

AN OPTIMIZATION PROCEDURE FOR CONCRETE BEAM-COLUMN JOINTS STRENGTHENED WITH FRP

V. Khashi¹, H. Dehghani^{*,†,2} and A.A Jahanara¹

¹Department of Civil Engineering, Islamic Azad University - Bam Branch, Bam, Iran

²Department of Civil Engineering, Higher Education Complex of Bam, Bam, Iran

ABSTRACT

This paper illustrates an optimization procedure of concrete beam-column joints subjected to shear that are strengthened with fiber reinforced polymer (FRP). For this aim, five different values have been considered for length, width and thickness of the FRP sheets which created 125 different models to strengthen of concrete beam-column joints. However, by using response surface methodology (RSM) in design expert software the number of these models is reduced to 20. Then, each of 20 models is simulated in ABAQUS finite element software and shear capacity is also determined. The relationship between different dimensions of the FRP sheets and shear capacity are specified by using RSM. Furthermore the optimum dimensions are determined by particle swarm optimization (PSO) algorithm.

Keywords: fiber reinforced polymer; beam-column joints; response surface methodology; optimization.

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1. INTRODUCTION

Repair and strengthening of existing concrete structures have become a major construction activity all over the world. Typical damaged concrete structure after an earthquake shows that the failure of beam-column joints is the major contributor for the collapse of concrete structures due to earthquake excitation. It needs for engineering approach to adopt efficient and economical methods to improve the joint performance. The use of FRP to strengthen has increased in popularity over the past few years. The lightweight and formability of FRP reinforcement make these systems easy to install. As the materials used in these systems are non-corrosive, non-magnetic, and generally resistant to chemicals, they are an excellent choice for external reinforcement [1, 2]. Due to the success of this technique, a number of

*Corresponding author: Department of Civil Engineering, Higher Education Complex of Bam, Bam, Iran

†E-mail address: hdehghani@bam.ac.ir (H. Dehghani)

researchers have investigated experimental, analytical and numerical the use of FRP materials for repair and strengthening of concrete beam-column joints [3-5]. Although the considerable advantages have been offered by the FRP sheets, there still exist significant challenges which must be solved. Among these challenges, the high cost is the most important. Thus, it is necessary to develop an optimization procedure to determine length, width and thickness of the FRP sheets which can be very helpful in reducing costs. In this regard, several studies have been performed for the optimization application of the structures strengthened with FRP which a number of them are mentioned in the following.

Perera *et al.* [6] were proposed a new method to estimate satisfactorily the shear capacity of reinforced concrete beams strengthened with FRP. Their model was based on an extension of the strut-and-tie models used for the shear strength design of RC beams to the case of shear strengthened beams with FRP. By the formulation of an optimization problem solved by using genetic algorithms, the optimal configuration of the strut-and-tie mechanism of an FRP shear strengthened RC beam was determined. Awad *et al.* [7] were provided a review on the available studies related to the design optimization of fibre composite structures used in civil engineering such as; plate, beam, box beam, sandwich panel, bridge girder, and bridge deck. Various optimization methods were presented and compared. In addition, the importance of using the appropriate optimization technique was discussed.

Bennegadi *et al.* [8] were developed a numerical model for the optimization of the external reinforcement of reinforced concrete beams by Hybrid Fiber Reinforced Polymer (HFRP) Plate. Their study provided a finite element method by ANSYS. Parametric study was made to evaluate both effects of height and width of the HFRP plate on the retrofitted beam. Finally, their model was used to optimize the volume of the HFRP plate which is bonded externally to the concrete beam. Numerical optimization of strengthening disturbed regions of dapped-end beams using near surface mounted and externally bonded carbon fiber-reinforced polymers was studied by Sas *et al.* [9]. Bruggi and Milani [10] studied the optimal FRP reinforcement of masonry walls out-of-plane loaded. Topology optimization was then applied to the investigation of the optimal reinforcement of plain and windowed panels, comparing the conventional energy based method and the proposed stress-based approach. The present paper attempts to study numerical analysis and to propose a procedure to optimize the dimensions of the FRP sheets bonded externally to a concrete beam-column joint. Then finite element modeling is integrated into the commercial software ABAQUS to determine the shear capacity. The PSO algorithm is carried out to obtain optimal dimensions of the FRP sheets used for concrete beam-column joint. To achieve this aim, the relationships between dimensions of the FRP sheets and shear capacity have been determined, and then these relationships are optimized.

2. RESPONSE SURFACE METHOD

Response surface methodology (RSM) method is introduced by Box and Wilson [11]. Generally, response surface methodology is a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response. RSM has been successfully applied in various fields such as in the chemical industry, computer

simulation and concrete industry for optimization and processing purposes [12]. During recent decades, Researchers began to identify the use of approximation concepts as a device to reduce the number of structural analyses. RSM, as a robust global approximation method, is more capable of satisfactorily predicting structural response over a wide range of design space. It has been reported as a potentially useful approach which is able to provide a suitable functional relationship between the responses and the factors (i.e. input parameters). The function can be expressed as follow,

$$y = f(x_1, x_2, \dots, x_n) + e \quad (1)$$

where f is called response surface or response function. x_1, x_2, \dots, x_n are quantitative process variables and e measures the experimental error. In this paper, RSM is used to establish the relationships between dimensions of the FRP sheets and shear capacity. Representing the shear capacity by R , the response is a function of length (L), width (W) and thickness (T) of FRP sheets, and it can be expressed as follow,

$$R = f(L, W, T) \quad (2)$$

The general quadratic response-surface model, used to evaluate the parametric effects is as follows [13]:

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i=1}^k \sum_{j=1}^k \beta_{ij} x_i x_j + e_i \quad (3)$$

where k is the number of factors studied and optimized in the experiment, β_0 is the coefficient for constant term and $\beta_i, \beta_{ii}, \beta_{ij}$ are the coefficients for linear, square and interaction terms respectively. For the convenience of recording and processing the experimental data, the upper and lower levels of the parameters are coded as +2 and -2. The coded value of any intermediate levels can be calculated by using the following expression.

$$x_i = \frac{[2x - (x_{max} + x_{min})]}{\left[\frac{x_{max} - x_{min}}{2} \right]} \quad (4)$$

where x_{max} is the upper level of the parameter, x_{min} is the lower level of the parameter and x_i is the required coded values of the parameter of any value of X from x_{min} to x_{max} . In the present work, Design Expert software (Stat-Ease Inc., Minneapolis, USA) is used to perform these calculations.

3. PARTICLE SWARM OPTIMIZATION ALGORITHMS

Particle swarm optimization (PSO) is one of the most successful evolutionary algorithms used to solve multi-objective optimization problems because of its high speed of

convergence. The PSO algorithm is introduced by Kennedy and Eberhart [14] in the mid 1990s. PSO was first intended for simulating social behavior as a stylized representation of the movement of organisms, for example in a bird flock. A swarm of particles is considered, each particle represents a bird in search-space. The algorithm promotes the swarm to optimal solution by updating the position of particles based on their fitness. The algorithm initiates with a candidate group of random solutions, then by updating the position and velocity of particles, the algorithm searches for optimal solution in the space of problem. Each particle is characterized by X and V values which denote the position and velocity respectively. The position of the particles is the desired answer of our problem and their velocity implies the rate of their position variations. The larger velocity values suggest that the current position is not favorable and it has a noticeable distance to optimal position. In each movement of the swarm, position and velocity of each particle is updated based on local and global values.

The best local value, denoted as P_{best} , is the solution that has the most fitness, and is obtained individually for each particle. The best global position is the best value that is achieved among the whole particles and is denoted as G_{best} . The new velocity and position of the i th particle in the k th iteration are updated as follows,

$$v_i^{k+1} = wv_i^k + c_1r_1(p_i^k - x_i^k) + c_2r_2(p_g^k - x_i^k) \quad (5)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (6)$$

where v_i^k is the velocity vector in the k th iteration, r_1 and r_2 are two random numbers between one and ten, p_i^k stands for best position of the i th particle and p_g^k is the position of the best particle up to the k th iteration. c_1 and c_2 are personal and social learning factors, that are also called acceleration coefficients. c_1 and c_2 take the values between 1.5 and 2, but the best value for these two parameters is 2. w is the weight inertia parameter. For large values of w the velocity increases and the steps become larger, and as the w decreases the steps become smaller. This would be helpful for convergence to optimal solution in last steps. Therefore the constant value of w is replaced by following relationship,

$$w_{k+1} = w_{max} - \frac{w_{max} - w_{min}}{k_{max}} k \quad (7)$$

where k_{max} is the maximum number of iterations, w_{max} and w_{min} are equal to 0.9 and 0.4 respectively [15]. In this paper, the PSO algorithm is used to obtain an optimal volume for dimensions of the FRP sheets

4. SIMULATION BY FINITE ELEMENT METHOD

Before the optimization procedure, it is necessary to develop a numerical model for the strengthening concrete beam-column joints by FRP sheets. Because experimental tests require a great amount of time and cost, many research works have also been assigned to the development of various analytical methods as well as the finite element method, which is a very powerful numerical method for simulating FRP- strengthening reinforced concrete beam-column joints. The numerical model has been developed using the commercial

software ABAQUS.

There are several effective components of the behavior beam-column joints which are concrete columns, concrete beams, bars and FRP sheets. Three models (joint2, joint3 and joint4) of the current study were simulated from the beam-column joints of a moment-resisting frame reinforced concrete building. Fig. 1 shows the schematic view of these models. Details of beam and column section are also given in table 1.

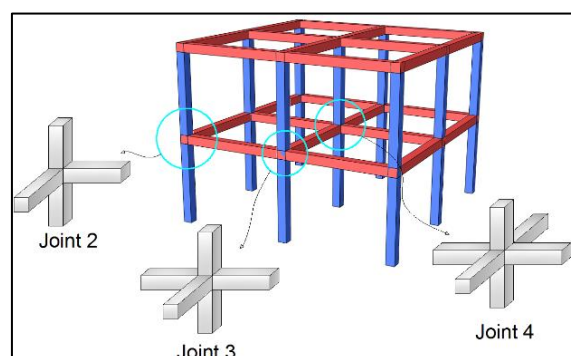
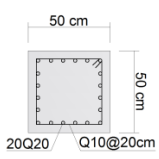
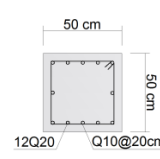


Figure 1. Schematic view of three models considered for beam-column joints

Table 1: Details of reinforced concrete column and beam element

Type of element	Column	Beam
Section		
Relative steel percent	2.5%	1.5%

FRP sheets dimensions were not considered exceed the width of beam. Change step of length and width of these sheets were considered 10 cm and change in thickness step was also considered 0.50 mm. five different values for each of the variables (length, width, and thickness) of the FRP sheet are considered, as shown in Fig. 2. In the following a detailed description of the modeling approach for each constitutive material in the FRP strengthened reinforced concrete beam-column joints is presented.

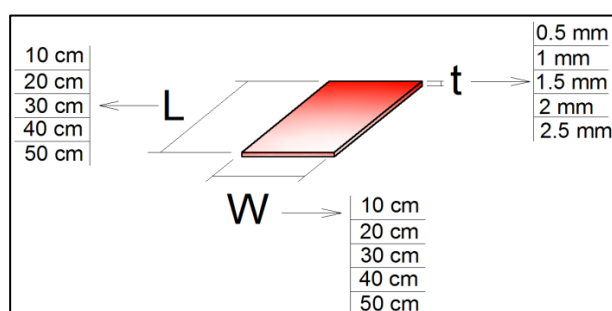


Figure 2. Variable dimensions of the FRP sheet

4.1 Concrete

Concrete is quasi brittle material and has very different behaviors in compression and tension.

The damage plasticity approach is adopted to model the concrete. Isotropic and linear elastic behavior of concrete both in compression and tension are defined using Young's modulus and Poisson's ratio. In this study, the pre-peak stress-strain of concrete was defined based on the Hognestad model [16], while the post peak behavior was based on the modified Kent and Park model [17]. Concrete is modeled using three dimensional eight node solid brick elements with three translational degrees of freedom at each node (C3D8R). Failure ratios for concrete used in the model are reported in Table 2.

Table 2: Parameters for finite element modeling

Dilation angle	Eccentricity	Biaxial to uniaxial compressive strength ratio	K	Viscosity parameter
36	0.1	1.16	0.67	0

4.2 Steel

Longitudinal and transverse reinforcements are modelled with three dimensional, two noded truss elements (T2D3). Elastic perfectly plastic stress strain relationship is assumed for steel under both compression and tension. Steel being an isotropic material, linear elastic behavior is defined by elastic modulus and Poisson's ratio. Perfectly plastic behavior is defined using any two points on the yield line in terms of inelastic strain and yield stress. The material properties of steel used in present study are listed in Table 3.

Table 3: Steel properties

Modulus of elasticity (E_s)	Poisson ratio (ν)	yield stress (f_{ys})	Density (ρ)	Ultimate stress (f_{us})	Ultimate strain (ϵ_{us})
200 Gpa	0.3	300 Mpa	7850 kg/m ³	400 Mpa	0.35

4.3 FRP

FRP is generally considered as transversely isotropic material which is a subset of an orthotropic material. Lamina behavior in ABAQUS defines transversely isotropic material that requires five constitutive constants to define stress strain relationship unlike nine constants in orthotropic material. FRP (S4R) is modeled using three dimensional shell element. The mechanical properties of FRP sheets are listed in Tables4.

Table 4: Summary of material properties of the FRP sheet

Longitudinal modulus of elasticity E_x (MPa)	Transverse modulus of elasticity E_y (MPa)	Poisson's ratio (ν)	Shear modulus G (MPa)
39600	3065.85	0.22	2075.14

4.4 Finite element meshing

In finite element modeling, a finer mesh typically results in a more accurate solution. However, as the mesh is made finer, the computation time increases. To manage the accuracy and computing resource, a mesh convergence study is performed. Initially, a mesh is created with arbitrary number of elements and the model is analyzed. Subsequently, the mesh is recreated with a denser element distribution and re-analyzed. The results obtained are compared to those of the previous mesh. The mesh density is increased repeatedly and the model is re-analyzed until the results converge satisfactorily. In this study, the models are meshed by 50 by 50 mm divisions for convergent studies as shown in the Fig. 3.

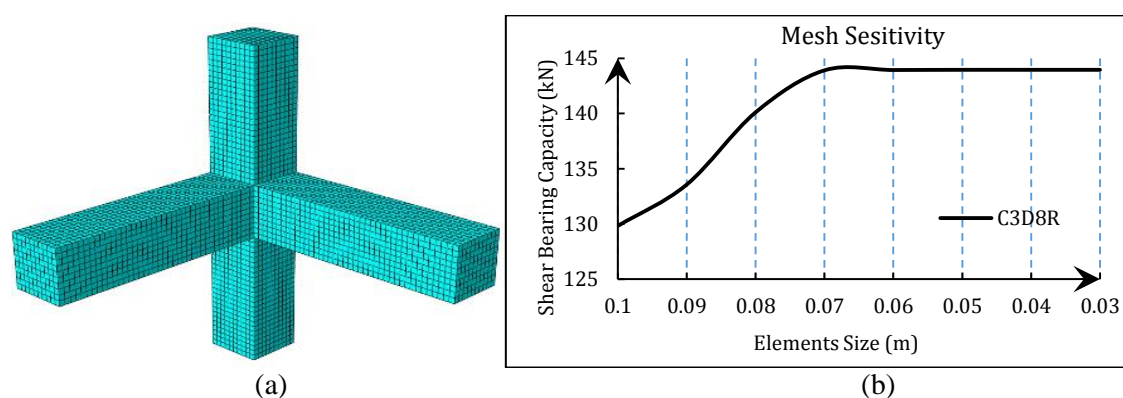


Figure 3. (a) Meshing of the model and (b) model sensitivity to the dimensions of elements

5. RESULTS AND DISCUSSIONS

Three models in the concrete frame structure, as shown in Fig. 1, are used for simulating FRP-strengthening reinforced concrete beam-column joints. In order to investigate the connections, the joints are extended to the column mid-height and beam mid-span. Also three parameters including length, width and thickness are considered as variables in simulation of FRP sheets. Five different values for each of the variables (length, width, and thickness) of the FRP sheet are considered, so that we have 125 models of FRP layout. Among these models, only 20 models have been selected by response surface method to reduce the number of tests. The proposed dimensions of FRP by response surface method are introduced by Design Expert software which is presented in table 5. The nominating of models is based on the name of joints (Joint2, Joint3, and Joint4) then the proposed FRP layout number of response surface method will be stated. For example model of Joint'3-FRP12' is related to side column with three beams and number 12 indicated the specific model which its' dimensions are shown in Table 5.

Schematic figure of proposed models of response surface method in ABACUS software have been simulated which are presented in Figs. 4-a to 4-c.

In numerical investigation, the analysis is stopped while the model is damaged. Load-displacement curve of model response was obtained. Due to large number of curve, only three different connections are shown in Fig. 5.

Table 5: Proposed FRP dimensions by response surface method

Model number	1	2	3	4	5	6	7	8	9	10
FRP length (mm)	30	30	30	20	20	30	30	30	10	30
FRP width (mm)	30	30	30	40	20	30	30	10	30	30
FRP thickness(mm)	0.50	2.50	1.50	2.00	1.00	1.50	1.50	1.50	1.50	1.50
Model number	11	12	13	14	15	16	17	18	19	20
FRP length (mm)	30	30	40	30	40	50	20	40	20	40
FRP width (mm)	30	50	20	30	40	30	40	20	20	40
FRP thickness (mm)	1.50	1.50	1.00	1.50	2.00	1.50	1.00	2.00	2.00	1.00

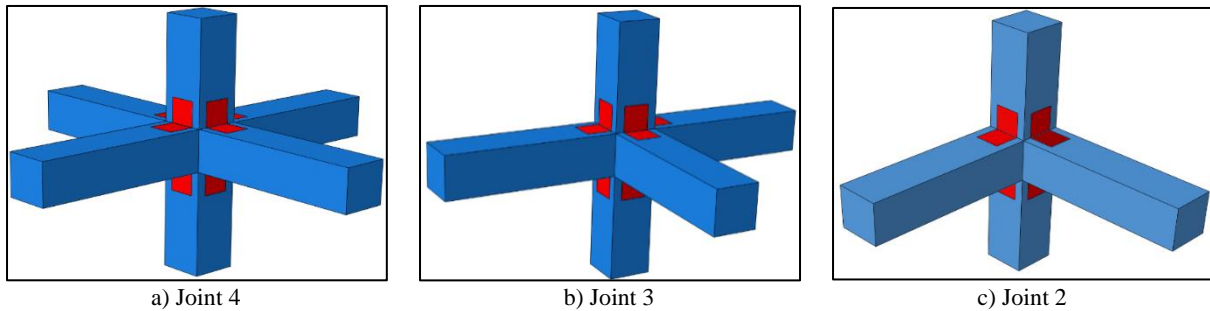


Figure 4. Assembled models in ABACUS software

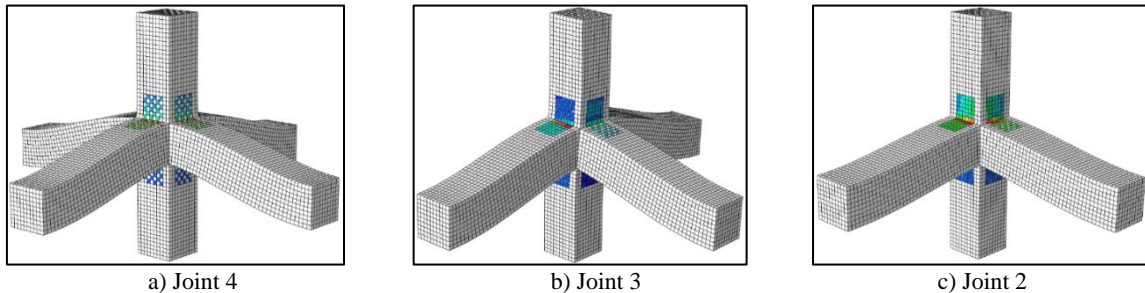


Figure 5. Numerical of failure modes the strengthened of concrete beam-column joints

The obtained results related to ABACUS software is saved for each model. Maximum value of shear capacity model is entered into the Design Expert software and the equation related to each joint is extracted from this software. Finally, the optimization is performed with the number of particles and different repetitions of PSO algorithm. This will be repeated for each type of three joint. The maximum shear capacity of models related to edge column shown in Table 6.

Table 6: Maximum shear capacity of models related to edge column (kN)

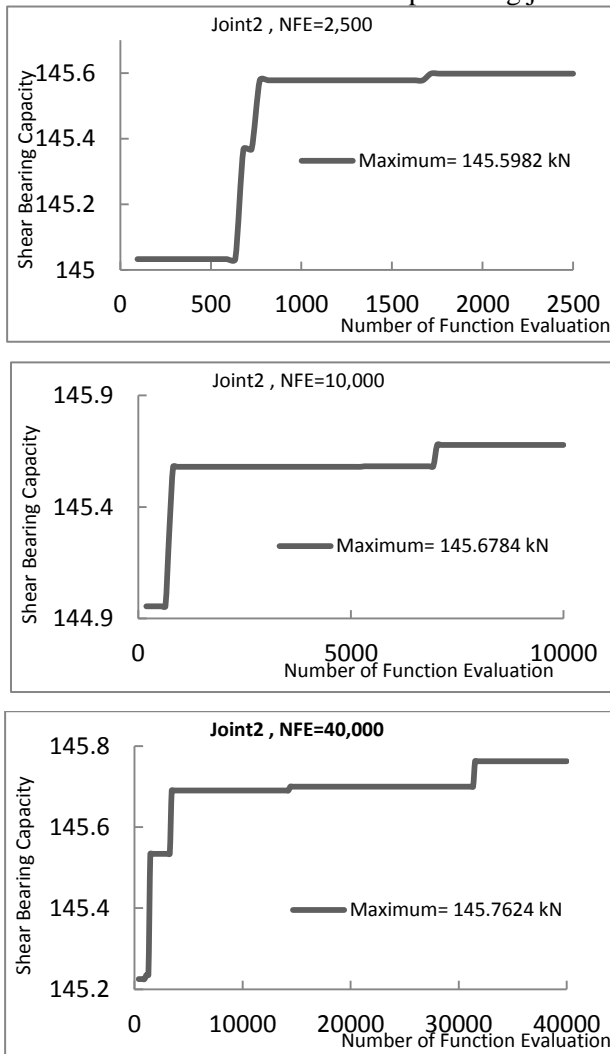
Joint2FRP0	Joint2FRP1	Joint2FRP2	Joint2FRP3	Joint2FRP4	Joint2FRP5	Joint2FRP6
143.96	143.96	144.31	144.08	143.96	143.96	144.14
Joint2FRP7	Joint2FRP8	Joint2FRP9	Joint2FRP10	Joint2FRP11	Joint2FRP12	Joint2FRP13
144.58	143.96	144.62	144.12	144.43	144.30	144.38
Joint2FRP14	Joint2FRP15	Joint2FRP16	Joint2FRP17	Joint2FRP18	Joint2FRP19	Joint2FRP20
144.34	144.49	144.75	143.96	144.18	144.65	143.96

The presented equation by response surface for edge column is presented as follow,

$$R_{J2} = 146.97727 - [0.077159 * L] - [0.065534 * W] - [1.72568 * T] - [2 * 10^{-4} * L * W] + [0.0105 * L * T] + [0.0125 * W * T] + [1.169332 * 10^{-3} * L^2] + [1.08182 * 10^{-3} * W^2] + [0.43773 * T^2] \quad (8)$$

in which L , W and T are length, width and thickness of FRP sheets respectively and R_{J2} is shear capacity. After optimization with PSO algorithm and with different number of particles and repetitions, equation 8 is optimized and its details are expressed in the following.

Table 7: Details and results of optimizing joint2 with different repetitions and particles



Particle number=50		
Repetition number=50		
Objective function call number=2500		
Optimum location		
T (mm)	W (cm)	L (cm)
2.4934	47.3315	10.0301
Optimum amount of bearing capacity (KN)		
145.5982		
Optimization implementation times=0.14519		
Particle number=100		
Repetition number=100		
Objective function call number = 10.000		
Optimum location		
T (mm)	W (cm)	L (cm)
2.4898	48.59	10.0387
Optimum amount of bearing capacity (KN)		
145.6784		
Optimization implementation times=0.32435		
Particle number=200		
Repetition=200		
Objective function call number=40.000		
Optimum location		
T (mm)	W (cm)	L (cm)
2.4882	49.9756	12.282
Optimum amount of bearing capacity (KN)		
145.7624		
Optimization implementation times=1.0343		

For side columns the maximum shear capacity is calculated by finite element model and presented in Table 8.

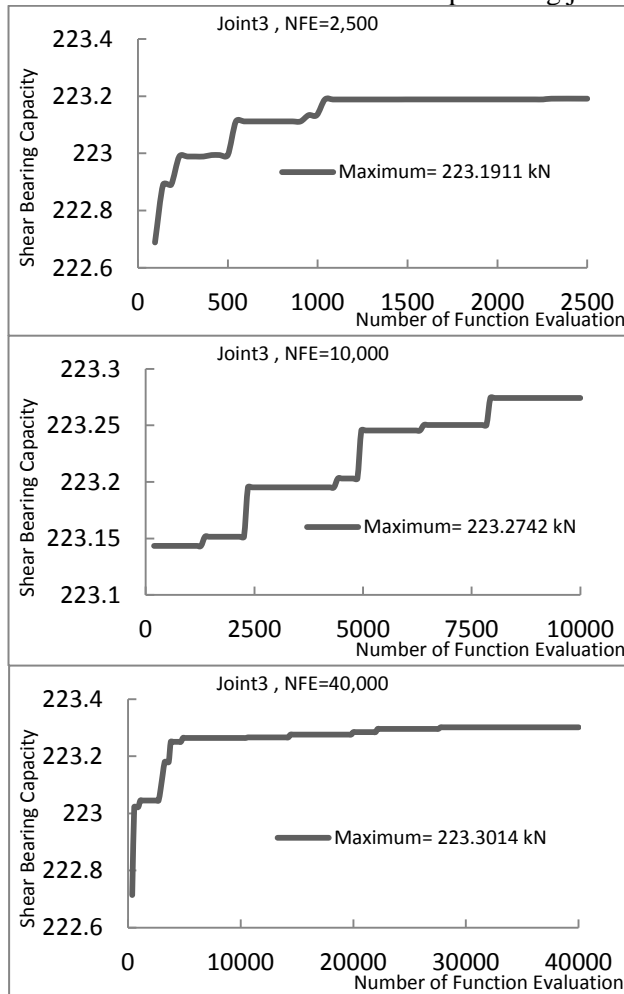
Table 8: Maximum shear capacity of models related to side column (KN)

Joint3FRP0	Joint3FRP1	Joint3FRP2	Joint3FRP3	Joint3FRP4	Joint3FRP5	Joint3FRP6
216.17	218.46	218.47	216.94	218.46	218.46	217.18
Joint3FRP7	Joint3FRP8	Joint3FRP9	Joint3FRP10	Joint3FRP11	Joint3FRP12	Joint3FRP13
220.20	218.46	219.88	217.01	218.29	217.89	217.95
Joint3FRP14	Joint3FRP15	Joint3FRP16	Joint3FRP17	Joint3FRP18	Joint3FRP19	Joint3FRP20
218.33	218.41	220.31	218.46	217.21	220.21	218.46

The determined equation by response surface for side column is presented as follow,

$$R_{j3} = 216.80182 - [5.22727 * 10^{-4} * L] - [0.020727 * W] - [0.46955 * T] - [2.50 * 10^{-5} * L * W] + [4.50 * 10^{-3} * L * T] + [0.061 * W * T] + [1.04545 * 10^{-4} * L^2] + [2.32955 * 10^{-4} * W^2] - [0.036818 * T^2] \quad (9)$$

Table 9: Details and results of optimizing joint3 with different repetitions and particles



Particle number =50
 Repetition number=50
 Objective function call number=2500
 Optimum location
 T (mm) W (cm) L (cm)
 2.4862 49.5569 39.5426
 Optimum amount of bearing capacity (KN)
 223.1911

Optimization implementation times=0.14235

Particle number=100
 Repetition number=100
 Objective function call number=10.000
 Optimum location
 T (mm) W (cm) L (cm)
 2.4966 49.8869 39.8424
 Optimum amount of bearing capacity(KN)
 223.2742

Optimization implementation times=0.34986

Particle number=200
 Repetition number=200
 Objective function call number=40.000
 Optimum location
 T (mm) W (cm) L (cm)
 2.4999 49.9849 39.9891
 Optimum amount of bearing capacity (KN)
 223.3014

Optimization implementation times=1.1029

For middle column with 4 connected beams, maximum shear capacity value is extracted (see Table 10).

Table 10: Maximum shear capacity of models related to side column (kN)

Joint4FRP0	Joint4FRP1	Joint4FRP2	Joint4FRP3	Joint4FRP4	Joint4FRP5	Joint4FRP6
307.26	311.72	311.46	309.28	311.72	311.72	309.77
Joint4FRP7	Joint4FRP8	Joint4FRP9	Joint4FRP10	Joint4FRP11	Joint4FRP12	Joint4FRP13
314.03	311.72	313.80	309.44	311.59	311.11	311.31
Joint4FRP14	Joint4FRP15	Joint4FRP16	Joint4FRP17	Joint4FRP18	Joint4FRP19	Joint4FRP20
311.39	311.90	314.49	311.72	309.87	313.61	311.72

The presented equation by response surface for middle column is as follow:

$$R_{j4} = 307.97511 - [0.013795 * L] + [0.03308 * W] + [4.09091 * 10^{-3} * T] + [4.50 * 10^{-4} * L * W] + [9 * 10^{-3} * L * T] + [0.069 * W * T] - [2.84091 * 10^{-5} * L^2] - [6.15909 * 10^{-4} * W^2] - [0.071364 * T^2] \tag{10}$$

The results of optimization using different number of particles for middle column have been presented in Table 11.

Table 11: Details and results of optimizing joint4 with different repetitions and particles

	Particle number=50		
	Repetition number=50		
	Objective function call number=2500		
	Optimum location		
	T (mm)	W (cm)	L (cm)
	2.4916	48.9659	40.1812
	Optimum amount of bearing capacity (KN)		
	317.5151		
	Optimization implementation times=0.14506		
		Particle number=100	
Repetition number=100			
Objective function call number=10.00			
Optimum location			
T (mm)		W (cm)	L (cm)
2.4949		49.8788	40.8333
Optimum amount of bearing capacity (KN)			
317.7392			
Optimization implementation times=0.33631			
		Particle number=200	
	Repetition number=200		
	Objective function call number=40.000		
	Optimum location		
	T (mm)	W (cm)	L (cm)
	2.4999	49.9299	40.8857
	Optimum amount of bearing capacity (KN)		
	317.7486		
	Optimization implementation times=1.1313		

From Tables 7, 9 and 11, it can be concluded that the optimization of FRP sheets of the strengthening beam-column joints is very important, hence it enable us to find the optimal dimensions of the FRP sheets leading to the low cost. The results also show that the most important optimization factors are the thickness and width of the FRP sheet. As, they take the highest values of the predetermined values. The increase length of the FRP has less significant contributions to the shear capacity.

6. CONCLUSION

A numerical model is developed for the optimization of dimensions of the FRP sheets bonded externally to a concrete beam-column joint. The model used a finite element method adopted by ABAQUS. To achieve this aim, a 3D finite-element model is developed to calculate the capacity shear. Response surface methodology is used to evaluate effects of height, width and thickness of the FRP sheet on shear capacity. Next, our model is used to optimize the volume of the FRP plate which is bonded externally to the concrete beam-column joint. To obtain an optimal volume for dimensions of the FRP sheets, PSO algorithm is used. The results showed that the most important optimization factors were the thickness and width of the FRP sheet.

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