



## STRUCTURAL DAMAGE DETECTION BY USING TOPOLOGY OPTIMIZATION FOR PLANE STRESS PROBLEMS

S.M. Eslami, F. Abdollahi, J. Shahmiri and S.M. Tavakkoli<sup>\*,†</sup>  
*Civil Engineering Department, Shahrood University of Technology, Shahrood, Iran*

### ABSTRACT

This paper aims to introduce topology optimization as a robust tool for damage detection in plane stress structures. Two objective functions based on natural frequencies and shape modes of the structure are defined to minimize discrepancy between dynamic specifications of the real damaged structure and the updating model. Damage area is assumed as a porous material where amount of porosity signifies the damage intensity. To achieve this, Solid Isotropic Material with Penalization (SIMP) model is employed. Sensitivity analysis is achieved and a mathematical based method is used for solving the optimization problems. In order to demonstrate efficiency and robustness of the method to identify various type of damages in terms of both location and intensity, several numerical examples are presented and the results are discussed.

**Keywords:** damage detection; topology optimization; structural health monitoring.

Received: 12 March 2018; Accepted: 20 June 2018

### 1. INTRODUCTION

Damage in a structure, depending on its location and intensity, affects both static and dynamic characteristics of the structure. However, dynamic responses are more sensitive and change during the early stages of a progressive damage, which is crucial to retrofit and extend the service life of the structure. Therefore, dynamic response based methods has received more attention in structural health monitoring community. These methods may be divided into two groups of input-output and output based methods. Since it is not an easy task to find input data, for instance, in structures under ambient vibrations, free vibration characteristics such as natural frequencies and shape modes are usually employed in damage identification process. In order to identify damages in structures, several methods have been introduced based on detecting changes in modal parameters such as natural frequency [1–4],

---

\*Corresponding author: Civil Engineering Department, Shahrood University of Technology, Shahrood, Iran

s.m.tavakkoli@gmail.com (S.M. Tavakkoli)

modal strain energy [5,6], mode shape derivatives [7–9], residual force vector [10,11] and wavelet transform of measured modal shapes [12,13].

Damage might be considered as a reduction of stiffness and mass of a part of a structure. The damage parameters can be identified by minimizing an error function defined as the difference between the computed and measured dynamic properties in the computer model and the real damaged structure, respectively [14–16]. However, modal data of the real structure cannot be derived directly and therefore, a modal identification process is involved. Any mathematical based or metaheuristic iterative optimization methods can be employed in order to update the model towards the real damaged structure [17–25]. An excellent review of history of the metaheuristic based methods is given in the books by Kaveh [26,27].

Structural topology optimization is employed to specify the optimum material distribution in the structure. This approach has received enormous attention since the introduction of the homogenization approach to topology optimization by Bendsøe and Kikuchi in 1988 [28]. Several methods have been developed since then mainly within two types of element-based methods, such as the Solid Isotropic Material with Penalization (SIMP) [29–31], and using a level of an implicit function to represent the boundaries [32–35].

Structural damage diagnosis might be seen as a topology optimization problem by considering damages as void (zero stiffness) or porous material (reduced stiffness) regions in the layout of the structure. Using element-based topology optimization methods, Lee *et al.* utilized the Methods of Moving Asymptotes (MMA) to minimize discrepancy between some points of frequency response functions of the model and the real structure [36]. Nishizu *et al.* also exploited topology optimization to identify defects based on natural frequencies and by using MMA [37]. Recently, a level set based approach is proposed by Zhang *et al.* to find the location of damages again by considering natural frequencies [38].

This paper aims to utilize topology optimization for damage identification in plane structures. Two optimization problems are constructed based on natural frequencies and mode shapes under design variables restrictions. Since both location and intensity of damage areas are significant issues in structural health monitoring, the SIMP approach is chosen to provide the possibility of having porous material regions (gray areas) in the structure. It is noted that in level set based topology optimization methods the severity of the damage cannot be attained. A simple steepest descent algorithm is employed to solve the unconstrained optimization problem. Several examples considering different structures as well as various local damages in terms of both location and intensity are studied in order to demonstrate efficiency and robustness of the method.

Optimization problems considered in this paper are defined in Section 2. Section 3 is devoted to parametrize the problems using SIMP approach. The sensitivity analyses and optimization algorithm are presented in Sections 4 and 5, respectively. Finally, numerical examples and concluding remarks are provided in Sections 5 and 6.

## 2. OPTIMIZATION PROBLEMS FOR DAMAGE DETECTION

Damage detection optimization problem can be defined based on updating a numerical model so that the difference between the model data and the measured data in real structure

is minimized. As it mentioned before, the modal data are sensitive to defects in the structure. In this paper, two different objective functions are considered based on modal data which are defined as follows.

For the first objective function, natural frequencies discrepancy in numerical model and real damaged structure might be considered as follows

$$f(\xi) = \sum_{i=1}^{nm} w_i^\lambda \left( \frac{\lambda_i(\xi) - \lambda_i^*}{\lambda_i^*} \right)^2 \quad (1)$$

where  $\xi$  is the design variables vector,  $\lambda_i(\xi)$  and  $\lambda_i^*$  are the  $i$ th eigenvalues, derived from numerical model and measured in damaged structure, respectively.  $w_i^\lambda$  are the weights for each natural frequency and  $nm$  is the number of modes which is taken into consideration. The weights are taken as unity in this research, although they can be different if required. The main disadvantage of using eigenvalue changes are that the damages in symmetric locations cannot be differentiated for a symmetric structure. In such cases mode shapes might be incorporated.

If mode shapes are identified and given in  $np$  points, the objective function for all  $nm$  mode shapes takes the form

$$f(\xi) = \sum_{i=1}^{nm} w_i^\varphi \sum_{j=1}^{np} \left( |\varphi_{ij}(\xi)| - |\varphi_{ij}^*| \right)^2 \quad (2)$$

where  $\varphi_{ij}$  and  $\varphi_{ij}^*$  are the  $j$ th components of the  $i$ th normalized eigenvectors in the model and the real structure, respectively. It is noted that  $\varphi_{ij}^*$  are constant during the optimization process. Also,  $w_i^\varphi$  is the weight parameter related to the  $i$ th mode.

### 3. DAMAGE DETECTION BY USING TOPOLOGY OPTIMIZATION

In general, structural topology optimization seeks material distribution within a defined design domain for a given set of loads and boundary conditions so that the objective function is minimized. Different optimization problems have been introduced in this context. Among them, minimization of mean compliance with a certain amount of material and weight minimization considering stress constraints can be mentioned [30,31]. In this research, topology optimization is used as a tool to find damaged regions in the structure. To achieve this, damage is assumed as a stiffness reduction and, therefore, density of material is decreased in damaged areas. Topology optimization methods can be used to find the material distribution throughout the design domain. For this purpose, an SIMP model can be developed as follows.

Considering damage as a porous material region, the structure can be described by a function  $\zeta(\mathbf{x})$ , defined at each point  $\mathbf{x}$  as

$$\xi(\mathbf{x}) = \begin{cases} 1 & \mathbf{x} \in \Omega_s \quad \text{no damage} \\ 0 < \xi_{\min} \leq \xi(\mathbf{x}) < 1 & \mathbf{x} \in \Omega \setminus \Omega_s \quad \text{damage} \end{cases} \quad (3)$$

where  $\xi_{\min}$  is assumed more than zero value to avoid singularity. It is noted that intermediate values of  $\xi(\mathbf{x})$  show gray areas and extent of damage in the structure.

Therefore, density and elasticity matrix for each point  $\mathbf{x}$  of the structure can be written as

$$\rho(\mathbf{x}) = \xi(\mathbf{x})\rho^0 \quad (4)$$

$$\mathbf{C}(\mathbf{x}) = \xi^p(\mathbf{x})\mathbf{C}^0 \quad p \geq 1 \quad (5)$$

where  $\rho^0$  and  $\mathbf{C}^0$  are the density and elasticity matrix of solid materials. Also,  $p$  is the penalty factor. The penalty factor is typically used to suppress the gray areas in standard topology optimization problems. Here, since gray areas show the intensity of damages, the material model should not be penalized. On the other hand, penalizing the model make the solution more sensitive to defects in the structure. Therefore, the continuation method [31] is used in this research. It means, the penalty parameter varies during the optimization process from  $p > 1$ , for instance  $p = 3$ , and ends up to one at the last iterations in order to find extent of the damage, appropriately.

To be more practical, damage at each point  $\mathbf{x}$  or function  $\xi(\mathbf{x})$  is assumed to be constant in each finite element, shown by  $\xi_e$  and called damage index of element  $e$ . The element stiffness matrix can be written as

$$\mathbf{K}_e(\xi_e) = \int_{\Omega^e} \mathbf{B}^T \mathbf{C}(\xi_e) \mathbf{B} d\Omega \quad (6)$$

where  $\mathbf{B}$  is the strain-displacement matrix and  $\Omega^e$  is the finite element volume. Also, the element mass matrix may be written as

$$\mathbf{M}_e(\xi_e) = \rho_e(\xi_e) \int_{\Omega^e} \mathbf{N}^T \mathbf{N} d\Omega = \xi_e \mathbf{M}_e^0 \quad (7)$$

where  $\mathbf{N}$  is the element shape functions matrix and  $\mathbf{M}_e^0$  represents the mass matrix for fully solid material. It is important to note that artificial localized eigenmodes may be occurred in low density material areas, where the ratio between the stiffness and the mass is very small. To avoid the numerical singularity, Equation (10) might be rewritten as [39]

$$\mathbf{M}_e(\xi_e) = \begin{cases} \xi_e \mathbf{M}_e^0 & \xi_e > 0.1 \\ (c_1 \xi_e^6 + c_2 \xi_e^7) \mathbf{M}_e^0 & \xi_e \leq 0.1 \end{cases} \quad (8)$$

where the coefficients  $c_1$  and  $c_2$  are set to be  $6 \times 10^5$  and  $-5 \times 10^6$ , respectively. By using the modified SIMP model as stated above, the artificial modes are eliminated.

The global stiffness and mass matrices can be expressed as

$$\mathbf{K} = \sum_{e=1}^{ne} \xi_e^p \mathbf{K}_e^0, \quad \mathbf{M} = \sum_{e=1}^{ne} \xi_e \mathbf{M}_e^0 \quad (9)$$

where  $\mathbf{K}_e^0$  and  $\mathbf{M}_e^0$  are, respectively, stiffness and mass matrices corresponding to a finite element with fully solid material.  $ne$  is the number of finite elements in the model.

Using stiffness and mass matrices in (9), natural frequencies of the structure  $\omega_i$  can be obtained by solving the dynamic equilibrium equation that governs free vibration of a typical mode  $\boldsymbol{\varphi}_i$  as follows

$$(\mathbf{K} - \lambda_i \mathbf{M}) \boldsymbol{\varphi}_i = 0 \quad (10)$$

where  $\lambda_i = \omega_i^2$ .

#### 4. SENSITIVITY ANALYSIS

To obtain derivative of eigenvalues with respect to the design variable  $\xi_e$ , Equation (10) can be differentiated as

$$\left( \frac{\partial \mathbf{K}}{\partial \xi_e} - \lambda_i \frac{\partial \mathbf{M}}{\partial \xi_e} \right) \boldsymbol{\varphi}_i - \frac{\partial \lambda_i}{\partial \xi_e} \mathbf{M} \boldsymbol{\varphi}_i + (\mathbf{K} - \lambda_i \mathbf{M}) \frac{\partial \boldsymbol{\varphi}_i}{\partial \xi_e} = 0 \quad (11)$$

where the third term is zero based on Equation (10). Pre-multiplying both sides of Equation (11) by  $\boldsymbol{\varphi}_i^T$  and using mass-orthogonal eigenvectors ( $\boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_i = 1$ ), eigenvalue derivative with respect to  $\xi_e$  is given by [40]

$$\frac{\partial \lambda_i}{\partial \xi_e} = \boldsymbol{\varphi}_i^T \left( \frac{\partial \mathbf{K}}{\partial \xi_e} - \lambda_i \frac{\partial \mathbf{M}}{\partial \xi_e} \right) \boldsymbol{\varphi}_i \quad (12)$$

Therefore, derivative of the first objective function based on natural frequencies, defined in Section 2, with respect to design variables is obtained as follows

$$\frac{\partial f(\boldsymbol{\xi})}{\partial \xi_e} = \sum_{i=1}^{nm} 2w_i^\lambda \left( \frac{\partial \lambda_i}{\partial \xi_e} \right) \left( \frac{\lambda_i(\boldsymbol{\xi}) - \lambda_i^*}{\lambda_i^{*2}} \right) \quad (13)$$

In order to find derivative of eigenvectors with respect to design variable  $\xi_e$ , again differentiating equation (10) gives

$$\mathbf{F}_i \frac{\partial \boldsymbol{\varphi}_i}{\partial \xi_e} = -\frac{\partial \mathbf{F}_i}{\partial \xi_e} \boldsymbol{\varphi}_i \quad (14)$$

where  $\mathbf{F}_i = \mathbf{K} - \lambda_i \mathbf{M}$ . Also, by differentiating both sides of the equation  $\boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_i = 1$ , we obtain

$$2\boldsymbol{\varphi}_i^T \mathbf{M} \frac{\partial \boldsymbol{\varphi}_i}{\partial \xi_e} = -\boldsymbol{\varphi}_i^T \frac{\partial \mathbf{M}}{\partial \xi_e} \boldsymbol{\varphi}_i \quad (15)$$

Since  $\mathbf{F}_i$  is a singular matrix, linearly independent equations of (15) might be added to equation (14) as [40]

$$\begin{bmatrix} \mathbf{F}_i \\ 2\boldsymbol{\varphi}_i^T \mathbf{M} \end{bmatrix} \frac{\partial \boldsymbol{\varphi}_i}{\partial \xi_e} = -\begin{bmatrix} \partial \mathbf{F}_i / \partial \xi_e \\ \boldsymbol{\varphi}_i^T (\partial \mathbf{M} / \partial \xi_e) \end{bmatrix} \boldsymbol{\varphi}_i \quad (16)$$

Pre-multiplying Eq. (16) by  $[\mathbf{F}_i / \boldsymbol{\varphi}_i^T \mathbf{M}]^T$  we can obtain

$$\frac{\partial \boldsymbol{\varphi}_i(\boldsymbol{\xi})}{\partial \xi_e} = -[\mathbf{F}_i \mathbf{F}_i + 2\mathbf{M} \boldsymbol{\varphi}_i \boldsymbol{\varphi}_i^T \mathbf{M}]^{-1} \left[ \mathbf{F}_i \frac{\partial \mathbf{F}_i}{\partial \xi_e} + \mathbf{M} \boldsymbol{\varphi}_i \boldsymbol{\varphi}_i^T \frac{\partial \mathbf{M}}{\partial \xi_e} \right] \boldsymbol{\varphi}_i \quad (17)$$

where

$$\frac{\partial \mathbf{F}_i}{\partial \xi_e} = \left[ \frac{\partial \mathbf{K}}{\partial \xi_e} - \lambda_i \frac{\partial \mathbf{M}}{\partial \xi_e} - \frac{\partial \lambda_i}{\partial \xi_e} \mathbf{M} \right] \quad (18)$$

Therefore, derivative of the second objective function based on eigenvectors, mentioned in Section 2, is obtained by the following formulation

$$\frac{\partial f(\boldsymbol{\xi})}{\partial \xi_e} = \sum_{i=1}^{nm} w_i^\phi \sum_{j=1}^{np} 2 \frac{\partial \boldsymbol{\varphi}(\boldsymbol{\xi})}{\partial \xi_e} \Big|_{ij} \frac{\boldsymbol{\varphi}_{ij}(\boldsymbol{\xi})}{|\boldsymbol{\varphi}_{ij}(\boldsymbol{\xi})|} \left( |\boldsymbol{\varphi}_{ij}(\boldsymbol{\xi})| - |\boldsymbol{\varphi}_{ij}^*(\boldsymbol{\xi})| \right) \quad (19)$$

## 5. OPTIMIZATION ALGORITHM

The steepest descent method is used for solving the unconstraint damage detection optimization problem, proposed in this paper. Thus, the optimum search direction in iteration  $k$ ,  $\mathbf{d}^k$ , is chosen to be

$$\mathbf{d}^k = -\nabla f(\boldsymbol{\xi}^k) \quad (20)$$

where  $\nabla f(\xi^k)$  is the gradient of objective function, derived from sensitivity analysis. Also, the step size  $\alpha^k$  is calculated by using the golden search method [41,42]. Finally, design variables of the optimization problem are updated by  $\xi^{k+1} = \xi^k + \alpha^k \mathbf{d}^k$ .

The steepest-descent algorithm for damage detection problem, used in this research, is stated as follows

*Step 1.* It is assumed that there is no damage in the structure for the first step and therefore, starting design variables are set to be  $\xi^0 = \mathbf{1}$ . Also, a convergence parameter  $\varepsilon > 0$  is considered,

*Step 2.* The stiffness matrix, mass matrix, eigenvalues and normalized eigenvectors are calculated. Also, the objective function is evaluated,

*Step 3.* Sensitivity analysis is achieved based on derived equations in Section 4 and search direction is set to be  $\mathbf{d}^k = -\nabla f(\xi^k)$ ,

*Step 4.* If  $\|\nabla f(\xi^k)\| < \varepsilon$ , the optimization iterative process is stopped,

*Step 5.* The step size  $\alpha^k$  is calculated based on minimizing  $f(\alpha) = f(\xi^k + \alpha \mathbf{d}^k)$  in the direction  $\mathbf{d}^k$ ,

*Step 6.* The design variables are updated using  $\xi^{k+1} = \xi^k + \alpha^k \mathbf{d}^k$ . Then, the design variables restrictions  $0 \leq \xi_e \leq 1$  is imposed. Using new design variables, the iterative process continues from step 2 until it is stopped in step 4 or after a certain number of iterations.

## 6. NUMERICAL EXAMPLES

In this section, five numerical examples are presented to demonstrate the capability of the implemented optimization procedure. First example is devoted to natural frequency based optimization and the second objective function created by shape modes is considered for other examples. In all examples, modulus of elasticity and Poisson's ratio are considered to be 2 Pa and 0.3, respectively. The density and thickness are also assumed 0.00785 kg/cm<sup>3</sup> and 1cm, respectively. Also, it should be noted that initial design variables are chosen to be one which means no damage is considered for starting point of optimization process. Using the continuation method, in all examples, penalty factor  $p$  changes from 3 to 1 gradually during the optimization iterations.

**Example 1.** A cantilever beam with a damage in the middle, shown in Fig. 1, is considered. To solve this problem, the beam structure is discretized into 30×30 linear finite elements. The damage is assumed in 24 elements with 100% intensity as it is indicated in Fig. 1. Damage is sought by updating the model to minimize the objective function defined in (1) considering the first five natural frequencies of the structure. The optimum configuration after 300 iterations and iteration history are shown in Figs. 2(a) and 2(b), respectively. The damage location is approximately identified and some fictitious defects are also appeared in other parts of the structure.

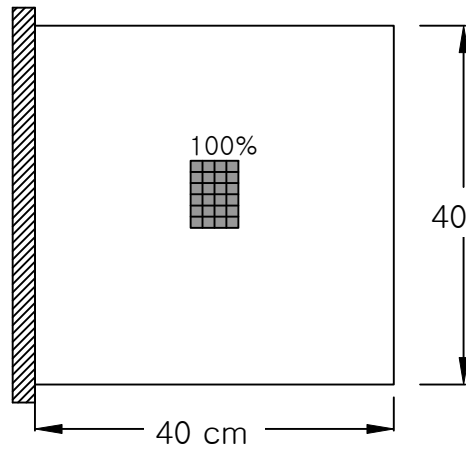


Figure 1. Problem definition in Example 1

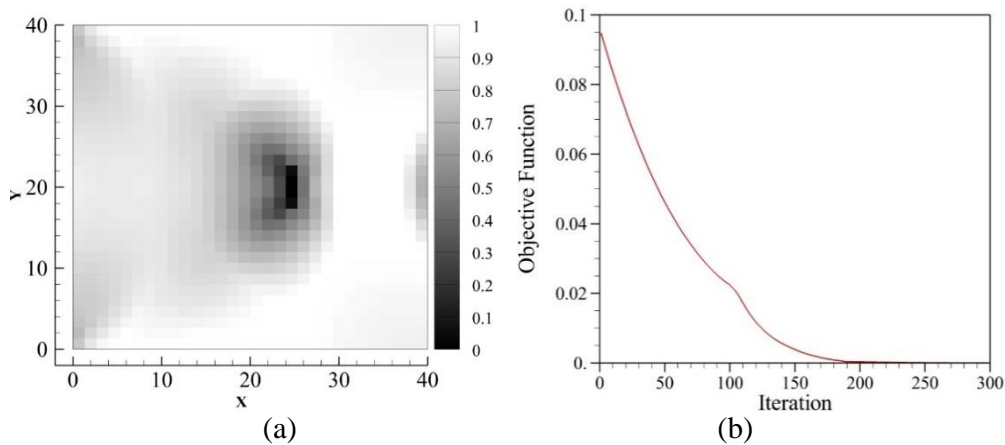


Figure 2. (a) Optimum material distribution (b) Iteration history for Example 1

**Example 2.** Another cantilever beam with different aspect ratio and more complex damage is considered as indicated in Fig. 3. Discrepancy between given and model mode shapes is minimized as defined in Equation (2). The beam is discretized into  $45 \times 30$  linear finite elements. A diagonal damage including 10 finite elements with 15% intensity is considered at the corner.

Obtained results after 25, 50, 75 and 100 iterations are depicted in Fig. 4. The result shows, exact damage location has been detected. Given material density in damage area is 15% and the obtained average density is 12.01% in 100 iterations which makes around 20% error in terms of finding the damage intensity. Fig. 5 shows iterations history in which the objective function has closely converged to zero.



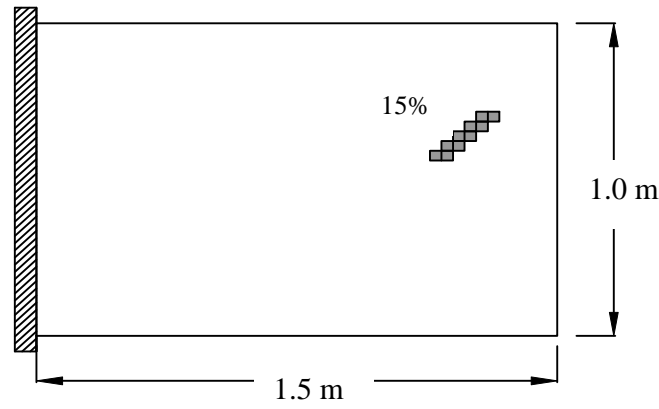


Figure 3. Problem definition for Example 2

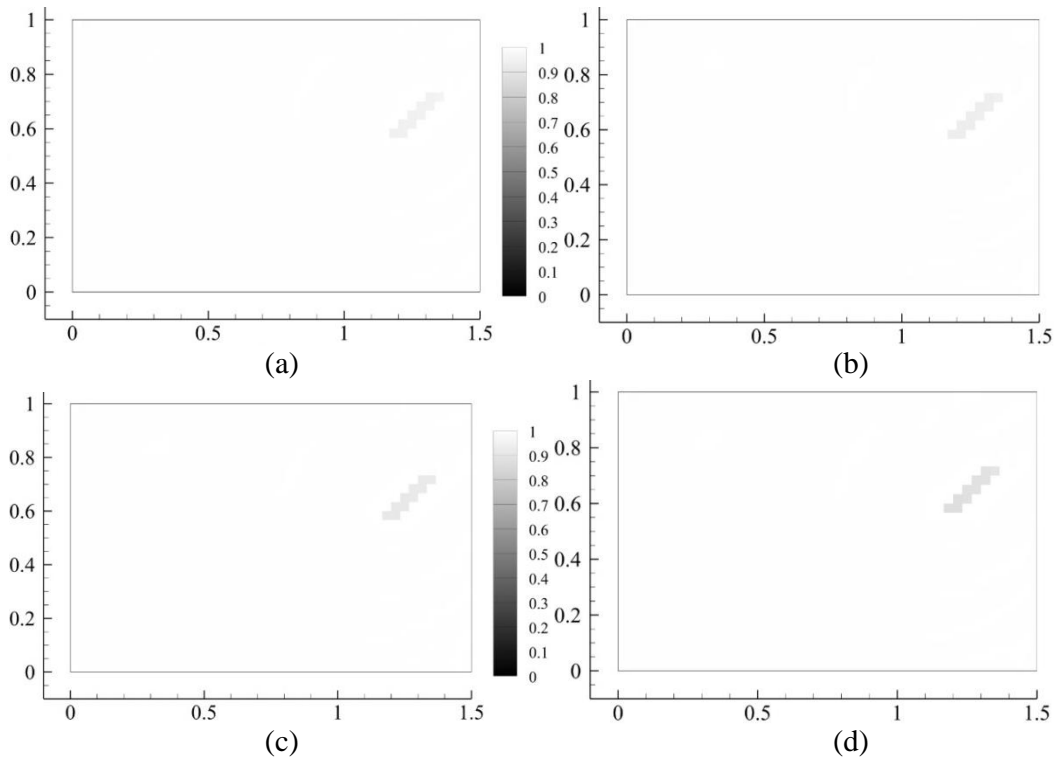


Figure 4. Damaged area, detected in iteration (a) 25, (b) 50, (c) 75, (d) 100

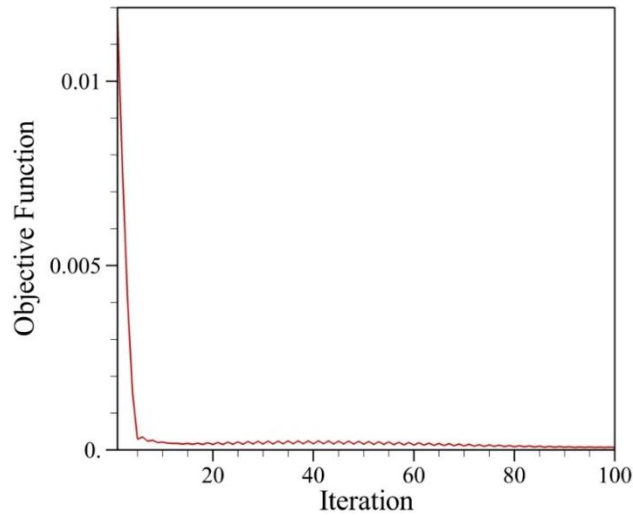


Figure 5. Iteration history for Example 2

**Example 3.** A simple supported beam with 10m length and 1m height is considered as shown in Fig. 6. The objective function is to minimize the difference between three given and the model eigenvectors. The beam is discretized into  $100 \times 10$  linear finite elements. Two full damaged elements are assumed at the right bottom of the beam. The obtained results for 50, 100, 150 and 200 optimization iterations are illustrated in Fig. 7. Iteration history is also depicted in Fig. 8.

Location of damage is precisely evaluated from the first iterations and average damage intensity (material density) in both elements is obtained 95.5% after 200 iterations which is close to existing damage. Some false defects are also emerged with low material density around true damage and in the middle of the beam.

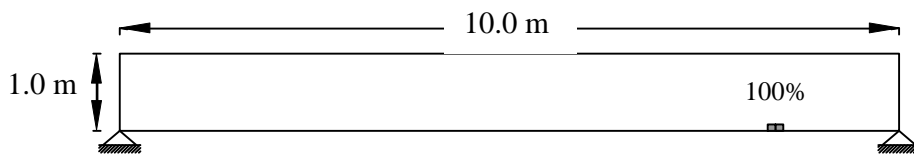
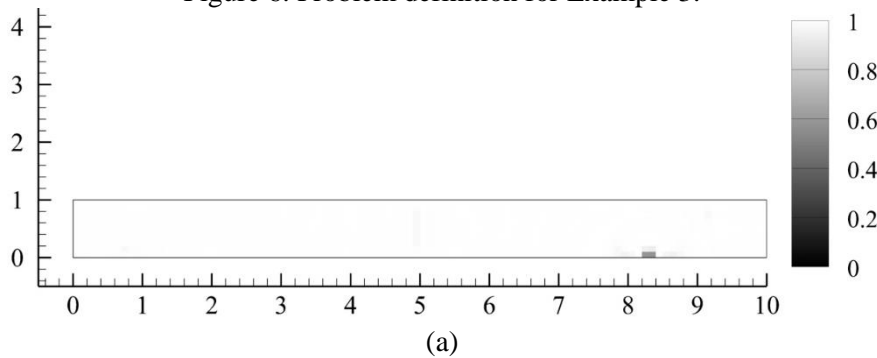


Figure 6. Problem definition for Example 3.



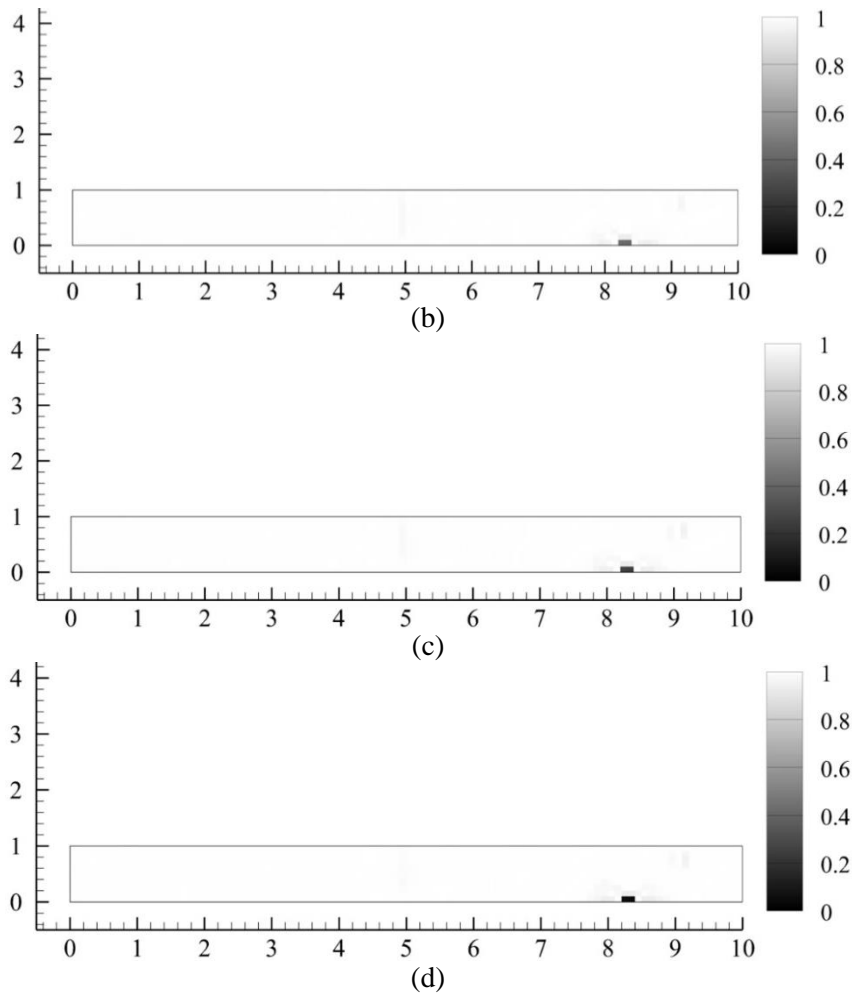


Figure 7. Identified Damages in iteration (a) 50 (b) 100 (c) 150 (d) 200

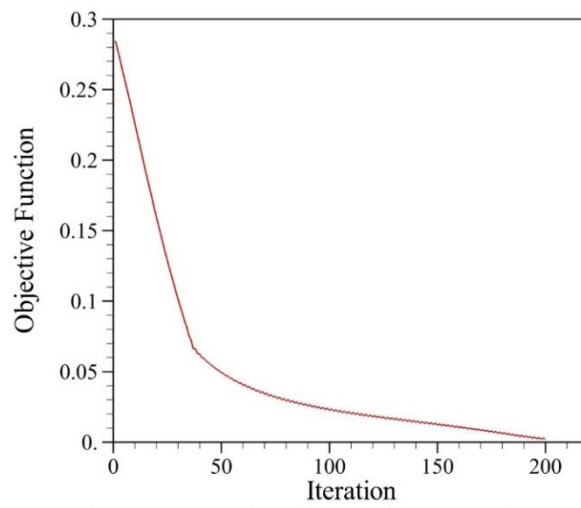


Figure 8. Iteration history for Example 3

**Example 4.** In this example an L-shaped beam with a damage that comprises different intensities is considered as illustrated in Fig. 9. The beam is discretized into 1200 linear finite elements. The main goal of this example is to identify a damage with different intensity. Damage is assigned as if it is a crack at edge of the beam. The intensities are 20%, 40% and 60% as it reaches to the middle of the crack.

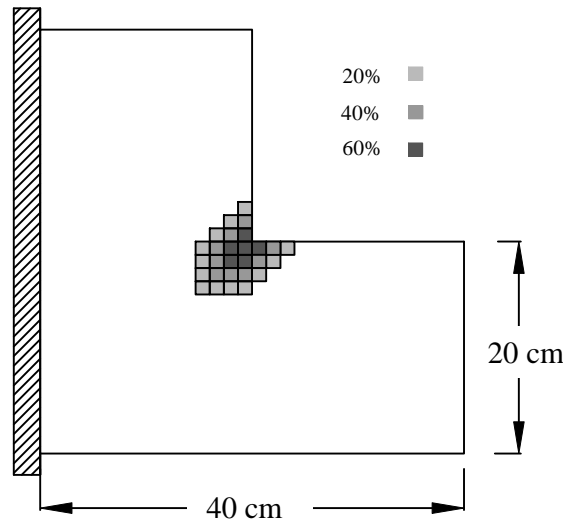


Figure 9. Geometry, boundary conditions and assumed damage in L-shaped beam

Results for iterations 50, 100, 150 and 200 are shown in Fig. 10. As it is indicated, the optimization procedure has effectively detected the damage. There are some extra damages appeared in optimum layout that can be because of complexity in geometry of structure and different intensities in assigned damage. The average damage for each intensity is computed as 13.5%, 32.7% and 53.2% which are compared with original damages in Table 1. It should be noted that extents of damage are predicted less than exact values and also more error obtained for less damage intensity areas. The objective function is converged as illustrated in Fig. 11.

Table 1: Damage intensity error for Example 4

Given damage intensity (%)	Obtained damage intensity (%)	Error (%)
20	13.5	32.5
40	32.7	18.25
50	53.2	6.4

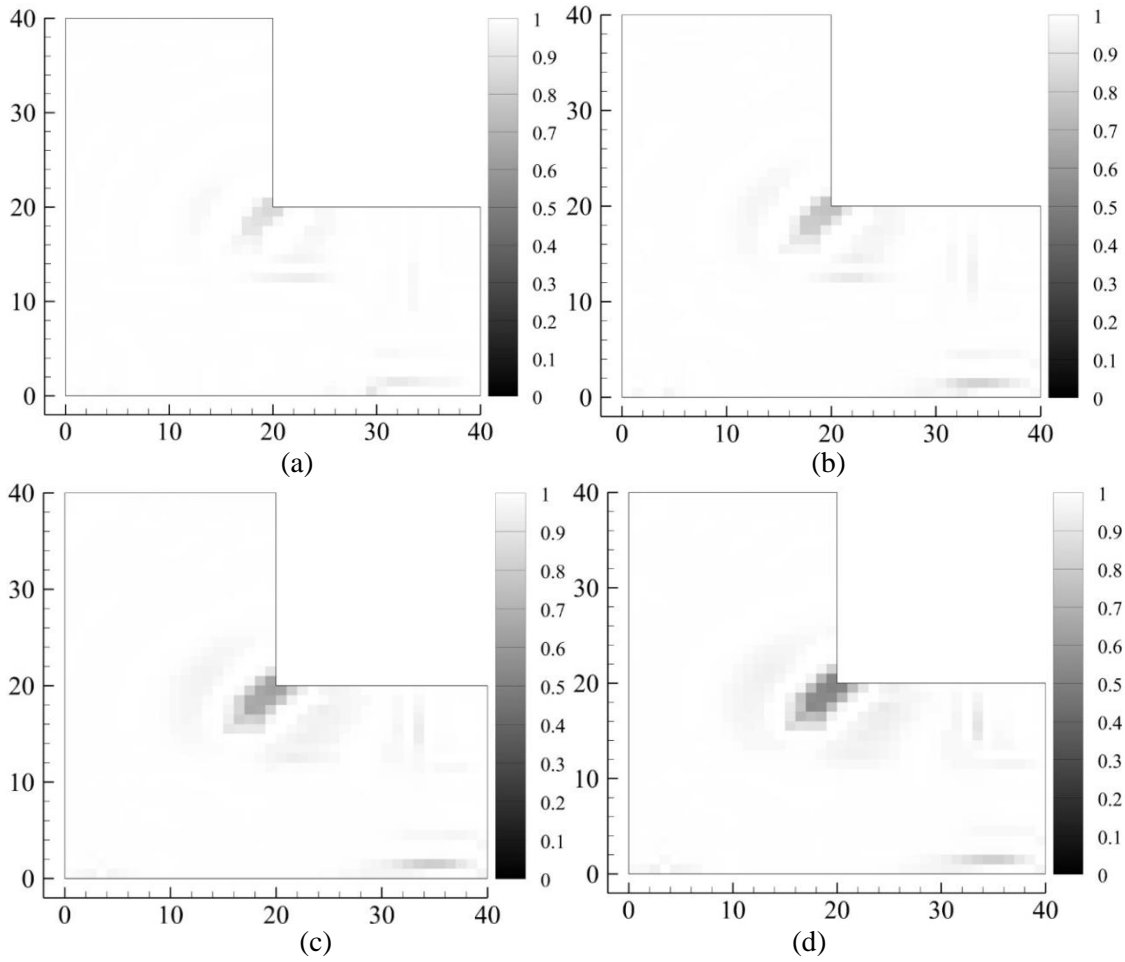


Figure 10. Identified damage areas in iteration (a) 50, (b) 100, (c) 150, (d) 200

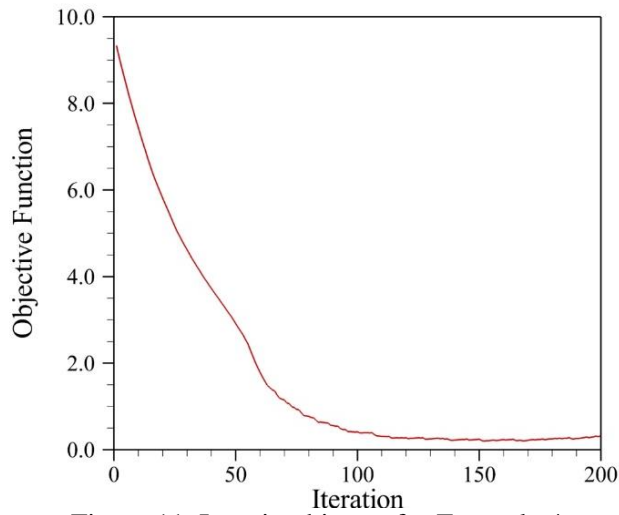


Figure 11. Iteration history for Example 4

**Example 5.** In order to check efficiency of the proposed method, more complicated design domain is given in this example. To achieve this, a bridge shape frame is considered that consists a deck and two diagonal columns with fixed and pin supports as shown in Fig. 12. The frame is discretized into 1024 linear finite elements. Damages are assigned in three different regions, first, at left column including two elements with intensity of 20%. Second damage is in the middle zone of the deck containing two elements with 30% damage and the third damage is located at left side of deck which has 15% intensity. First three mode shapes of the structure are considered for constructing the objective function.

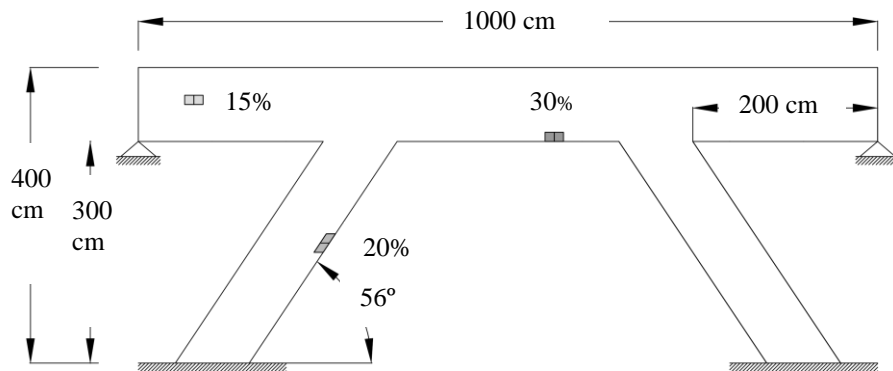
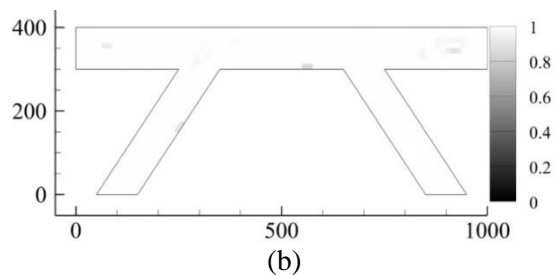
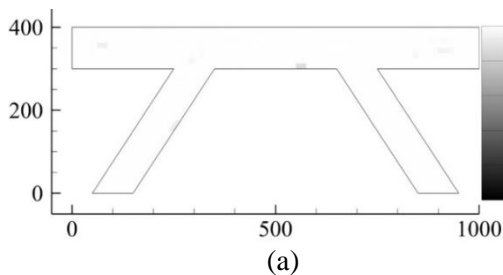


Figure 12. Problem definition for Example 5

Fig. 13 demonstrates the results of damage detection procedure. It is observed that both location and severity of damages are appropriately detected. Also, extra damages are appeared in optimum layout. The average density of material in damages are obtained as 13%, 16.5%, and 27.5% for 15%, 20%, and 30% damages, respectively. Fig.14 indicates that the objective function is minimized and oscillates during optimization iterations. However, the oscillations are decreased in the last iterations and the objective function is almost converged.

Table 2: Damage intensity error for Example 5

Given damage intensity (%)	Obtained damage intensity (%)	Error (%)
15	13	13.34
20	16.5	17.5
30	27.5	8.33



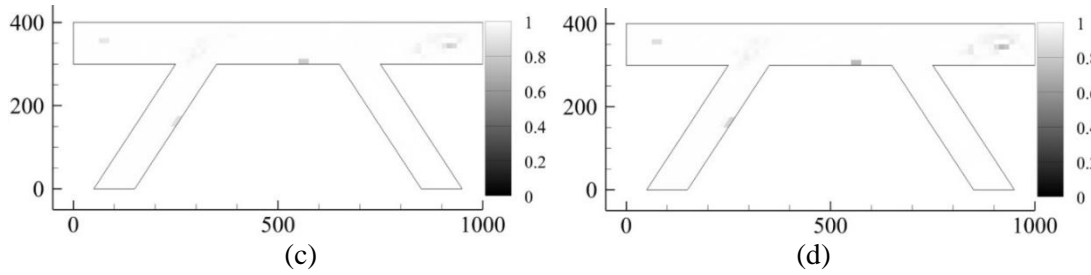


Figure 13. Detected damages in iteration (a) 30, (b) 60, (c) 90, (d) 120

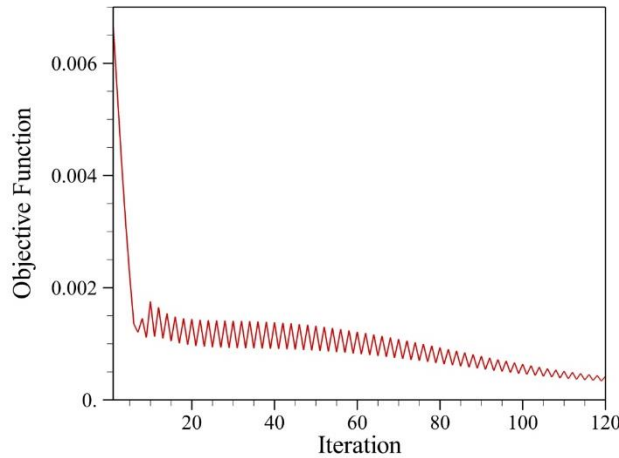


Figure 14. Iteration history for Example 5

## 7. CONCLUSION

Topology optimization is utilized for damage identification in plane elasticity problems. Damage is simulated by a material density reduction in the structure. An SIMP based material model is defined to parametrize the optimization problem. Discrepancy of modal data between real structure and the model is minimized to find the material distribution in the design domain. Eigenvalues based objective function is employed in example 1 and the defect is approximately detected. Fictitious defect areas are also identified in other regions of the design domain. Four examples are also achieved considering eigenvectors as the objective function. According to the results, depends on complexity in geometry of the structure as well as location and extent of damages, different precision is obtained. Location of damages is often identified precisely, but some extra untrue defects are also identified during the optimization process. In addition, reasonable intensity can be found which is frequently less than true solution. More error is found for detecting damages with less intensity in the examples.

## REFERENCES

1. Paolozzi A, Peroni I. Detection of debonding damage in a composite plate through natural frequency variations, *J Reinf Plast Compos* 1990; **9**: 369-89. doi:10.1177/073168449000900405.
2. Dems K, Mróz Z. Identification of damage in beam and plate structures using parameter-dependent frequency changes, *Eng Comput* 2001; **18**: 96-120. doi:10.1108/02644400110365833.
3. Yang Z, Chen X, Yu J, Liu R, Liu Z, He Z. A damage identification approach for plate structures based on frequency measurements, *Nondestruct Test Eval* 2013; **28**: 321-41. doi:10.1080/10589759.2013.801472.
4. Cawley P, Adams RD. A vibration technique for non - destructive testing of fibre composite structures, *J Compos Mater* 1979; **13**: 161-75. doi:10.1177/002199837901300207.
5. Haute T, Alamos L. Application of the strain energy damage detection method to plate, *Like Struct* 1999; **224**.
6. Li YY, Cheng L, Yam LH, Wong WO. Identification of damage locations for plate-like structures using damage sensitive indices: Strain modal approach, *Comput Struct* 2002; **80**: 1881-94. doi:10.1016/S0045-7949(02)00209-2.
7. Qiao P, Lu K, Lestari W, Wang J. Curvature mode shape-based damage detection in composite laminated plates, *Compos Struct* 2007; **80**: 409-28. doi:10.1016/j.compstruct.2006.05.026.
8. Moreno-García P, Araújo dos Santos J V., Lopes H. A new technique to optimize the use of mode shape derivatives to localize damage in laminated composite plates, *Compos Struct* 2014; **108**: 548-54. doi:10.1016/j.compstruct.2013.09.050.
9. Navabian N, Bozorgnasab M, Taghipour R, Yazdanpanah O. Damage identification in plate-like structure using mode shape derivatives, *Arch Appl Mech* 2016; **86**: 819-30. doi:10.1007/s00419-015-1064-x.
10. Beskhyroun S, Wegner LD, Sparling BF. New methodology for the application of vibration-based damage detection techniques, *Struct Control Heal Monit* 2011; **19**: 88-106. doi:10.1002/stc.
11. Eraky A, Saad A, Anwar AM, Abdo A. Damage detection of plate-like structures based on residual force vector, *HBRC J* 2016; **12**: 255-62. doi:10.1016/j.hbrj.2015.01.005.
12. Katunin A. Vibration-based damage identification in composite circular plates using polar discrete wavelet transform, *J Vibroeng* 2013; **15**: 355-63.
13. Cao MS, Xu H, Bai RB, Ostachowicz W, Radziński M, Chen L. Damage characterization in plates using singularity of scale mode shapes, *Appl Phys Lett* 2015; **106**. doi:10.1063/1.4916678.
14. Hajela P, Soeiro FJ. Structural damage detection based on static and modal analysis, *AIAA J* 1990; **28**: 1110-5. doi:10.2514/3.25174.
15. Hassiotis S, Jeong GD. Identification of stiffness reductions using natural frequencies, *J Eng Mech* 1995; **121**: 1106-13. doi:10.1061/(ASCE)0733-9399(1995)121:10(1106).
16. Morassi A, Rovere N. Localizing a notch in a steel frame from frequency measurements, *J Eng Mech ASCE*, 1997; **123**: 422-32.



17. Majumdar A, Maiti DK, Maity D. Damage assessment of truss structures from changes in natural frequencies using ant colony optimization, *Appl Math Comput* 2012; **218**: 9759–72. doi:10.1016/j.amc.2012.03.031.
18. Hao H, Xia Y. Vibration-based damage detection of structures by genetic algorithm, *J Comput Civ Eng* 2002; **16**: 222–9. doi:10.1061/(ASCE)0887-3801(2002)16:3(222).
19. Teughels A, Maeck J, De Roeck G. Damage assessment by FE model updating using damage functions, *Comput Struct* 2002; **80**: 1869–79. doi:10.1016/S0045-7949(02)00217-1.
20. Kaveh A, Dadras A. Structural damage identification using an enhanced thermal exchange optimization algorithm, *Eng Optim* 2017; **0**: 1–22. doi:10.1080/0305215X.2017.1318872.
21. Kaveh A, Javadi SM, Maniat M. Damage assessment via modal data with a mixed particle swarm strategy, ray optimizer, and harmony search, *Asian J Civil Eng* 2014; **15**: 95–106.
22. Kaveh A, Zolghadr A. An improved CSS for damage detection of truss structures using changes in natural frequencies and mode shapes, *Adv Eng Softw* 2015; **80**: 93–100. doi:10.1016/j.advengsoft.2014.09.010.
23. Kaveh A, Vaez SRH, Hosseini P, Fallah N. Detection of damage in truss structures using Simplified Dolphin Echolocation algorithm based on modal data, *Smart Struct Syst* 2016; **18**: 983–1004. doi:10.12989/sss.2016.18.5.983.
24. Kaveh A, Zolghadr A. Cyclical Parthenogenesis Algorithm for guided modal strain energy based structural damage detection, *Appl Soft Comput J* 2017; **57**: 250–64. doi:10.1016/j.asoc.2017.04.010.
25. Kaveh A, Zolghadr A. Guided Modal strain energy-based approach for structural damage identification using tug-of-war optimization algorithm, *J Comput Civil Eng* 2017; **31**: 4017016. doi:10.1061/(ASCE)CP.1943-5487.0000665.
26. Kaveh A. *Applications of Metaheuristic Optimization Algorithms in Civil Engineering*, 2016. doi:10.1007/978-3-319-48012-1.
27. Kaveh A. *Advances in Metaheuristic Algorithms for Optimal Design of Structures*, second edition, 2016. doi:10.1007/978-3-319-46173-1.
28. Bendsøe MP, Kikuchi N. Generating optimal topologies in structural design using a homogenization method, *Comput Meth Appl Mech Eng* 1988; **71**: 197–224. doi:10.1016/0045-7825(88)90086-2.
29. Rozvany GIN, Zhou M. The COC algorithm, part I: Cross-section optimization or sizing, *Comput Methods Appl Mech Eng* 1991; **89**: 281–308. doi:10.1016/0045-7825(91)90045-8.
30. Hassani B, Hinton E. *Homogenization and Structural Topology Optimization: Theory, Practice and Software*, London, UK, Springer, 1999.
31. Bendsøe MP, Sigmund O. *Topology Optimization: Theory, Methods, and Applications*, 2nd Editio, 2003. doi:10.1063/1.3278595.
32. Wang MY, Wang X, Guo D. A level set method for structural topology optimization, *Comput Methods Appl Mech Eng* 2003; **192**: 227–46. doi:10.1016/S0045-7825(02)00559-5.
33. Allaire G, Jouve F, Toader AM. Structural optimization using sensitivity analysis and a level-set method, *J Comput Physic* 2004; **194**: 363–393. doi:10.1016/j.jcp.2003.09.032.

34. Dunning PD, Kim HA. Introducing the sequential linear programming level-set method for topology optimization, *Struct Multidis Optim* 2015; **51**: 631–43. doi:10.1007/s00158-014-1174-z.
35. Jahangiry HA, Tavakkoli SM. An isogeometrical approach to structural level set topology optimization, *Comput Meth Appl Mech Eng* 2017; **319**. doi:10.1016/j.cma.2017.02.005.
36. Lee JS, Kim JE, Kim YY. Damage detection by the topology design formulation using modal parameters, *Int J Numer Meth Eng* 2007; **69**: 1480–98. doi:10.1002/nme.1817.
37. Nishizu T, Takezawa A, Kitamura M. Eigenfrequency-based damage identification method for non-destructive testing based on topology optimization, *Eng Optim* 2017; **49**: 417–33. doi:10.1080/0305215X.2016.1190350.
38. Zhang W, Du Z, Sun G, Guo X. A level set approach for damage identification of continuum structures based on dynamic responses, *J Sound Vib* 2017; **386**: 100–15. doi:10.1016/j.jsv.2016.06.014.
39. Du J, Olhoff N. Topological design of freely vibrating continuum structures for maximum values of simple and multiple eigenfrequencies and frequency gaps, *Struct Multidis Optim* 2007; **34**: 91–110. doi:10.1007/s00158-007-0101-y.
40. Fox RL, Kapoor MP. Rates of Change of Eigenvalues and Eigenvectors, *Aiaa J* 1968; **6**: 2426–9.
41. Arora JS. *Introduction to Optimum Design*, 2004. doi:10.1016/B978-0-12-064155-0.X5000-9.
42. Haftka RT, Gürdal Z. Elements of structural optimization, *Elem Struct Optim* 1992; **11**: 481. doi:10.1002/nme.2403.