

Modified Sine-Cosine Algorithm for Sizing Optimization of Truss Structures with Discrete Design Variables

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ABSTRACT

This paper proposes a modified sine cosine algorithm (MSCA) for discrete sizing optimization of truss structures. The original sine cosine algorithm (SCA) is a population-based metaheuristic that fluctuates the search agents about the best solution based on sine and cosine functions. The efficiency of the original SCA in solving standard optimization problems of well-known mathematical functions has been demonstrated in literature. However, its performance in tackling the discrete optimization problems of truss structures is not competitive compared with the existing metaheuristic algorithms. In the framework of the proposed MSCA, a number of worst solutions of the current population is replaced by some variants of the global best solution found so far. Moreover, an efficient mutation operator is added to the algorithm that reduces the probability of getting stuck in local optima. The efficiency of the proposed MSCA is illustrated through multiple benchmark optimization problems of truss structures.

Keywords: discrete optimization; sizing optimization; truss structures; metaheuristic; sine cosine algorithm.

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1. INTRODUCTION

The material cost is an important factor in the construction of structures and from an economical viewpoint, the design of a minimum weight is the best structure. In order to find such designs, structural optimization techniques can be effectively used. In the last decade, many optimization techniques have been developed and successfully applied to a wide range of structural optimization problems including sizing, layout and topology optimization problems [1-3]. Metaheuristics are the most general kinds of stochastic optimization algorithms and they are now recognized as one of the most practical approaches for solving

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a wide range of optimization problems. The main idea behind designing these metaheuristic algorithms is to solve complex optimization problems where other optimization methods have failed to be effective. The practical advantage of metaheuristics lies in both their effectiveness and general applicability. In recent years, metaheuristic algorithms are emerged as the global search approaches which are responsible to tackle the complex optimization problems.

Most of the metaheuristic algorithms are developed based on natural phenomena. Every metaheuristic method consists of a group of search agents that explore the design space based on randomization and some specified rules inspired the laws of natural phenomena. For example, Genetic Algorithms (GA) [4], Biogeography-Based Optimization (BBO) [5], and Differential Evolution (DE) [6] are developed based on the Darwin's principle of survival of the fittest. Gravitational Search Algorithm (GSA) [7], Colliding Bodies Optimization (CBO) [8] and Center of Mass Optimization (CMO) [9] are Physics-based metaheuristic algorithms. Particle Swarm Optimization [10] (PSO), Ant Colony Optimization [11] (ACO), Bat algorithm [12] (BA) and Dolphin Echolocation Algorithm (DEA) [13] are recognized as popular Swarm intelligence metaheuristics. One of the newly developed metaheuristic algorithms is the Sine Cosine Algorithm (SCA), which is proposed by Mirjalili [14]. The SCA requires that the generated solutions fluctuate outwards or towards the best solution found so far using sine and cosine functions. It was demonstrated in [14] that the SCA is able to effectively solve the continuous optimization problems.

Optimization of truss structures is very popular in the area of structural optimization and over the last decades, various algorithms have been proposed for solving these problems. There is a significant number of metaheuristics employed for truss optimization with discrete variables in the literature such as: Discrete Heuristic Particle Swarm Ant Colony Optimization (DHPSACO) [15], Improved Dolphin Echolocation Algorithm (IDEA) [16], Improved Mine Blast Algorithm (IMBA) [17], Adaptive Elitist Differential Evolution (AEDE) [18], and Improved Fireworks Algorithm (IFWA) [19]. In the present study, SCA is focused and a modified Sine Cosine algorithm (MSCA) is proposed to handle the truss structures optimization with discrete design variables. In the MSCA two main strategies are followed for the exploration and exploitation of the design space. In the first strategy, some of worst solutions in each iteration are removed and the same number of variants of the best solution is added to the population. In the second strategy, a mutation operator is added to the algorithm. Five benchmark optimization problems of truss structures with discrete variables are presented and the results of MSCA are compared with literature.

2. TRUSS OPTIMIZATION PROBLEM

For the optimization problem of trusses, objective function is the structural weight and some limitations are usually considered on nodal displacements and element stress as the design constraints. Formulation of truss structures optimization problem is as follows:

$$\text{Minimize: } W = \sum_{i=1}^n \gamma_i l_i X_i, \quad i = 1, 2, \dots, n \quad (1)$$

$$\text{Subject to: } \begin{cases} g_j^d = \frac{d_j}{\bar{d}_j} - 1, & j = 1, 2, \dots, m \\ g_k^s = \frac{\sigma_k}{\bar{\sigma}_k} - 1, & k = 1, 2, \dots, n \end{cases} \quad (2)$$

$$X_i^L \leq X_i \leq X_i^U \quad (3)$$

where W is structural weight; γ_i , l_i and X_i are the density of material, element length and cross-sectional area of i th element, respectively; displacement and stress constraints are represented by g_j^d and g_k^s , respectively; d_j and σ_k are j th node displacement and k th element stress, respectively; \bar{d}_j and $\bar{\sigma}_k$ are their allowable values; n and m are numbers of elements and nodes, respectively.

The following exterior penalty function (EPF) is employed to handle the constraints of the above constrained optimization problem.

$$\Phi = W \times \left(1 + r_p \sum_{j=1}^m (\max\{0, g_j^d\})^2 + r_p \sum_{k=1}^n (\max\{0, g_k^s\})^2 \right) \quad (4)$$

where Φ is pseudo unconstrained objective function; and r_p is a penalty parameter. In this study, r_p is linearly increased from 1.0 at the first iteration to 10^6 at the last one during the optimization process.

3. SINE COSINE ALGORITHM

In essence, all population-based metaheuristic algorithms explore the design space using a number of search agents which follow a set of updating rules. These updating rules play an important role in performance of the metaheuristic algorithms. In the Sine Cosine Algorithm (SCA) [14] the following equation is used as the updating rule of position of population in the design space:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \times \sin(r_2) \times |r_3 \times P_j^t - X_{i,j}^t|, & r_4 < 0.5 \\ X_{i,j}^t + r_1 \times \cos(r_2) \times |r_3 \times P_j^t - X_{i,j}^t|, & r_4 \geq 0.5 \end{cases} \quad (5)$$

$$r_1 = a \times \left(1 - \frac{t}{t_{\max}} \right) \quad (6)$$

where $X_{i,j}^{t+1}$ and $X_{i,j}^t$ are the j th design variable of the i th solution at iterations $t+1$ and t , respectively; $\sin(\cdot)$ and $\cos(\cdot)$ represent the sine and cosine mathematical functions; P_j^t is the j th design variable of the best solution; a is a constant and in this study $a = 2.0$; t_{\max} is the

maximum number of iterations; r_2 and r_4 are random numbers in $[0, 2\pi]$ and $[0,1]$, respectively; and r_3 is a random number that $r_3 > 1$ and $r_3 < 1$ emphasize and deemphasize the effect of the best solution in defining the distance.

It was demonstrated in [14] that the original SCA has proper performance in solving standard optimization problems of well-known mathematical functions. The computational experience of the present study however reveals that the SCA is not an efficient metaheuristic algorithm for discrete sizing optimization of truss structures.

4. MODIFIED SINE COSINE ALGORITHM

In order to improve the performance of the SCA in dealing with the discrete sizing optimization problems of truss structures two computational strategies are implemented and the improved metaheuristic is named as modified Sine Cosine algorithm (MSCA). The proposed strategies, termed here as Regeneration and Mutation, are described below. In addition, updating rule of position of population in the discrete MSCA is as follows:

$$X_{i,j}^{t+1} = \begin{cases} \text{round}(X_{i,j}^t + r_1 \times \sin(r_2) \times |r_3 \times P_j^t - X_{i,j}^t|), & r_4 < 0.5 \\ \text{round}(X_{i,j}^t + r_1 \times \cos(r_2) \times |r_3 \times P_j^t - X_{i,j}^t|), & r_4 \geq 0.5 \end{cases} \quad (7)$$

where $\text{round}(\cdot)$ rounds numbers to their nearest integer.

Regeneration: in each iteration of the optimization process, the population, including np particles, is sorted in an ascending order based on the objective function values of particles as represented below:

$$\text{sort}(\mathbf{X}^t) = [X_1^t \ X_2^t \ \dots \ X_i^t \ \dots \ X_{k-1}^t \ X_k^t \ X_{k+1}^t \ \dots \ X_l^t \ \dots \ X_{np-1}^t \ X_{np}^t] \quad (8)$$

where $\text{sort}(\mathbf{X}^t)$ is the sorted current population; and X_k^t to X_{np}^t are the worst solutions at iteration t that should be regenerated.

Then, a number of $\lambda \times np$ worst particles (i.e. X_k^t to X_{np}^t) are removed from the population and instead, the best solution found so far, $\mathbf{X}^* = [X_1^* \ X_2^* \ \dots \ X_j^* \ \dots \ X_n^*]^T$, is copied $\lambda \times np$ times in the population. In these solutions, except the last one, one randomly selected design variable is regenerated in the design space on a random basis as follows:

$$X_l^t \rightarrow [X_1^* \ X_2^* \ \dots \ X_j^* \ \dots \ X_n^*]^T, \quad l = k, \dots, np - 1 \quad (9)$$

in which

$$X_j^* = \text{round}(X_j^L + r \times (X_j^U - X_j^L)), \quad j \in [1, 2, \dots, n] \quad (10)$$

where X_j^L and X_j^U are lower and upper bounds of the j th design variable; and r is a random

number in [0,1].

The regenerated design variables of particles X_k^t to X_{np-1}^t are substituted in the last particle (X_{np}^t). This strategy will increase the probability of finding the promising regions of the design space.

Mutation: in the framework of MSCA, a mutation operation is implemented to reduce the probability of trapping into local optima. In this way, a mutation rate of mr is considered and for each particle (X_i , $i=1,2,\dots,np$) a random number in [0,1] is selected in each iteration. If for the i th particle, the selected random number is less than mr , X_i will be regenerated in the design space as follows:

$$X_i^{t+1} = \text{round}\left(X_i^t + \left(\frac{t}{t_{\max}}\right) \times R^t \otimes (X_{\text{best}}^t - X_r^t)\right) \tag{11}$$

where in iteration t , R^t is a vector of random numbers in [0, 1]; X_{best}^t is the best particle of the current population; and X_r^t is a randomly selected particle from the current population.

In the framework of MSCA, a simple mechanism is employed to return into the feasible region the agents that violate side constraints. During the optimization process, if a design variable violates the side constraints, it will be replaced by the lower/upper bound as follows:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^L & \text{if } X_{i,j}^{t+1} < X_{i,j}^L \\ X_{i,j}^U & \text{if } X_{i,j}^{t+1} > X_{i,j}^U \end{cases} \tag{12}$$

where $X_{i,j}^L$ and $X_{i,j}^U$ are respectively the lower and upper bounds of the j th design variable of the i th solution.

The best combination of internal parameters λ and mr , is determined by performing sensitivity analysis. In this way, $\lambda \in \{0.1, 0.2, 0.3\}$ and $mr \in \{0.01, 0.05, 0.10\}$ are considered and for each combination of these two parameters, 20 independent optimization runs are conducted. The results of this study demonstrate that the best combination is $\lambda=0.2$ and $mr=0.05$.

The flowchart of MSCA is depicted in Fig. 1.

5. NUMERICAL RESULTS

In order to illustrate the merit of the proposed MSCA, a number of popular discrete benchmark truss optimization problems are presented and the obtained results are compared with those of literature. For the presented examples, 20 independent optimization runs are performed and the best weight (Best), average weight (Average) and the standard deviation (SD) of optimal weights are reported.

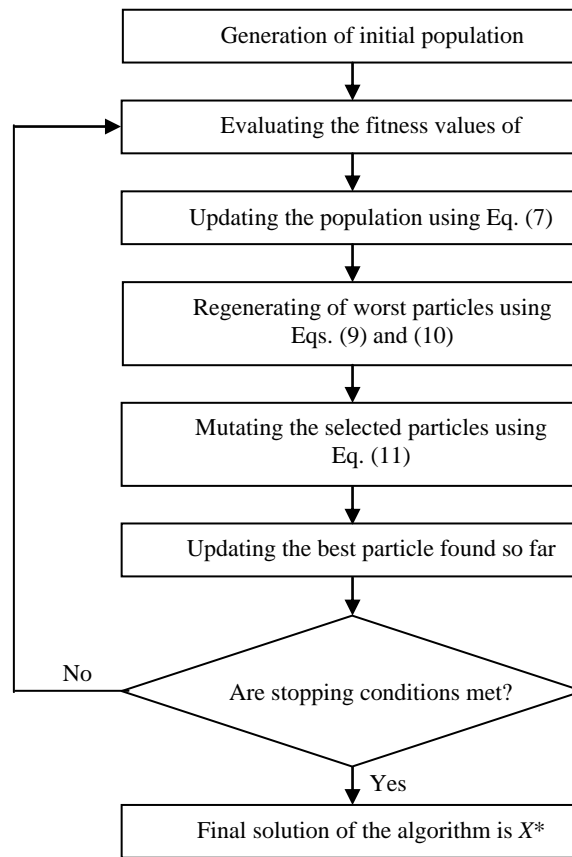


Figure 1. Flowchart of MSCA

5.1 Example 1: 10-bar planar truss

The 10-bar truss shown in Fig. 2 is one of the most extensively studied problems. The vertical load in nodes 2 and 4 is equal to 10^5 lb. The Young's modulus and density of material are 10^4 ksi and 0.1 lb/in³, respectively.

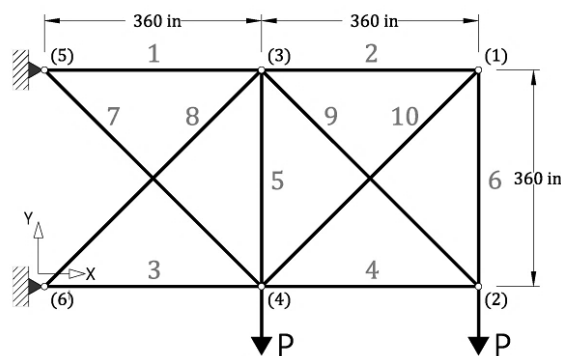


Figure 2. 10-bar truss

The allowable stress for all members is specified as 25 ksi both in tension and

compression. The maximum displacements of all free nodes in the x and y directions are limited to ± 2 in. In this example, the discrete design variables are selected from the following list: [1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50] (in^2).

Cross-sectional areas of elements 1 to 10 (i.e. A_1 to A_{10}) are considered as the design variables. In the optimization process, 50 particles are involved and the maximum number of iterations is chosen to be 200.

The optimization results of SCA and MSCA are compared with those of HPSO [20], HHS [21], and AEDE [18] in Table 1. In addition, the convergence curves of SCA and MSCA are compared in Fig. 3.

Table 1: Results of optimization for the 10-bar truss

Design variables	HPSO	HHS	AEDE	SCA	MSCA
A_1	30.0	33.5	33.5	26.5	33.5
A_2	1.62	1.62	1.62	2.62	1.62
A_3	22.9	22.9	22.9	26.5	22.9
A_4	13.5	14.2	14.2	18.8	14.2
A_5	1.62	1.62	1.62	1.62	1.62
A_6	1.62	1.62	1.62	2.38	1.62
A_7	7.97	7.97	7.97	11.5	7.97
A_8	26.5	22.9	22.9	22.0	22.9
A_9	22.0	22.0	22.0	19.9	22.0
A_{10}	1.80	1.62	1.62	1.80	1.62
Best (lb)	5531.98	5490.74	5490.74	5633.44	5490.74
Average (lb)	N/A	5493.49	5502.62	5838.26	5492.64
SD (lb)	N/A	10.46	20.78	220.39	2.42
Analyses	50000	5000	2550	10000	10000

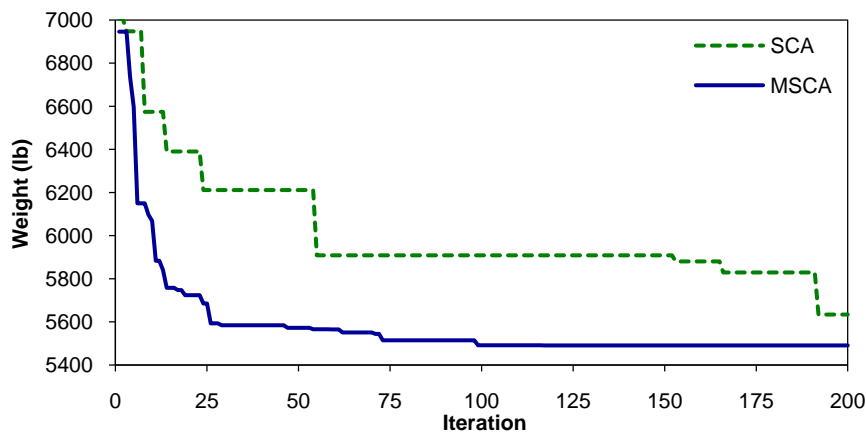


Figure 3. Convergence histories of SCA and MSCA for 10-bar truss

It can be observed that SCA could not provide competitive results compared with the other algorithms. MSCA, AEDE and HHS find the best optimal design among other algorithms. However, the statistical results of MSCA, in terms of Average and SD are very better than those of AEDE and HHS. Moreover, the convergence rate of the MSCA is considerably better than that of the original SCA.

5.2 Example 2: 25-bar spatial truss

The 25-bar spatial truss structure, shown in Fig. 4, is one of the popular design examples in literature. The material density is 0.1 lb/in^3 and the modulus of elasticity is 10^4 ksi . The structure includes 25 members, which are divided into eight groups, as follows: (1) A_1 , (2) A_2 – A_5 , (3) A_6 – A_9 , (4) A_{10} – A_{11} , (5) A_{12} – A_{13} , (6) A_{14} – A_{17} , (7) A_{18} – A_{21} and (8) A_{22} – A_{25} . The allowable stress of the members is $\pm 40 \text{ ksi}$ and all nodes are subjected to displacement limitation of $\pm 0.35 \text{ in}$.

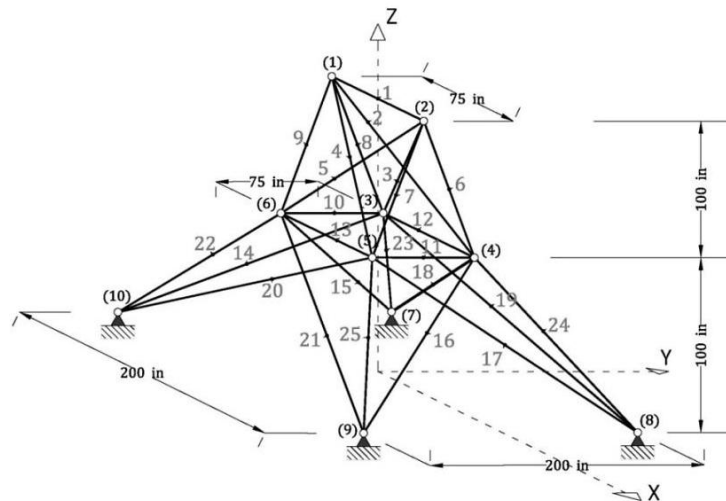


Figure 4. 25-bar spatial truss

The design variables will be selected from the set: [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4] (in^2). The loads applied to the truss are given in Table 2.

Table 2: Loading conditions for the 25-bar truss (kips)

Node	F_x	F_y	F_z
1	1.0	-10.0	-10.0
2	0.0	-10.0	-10.0
3	0.5	0.0	0.0
6	0.6	0.0	0.0

The optimization results of this study, by considering 50 particles and 100 iterations, are compared with those of HHS [21], AEDE [18] and ECBO [22] in Table 3. In addition, Fig. 5 compares the convergence curves of SCA and MSCA.

Table 3: Results of optimization for the 25-bar truss

Design variables	HPSO	ECBO	AEDE	SCA	MSCA
A_1	0.1	0.1	0.1	0.3	0.1
A_2-A_5	0.3	0.3	0.3	0.1	0.3
A_6-A_9	3.4	3.4	3.4	3.4	3.4
$A_{10}-A_{11}$	0.1	0.1	0.1	0.1	0.1
$A_{12}-A_{13}$	2.1	2.1	2.1	1.4	2.1
$A_{14}-A_{17}$	1.0	1.0	1.0	1.2	1.0
$A_{18}-A_{21}$	0.5	0.5	0.5	0.7	0.5
$A_{22}-A_{25}$	3.4	3.4	3.4	3.4	3.4
Best (lb)	484.85	484.85	484.85	486.29	484.85
Average (lb)	-	485.89	485.01	491.17	484.94
SD (lb)	-	-	0.273	2.55	0.22
Analyses	25000	7050	1678	5000	5000

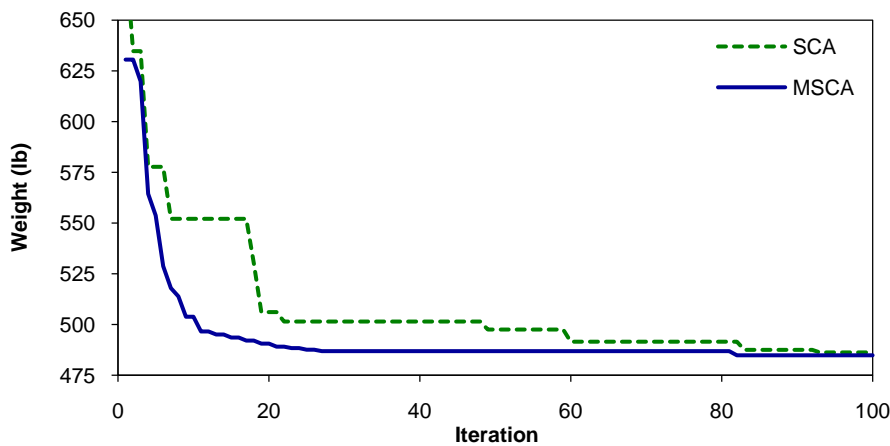


Figure 5. Convergence histories of SCA and MSCA for 10-bar truss

The numerical results indicate that, all algorithms, except SCA, converge to the best optimal design however, the Average, SD and convergence rate of MSCA are better in comparison with other algorithms. The computational effort of HPSO is significantly more than that of other algorithms. As shown in Fig. 5, the MSCA present better convergence behavior in comparison with the original SCA.

5.3 Example 3: 52-bar planar truss

Another popular benchmark truss optimization problem is the 52-bar truss shown in Fig. 6 in which $P_x=100$ kN and $P_y=200$ kN. The Young's modulus, the material density and the allowable stress are 207 GPa, 7860 kg/m³ and ± 180 MPa, respectively. Element groups are as: (1) A_1-A_4 , (2) A_5-A_{10} , (3) $A_{11}-A_{13}$, (4) $A_{14}-A_{17}$, (5) $A_{18}-A_{23}$, (6) $A_{24}-A_{26}$, (7) $A_{27}-A_{30}$, (8) $A_{31}-A_{36}$, (9) $A_{37}-A_{39}$, (10) $A_{40}-A_{43}$, (11) $A_{44}-A_{49}$, and (12) $A_{50}-A_{52}$ which are selected from Table 4 during the optimization process. In this example, population size and maximum number of iterations are 50 and 200, respectively.

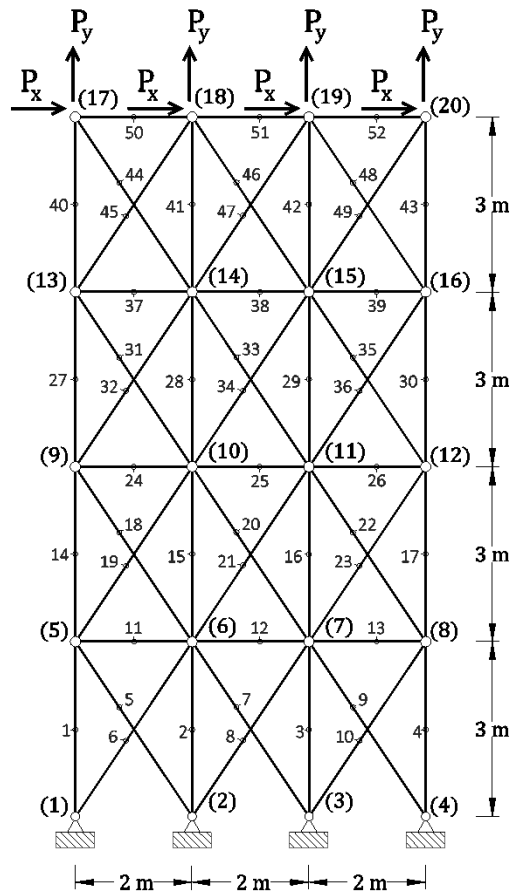


Figure 6. 52-bar truss

Table 4: Available cross-sectional areas of the AISC

No.	mm ²	in ²	No.	mm ²	in ²	No.	mm ²	in ²	No.	mm ²	in ²
1	71.613	0.111	17	1008.385	1.563	33	2477.414	3.84	49	7419.340	11.5
2	90.968	0.141	18	1045.159	1.62	34	2496.769	3.87	50	8709.660	13.5
3	126.451	0.196	19	1161.288	1.80	35	2503.221	3.88	51	8967.724	13.9
4	161.290	0.250	20	1283.868	1.99	36	2696.769	4.18	52	9161.272	14.2
5	198.064	0.307	21	1374.191	2.13	37	2722.575	4.22	53	9999.980	15.5
6	252.258	0.391	22	1535.481	2.38	38	2896.768	4.49	54	10322.560	16.0
7	285.161	0.442	23	1690.319	2.62	39	2961.284	4.59	55	10903.204	16.9
8	363.225	0.563	24	1696.771	2.63	40	3096.768	4.80	56	12129.008	18.8
9	388.386	0.602	25	1858.061	2.88	41	3206.445	4.97	57	12838.684	19.9
10	494.193	0.766	26	1890.319	2.93	42	3303.219	5.12	58	14193.520	22.0
11	506.451	0.785	27	1993.544	3.09	43	3703.218	5.74	59	14774.164	22.9
12	641.289	0.994	28	2019.351	3.13	44	4658.055	7.22	60	15806.420	24.5
13	645.160	1.0	29	2180.641	3.38	45	5141.925	7.97	61	17096.740	26.5
14	792.256	1.228	30	2238.705	3.47	46	5503.215	8.53	62	18064.480	28.0
15	816.773	1.266	31	2290.318	3.55	47	5999.988	9.30	63	19354.800	30.0
16	939.998	1.457	32	2341.931	3.63	48	6999.986	10.85	64	21612.860	33.5

Table 5 compares the optimization results of the present study and those of obtained by HPSO [20], IMBA [17] and AEDE [18]. Comparison of convergence curves of SCA and MSCA is shown in Fig. 7.

Table 5: Results of optimization for the 52-bar truss

Design variables	HPSO	IMBA	AEDE	SCA	MSCA
A_1-A_4	4658.055	4658.055	4658.055	4658.055	4658.055
A_5-A_{10}	1161.288	1161.288	1161.288	1161.288	1161.288
$A_{11}-A_{13}$	363.225	494.193	494.193	363.225	494.193
$A_{14}-A_{17}$	3303.219	3303.219	3303.219	3303.219	3303.219
$A_{18}-A_{23}$	939.998	939.998	939.998	1045.159	939.998
$A_{24}-A_{26}$	494.193	494.193	494.193	506.451	494.193
$A_{27}-A_{30}$	2238.705	2238.705	2238.705	2238.705	2238.705
$A_{31}-A_{36}$	1008.385	1008.385	1008.385	1008.385	1008.385
$A_{37}-A_{39}$	388.386	494.193	494.193	641.289	494.193
$A_{40}-A_{43}$	1283.868	1283.868	1283.868	1690.319	1283.868
$A_{44}-A_{49}$	1161.288	1161.288	1161.288	1045.159	1161.288
$A_{50}-A_{52}$	792.256	494.193	494.193	645.160	494.193
Best (kg)	1905.49	1902.605	1902.605	1947.535	1902.605
Average (kg)	-	1903.076	1906.735	1958.564	1904.129
SD (kg)	-	1.13	6.679	9.37	2.67
Analyses	100000	4750	3402	10000	10000

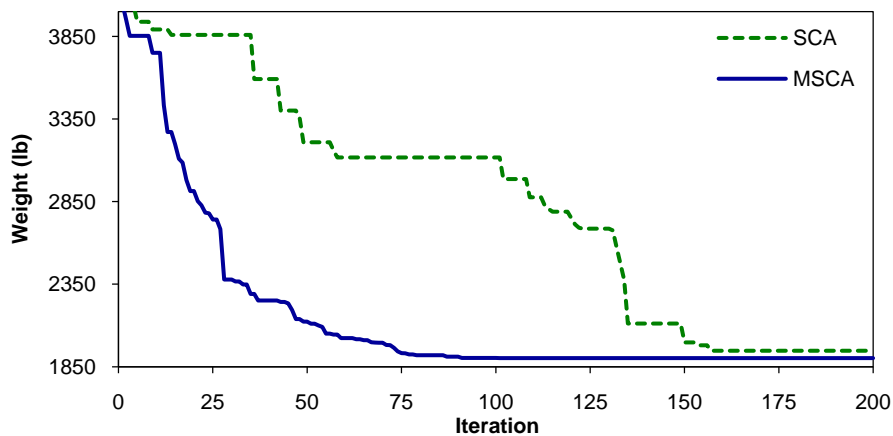


Figure 7. Convergence histories of SCA and MSCA for 52-bar truss

It can be seen that IMBA, AEDE and MSCA converge to the same best solution and the SCA is not competitive with the other algorithms. In this example, IMBA is the best algorithm in terms of Average and SD and the second best algorithm is MSCA. Furthermore, Fig. 7 reveals that the convergence rate of the MSCA is very better than that of the original SCA.

5.4 Example 4: 72-bar spatial truss

The 72-bar spatial truss is shown in Fig. 8. In this example, there are 16 groups of elements as follows: (1) A_1 – A_4 , (2) A_5 – A_{12} , (3) A_{13} – A_{16} , (4) A_{17} – A_{18} , (5) A_{19} – A_{22} , (6) A_{23} – A_{30} (7) A_{31} – A_{34} , (8) A_{35} – A_{36} , (9) A_{37} – A_{40} , (10) A_{41} – A_{48} , (11) A_{49} – A_{52} , (12) A_{53} – A_{54} , (13) A_{55} – A_{58} , (14) A_{59} – A_{66} (15) A_{67} – A_{70} , (16) A_{71} – A_{72} . The modulus of elasticity and material density are 10^4 ksi and 0.1 lb/in³, respectively. During the optimization process the design variables are selected from the data base of Table 4. The allowable stress in elements is ± 25 ksi and the allowable horizontal displacement is ± 0.25 in. In addition, there are two loading conditions given in Table 6.

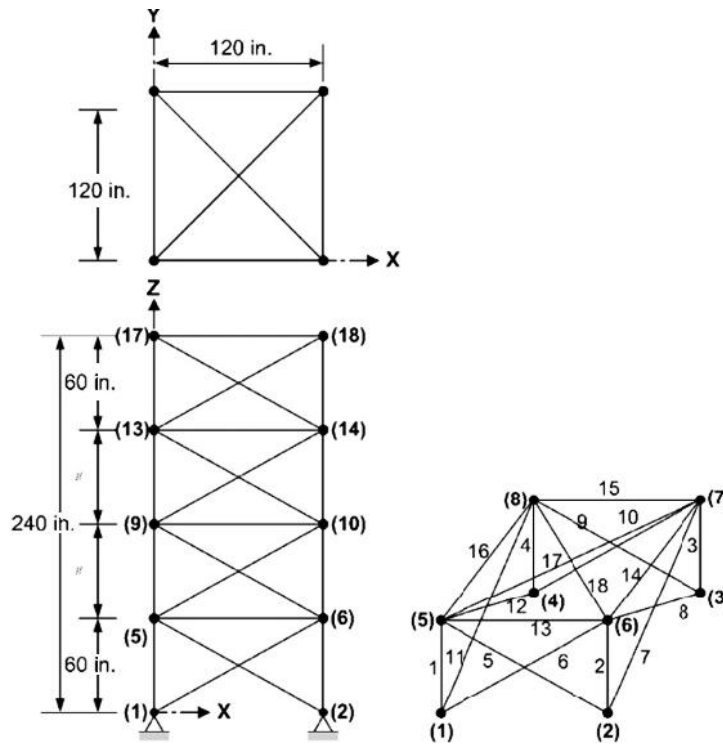


Figure 8. 72-bar truss

Table 6: Loading conditions for the 72-bar truss

Node	Loading condition 1 (kips)			Loading condition 2 (kips)		
	F_x	F_y	F_z	F_x	F_y	F_z
17	5.0	5.0	-5.0	0.0	0.0	-5.0
18	0.0	0.0	0.0	0.0	0.0	-5.0
19	0.0	0.0	0.0	0.0	0.0	-5.0
20	0.0	0.0	0.0	0.0	0.0	-5.0

In the optimization process the population size and maximum number of iterations are considered to be 50 and 200, respectively. The results obtained in the present study are compared with those of HPSO [20], IMBA [17] and AEDE [18] in Table 8. Furthermore, convergence curves of SCA and MSCA are compared in Fig. 9.

Table 8: Results of optimization for the 72-bar truss

Design variables	HPSO	IMBA	AEDE	SCA	MSCA
A_1-A_4	4.97	1.990	1.990	3.130	1.990
A_5-A_{12}	1.228	0.442	0.563	0.563	0.563
$A_{13}-A_{16}$	0.111	0.111	0.111	0.111	0.111
$A_{17}-A_{18}$	0.111	0.111	0.111	0.111	0.111
$A_{19}-A_{22}$	2.88	1.228	1.228	1.266	1.228
$A_{23}-A_{30}$	1.457	0.563	0.442	0.442	0.442
$A_{31}-A_{34}$	0.141	0.111	0.111	0.111	0.111
$A_{35}-A_{36}$	0.111	0.111	0.111	0.111	0.111
$A_{37}-A_{40}$	1.563	0.563	0.563	0.563	0.563
$A_{41}-A_{48}$	1.228	0.563	0.563	0.442	0.563
$A_{49}-A_{52}$	0.111	0.111	0.111	0.111	0.111
$A_{53}-A_{54}$	0.196	0.111	0.111	0.111	0.111
$A_{55}-A_{58}$	0.391	0.196	0.196	0.196	0.196
$A_{59}-A_{66}$	1.457	0.563	0.563	0.563	0.563
$A_{67}-A_{70}$	0.766	0.391	0.391	0.442	0.391
$A_{71}-A_{72}$	1.563	0.563	0.563	0.442	0.563
Best (lb)	933.09	389.33	389.33	402.96	389.33
Average (lb)	N/A	389.82	390.91	412.62	390.67
SD (lb)	N/A	0.84	1.161	11.73	1.063
Analyses	50000	50000	4160	10000	10000

These results reveal that, MSCA is competitive in comparison with other algorithms of literature. The statistical results of IMBA are slightly better than those of MSCA however at very high computational effort. In addition, it is demonstrated that the performance of MSCA is better than that of SCA.

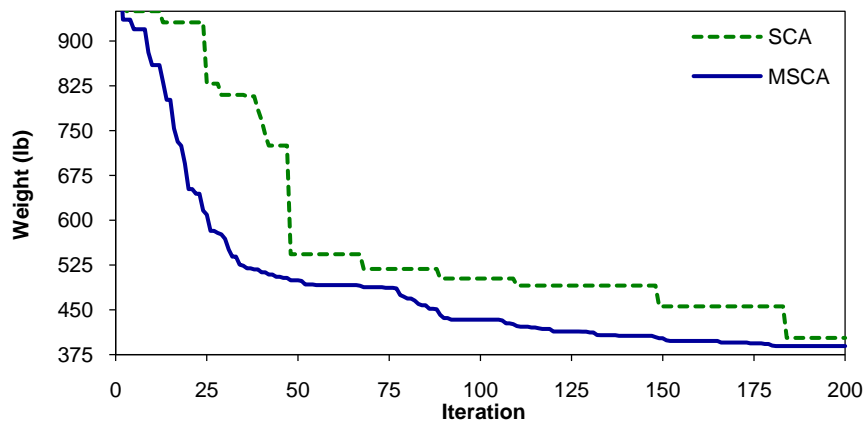


Figure 9. Convergence histories of SCA and MSCA for 72-bar truss

5. 5 Example 5: 200-bar planar truss

The 200-bar truss shown in Fig. 10 is one of the challenging truss optimization problems.

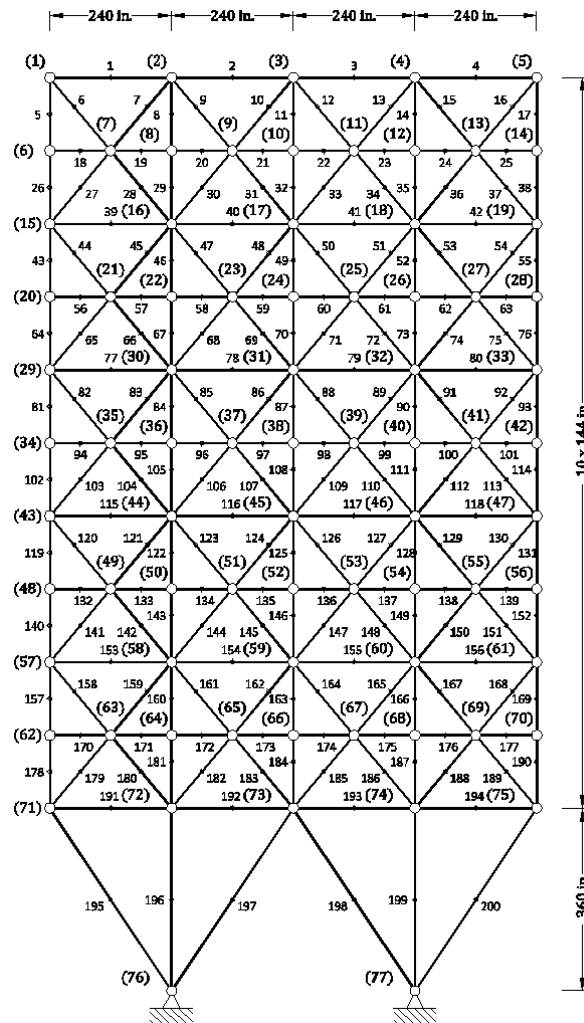


Figure 10. 200-bar truss

The material density, modulus of elasticity and stress limitations of the members are 0.283 lb/in^3 , 30 Msi, and $\pm 10 \text{ ksi}$, respectively. There are three loading conditions: (1) 1 kip acting in the positive x direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62, and 71; (2) 10 kips acting in the negative y direction at nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 70, 71, 72, 73, 74, and 75; and (3) loading conditions 1 and 2 acting together. The variables are selected from the following database $S = \{0.100, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180, 23.680, 28.080, 33.700\} \text{ in}^2$.

In this example, 60 particles at 200 iterations are employed during the optimization process. The results obtained in the present study are compared with those of improved genetic algorithm (IGA) [23], elitist self-adaptive step-size search (ESASS) [24] and AEDE [18] in Table 9. Furthermore, convergence curves of SCA and MSCA are compared in Fig. 11.

Table 9: Optimization results of the 200-bar truss

Design variables	Members in the group	IGA	DE	AEDE	SCA	MSCA
1	1, 2, 3, 4	0.347	0.1	0.1	0.1	0.1
2	5, 8, 11, 14, 17	1.081	0.954	0.954	0.954	0.954
3	19, 20, 21, 22, 23, 24	0.1	0.1	0.347	0.1	0.1
4	18, 25, 56, 63, 94, 101, 132, 139, 170, 177	0.1	0.1	0.1	0.1	0.1
5	26, 29, 32, 35, 38	2.142	2.142	2.142	2.142	2.142
6	6, 7, 9, 10, 12, 13, 15, 16, 27, 28, 30, 31, 33,34,36,37	0.347	0.347	0.347	0.347	0.347
7	39, 40, 41, 42	0.1	0.1	0.1	0.1	0.1
8	43, 46, 49, 52, 55	3.565	3.131	3.131	3.813	3.131
9	57, 58, 59, 60, 61, 62	0.347	0.1	0.347	0.1	0.1
10	64, 67, 70, 73, 76	4.805	4.805	4.805	4.805	4.805
11	44, 45, 47, 48, 50, 51, 53, 54, 65, 66, 68, 69, 71, 72, 74, 75	0.44	0.347	0.539	0.347	0.347
12	77, 78, 79, 80	0.44	0.1	0.347	0.1	0.954
13	81, 84, 87, 90, 93	5.952	5.952	5.952	4.805	5.952
14	95,96, 97, 98, 99, 100	0.347	0.1	0.1	1.764	0.1
15	102, 105, 108, 111, 114	6.572	6.572	6.572	5.952	6.572
16	82, 83, 85, 86, 88, 89, 91, 92, 103, 104, 106, 107, 109, 110, 112, 113	0.954	0.44	0.954	1.174	0.954
17	115, 116, 117, 118	0.347	0.539	0.44	0.347	0.347
18	119, 122, 125, 128, 131	8.525	7.192	8.525	8.525	8.525
19	133, 134, 135, 136, 137, 138	0.1	0.44	0.1	1.081	0.1
20	140, 143, 146, 149, 152	9.3	8.525	9.3	8.525	9.3
21	120, 121, 123, 124, 126, 127, 129, 130, 141, 142, 144, 145, 147, 148, 150, 151	0.954	0.954	0.954	1.081	1.174
22	153, 154, 155, 156	1.764	1.174	1.081	2.697	1.081
23	157, 160, 163, 166, 169	13.3	10.85	13.33	10.85	13.33
24	171, 172, 173, 174, 175, 176	0.347	0.44	0.539	2.697	0.44
25	178, 181, 184, 187, 190	13.3	10.85	14.29	13.33	13.33
26	158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189	2.142	1.764	2.142	3.131	2.142
27	191, 192, 193, 194	4.805	8.525	3.813	4.805	3.813
28	195, 197, 198, 200	9.3	13.33	8.525	9.3	8.525
29	196, 199	17.17	13.33	17.17	17.17	17.17
Best (lb)		28544	28075.	27858.	29611.	27693.6
		.01	49	50	51	8
Average (lb)		-	-	28425.	31820.	28358.4
				87	84	6
SD (lb)		-	-	481.59	1326.7	456.02
				0	9	
Analyses		51360	-	11644	12000	12000

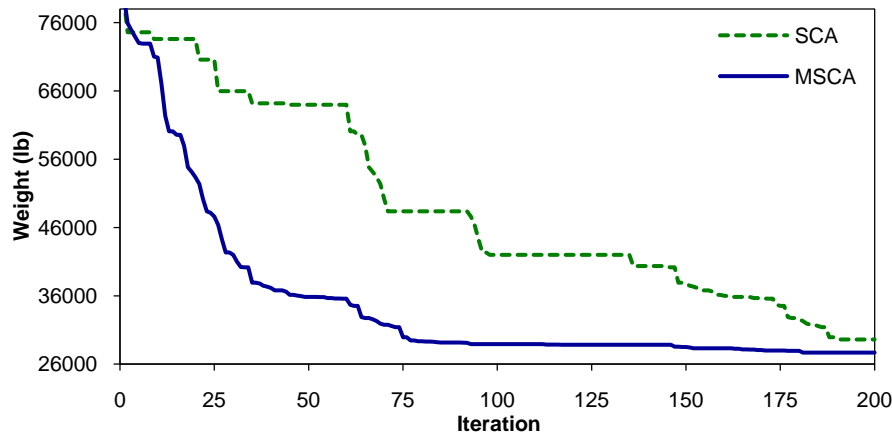


Figure 11. Convergence histories of SCA and MSCA for 200-bar truss

The optimization results demonstrate that the proposed MSCA outperforms all other algorithms in terms of Best, Average and SD of the obtained optimal weights. Moreover, it can be observed from Fig. 11 that the convergence rate of MSCA is very better than that of the SCA.

6. CONCLUDING REMARKS

The present study focuses on a newly developed sine cosine algorithm (SCA) and proposes a modified SCA (MSCA) as the original version of this metaheuristic seriously suffers from the slow convergence rate when dealing with the discrete truss optimization problems. The proposed MSCA integrates two computational strategies during its search process. In the first strategy, named as Regeneration, a kind of elitism is utilized by substituting a number of worst solutions of the current population with some variants of the global best solution. In the second strategy, named as Mutation, a mutation operation is performed to increase the probability of finding the global optimum or near global optima.

In order to illustrate the efficiency of the MSCA, a sort of well-known discrete benchmark truss optimization problems, including 10-, 25-, 52-, 72- and 200-bar trusses, are presented and the results of MSCA are compared with those of HPSO, HHS, AEDE, ECBO, IMBA, IGA, ESASS and SCA. The numerical results demonstrate that the original SCA is not competitive with the mentioned algorithms and consequently cannot converge to appropriate solutions. In contrast, in the most cases, the proposed MSCA outperforms other algorithms and presents an appropriate performance.

REFERENCES

1. Kaveh A, Talatahari S. Size optimization of space trusses using Big Bang-Big Crunch algorithm, *Comput Struct* 2009; **87**: 1129-40.

2. Gholizadeh S. Layout optimization of truss structures by hybridizing cellular automata and particle swarm optimization, *Comput Struct* 2013; **125**: 86-99.
3. Zhu JH, He F, Liu T, FH, Zhang WH, Liu Q, Yang C. Structural topology optimization under harmonic base acceleration excitations, *Struct Multidiscip Optim* 2018; **57**: 1061-1078.
4. Holland JH, Reitman JS. Cognitive systems based on adaptive algorithms, *ACM Sigart Bull* 1977; **63**: 49.
5. Simon D. Biogeography-based optimization, *IEEE Trans Evol Comput* 2008; **12**: 702-13.
6. Storn R, Price K. Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces, *J Global Optim* 1997; **11**: 341-59.
7. Rashedi E, Nezamabadi-Pour H, Saryazdi S. GSA: a gravitational search algorithm, *Inform Sci* 2009; **179**: 2232-48.
8. Kaveh A, Mahdavi VR. Colliding Bodies Optimization method for optimum discrete design of truss structures, *Comput Struct* 2014; **139**: 43–53.
9. Gholizadeh S, Ebadijalal M. Performance based discrete topology optimization of steel braced frames by a new metaheuristic, *Adv Eng Softw* 2018; **123**: 77–92.
10. Eberhart RC, Kennedy J. A new optimizer using particle swarm theory, in: *Proceedings of the Sixth International Symposium on Micro Machine and Human Science* 1995, pp. 39–43.
11. Dorigo M, Birattari M. Ant colony optimization, *Encyclopedia of Machine Learning*, Springer, 2010, pp. 36–39.
12. Yang XS. *A New Metaheuristic Bat-Inspired Algorithm*. *Nature Inspired Cooperative Strategies for Optimization, Studies in Computational Intelligence*, J. R. Gonzalez, ed., Springer, Berlin, 2010, **284**: pp. 65–74.
13. Kaveh A, Farhoudi N. A new optimization method: Dolphin echolocation, *Adv Eng Softw* 2013; **59**: 53–70.
14. Mirjalili S. SCA: A Sine Cosine Algorithm for solving optimization problems, *Knowl Based Syst* 2016; **96**: 120-133.
15. Kaveh A, Talatahari S. A particle swarm ant colony optimization for truss structures with discrete variables, *J Constr Steel Res* 2009; **65**: 1558-68.
16. Gholizadeh S, Poorhoseini H. Seismic layout optimization of steel braced frames by an improved dolphin echolocation algorithm, *Struct Multidisc Optim* 2016; **54**: 1011–29.
17. Sadollah A, Eskandar H, Bahreininejad A, Kim JH. Water cycle, mine blast and improved mine blast algorithms for discrete sizing optimization of truss structures, *Comput Struct* 2015; **149**: 1-16.
18. Ho-Huu V, Nguyen-Thoi T, Vo-Duy T, Nguyen-Trang T. An adaptive elitist differential evolution for optimization of truss structures with discrete design variables, *Comput Struct* 2016; **165**: 59-75.
19. Gholizadeh S, Milany A. An improved fireworks algorithm for discrete sizing optimization of steel skeletal structures, *Eng Optim* 2018; doi.org/10.1080/0305215X.2017.1417402.
20. Li LJ, Huang ZB, Liu F. A heuristic particle swarm optimization method for truss structures with discrete variables, *Comput Struct* 2009; **87**: 435-43.
21. Cheng MY, Prayogo D, Wu YW, Lukito MM. A hybrid harmony search algorithm for discrete sizing optimization of truss structure, *Automat Constr* 2016; **69**: 21-33.

22. Kaveh A, Ilchi Ghazaan M. A comparative study of CBO and ECBO for optimal design of skeletal structures, *Comput Struct* 2015; **153**: 137-47.
23. Togan V, Daloglu AT. An improved genetic algorithm with initial population strategy and selfadaptive member grouping, *Comput Struct* 2008; **86**: 1204-18.
24. Kazemzadeh Azad S, Hasançebi O. An elitist self-adaptive step-size search for structural design optimization, *Appl Soft Comput* 2014; **19**: 226-35.