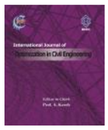
INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING
Int. J. Optim. Civil Eng., 2019; 9(1):65-77



SIMULATION OF ENDURANCE TIME EXCITATIONS USING INCREASING SINE FUNCTIONS

M. Mashayekhi[*, †], H. E. Estekanchi and H. Vafai
Sharif University of Technology

ABSTRACT

Endurance Time method is a time history dynamic analysis in which structures are subjected to increasing excitations. These excitations are known as endurance time excitation functions (ETEF). This study proposes a new method for generating ETEFs. In the proposed method, a new basis function for representing ETEFs is introduced. This type of ETEFs representation creates an intelligent space for this ETEFs simulating optimization problem. The proposed method is then applied in order to simulate new ETEFs. To investigate the efficiency of this proposed optimization space, newly generated ETEFs are compared with those simulated by conventional approaches. Results show an improvement in the accuracy of ETEFs as well as the reduction in the required computational time.

Keywords: endurance time method; time history dynamic analysis; unconstrained nonlinear optimization, artificial motion.

Received: 12 March 2018; Accepted: 25 May 2018

1. INTRODUCTION

Endurance Time (ET) method is an efficient tool for structural seismic analysis in which structures are subjected to intensifying acceleration time histories; these excitations are also known as endurance time excitation functions (ETEF). The need for more accurate structural analysis tools is currently growing owing to emergence of more complex structures due to modern architectural designs and using complicated seismic mitigation apparatus. Time history dynamic analysis in which all sort of complicated geometry and material can be incorporated is known as most accurate method. In spite of the accuracy of time history analysis, its extensive computational demand has prohibited the widespread application of such analyses in practice. As an alternative to conventional time history analysis, the ET method is proposed to overcome the mentioned corresponding computational demand.

---

[*]Corresponding author: Department of Civil Engineering, Sharif University of Technology
mmreza@mehr.sharif.ir (M. Mashayekhi)

The concept of the ET method is quite straightforward and is similar to well-known exercise test in medicine. In analogy with exercise test, structures are subjected to increasing demands and the corresponding responses are monitored during analysis. As the intensity of motions increase with time, different hazard levels are covered. Structural responses in the ET method are used to evaluate the performance of structures at different earthquake levels The ET method illustrates the performance of structures in term of correlation between structural seismic demands such as peak inter-story drift versus a measure of earthquake intensities such as peak acceleration spectra at the first mode of structures.

The ET method has been widely used in different area of earthquake engineering. For example, Estekanchi et al. [1] investigated the application of ET method in linear seismic assessment. Riahi et al. [2] investigated potential and limitation of ET method in nonlinear seismic analysis of single degree of freedom (SDOF) structures. SDOF structures with different strength ratio, different ductility, and damping ratio are considered. Stiffness degradation and deterioration is also considered in SDOF structures. The comparison of ET approximation and the time history analysis show that ET approximation are in good agreement with time history analysis. Riahi et al. [3] applied ET method for seismic assessment of steel frames with different stories and different bays. Drift ratio, plastic hinge rotation of ET method is compared with those of ground motions. Moreover, approximation of location of plastic hinges are considered. Mirzaei and Estekanchi [4] developed an ET method-based methodology for performance-based retrofitting of typical steel frames. For retrofitting a structure, there are several alternative options and each option also has different design alternative that should be specified. Versatility of ET method for overcoming the aforesaid issues are demonstrated. Rahimi and Estekanchi [5] applied ET method for collapse assessment of buildings. The results of ET methods are compared with incremental dynamic analysis (IDA). The results show the considerable decreasing in computational demand in spite of having acceptable accuracy. Basim and Estekanchi [6] investigated the application of ET method in performance-based design of structures and proposed practical optimum design procedure.

ETEFs are central part of the ET method. Simulating efficient ETEFs is essential for the development of the ET method. It is expected that structural responses under ETEFs are consistent with responses when those structures are subjected to real earthquakes. In order to achieve this expectation, intensity measures of ETEFs must be compatible with real earthquakes and also increase with time. Acceleration spectrum is a common intensity measure considered in simulating ETEFs. However, several studies have considered the significance of duration related parameters in the ET method [7, 8, 9].

Simulating ETEFs with these mentioned properties leads to a complicated large-scale optimization problem. In optimization problems, equations are defined by objective functions. Several studies have been aimed to simulate ETEFs [10, 11, 12].

In simulating ETEFs, signals can be represented by using different approaches. Optimization variable definitions differ in various signal representations. In a case that signals are represented by acceleration values, optimization variables are corresponding acceleration values. It is well known that acceleration values do not express dynamic characteristics of signals, such as frequency content, individually. Dynamic characteristics of signals can be extracted by means of signal processing tools such as Fourier and wavelet transforms. Fourier transform decomposes a signal into its frequency components. The drawback of the Fourier transform is that it cannot extract frequency variations during signals. Thus, the Fourier transform is suitable for stationary signals. On the other hand, the wavelet transform represents a signal in a time-frequency domain. The main difference between these two transforms is that time variation of frequency can be detected by the wavelet transform. In fact, the wavelet transform can extract temporal variations of signals and is applicable to the non-stationary signals such as ground motions. When signals are represented by Fourier transform or wavelet transform, coefficients of associated basis functions are considered as optimization variables. In fact, simulating ETEFs is to find best values of these coefficients.

In this study, a new optimization space for simulating ETEFs is introduced. In the proposed method, increasing sine functions are introduced and are employed as basis functions. New ETEFs are simulated by using the proposed optimization space. Newly ETEFs simulated are then compared with ETEFs simulated by conventional approaches regarding the accuracy and the computational time of simulating.

2. ENDURANCE TIME EXCITATION SIMULATION

ETEFs are acceleration time histories the intensity of which increases with time. They are generated so that the response of structures under the ETEFs will be compatible with the responses when they are subjected to real recorded ground motions. In order to meet this expectation, dynamic characteristics of ETEFs must be compatible with those associated with recorded ground motions. One of the most prominent dynamic characteristics that can be considered is the acceleration spectra.

Acceleration spectra of ETEFs are expected to increase with time, while are compatible with ground motions. In this regard, objective function of Equation (1) must be solved. This equation computes the discrepancy between the ETEFs acceleration spectra and targets.

$$F_{\mathrm{ETEF}}\left(a_g\right)=\int_{T_{\min}}^{T_{\max}}\int_{0}^{t_{\max}}\left\{\left[S_a\left(T,t\right)-S_{aC}\left(T,t\right)\right]^2\right\}\mathrm{d}t\mathrm{d}T \tag{1}$$

where S_a (t,T) denotes acceleration spectra produced by time window [0,t] of ETEFs at period of T. S_{aC} is target acceleration spectra of ETEFs. $t_{\max}$ is duration of ETEFs. And also, $T_{\min}$ and $T_{\max}$ are the minimum and maximum of the periods considered in the generating process, respectively.

S_{ac} (t,T) is target acceleration spectra of simulating ETEFs and is calculated by Equation (2):

$$S_{ac}\left(t,T\right)= t/t_{\mathrm{target}} * S_a^{\mathrm{Target}}\left(T\right) \tag{2}$$

where t_{target} is the time at which ETEFs are compatible with normalized ground motions. S_a^{target} is the median acceleration spectra of normalized ground motions. In this study, the ground motions suite suggested by FEMA P-695 [13] is used. The normalization procedure is employed by peak ground velocity (PGV) according to FEMA P-695 guideline. Median acceleration spectra of these ground motions are depicted in Fig. 1. It should be noted that median acceleration spectra of these motions are smoothed by using Spline functions. Explanation of this procedure is beyond the scope of this paper.

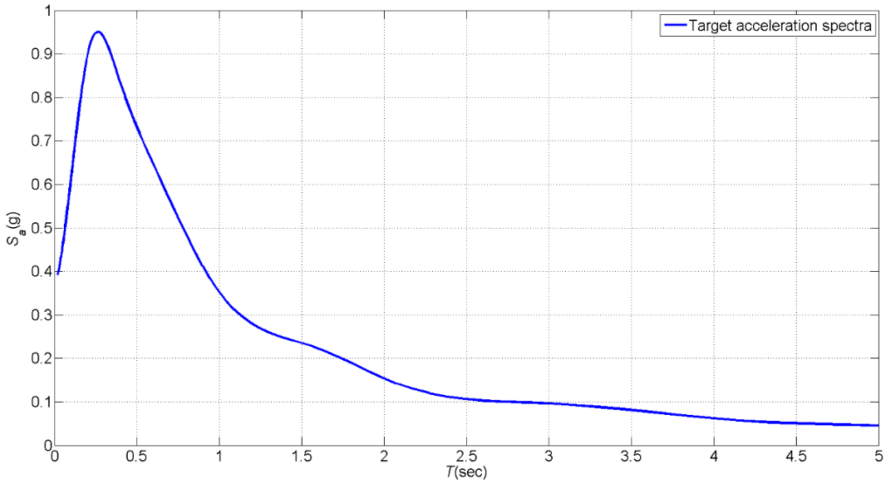


Figure 1. Target acceleration spectra of generating ETEFs

$S_a(t,T)$ is calculated by Equation (3).

$$S_a(t,T) = \max\left(\left|\ddot{x}(\tau) + a_g(\tau)\right|\right) \quad 0 \le \tau \le t \tag{3}$$

where $\ddot{x}(\tau)$ is the relative acceleration response of an SDOF with a period of T and damping ratio of 5% under the ETEFs, while $a_g(\tau)$ is the acceleration time history of ETEFs.

To solve the abovementioned equations, unconstrained nonlinear optimization procedure is employed. Note that discretization is required in solving such objective functions; not to mention that the type of discretization to be used can impact the results. Given that times are sampled at n points t_j (j=1:n), and periods are sampled at m points T_i (i=1:m); after discretization is applied, objective function of Equation (1) converts double integrals to double summations, as stated in Equation (4).

$$F_{\mathrm{ETEF}}(a_g) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\left[S_a(T_i,t_j) - S_{aC}(T_i,t_j)\right]^2\right\} \tag{4}$$

In this study, 120 periods with a logarithmic distribution between 0.02 seconds and 5seconds are opted. The logarithmic distribution produces more data in the low period region where fluctuation of acceleration spectra is considerably higher than the high period region. t is sampled at 2048points with equal intervals of 0.01seconds.

3. METHODOLOGY

Simulating ETEFs is an optimization problem which intends to find optimum values of decision variables. In the simulation process, ETEFs can be represented in several ways. In each way, optimization variable definitions differ. In this study, ETEFs are represented by using increasing sine functions. In fact, this study uses increasing sine functions as basis function. The schematic of this basis function is shown in Fig. 2. This signal representation is presented in Equation (5).

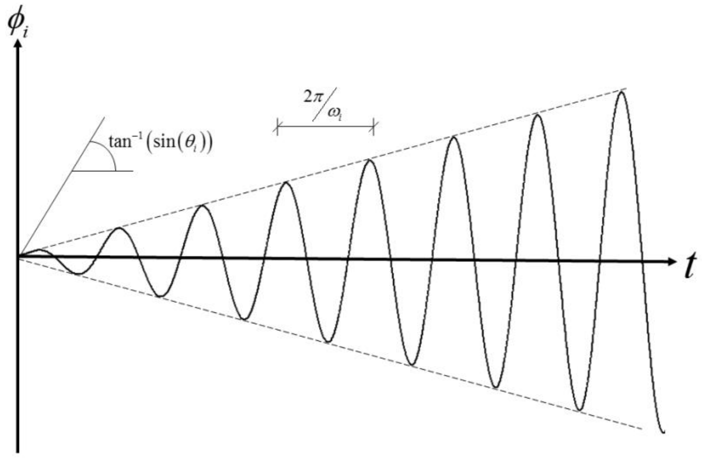


Figure 2. Increasing sine basis function

$$a_g(t) = \sum_{i=1}^{n_f} a_i \phi_i(t) = \sum_{i=1}^{n_f} a_i t \sin(\omega_i t + \theta_i) \tag{5}$$

where $\phi_i(t)$ is i-th basis function which is an increasing sine function shown in Fig. 2, a_i is the contributing factor of i-th basis function in acceleration time history of ETEFs that must be determined during optimization process, ω_i and θ_i are the angular frequency and the phase angles of i-th increasing sine functions, respectively, n_f is the number of considered increasing sine functions.

The frequency of i-th increasing sine function is calculated according to Equation (6). In this equation, frequencies of considered increasing sine functions are distributed logarithmically between the maximum and the minimum considered frequencies.

$$\omega_i = 10^{\frac{\log(\omega_{\min})\times(n_f - i) + \log(\omega_{\max})\times(i-1)}{n_f - 1}} \tag{6}$$

where $\omega_{\min}$ and $\omega_{\max}$ denote the minimum and the maximum considered frequencies for increasing sine functions.

Simulating ETEFs is to find values of these coefficients so that minimize the objective function of the problem. Objective function value is also denoted as cost function value in this paper. The problem of simulating ETEFs is summarized as below:

Find $\{x\}_{1\times 2n_f} = \left[a_1, a_2, ..., a_{n_f}, \phi_1, \phi_2, ..., \phi_{n_f} \right]$

To Minimize $F_{ETEF}\{x\}$

The algorithm for implementing the new optimization space is depicted in Fig. 3.

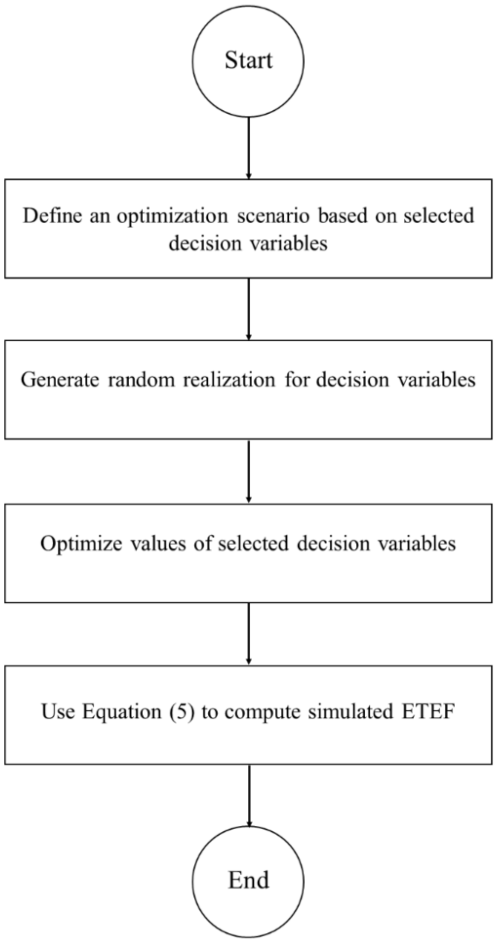


Figure 3. The proposed algorithm for implementation of new basis function in simulating ETEFs

4. PARAMETER TUNING

In this section, 40second ETEFs are simulated by using the proposed method. One advantage of the proposed method over conventional ETEFs simulating procedure is the possibility to adjust the number of optimization variables. In this study, eight optimization scenarios are defined. The number of optimization variables is proportional to the number of considered increasing sine functions. If phase angle is not considered in the process, the number of optimization variables will be equal to the number of considered increasing sine functions. Otherwise, the number of optimization variables will be equal to twice the number of considered increasing sine functions. In this study, 1.3 rad/sec and 314.2 rad/sec are assigned to ω_{min} and ω_{max}, respectively. The considered scenarios are presented in Table 1.

Table 1: Optimization scenarios for simulating ETEFs based on the proposed method

scenario	n_f	Phase angle inclusion	Optimization variable number
ETEF-IS-01	300	No	300
ETEF-IS-02	600	No	600
ETEF-IS-03	900	No	900
ETEF-IS-04	1200	NO	1200
ETEF-IS-05	300	Yes	600
ETEF-IS-06	600	Yes	1200
ETEF-IS-07	900	Yes	1800
ETEF-IS-08	1200	Yes	2400

For each scenario, the optimization algorithm depicted in Fig. 3 is executed by using three different initial random motions. The used initial motions are generated randomly and associated probability distribution parameters are presented in Table 2. Results of these runs are summarized in Table 3. Results show that ETEF-IS-04 brings about more accurate ETEFs. In addition, it can be observed that including phase angles as optimization variables not only does not improve the accuracy of simulated ETEFs but also increases the cost function values.

Table 2: The distribution functions of variables for generating initial random motions

Variables	Distribution type	Distribution parameters	
		Parameter	Value
Amplitudes	Normal	Median	0
		Standard deviation	0.005
Phase angles	Uniform	Lower bound	0
		Upper bound	6.28

Table 3: Simulated ETEFs results of defined scenarios

Scenario	Run number	Cost function		Computational time (sec)	
		Values	average	Values	average
ETEF-IS-01	01	347.2	341.2	6710	8522
	02	360.1		6239	
	03	316.4		12618	
ETEF-IS-02	01	265.4	251.3	15294	14185
	02	265.3		15345	
	03	223.1		11917	
ETEF-IS-03	01	212.3	216.0	17250	19915
	02	221.4		21926	
	03	214.3		20569	

	01	173.8		15802	
ETEF-IS-04	02	214.1	198.8	14945	16473
	03	208.6		18672	
	01	451.8		58280	
ETEF-IS-05	02	379.2	396.8	58146	58145
	03	359.3		58010	
	01	390.1		102538	
ETEF-IS-06	02	332.5	369.2	102842	120741
	03	385.1		156843	
	01	336.6		167256	
ETEF-IS-07	02	426.4	410.1	166216	166666
	03	467.3		166528	
	01	373.3		462412	
ETEF-IS-08	02	367.4	364.4	465357	464456
	03	352.4		465599	

In order to investigate the efficiency of the proposed method, three ETEFs are simulated in time domain. It should be considered that simulating ETEFs is currently performed in time-domain. Further investigation is conducted by comparing the proposed method results with simulating ETEFs in wavelet transform space optimization. Simulating ETEFs in discrete wavelet transform is proposed by Mashayekhi and Estekanchi [14]. ETEFs simulated in time domain are hereafter denoted by ETEF-T and ETEFs simulated in discrete wavelet transform space is hereafter denoted by ETEF-W. Two scenarios are defined in discrete wavelet transform space, namely ETEF-W-01 and ETEF-W-02. In ETEF-W-01, first 512 wavelet coefficients are considered as optimization variables, while in ETEF-W-02 first 1024 wavelet coefficients are considered as optimization variables. Simulated ETEF results are summarized in Table 4. Results shown that the scenario ETEF-W-02 creates more accurate ETEFs among the considered conventional scenarios.

Table 4: Results of ETEFs simulated in time domain and discrete wavelet transform space

Scenario	Run number	Cost function		Computational time	
		Values	Average	Values	Average
	01	5964.4		242367	
ETEF-T	02	4704.8	4448.3	244227	242654
	03	2675.6		241369	
	01	366.2		31644	
ETEF-W-01	02	309.3	320.4	21777	25901
	03	285.6		24282	
	01	232.6		56697	
ETEF-W-02	02	248.6	235.3	59325	56302
	03	224.7		52885	

Table 5 compares the proposed method results with the conventional approach ones. Results show that the proposed method improve the accuracy of simulated ETEFs by 15.5%.

The proposed method also decreases the required computational time by 71%. However, the standard deviation of cost functions associated with the proposed method increases by 44%.

Table 5: The proposed optimization space vs. the conventional approach

Parameter	Conventional approach	The proposed method
Average cost function	235.3	198.8
Standard deviation cost function	12.2	21.8
Computational time (sec)	56302	16473
Best cost	224.7	173.8
Worst cost	248.6	214.1

In order to quantify the accuracy of the simulated ETEFs, normalized residuals are defined as in Equations (7), (8), (9). NRR integrates residuals over at all times and periods. The residuals at each time are integrated at all periods and then are normalized. This normalization method avoids residuals domination where response spectra values are little and division by little numbers occurs. Normalized residuals express the accuracy of ETEFs in percent. Therefore, it is an acceptable measure to investigate the efficiency of ETEFs.

$$\mathrm{NRR}_{\mathrm{S}_a} = \frac{1}{t_{\max}} \int_0^{t_{\max}} \left(\frac{\int_{T_{\min}}^{T_{\max}} \left| \left(S_a(T,t) - S_{ac}(T,t) \right) \right| dT}{\int_{T_{\min}}^{T_{\max}} S_{ac}(T,t)\, dT} \right) dt \tag{7}$$

$$\mathrm{NRR}_{\mathrm{S}_d} = \frac{1}{t_{\max}} \int_0^{t_{\max}} \left(\frac{\int_{T_{\min}}^{T_{\max}} \left| \left(S_d(T,t) - S_{dc}(T,t) \right) \right| dT}{\int_{T_{\min}}^{T_{\max}} S_{dc}(T,t)\, dT} \right) dt \tag{8}$$

$$\mathrm{NRR}_{\mathrm{S}_v} = \frac{1}{t_{\max}} \int_0^{t_{\max}} \left(\frac{\int_{T_{\min}}^{T_{\max}} \left| \left(S_v(T,t) - S_{vc}(T,t) \right) \right| dT}{\int_{T_{\min}}^{T_{\max}} S_{vc}(T,t)\, dT} \right) dt \tag{9}$$

where NRR_{Sa}, NRR_{Sd}, and NRR_{Sv} are normalized residuals associated with acceleration spectra, displacement spectra and velocity spectra, respectively.

Table 6 compares normalized residuals of ETEFs simulated by the proposed method and ETEFs simulated by conventional approaches. Results show somewhat improvement in ETEFs simulated by the proposed method.

Table 6: Normalized residuals of ETEFs simulated by the proposed method vs. conventional approaches

Parameter	The proposed method	Conventional approach
NRR_{Sa}	6.4%	7.8%
NRR_{Sd}	22.5%	48%
NRR_{Sv}	13.4%	32.8%

5. ACCURACY OF ETEFS SIMULATED WITH THE PROPOSED METHOD

In this section, ETEFs simulated by the proposed method (ETEF-IS-04) are presented and are explored. These ETEF are denoted by ETEF-IS. Convergence history of simulating ETEF-IS are depicted in Fig. 4. It should be mentioned that the scale of ordinate and abscissa of this figure is logarithmic. Acceleration time history of ETEF-IS is shown in Fig. 5Figure 5. Acceleration spectra of ETEF-IS is compared with targets in Fig. 6 at four times, *t* = 15 second, 25 second, 35 second, and 40 second. This figure shows the acceptable correspondence between ETEF-IS acceleration spectra and targets. This fact proves the efficiency of the proposed method.

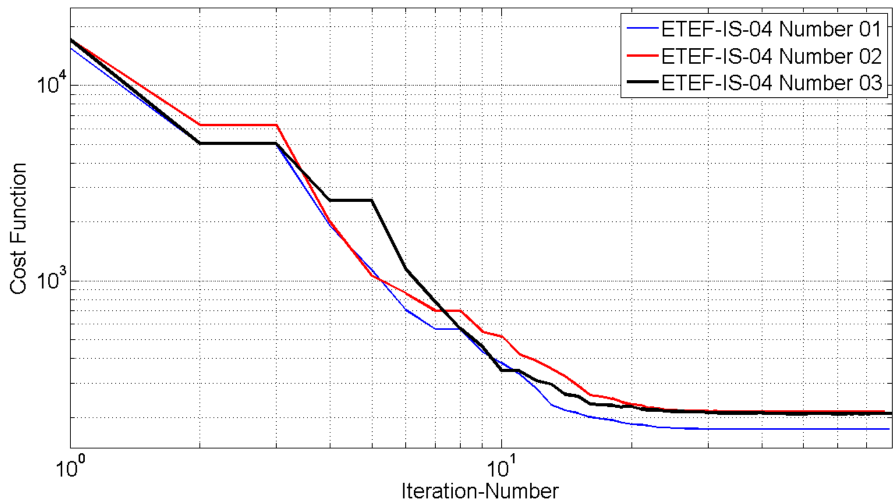


Figure 4. Convergence history of simulating ETEF-IS

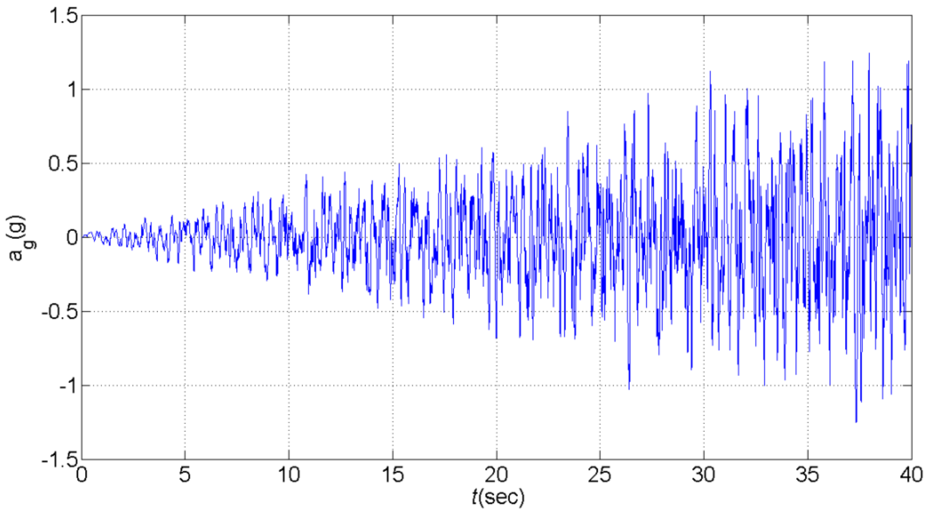


Figure 5. Acceleration time history of ETEF-IS

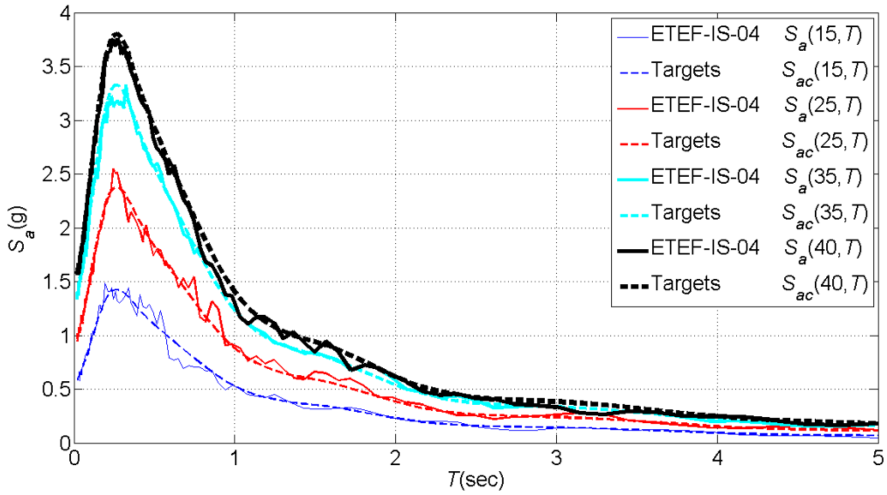


Figure 6. Comparison of acceleration spectra of ETEF-IS with targets at time t=15second, t=25second, t=35second, and t=40second

6. CONCLUSION

Endurance Time Method (ET) is a time history dynamic analysis in which structural seismic assessment is performed with considerably low computational time as compared to conventional time history analysis. In the ET method, structures are subjected to increasing acceleration time histories which are also known as endurance time excitation functions (ETEF). Therefore, ETEFs are the central part of the ET method. Unconstrained nonlinear optimization is utilized to simulate ETEFs. In this paper, a new optimization space based on increasing sine functions is introduced. New ETEFs are generated in the proposed optimization space. The best parameters for defining this proposed optimization space is determined. A comparative study is conducted to investigate the efficiency of the proposed method in comparison with conventional approaches. Results are listed below:

1- It is shown that including phase angles of increasing sine functions does not improve the accuracy of simulated ETEFs. However, this inclusion only increases the required computational time of simulating ETEFs.
2- The accuracy of simulated ETEFs in the proposed optimization space is increased about 15.5% as compared to conventional simulating ETEFs approaches. It is also observed that the required computational demand for simulating ETEFs by the proposed method is decreased about 71%.
3- Acceleration spectra of simulated ETEFs by the proposed method are compared with target acceleration spectra. This comparison shows well consistency between acceleration spectra of simulated ETEFs with targets. This fact proves the efficiency of the proposed method.
4- It is observed that the optimization space considerably influences the accuracy of simulated ETEFs. This paper introduced a novel method for defining optimization

variables. It is proposed that further studies should be performed in order to find an appropriate optimization space for simulating new ETEFs.

Acknowledgment: The authors would like to thank Sharif University of Technology Research Council for their support of this research.

NOMENCLATURE

$a_g(\tau)$	acceleration time history of an Endurance Time excitation
ET	endurance time method
ETEF	endurance time excitation function
F_{ETEF}	objective functions of simulating endurance time excitations
m	sample number of periods
n	sample number of time
n_f	number of considered increasing sine function
$\mathrm{NRR}_{\mathrm{Sa}}$	Normalized residuals of acceleration spectra
$\mathrm{NRR}_{\mathrm{Sd}}$	Normalized residuals of displacement spectra
$\mathrm{NRR}_{\mathrm{Sv}}$	Normalized residuals of velocity spectra
SDOF	single degree of freedom
$S_a(t,T)$	acceleration spectra produced by ETEFs at time t and period T
$S_{aT}(t,T)$	target acceleration spectra
$S_a^{\mathrm{target}}(T)$	target acceleration spectra
T	period
t	time
$T_{\max}$	maximum period considered in the simulation process
$t_{\max}$	duration of endurance time exitations
$T_{\min}$	minimum period considered in the simulation process
x_i	optimization variable
$\ddot{x}(\tau)$	acceleration response of single degree of freedom systems
$x_{i,\max}$	upper bound of optimization variables
$x_{i,\min}$	lower bound of optimization variables
θ	phase angle
ϕ	basis functions
ω	angular frequency
$\omega_{\max}$	maximum considered frequency for increasing sine functions
$\omega_{\min}$	minimum considered frequency for increasing sine functions

REFERENCES

1. Estekanchi HE, Valamanesh V, Vafai A. Application of endurance time method in linear seismic analysis, *Eng Struct* 2007; **29**(10): 2551-62.
2. Riahi HT, Estekanchi HE, Vafai A. Application of Endurance Time method in nonlinear seismic analysis of SDOF systems, *J Appl Sci* 2009; **9**(10): 1817-32.

3. Riahi HT, Estekanchi HE, Vafai A. Seismic assessment of steel frames with the endurance time method, *J Constr Steel Res* 2010; **66**(6): 780-92.
4. Mirzaee A, Estekanchi HE. Performance-based seismic retrofitting of steel frames by the endurance time method, *Earthq Spectra* 2015; **31**(1): 383-402.
5. Rahimi E, Estekanchi HE.Collapse assessment of steel moment frames using endurance time method, *Earthq Eng Eng Vib* 2015; **14**(2): 347–360.
6. Basim MC, Estekanchi HE, Vafai A. A methodology for value based seismic design of structures, *Sci Iran* 2016; **23**(6): 2514-27.
7. Mashayekhi M, Estekanchi HE. Investigation of non-linear cycles' properties in structures subjected to endurance time excitation functions, *Int J Optim Civil Eng* 2013; **3**(2): 239-57.
8. Mashayekhi M, Estekanchi HE. Investigation of strong-motion duration consistency in endurance time excitation functions, *Sci Iran* 2013; **20**(4): 1085-93.
9. Mashayekhi M, Estekanchi HE. Significance of effective number of cycles in endurance time analysis, *Asian J Civil Eng (Building Housing)* 2012; **13**(5): 647-57.
10. Nozari A, Estekanchi HE. Optimization of endurance time acceleration functions for seismic assessment of structures, *Int J Optim Civil Eng* 2011; **2**: 257-77.
11. Kaveh A, Kalateh M, Estekanchi HE. Production of endurance time excitation function : the CMA evolution strategy approach, *Iran J Sci Technol Trans Civil Eng* 2013; **37**(C): 383-94.
12. Kaveh A, Mahdavi VR, Estekanchi HE. Generation of endurance time acceleration functions using the wavelet transform, *Int J Optim Civil Eng* 2012; **2**(2): 203-19.
13. FEMA P-695, .Quantification of building seismic performance factors, Federal Emergency Management Agency: Washington, DC, 2009.
14. Mashayekhi M, Estekanchi HE, Vafai A. Simulation of endurance time excitations via wavelet transform, submitted for publication 2018.