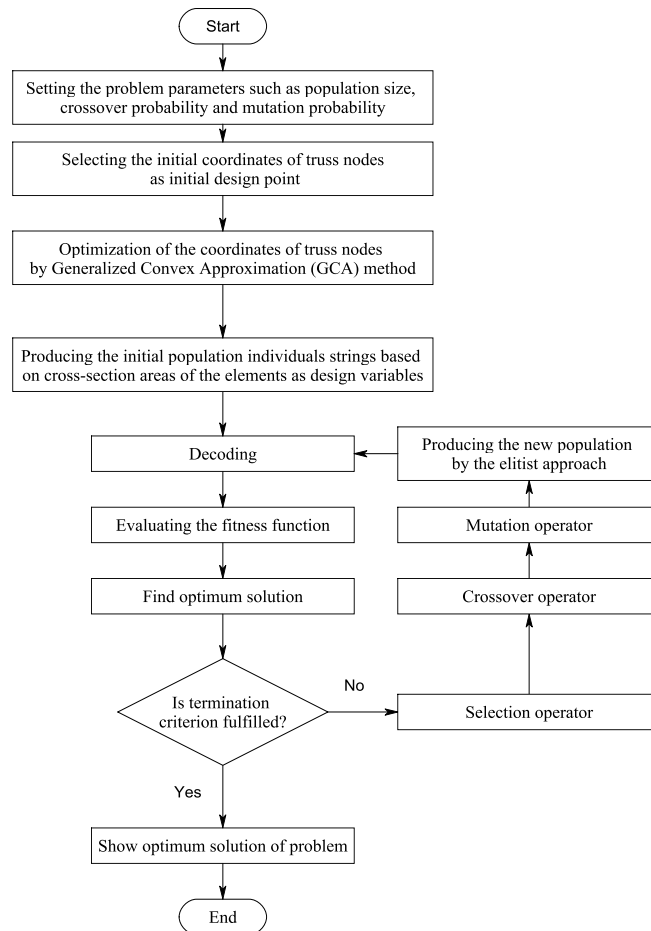


Figure 7. The flowchart of optimization of trusses using combined algorithm DNA-GCA

## 7. NUMERICAL EXAMPLES

In this research, five problems about minimizing the weight of truss structures in order to control the numerical efficacy of the combination of DNA computing algorithm and Generalized Convex Approximation (GCA) method have been solved. This method has been coded in MATLAB software. The truss structures have been analyzed by using the stiffness method. The results of optimization have been compared with the results of different references. The parameters set for solving the problems are summarized in Table 1.

Table 1: The parameters set for solving the problems

| Parameter                          | Value |
|------------------------------------|-------|
| Population size                    | 100   |
| Crossover probability ( $\rho_c$ ) | 80%   |
| Mutation probability ( $\rho_m$ )  | 10%   |

Since the discussed method for optimization process is stochastic, the optimization program has been run 10 times and the average of 10 runs and also the best run have been reported.

### 7.1 Twenty five-bar truss

A twenty five-bar truss as Fig. 8 has been considered. The coordinates of truss nodes have been shown in Table 2. The grouping of truss elements as the size variables has been shown in Table 3. The data to design truss has been shown in Table 4. Modulus of elasticity of the material is 68.95 Gpa and the specific weight is 2720 kg/m<sup>3</sup>. The results of size and geometry optimization of the truss have been compared with other references in Table 5. The stresses of the elements have been shown in Table 6. The displacements of truss nodes have been shown in Table 7. The optimum geometry of the truss has been shown in Fig. 9. Also, the convergence process of truss weight has been shown in Fig. 10.

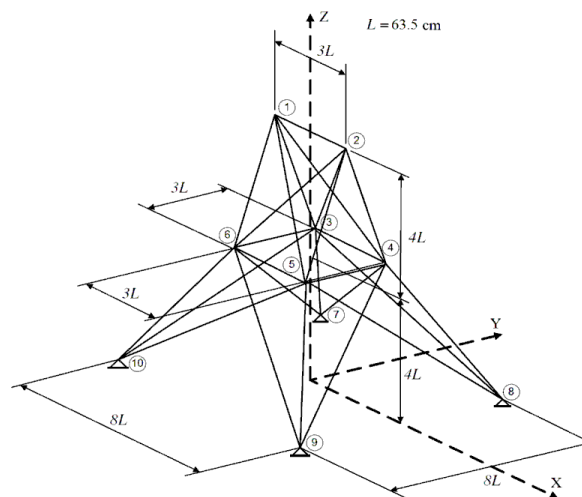


Figure 8. Twenty five-bar truss

Table 2: The coordinates of truss nodes for twenty five-bar truss

| Node | $x(cm)$ | $y(cm)$ | $z(cm)$ |
|------|---------|---------|---------|
| 1    | -95.25  | 0       | 508     |
| 2    | 95.25   | 0       | 508     |
| 3    | -95.25  | 95.25   | 254     |
| 4    | 95.25   | 95.25   | 254     |
| 5    | 95.25   | -95.25  | 254     |
| 6    | -95.25  | -95.25  | 254     |
| 7    | -254    | 254     | 0       |
| 8    | 254     | 254     | 0       |
| 9    | 254     | -254    | 0       |
| 10   | -254    | -254    | 0       |

Table 3: The grouping of truss elements for twenty five-bar truss

| Group | Members (end nodes)              |
|-------|----------------------------------|
| $A_1$ | 1(1,2)                           |
| $A_2$ | 2(1,4),3(2,3),4(1,5),5(2,6)      |
| $A_3$ | 6(2,5),7(2,4),8(1,3),9(1,6)      |
| $A_4$ | 10(3,6),11(4,5)                  |
| $A_5$ | 12(3,4),13(5,6)                  |
| $A_6$ | 14(3,10),15(6,7),16(4,9),17(5,8) |
| $A_7$ | 18(3,8),19(4,7),20(6,9),21(5,10) |
| $A_8$ | 22(3,7),23(4,8),24(5,9),25(6,10) |

Table 4: The data to design twenty five-bar truss

|   | Node  | $F_x (KN)$ | $F_y (KN)$ | $F_z (KN)$ |
|---|---|------------|------------|------------|
| Loading data                                      | 1   | 4.454      | -44.537    | -44.537    |
|   | 2   | 0          | -44.537    | -44.537    |
|   | 3   | 2.227      | 0          | 0          |
|   | 6   | 2.672      | 0          | 0          |
| Design variables                                  | Size variables  |            |            |            |
|   | $A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8$                                  |            |            |            |
|   | Geometry variables  |            |            |            |
| Constraint data                                   | $x_4 = x_5 = -x_3 = -x_6; y_4 = y_3 = -y_5 = -y_6; z_4 = z_3 = z_5 = z_6$ |            |            |            |
|   | $x_8 = x_9 = -x_7 = -x_{10}; y_8 = y_7 = -y_9 = -y_{10}$                  |            |            |            |
|   | Stress constraints  |            |            |            |
|   | $(\sigma_i)_i \leq 275.8 \text{ Mpa}; \quad i=1,2,\dots,25$               |            |            |            |
|   | $ (\sigma_c)_i  \leq 275.8 \text{ Mpa}; \quad i=1,2,\dots,25$             |            |            |            |
|   | Displacement constraints  |            |            |            |
| $ A_i  \leq 0.89 \text{ cm}; \quad i=1,2,\dots,6$ |   |            |            |            |
| Side constraints of geometry variables            |   |            |            |            |
| $50.8 \text{ cm} \leq x_4 \leq 152.4 \text{ cm}$  |   |            |            |            |
| $101.6 \text{ cm} \leq y_4 \leq 203.2 \text{ cm}$ |   |            |            |            |
| $228.6 \text{ cm} \leq z_4 \leq 330.2 \text{ cm}$ |   |            |            |            |
| $101.6 \text{ cm} \leq x_8 \leq 203.2 \text{ cm}$ |   |            |            |            |

|                                |   |
|--------------------------------|---|
|                                | $254\text{cm} \leq y_8 \leq 355.6\text{cm}$   |
| List of the available profiles | $A_i \in S = \{0.645I \ (I=1,2,\dots,26), 18.064, 19.355, 20.645, 21.935\}\text{cm}^2 \quad i=1,2,\dots,25$ |

Table 5: The optimization results of the size and geometry of twenty five-bar truss

| Design variables                    | Wu and Chow [20] | Kaveh and Kalatjari [21] | Rahami et al [22] | Present work |         |
|-------------------------------------|------------------|--------------------------|-------------------|--------------|---------|
| Size variables<br>( $\text{cm}^2$ ) | $A_1$            | 0.645                    | 0.645             | 0.645        | 0.645   |
|                                     | $A_2$            | 1.29                     | 0.645             | 0.645        | 0.645   |
|                                     | $A_3$            | 7.097                    | 7.097             | 7.097        | 5.805   |
|                                     | $A_4$            | 1.29                     | 0.645             | 0.645        | 0.645   |
|                                     | $A_5$            | 1.935                    | 0.645             | 0.645        | 0.645   |
|                                     | $A_6$            | 0.645                    | 0.645             | 0.645        | 0.645   |
|                                     | $A_7$            | 1.29                     | 0.645             | 1.29         | 0.645   |
|                                     | $A_8$            | 5.806                    | 6.452             | 5.16         | 6.45    |
| Geometry variables<br>(cm)          | $x_4$            | 104.318                  | 92.024            | 83.9436      | 92.29   |
|                                     | $y_4$            | 135.814                  | 148.742           | 136.0584     | 143.093 |
|                                     | $z_4$            | 316.484                  | 293.599           | 329.9694     | 320.342 |
|                                     | $x_8$            | 129.032                  | 118.008           | 111.2078     | 129.108 |
|                                     | $y_8$            | 333.959                  | 324.993           | 347.569      | 347.53  |
| Weight (kg)                         | 61.83            | 56.29                    | 54.53             | 53.0067      |         |

Table 6: The stresses of the elements of twenty five-bar truss

| Member | Stress ( $\text{kg}/\text{cm}^2$ ) | Member | Stress ( $\text{kg}/\text{cm}^2$ ) |
|--------|------------------------------------|--------|------------------------------------|
| 1      | 135.74                             | 14     | -487.17                            |
| 2      | -211.50                            | 15     | 467.94                             |
| 3      | 383.53                             | 16     | -519.97                            |
| 4      | -960.02                            | 17     | 426.99                             |
| 5      | -427.34                            | 18     | -434.61                            |
| 6      | -1077.97                           | 19     | 629.03                             |
| 7      | 116.89                             | 20     | -1302.82                           |
| 8      | 168.65                             | 21     | -171.49                            |
| 9      | -1031.63                           | 22     | 241.71                             |
| 10     | 397.00                             | 23     | 63.21                              |
| 11     | 527.95                             | 24     | -1003.52                           |
| 12     | -143.53                            | 25     | -828.92                            |
| 13     | -379.01                            |        |                                    |

Table 7: The displacements of truss nodes for twenty five-bar truss

| Node | $\Delta x$ (cm) | $\Delta y$ (cm) | $\Delta z$ (cm) |
|------|-----------------|-----------------|-----------------|
| 1    | 0.8274          | -0.8900         | -0.4687         |
| 2    | 0.8649          | -0.8806         | -0.4616         |
| 3    | 0.6701          | -0.4239         | -0.1880         |
| 4    | 0.6316          | -0.3932         | -0.1366         |



|    |        |         |         |
|----|--------|---------|---------|
| 5  | 0.6353 | -0.6123 | -0.1985 |
| 6  | 0.7368 | -0.5887 | -0.2560 |
| 7  | 0.0000 | 0.0000  | 0.0000  |
| 8  | 0.0000 | 0.0000  | 0.0000  |
| 9  | 0.0000 | 0.0000  | 0.0000  |
| 10 | 0.0000 | 0.0000  | 0.0000  |

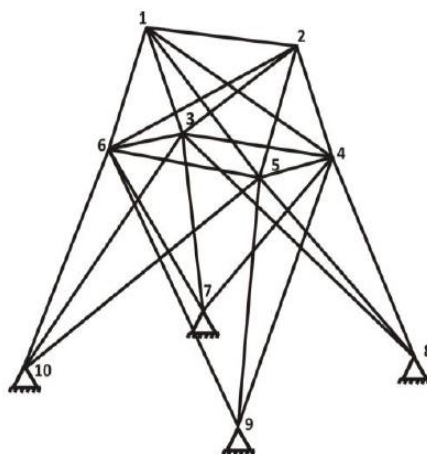


Figure 9. The optimum geometry of twenty five-bar truss

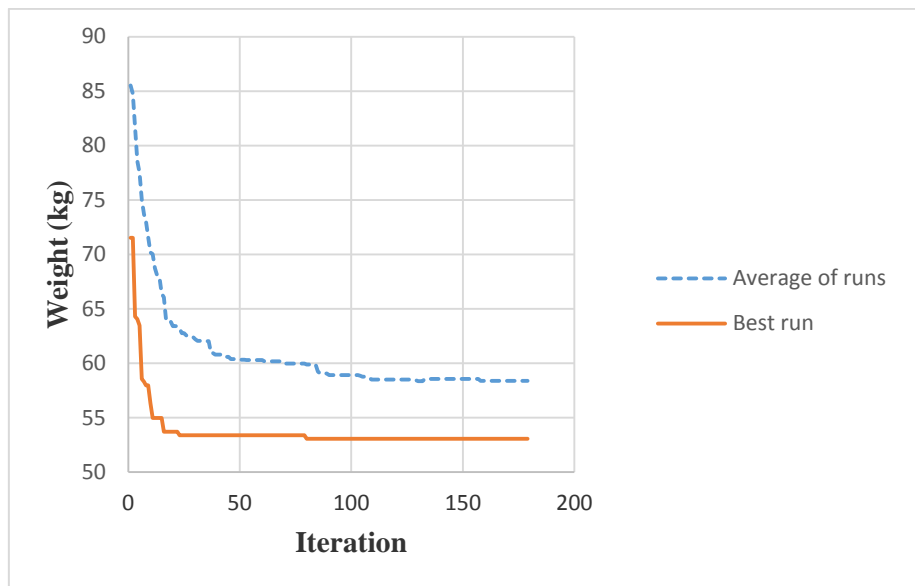


Figure 10. The weight convergence process of twenty five-bar truss

### 7.2 Eighteen-bar truss

An eighteen-bar truss as Fig. 11 has been considered. The data to design truss has been shown in Table 8. Modulus of elasticity of the material is 10000 ksi and the specific weight is 0.1 lb/in<sup>3</sup>. The results of size and geometry optimization of the truss have been compared

with other references in Table 9. The stresses of the elements have been shown in Table 10. The optimum geometry of the truss has been shown in Fig. 12. Also, the convergence process of truss weight has been shown in Fig. 13.

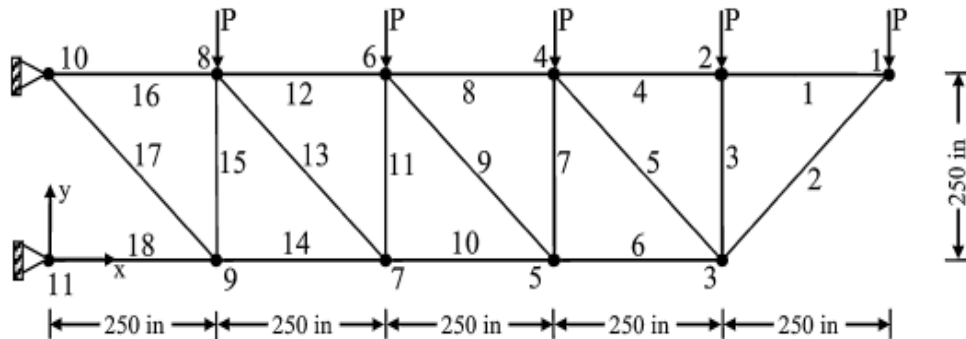


Figure 11. Eighteen-bar truss

Table 8: The data to design eighteen-bar truss

|  | Node   | $F_x$ (Kips) | $F_y$ (Kips) | $F_z$ (Kips) |
|--|--|--------------|--------------|--------------|
| Loading data                                   | 1  | 0            | -20          | 0            |
|  | 2  | 0            | -20          | 0            |
|  | 4  | 0            | -20          | 0            |
|  | 6  | 0            | -20          | 0            |
|  | 8  | 0            | -20          | 0            |
| Design variables                               | Size variables   |              |              |              |
|  | $A_1 = A_4 = A_8 = A_{12} = A_{16}; A_2 = A_6 = A_{10} = A_{14} = A_{18};$ |              |              |              |
|  | $A_3 = A_7 = A_{11} = A_{15}; A_5 = A_9 = A_{13} = A_{17}$                 |              |              |              |
| Constraint data                                | Geometry variables   |              |              |              |
|  | $x_3; y_3; x_5; y_5; x_7; y_7; x_9; y_9$                                   |              |              |              |
|  | Stress constraints   |              |              |              |
|  | $(\sigma_i)_i \leq 20 \text{ Ksi}; \quad i=1,2,\dots,18$                   |              |              |              |
|  | $ (\sigma_c)_i  \leq 20 \text{ Ksi}; \quad i=1,2,\dots,18$                 |              |              |              |
|  | Euler buckling stress constraints  |              |              |              |
|  | $ (\sigma_c)_i  \leq \alpha A_i E / L_i^2, \alpha=4; \quad i=1,2,\dots,18$ |              |              |              |
|  | Side constraints of geometry variables                                     |              |              |              |
|  | $775 \text{ in} \leq x_3 \leq 1225 \text{ in}$                             |              |              |              |
|  | $-225 \text{ in} \leq y_3 \leq 245 \text{ in}$                             |              |              |              |
| $525 \text{ in} \leq x_5 \leq 975 \text{ in}$  |  |              |              |              |
| $-225 \text{ in} \leq y_5 \leq 245 \text{ in}$ |  |              |              |              |
| $275 \text{ in} \leq x_7 \leq 725 \text{ in}$  |  |              |              |              |
| $-225 \text{ in} \leq y_7 \leq 245 \text{ in}$ |  |              |              |              |
| $25 \text{ in} \leq x_9 \leq 475 \text{ in}$   |  |              |              |              |
| $-225 \text{ in} \leq y_9 \leq 245 \text{ in}$ |  |              |              |              |
| List of the available profiles                 | $A_i \in S = \{2.00, 2.25, \dots, 21.50, 21.75\} \text{ in}^2$             |              |              |              |

Table 9: The optimization results of the size and geometry of eighteen-bar truss

| Design variables                     |                | Rajeev and Krishnamoorthy [23] | Yang [24] | Kang and Zong [25] | Present work |
|--------------------------------------|----------------|--------------------------------|-----------|--------------------|--------------|
| Size variables<br>(in <sup>2</sup> ) | A <sub>1</sub> | 12.50                          | 12.61     | 12.65              | 12.50        |
|                                      | A <sub>2</sub> | 16.25                          | 18.10     | 7.22               | 17.50        |
|                                      | A <sub>3</sub> | 8.00                           | 5.47      | 6.17               | 5.75         |
|                                      | A <sub>5</sub> | 4.00                           | 3.54      | 3.55               | 3.75         |
|                                      | x <sub>3</sub> | 891.90                         | 914.50    | 903.10             | 907.2491     |
| Geometry variables (in)              | y <sub>3</sub> | 145.30                         | 183.00    | 174.30             | 179.8672     |
|                                      | x <sub>5</sub> | 610.60                         | 647.00    | 630.30             | 636.7873     |
|                                      | y <sub>5</sub> | 118.20                         | 147.40    | 136.30             | 141.8272     |
|                                      | x <sub>7</sub> | 385.40                         | 414.20    | 402.10             | 407.9442     |
|                                      | y <sub>7</sub> | 72.50                          | 100.40    | 90.50              | 94.05591     |
|                                      | x <sub>9</sub> | 184.40                         | 200.00    | 195.30             | 198.7897     |
| Weight (Ib)                          |                | 23.40                          | 31.90     | 30.60              | 29.5158      |
|                                      |                | 4616.80                        | 4552.80   | 4515.6             | 4512.262     |

Table 10: The stresses of the elements of eighteen-bar truss

| Member | Stress (Ib/in <sup>2</sup> ) | Member | Stress (Ib/in <sup>2</sup> ) |
|--------|------------------------------|--------|------------------------------|
| 1      | 7819.47                      | 10     | -12808                       |
| 2      | -5701.06                     | 11     | -6906.20                     |
| 3      | -5767.01                     | 12     | 19168.13                     |
| 4      | 9935.48                      | 13     | 1753.73                      |
| 5      | 11214.86                     | 14     | -14608.41                    |
| 6      | -9383.01                     | 15     | -4395.81                     |
| 7      | -9347.36                     | 16     | 19999.99                     |
| 8      | 16116.99                     | 17     | 19911.73                     |
| 9      | 6103.33                      | 18     | -17330.79                    |

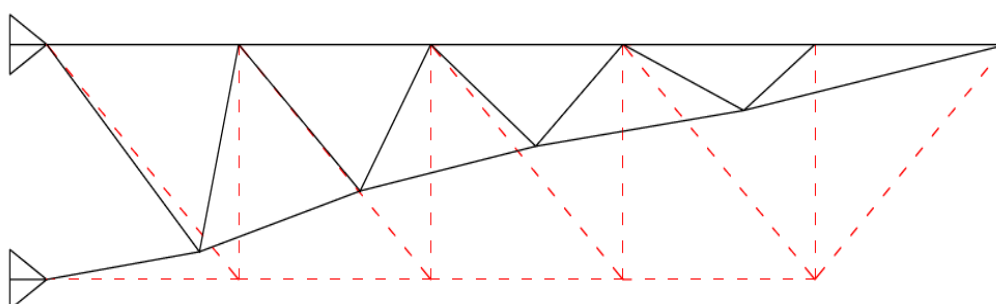


Figure 12. The optimum geometry of eighteen-bar truss

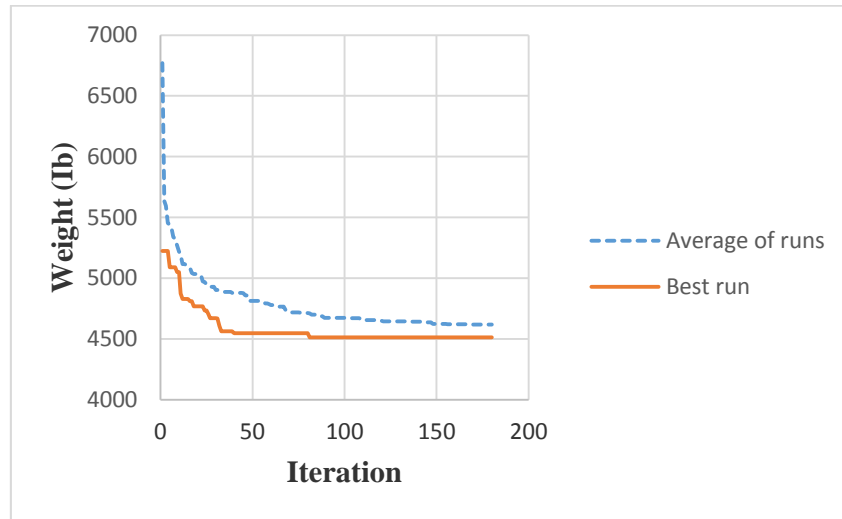


Figure 13. The weight convergence process of eighteen-bar truss

### 7.3 Fifteen-bar truss

A fifteen-bar truss as Fig. 14 has been considered. The data to design truss has been shown in Table 11. Modulus of elasticity of the material is 10000 ksi and the specific weight is 0.1 lb/in<sup>3</sup>. The results of size and geometry optimization of the truss have been compared with other references in Table 12. The stresses of the elements have been shown in Table 13. The optimum geometry of the truss has been shown in Fig. 15. Also, the convergence process of truss weight has been shown in Fig. 16.

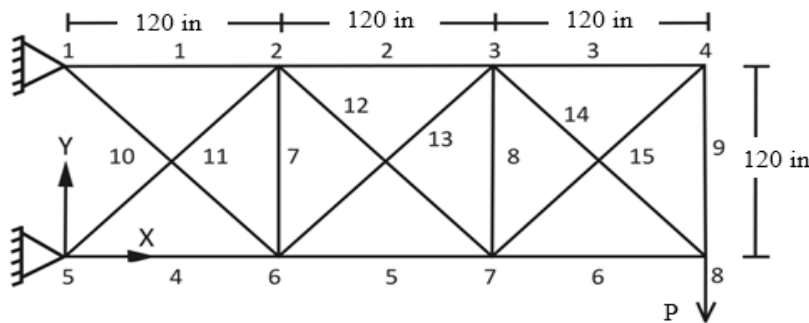


Figure 14. Fifteen-bar truss

Table 11: The data to design fifteen-bar truss

| Loading data     | Node  | $F_x$ (Kips) | $F_y$ (Kips) | $F_z$ (Kips) |
|------------------|---|--------------|--------------|--------------|
|                  | 8   | 0            | -10          | 0            |
| Design variables | Size variables  |              |              |              |
|                  | $A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8; A_9; A_{10}; A_{11}; A_{12}; A_{13}; A_{14}; A_{15}$ |              |              |              |
|                  | Geometry variables  |              |              |              |
|                  | $x_2 = x_6; x_3 = x_7; y_2; y_3; y_4; y_6; y_7; y_8$  |              |              |              |

|                                |   |
|--------------------------------|---|
| Constraint data                | Stress constraints  |
|                                | $(\sigma_i)_i \leq 25 \text{ Ksi}; \quad i=1,2,\dots,15$<br>$ (\sigma_c)_i  \leq 25 \text{ Ksi}; \quad i=1,2,\dots,15$  |
| List of the available profiles | Side constraints of geometry variables  |
|                                | $100 \text{ in} \leq x_2 \leq 140 \text{ in}$   |
|                                | $220 \text{ in} \leq x_3 \leq 260 \text{ in}$   |
|                                | $100 \text{ in} \leq y_2 \leq 140 \text{ in}$   |
|                                | $100 \text{ in} \leq y_3 \leq 140 \text{ in}$   |
|                                | $50 \text{ in} \leq y_4 \leq 90 \text{ in}$   |
|                                | $-20 \text{ in} \leq y_6 \leq 20 \text{ in}$  |
|                                | $-20 \text{ in} \leq y_7 \leq 20 \text{ in}$  |
|                                | $20 \text{ in} \leq y_8 \leq 60 \text{ in}$   |
|                                | $A_i \in S = \{0.111, 0.141, 0.174, 0.22, 0.27, 0.287, 0.347, 0.44, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.8, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.3, 10.85, 13.33, 14.29, 17.17, 19.18\} \text{in}^2$ |

Table 12: The optimization results of the size and geometry of fifteen-bar truss

| Design variables                  | Wu and Chow [26] | Hwang and He [27] | Tang et al [28] | Present work |          |
|-----------------------------------|------------------|-------------------|-----------------|--------------|----------|
| Size variables (in <sup>2</sup> ) | $A_1$            | 1.174             | 0.954           | 1.081        | 1.081    |
|                                   | $A_2$            | 0.954             | 1.081           | 0.539        | 0.539    |
|                                   | $A_3$            | 0.44              | 0.44            | 0.287        | 0.27     |
|                                   | $A_4$            | 1.333             | 1.174           | 0.954        | 0.954    |
|                                   | $A_5$            | 0.954             | 1.488           | 0.954        | 0.954    |
|                                   | $A_6$            | 0.174             | 0.27            | 0.22         | 0.22     |
|                                   | $A_7$            | 0.44              | 0.27            | 0.111        | 0.111    |
|                                   | $A_8$            | 0.44              | 0.347           | 0.111        | 0.111    |
|                                   | $A_9$            | 1.081             | 0.22            | 0.287        | 0.27     |
|                                   | $A_{10}$         | 1.333             | 0.44            | 0.22         | 0.287    |
|                                   | $A_{11}$         | 0.174             | 0.22            | 0.44         | 0.44     |
|                                   | $A_{12}$         | 0.174             | 0.44            | 0.44         | 0.287    |
|                                   | $A_{13}$         | 0.347             | 0.347           | 0.111        | 0.141    |
|                                   | $A_{14}$         | 0.347             | 0.27            | 0.22         | 0.27     |
|                                   | $A_{15}$         | 0.44              | 0.22            | 0.347        | 0.27     |
| Geometry variables (in)           | $x_2$            | 123.189           | 118.346         | 133.612      | 123.5286 |
|                                   | $x_3$            | 231.595           | 225.209         | 234.752      | 239.1095 |
|                                   | $y_2$            | 107.189           | 119.046         | 100.449      | 123.7912 |
|                                   | $y_3$            | 119.175           | 105.086         | 104.738      | 115.2112 |
|                                   | $y_4$            | 60.462            | 63.375          | 73.762       | 72.968   |
|                                   | $y_6$            | 16.728            | -20.0           | -10.067      | -8.153   |
|                                   | $y_7$            | 15.565            | -20.0           | -1.339       | 3.8959   |
|                                   | $y_8$            | 36.645            | 57.722          | 50.402       | 42.6028  |
| Weight (lb)                       | 120.52           | 104.573           | 79.82           | 79.8065      |          |

Table 13: The stresses of the elements of fifteen-bar truss

| Member | Stress (lb/in <sup>2</sup> ) | Member | Stress (lb/in <sup>2</sup> ) |
|--------|------------------------------|--------|------------------------------|
| 1      | 23249.16                     | 9      | 19291.09                     |
| 2      | 24547.4                      | 10     | 24498.17                     |
| 3      | 22192.73                     | 11     | -23161.28                    |
| 4      | -23953                       | 12     | 23733.03                     |
| 5      | -16427.66                    | 13     | -24216.56                    |
| 6      | -24835.33                    | 14     | 22481.39                     |
| 7      | 5040.43                      | 15     | -24129.05                    |
| 8      | -14690.79                    |        |                              |

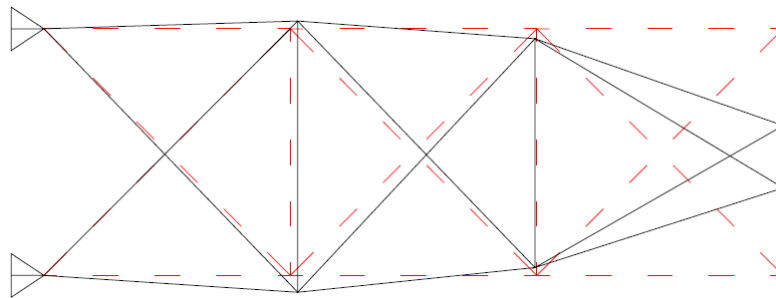


Figure 15. The optimum geometry of fifteen-bar truss

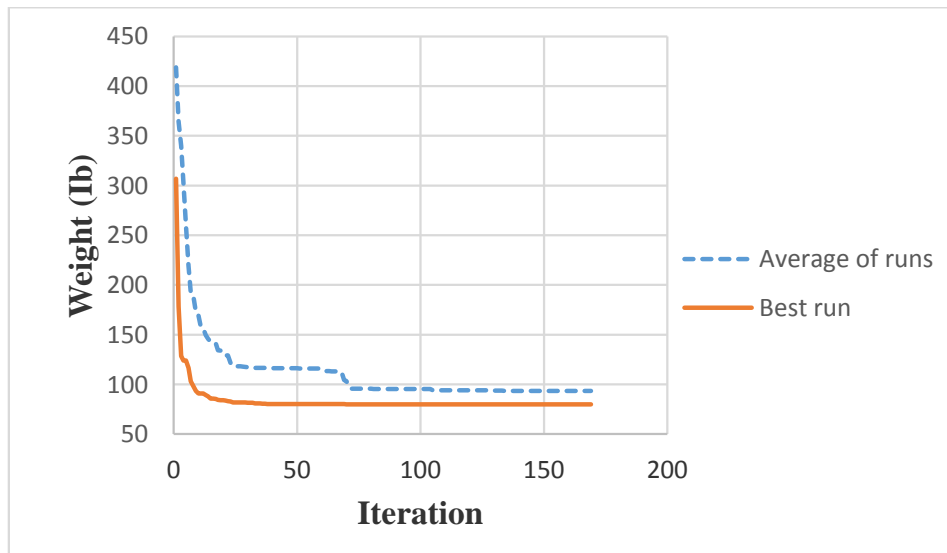


Figure 16. The weight convergence process of fifteen-bar truss

#### 7.4 Thirty nine-bar truss

A thirty nine-bar truss as Fig. 17 has been considered. The bottom and top nodes are fixed while all the intermediate node positions will be redesigned. The coordinates of three bottom nodes 1, 2, 3 and three top nodes 13, 14 and 15 have been shown in Table 14. The grouping of truss elements as the size variables has been shown in Table 15. The data to design truss

has been shown in Table 16. Modulus of elasticity of the material is 210 Gpa and the specific weight is 7800 kg/m<sup>3</sup>. The results of size and geometry optimization of the truss have been compared with other references in Table 17. The stresses of the elements have been shown in Table 18. The displacements of truss nodes have been shown in Table 19. The optimum geometry of the truss has been shown in Fig. 18. Also, the convergence process of truss weight has been shown in Fig. 19.

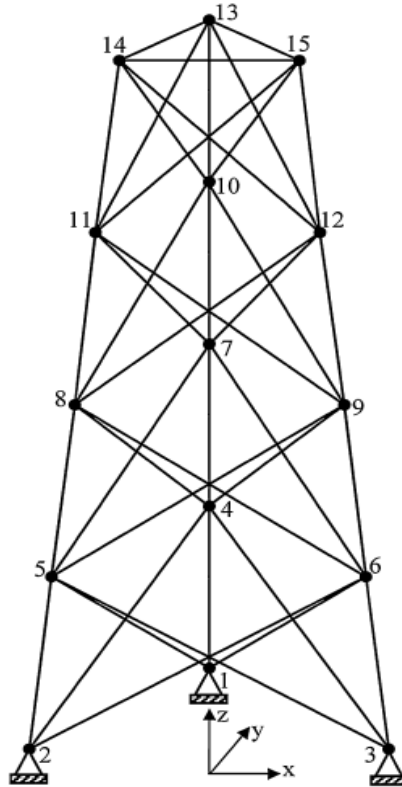


Figure 17. Thirty nine-bar truss

Table 14: The coordinates of truss fixed nodes for thirty nine-bar truss

| Node | $x(m)$                   | $y(m)$ | $z(m)$ |
|------|--------------------------|--------|--------|
| 1    | 0                        | 1      | 0      |
| 2    | $-\frac{\sqrt{3}}{2}$    | -0.5   | 0      |
| 3    | $\frac{\sqrt{3}}{2}$     | -0.5   | 0      |
| 13   | 0                        | 0.28   | 4      |
| 14   | $-\frac{0.42}{\sqrt{3}}$ | -0.14  | 4      |
| 15   | $\frac{0.42}{\sqrt{3}}$  | -0.14  | 4      |

Table 15: The grouping of truss elements for thirty nine-bar truss

| Group | Members (end nodes)             |
|-------|---------------------------------|
| $A_1$ | 1(1,4), 2(2,5), 3(3,6)          |
| $A_2$ | 4(4,7), 5(5,8), 6(6,9)          |
| $A_3$ | 7(7,10), 8(8,11), 9(9,12)       |
| $A_4$ | 10(10,13), 11(11,14), 12(12,15) |
| $A_5$ | Rest of the elements            |

Table 16: The data to design thirty nine-bar truss

|   | Node  | $F_x$ (KN) | $F_y$ (KN) | $F_z$ (KN) |
|---|---|------------|------------|------------|
| Loading data                              | 13  | 0          | 10         | 0          |
|   | 14  | 0          | 10         | 0          |
|   | 15  | 0          | 10         | 0          |
| Design variables                          | Size variables  |            |            |            |
|   | $A_1; A_2; A_3; A_4; A_5$                                   |            |            |            |
|   | Geometry variables  |            |            |            |
| Constraint data                           | $y_4; z_4; y_7; z_7; y_{10}; z_{10}$                        |            |            |            |
|   | Stress constraints  |            |            |            |
|   | $(\sigma_i)_i \leq 240 \text{ Mpa}; \quad i=1,2,\dots,39$   |            |            |            |
|   | $ (\sigma_c)_i  \leq 240 \text{ Mpa}; \quad i=1,2,\dots,39$ |            |            |            |
|   | Displacement constraints of node13 in direction y           |            |            |            |
|   | $ \Delta_{y,13}  \leq 4 \text{ mm};$                        |            |            |            |
|   | Side constraints of geometry variables                      |            |            |            |
|   | $0.28\text{m} \leq y_4 \leq 1\text{m}$                      |            |            |            |
|   | $0 \leq z_4 \leq 2\text{m}$                                 |            |            |            |
|   | $0.28\text{m} \leq y_7 \leq 1\text{m}$                      |            |            |            |
| $1\text{m} \leq z_7 \leq 3\text{m}$       |   |            |            |            |
| $0.28\text{m} \leq y_{10} \leq 1\text{m}$ |   |            |            |            |
| $2\text{m} \leq z_{10} \leq 4\text{m}$    |   |            |            |            |
| List of the available profiles            | $A_i \in S = \{0.1, 0.2, 0.3, \dots, 13\} \text{cm}^2$      |            |            |            |

Table 17: The optimization results of the size and geometry of thirty nine-bar truss

| Design variables                    | Wang et al [29] | Present work |        |
|-------------------------------------|-----------------|--------------|--------|
| Size variables<br>( $\text{cm}^2$ ) | $A_1$           | 11.01        | 12.7   |
|                                     | $A_2$           | 8.63         | 10.4   |
|                                     | $A_3$           | 6.69         | 7.2    |
|                                     | $A_4$           | 4.11         | 3.3    |
|                                     | $A_5$           | 4.37         | 1.7    |
| Geometry<br>variables (m)           | $y_4$           | 0.805        | 0.8894 |
|                                     | $z_4$           | 1.186        | 1.3405 |
|                                     | $y_7$           | 0.654        | 0.6816 |
|                                     | $z_7$           | 2.204        | 2.312  |
|                                     | $y_{10}$        | 0.466        | 0.4824 |
|                                     | $z_{10}$        | 3.092        | 3.3029 |
| Weight (kg)                         | 203.18          | 137.826      |        |



Table 18: The stresses of the elements of thirty nine-bar truss

| Member | Stress (kg/cm <sup>2</sup> ) | Member | Stress (kg/cm <sup>2</sup> ) |
|--------|------------------------------|--------|------------------------------|
| 1      | -567.45                      | 21     | 251.66                       |
| 2      | 283.74                       | 22     | 17.92                        |
| 3      | 283.74                       | 23     | 251.66                       |
| 4      | -535.83                      | 24     | 17.92                        |
| 5      | 267.89                       | 25     | -429.24                      |
| 6      | 267.89                       | 26     | -429.24                      |
| 7      | -557.11                      | 27     | 438.43                       |
| 8      | 278.62                       | 28     | -9.59                        |
| 9      | 278.62                       | 29     | 438.43                       |
| 10     | -483.88                      | 30     | -9.59                        |
| 11     | 241.99                       | 31     | -552.56                      |
| 12     | 241.99                       | 32     | -552.56                      |
| 13     | -379.70                      | 33     | 624.65                       |
| 14     | -379.70                      | 34     | -72.02                       |
| 15     | 557.91                       | 35     | 624.65                       |
| 16     | -178.43                      | 36     | -72.02                       |
| 17     | 557.91                       | 37     | -200.87                      |
| 18     | -178.43                      | 38     | -200.87                      |
| 19     | -269.32                      | 39     | 401.75                       |
| 20     | -269.32                      |        |                              |

Table 19: The displacements of truss nodes for thirty nine-bar truss

| Node | $\Delta x$ (cm) | $\Delta y$ (cm) | $\Delta z$ (cm) |
|------|-----------------|-----------------|-----------------|
| 1    | 0.0000          | 0.0000          | 0.0000          |
| 2    | 0.0000          | 0.0000          | 0.0000          |
| 3    | 0.0000          | 0.0000          | 0.0000          |
| 4    | 0.0000          | 0.1119          | -0.0272         |
| 5    | 0.0360          | 0.0494          | 0.0136          |
| 6    | -0.0360         | 0.0494          | 0.0136          |
| 7    | 0.0000          | 0.1538          | -0.0442         |
| 8    | -0.0210         | 0.1902          | 0.0221          |
| 9    | 0.0210          | 0.1902          | 0.0221          |
| 10   | 0.0000          | 0.3027          | -0.0416         |
| 11   | 0.0271          | 0.2558          | 0.0208          |
| 12   | -0.0271         | 0.2558          | 0.0208          |
| 13   | 0.0000          | 0.3998          | -0.0308         |
| 14   | -0.0046         | 0.4000          | 0.0154          |
| 15   | 0.0046          | 0.4000          | 0.0154          |

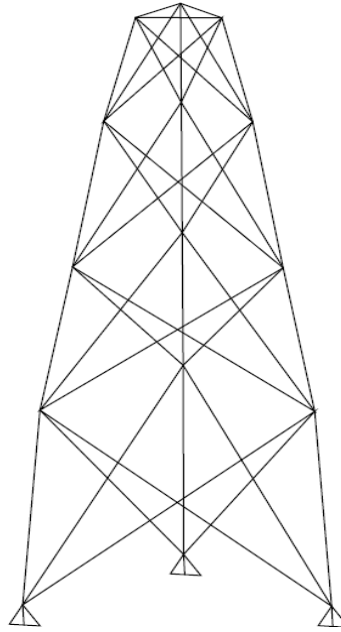


Figure 18. The optimum geometry of thirty nine-bar truss

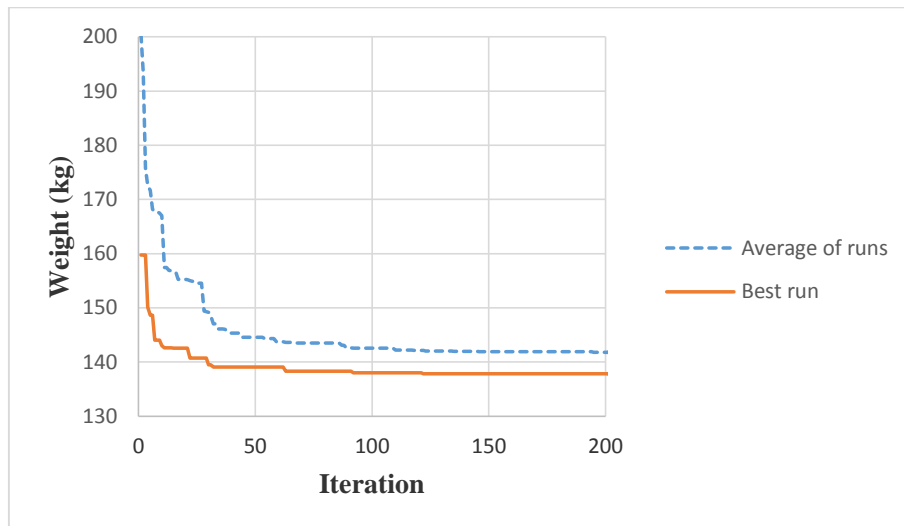


Figure 19. The weight convergence process of thirty nine-bar truss

### 7.5 Forty seven-bar truss

A forty seven-bar truss as Fig. 20 has been considered. The data to design truss has been shown in Table 20. Modulus of elasticity of the material is 30000 ksi and the specific weight is  $0.3 \text{ lb/in}^3$ . The results of size and geometry optimization of the truss have been compared with other references in Table 21. The stresses of the elements have been shown in Table 22. The optimum geometry of the truss has been shown in Fig. 21. Also, the convergence process of truss weight has been shown in Fig. 22.

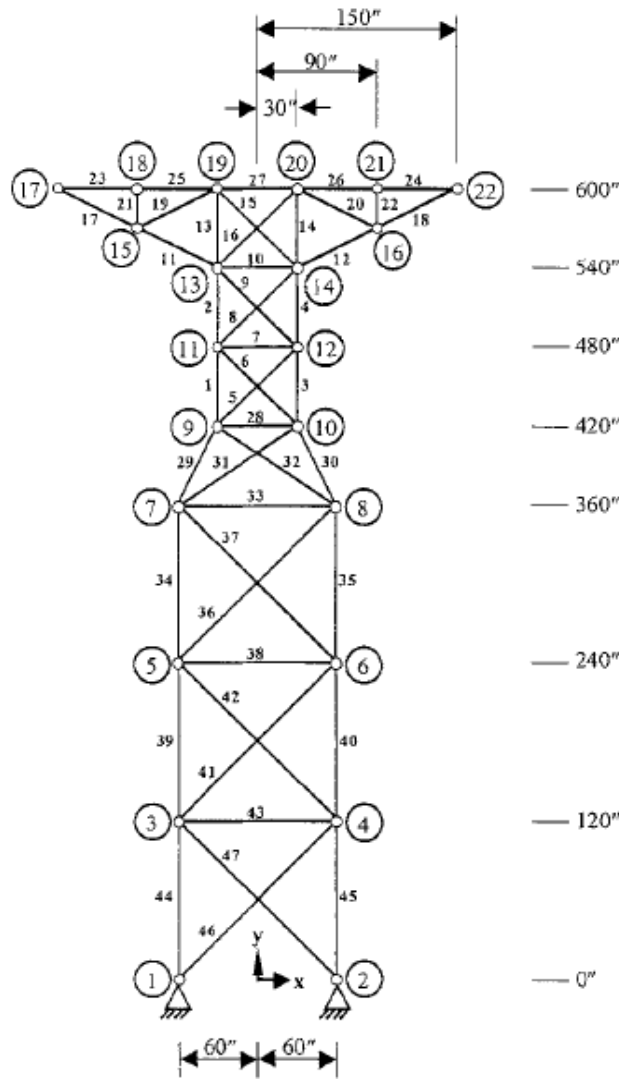


Figure 20. Forty seven-bar truss

Table 20: The data to design forty seven-bar truss

|                  | Node  | $F_x$ (Kips) | $F_y$ (Kips) | $F_z$ (Kips) |
|------------------|---|--------------|--------------|--------------|
| Loading data     | 17  | 6            | -14          | 0            |
|                  | 22  | 6            | -14          | 0            |
| Design variables | Size variables  |              |              |              |
|                  | $A_3 = A_1; A_4 = A_2; A_5 = A_6; A_7; A_8 = A_9; A_{10}; A_{12} = A_{11};$                             |              |              |              |
|                  | $A_{14} = A_{13}; A_{15} = A_{16}; A_{18} = A_{17}; A_{20} = A_{19}; A_{22} = A_{21}; A_{24} = A_{23};$ |              |              |              |
|                  | $A_{26} = A_{25}; A_{27}; A_{28}; A_{30} = A_{29}; A_{31} = A_{32}; A_{33}; A_{35} = A_{34}; A_{36} =$  |              |              |              |
|                  | $A_{37}; A_{38}; A_{40} = A_{39}; A_{41} = A_{42}; A_{43};$   |              |              |              |
|                  | $A_{45} = A_{44}; A_{46} = A_{47}$  |              |              |              |
|                  | Geometry variables  |              |              |              |
|                  | $x_2 = -x_1; x_4 = -x_3; y_4 = y_3; x_6 = -x_5; y_6 = y_5; x_8 = -x_7; y_8 = y_7;$                      |              |              |              |
|                  | $x_{10} = -x_9; y_{10} = y_9; x_{12} = -x_{11}; y_{12} = y_{11}; x_{14} = -x_{13}; y_{14} = y_{13};$    |              |              |              |

|                                |   |
|--------------------------------|---|
|                                | $x_{20}=-x_{19}; y_{20}=y_{19}; x_{21}=-x_{18}; y_{21}=y_{18}$              |
|                                | Stress constraints  |
|                                | $(\sigma_c)_i \leq 20 \text{ Ksi}; \quad i=1,2,\dots,47$                    |
|                                | $ (\sigma_c)_i  \leq 15 \text{ Ksi}; \quad i=1,2,\dots,47$                  |
|                                | Euler buckling stress constraints   |
|                                | $ (\sigma_c)_i  \leq \alpha A_i E/L_i^2, \alpha=3.96; \quad i=1,2,\dots,47$ |
|                                | Side constraints of geometry variables                                      |
|                                | $0 \leq x_2 \leq 150 \text{ in}$  |
|                                | $0 \leq x_4 \leq 150 \text{ in}$  |
|                                | $0 \leq y_4 \leq 240 \text{ in}$  |
|                                | $0 \leq x_6 \leq 150 \text{ in}$  |
|                                | $120 \text{ in} \leq y_6 \leq 360 \text{ in}$                               |
| Constraint data                | $0 \leq x_8 \leq 150 \text{ in}$  |
|                                | $240 \text{ in} \leq y_8 \leq 420 \text{ in}$                               |
|                                | $0 \leq x_{10} \leq 75 \text{ in}$  |
|                                | $360 \text{ in} \leq y_{10} \leq 480 \text{ in}$                            |
|                                | $0 \leq x_{12} \leq 75 \text{ in}$  |
|                                | $420 \text{ in} \leq y_{12} \leq 540 \text{ in}$                            |
|                                | $0 \leq x_{14} \leq 75 \text{ in}$  |
|                                | $480 \text{ in} \leq y_{14} \leq 600 \text{ in}$                            |
|                                | $0 \leq x_{20} \leq 75 \text{ in}$  |
|                                | $540 \text{ in} \leq y_{20} \leq 660 \text{ in}$                            |
|                                | $0 \leq x_{21} \leq 150 \text{ in}$   |
|                                | $540 \text{ in} \leq y_{21} \leq 660 \text{ in}$                            |
| List of the available profiles | $A_i c S = \{0.1, 0.2, 0.3, \dots, 5.0\} \text{in}^2$                       |

Table 21: The optimization results of the size and geometry of forty seven-bar truss

| Design variables | Hasancebi and Erbatur [30] | Salajegheh and Vanderplaats [31] | Hansen and Vanderplaats [32] | Present work |
|------------------|----------------------------|----------------------------------|------------------------------|--------------|
| $A_3$            | 2.50                       | 2.61                             | 2.42                         | 2.70         |
| $A_4$            | 2.20                       | 2.56                             | 2.35                         | 2.50         |
| $A_5$            | 0.70                       | 0.69                             | 0.82                         | 0.70         |
| $A_7$            | 0.10                       | 0.47                             | 0.10                         | 0.10         |
| $A_8$            | 1.30                       | 0.80                             | 0.86                         | 0.90         |
| $A_{10}$         | 1.30                       | 1.13                             | 1.15                         | 1.10         |
| $A_{12}$         | 1.80                       | 1.71                             | 1.77                         | 1.80         |
| $A_{14}$         | 0.50                       | 0.77                             | 0.67                         | 0.70         |
| $A_{15}$         | 0.80                       | 1.09                             | 0.86                         | 0.90         |
| $A_{18}$         | 1.20                       | 1.34                             | 1.24                         | 1.30         |
| $A_{20}$         | 0.40                       | 0.36                             | 0.33                         | 0.30         |
| $A_{22}$         | 1.20                       | 0.97                             | 1.22                         | 1.10         |
| $A_{24}$         | 0.90                       | 1.00                             | 0.93                         | 1.00         |
| $A_{26}$         | 1.00                       | 1.03                             | 0.86                         | 0.90         |
| $A_{27}$         | 3.60                       | 0.88                             | 0.69                         | 0.80         |
| $A_{28}$         | 0.10                       | 0.55                             | 0.15                         | 0.10         |
| $A_{30}$         | 2.40                       | 2.59                             | 2.46                         | 2.70         |
| $A_{31}$         | 1.10                       | 0.84                             | 0.90                         | 0.80         |
| $A_{33}$         | 0.10                       | 0.25                             | 0.10                         | 0.10         |

|                         |          |         |         |        |          |
|-------------------------|----------|---------|---------|--------|----------|
|                         | $A_{35}$ | 2.70    | 2.86    | 2.74   | 3.00     |
|                         | $A_{36}$ | 0.80    | 0.92    | 0.92   | 0.90     |
|                         | $A_{38}$ | 0.10    | 0.67    | 0.1    | 0.10     |
|                         | $A_{40}$ | 2.80    | 3.06    | 2.94   | 3.20     |
|                         | $A_{41}$ | 1.30    | 1.04    | 1.13   | 1.00     |
|                         | $A_{43}$ | 0.20    | 0.10    | 0.10   | 0.10     |
|                         | $A_{45}$ | 3.00    | 3.13    | 3.12   | 3.30     |
|                         | $A_{46}$ | 1.20    | 1.12    | 1.10   | 1.20     |
|                         | $x_2$    | 114.00  | 107.76  | 107.10 | 100.9724 |
|                         | $x_4$    | 97.00   | 89.15   | 91.20  | 80.4772  |
|                         | $y_4$    | 125.00  | 137.98  | 122.80 | 136.8699 |
|                         | $x_6$    | 76.00   | 66.75   | 74.20  | 64.3908  |
|                         | $y_6$    | 261.00  | 254.47  | 241.40 | 247.0491 |
|                         | $x_8$    | 69.00   | 57.38   | 65.50  | 55.2589  |
|                         | $y_8$    | 316.00  | 342.16  | 324.60 | 338.4534 |
|                         | $x_{10}$ | 56.00   | 49.85   | 57.10  | 48.7333  |
| Geometry variables (in) | $y_{10}$ | 414.00  | 417.17  | 400.40 | 409.738  |
|                         | $x_{12}$ | 50.00   | 44.66   | 49.30  | 43.4742  |
|                         | $y_{12}$ | 463.00  | 475.35  | 472.30 | 472.1479 |
|                         | $x_{14}$ | 54.00   | 41.09   | 47.40  | 44.8349  |
|                         | $y_{14}$ | 524.00  | 513.15  | 507.50 | 512.1903 |
|                         | $x_{20}$ | 1.00    | 17.90   | 3.90   | 3.8416   |
|                         | $y_{20}$ | 587.00  | 597.92  | 586.50 | 591.1449 |
|                         | $x_{21}$ | 99.00   | 93.54   | 83.30  | 84.50416 |
|                         | $y_{21}$ | 631.00  | 623.94  | 636.00 | 630.3472 |
| Weight (lb)             |          | 1925.79 | 1900.00 | 1850.4 | 1860.161 |

Table 22: The stresses of the elements of forty seven-bar truss

| Member | Stress (lb/in <sup>2</sup> ) | Member | Stress (lb/in <sup>2</sup> ) | Member | Stress (lb/in <sup>2</sup> ) |
|--------|------------------------------|--------|------------------------------|--------|------------------------------|
| 1      | 2333.29                      | 17     | -14980.40                    | 33     | 4712.87                      |
| 2      | 791.45                       | 18     | -10016.58                    | 34     | 5644.96                      |
| 3      | -12931.78                    | 19     | -4489.07                     | 35     | -14885.74                    |
| 4      | -10973.86                    | 20     | 19865.83                     | 36     | 3728.92                      |
| 5      | 8021.24                      | 21     | -9476.08                     | 37     | -4488.52                     |
| 6      | -6709.07                     | 22     | -14644.85                    | 38     | -444.23                      |
| 7      | 8073.72                      | 23     | 12584.71                     | 39     | 6299.89                      |
| 8      | 4487.23                      | 24     | 19449.1                      | 40     | -14991.32                    |
| 9      | -11371.45                    | 25     | 12938.38                     | 41     | 959.16                       |
| 10     | -14278.88                    | 26     | 19995.67                     | 42     | -1750.75                     |
| 11     | -13684.85                    | 27     | 18745.22                     | 43     | 4377.84                      |
| 12     | -14414.45                    | 28     | -1248.42                     | 44     | 6506.11                      |
| 13     | 12992.18                     | 29     | 4510.74                      | 45     | -14998.30                    |
| 14     | -4257.07                     | 30     | -14885.74                    | 46     | 581.01                       |
| 15     | -3472.72                     | 31     | 5019.47                      | 47     | -975.10                      |
| 16     | 11865.98                     | 32     | -5976.77                     |        |                              |

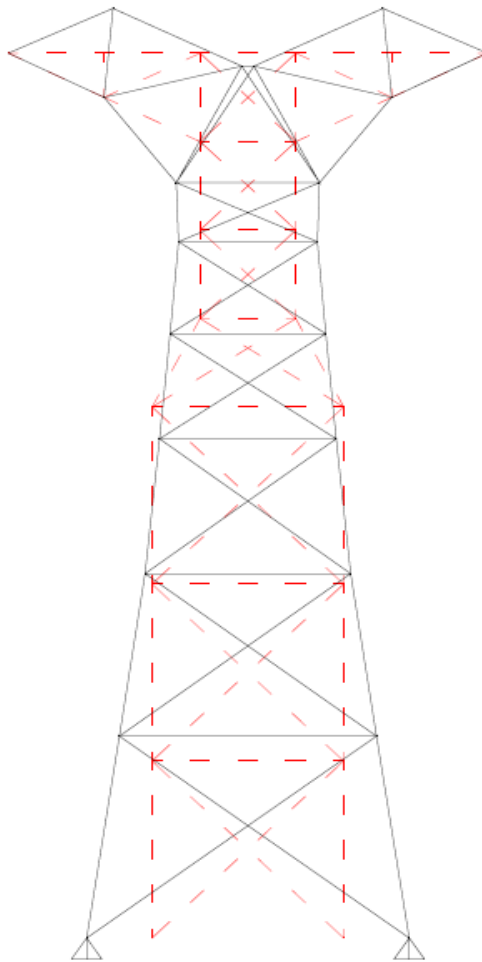


Figure 21. The optimum geometry of forty seven-bar truss

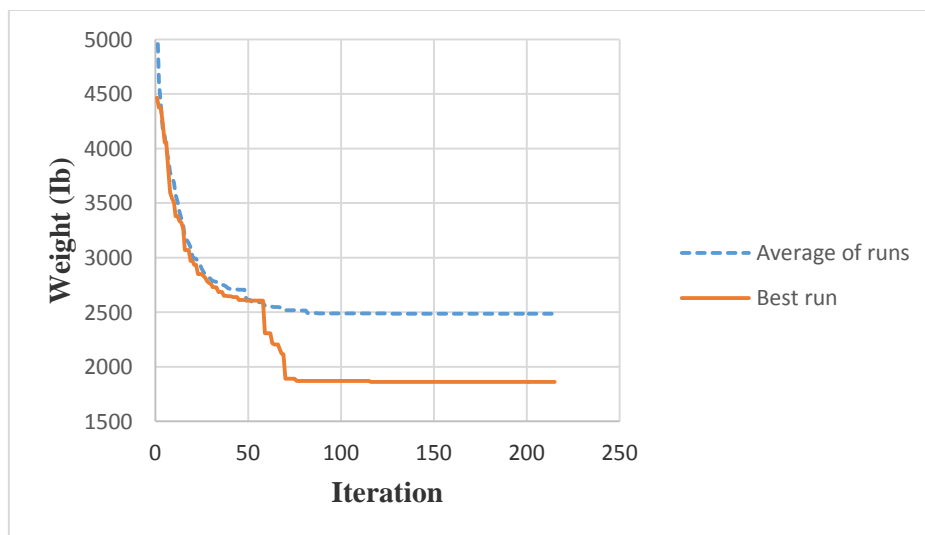


Figure 22. The weight convergence process of forty seven-bar truss

## 8. CONCLUSION

In this research, the combination of DNA computing algorithm and Generalized Convex Approximation (GCA) method has been used to optimize the size and geometry of truss structures under the constraints such as axial stresses of the elements, displacements in the degrees of freedom of truss nodes and slenderness ratios of the elements.

In this way, at the first stage by selecting appropriate and similar cross sections for truss elements, the coordinates of truss nodes as continuous variables have been optimized by Generalized Convex Approximation (GCA) method. Then, at the second stage with achieved coordinates from the first stage and by using DNA computing algorithm, the cross-section areas of the elements as discrete variables have been optimized.

According to the achieved results from numerical examples and comparing them with different references, it is concluded that the combination of DNA computing algorithm and Generalized Convex Approximation (GCA) method is an appropriate method to optimize truss structures.

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