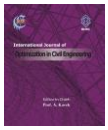
INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING
Int. J. Optim. Civil Eng., 2019; 9(2):233-250



MODELING OF FLOW NUMBER OF ASPHALT MIXTURES USING A MULTI–KERNEL BASED SUPPORT VECTOR MACHINE APPROACH

M. Araghi and M. Khatibinia[*, †]
Department of Civil Engineering, University of Birjand, Birjand, Iran

ABSTRACT

Flow number of asphalt–aggregate mixtures as an explanatory factor has been proposed in order to assess the rutting potential of asphalt mixtures. This study proposes a multiple–kernel based support vector machine (MK–SVM) approach for modeling of flow number of asphalt mixtures. The MK–SVM approach consists of weighted least squares–support vector machine (WLS–SVM) integrating two kernel functions in order to improve the learning and generalization ability of WLS–SVM. In the proposed method, a linear convex combination of the radial basis function (RBF) and Morlet wavelet kernel functions is adopted, which are considered as the most popular kernel functions. To validate the efficiency of the proposed method, experiments are conducted on a database including 118 uniaxial dynamic creep test results. The results of the statistical criteria show a good agreement between the predicted and measured flow number values. Further, the simulation results demonstrate that the proposed MK–SVM approach has more superior performance than the single kernel based WLS–SVM and other methods found in the literature.

Keywords: asphalt–aggregate mixture; flow number; multiple–kernel; weighted least squares–support vector machine; radial basis function; Morlet wavelet.

Received: 18 June 2018; Accepted: 12 August 2018

1. INTRODUCTION

Rutting in asphalt has been considered as the well–known cause of traffic loads which result in pavement permanent deformations. The rutting phenomenon is progressively developed by the accumulation of the deformations [1]. The reduction of useful service life in the pavement is concluded by the rutting phenomenon so that serious hazards are created for

---

[*]Corresponding author: Department of Civil Engineering, University of Birjand, Birjand, Iran
[†]E-mail address: m.khatibinia@birjand.ac.ir (M. Khatibinia)

highway users [2–4]. Also, the thickness can be reduced by the rutting and the occurrence of pavement failure can be increased through fatigue cracking [5]. Thus, the behavior of asphalt mixes subjected to repeated loading such as the permanent deformation should be investigated [2–4, 6].

More researchers have drawn attention to evaluating the rutting potential of asphalt mix over the last decades. The most permanent–deformation models were proposed as empirical or semi–mechanistic models with limited fundamental material characterization. The empirical models were obtained using limited sets which included materials and environmental conditions, while the correlations of the models could not satisfy the actual field performance. Therefore, the models cannot be generalized for other conditions [2–4]. In order to evaluate and predict the rutting, three main approaches have been proposed which include (1) mechanistic–empirical modeling approaches, (2) advanced constitutive modeling approaches, and (3) development of a simple performance test [7]. The comprehensive and complete review of these approaches has been presented in the literature [4].

According to the Superpave mixture design method, a direct test method has not still proposed in order to evaluate the permanent deformation resistance of mixtures. Hence, different researchers proposed an indicator parameter as rutting resistance. This parameter is called flow number, which can be measured using a permanent deformation test subjected to repeated load. In fact, the number of cycles occurred in asphalt after the tertiary deformation can be indicated by the flow number [5]. In order to determine the flow number, the dynamic tests as the very sensitive and inconvenient procedures should be performed. Therefore, the development of a precise model which denotes the flow number based on the mix design parameters of asphalts is necessary.

During the last two decades, soft computing techniques as the modern approach for constructing a computationally intelligent system have been successfully used for in civil engineering problems [8–15]. Hence, the techniques have been adopted for modeling the flow number based on the mix design parameters of asphalts. Mirzahosseini et al. [16] utilized multi expression programming (MEP) and multilayer perceptron (MLP) of artificial neural networks for modeling the rutting potential of dense asphalt–aggregate mixtures. The simulated results verified that the MEP–based straightforward formulas in compared with those of MLP are much more practical for modeling the rutting potential. Alavi et al. [3] proposed a high–precision model for the rutting resistance of asphalt mixtures. The model was developed based on the hybrid of genetic programing and simulated annealing. The results demonstrated that the proposed model was effectively utilized for evaluating the flow number of asphalt mixtures. In the work of Gandomi et al. [4], gene expression programming (GEP) was developed to predict the flow number of dense asphalt–aggregate mixtures. The results showed that the simple, straightforward, and particularly formulas could be obtained by using GEP. Recently, Alavi et al. [1] have proposed the multi–gene genetic programming (MGGP) for the determination of flow number. Based on the results of this study, the MGGP model can be considered as a more practical model in comparison with the existing models. In fact, the model consists of the effects of most of the parameters which are required for obtaining an optimal mix design.

Support vector machines (SVMs) were introduced as another primary class of soft–computing methods which were used for pattern recognition in large quantities of data [17]. The SVM approaches use a nonlinear kernel function to map the original parameter vectors into a higher dimensional feature space, referred to as primal space. This is followed by a procedure to find an optimum hyperplane that minimizes training error. Although Artificial Neural Networks (ANNs) have been implemented based on the empirical risk minimization, SVMs have used the structural risk minimization to eliminate the influence of the input space dimensionality on the computational complexity of the developed model. Furthermore, a solution of SVM is global and unique and can be geometrically interpreted, while ANNs can have multiple local minima [17]. In recent years, the SVM approaches have been successfully used in engineering problems[18–25].

The main contribution of this study is to propose a multiple–kernel based support vector machine (MK–SVM) approach in order to model and predict the flow number of asphalt mixtures. The MK–SVM approach consists of weighted least squares–support vector machine (WLS–SVM) which combines the radial basis function (RBF) and Morlet wavelet function as kernel function. In fact, the accuracy merits of two functions are integrated to improve the learning and generalization ability of WLS–SVM. The effectiveness and accuracy of the proposed method are investigated based on modeling results of a database including 118 uniaxial dynamic creep test results. The results reveal that the flow number can accurately be estimated by employing the proposed MK–SVM. Furthermore, the performance of the proposed MK–SVM is more superior than that of the single kernel based WLS–SVM and other methods found in the literature.

2. EMPIRICAL MODEL FOR THE FLOW NUMBER

According to several studies [1], the flow number (F_n) depends on some of the parameters including the percentages of coarse aggregate (*CP*), filler (*FP*), bitumen (*BP*), air voids (*Va*), voids in mineral aggregate (*VMA*) and Marshall stability to flow ratio (*M/F*). Consequently, the formulation was proposed as follows [1]:

$$Log\left(F_n\right) = f(CP, FP, BP, V_a, VMA, \frac{M}{F}) \tag{1}$$

where M_{TMD}, K_{TMD} and C_{TMD} are stiffness, mass and damper of TMD, respectively. $x_i(t)$and $a_i(t)$ are the displacement and acceleration of *i*th storey at the *t*th time, respectively. Furthermore, *N* is the number of structure storeris.

3. EXPERIMENTAL DATABASE

In order to obtain a general model for evaluating the flow number of asphalt, a database of laboratory including 118 uniaxial dynamic creep test results [4] is selected. The statistical properties of the database are shown in Table 1. In this study, the percentages of coarse aggregate (*CP*), filler (*FP*), bitumen (*BP*), air voids (*Va*), voids in mineral aggregate (*VMA*) and Marshall stability to flow ratio (*M/F*) are considered as input variables, while the flow number (F_n) is presented as output variable.

Table 1: Statistical properties of the parameters

Parameter	*CP* (%)	*FP* (%)	*BP* (%)	V_a (%)	*VMA* (%)	*M/F*	F_n
Mean	57.31	5.54	5.51	4.45	16.55	2.99	227
Standard error	1.32	0.29	0.07	0.14	0.13	0.07	13.25
Standard deviation	14.33	3.17	0.81	1.52	1.41	0.74	143.97
Minimum	33	1	4	1.71	13.20	0.61	22
Maximum	81	10	7	8.77	19.04	4.81	510

4. WLS–SVM APPROACH

Support vector machines (SVMs) have two inherent drawbacks: 1) incapability to accurately tune the penalty parameter or kernel parameter settings, and 2) their sole reliance on support vectors to determine the decision boundary. In case support vectors are formed by outliers in the training dataset, the latter drawback will cause the decision boundary to significantly deviate from the optimum hyperplane, making the SVM outputs very sensitive to outliers [26].

Suykens et al. [17] addressed these drawbacks by introducing weighted least squares support vector machines (WLS–SVM), where the robustness of a least squares SVM is improved by assigning weights to its error variables. Given a training dataset of N samples $\{(\boldsymbol{x}_k, y_k)\}_{k=1}^{N}$ with input data $\boldsymbol{x}_i \in \boldsymbol{R}^d$ and output data $y_i \in \boldsymbol{R}$, WLS–SVM regression is formulated as the following optimization problem in the primal weight space [27]:

$$\begin{aligned} &\text{Minimize}: J(\boldsymbol{w},\boldsymbol{e}) = \frac{1}{2}\boldsymbol{w}^T\boldsymbol{w} + \frac{1}{2}\gamma\sum_{i=1}^{N}\overline{v}_i e_i^2 \\ &\text{Subject to}: y_i = \boldsymbol{w}^T\phi(\boldsymbol{x}_i) + b + e_i\ , \qquad i = 1,2,\ldots,N \end{aligned} \tag{2}$$

where $\phi(.):\boldsymbol{R}^d \to \boldsymbol{R}^{\tilde{d}}$ is an operator mapping the input space into a higher dimensional space; $\boldsymbol{w} \in \boldsymbol{R}^d$ represents weight vector in primal weight space; and $e_i \in \boldsymbol{R}$ and $b \in \boldsymbol{R}$ represent error variable and bias term, respectively.

In primal weight space, the model of WLS–SVM is expressed by the optimization problem (Eq. 2) and the training set as:

$$y(\boldsymbol{x}) = \boldsymbol{w}^T\phi(\boldsymbol{x}) + b \tag{3}$$

Generally, the structure of the function $\varphi(\boldsymbol{x})$ is unknown. Hence, it is impossible to indirectly calculate $\boldsymbol{w}$ from Eq. (2). Therefore, the solutions of WLS–SVM regression are obtained by constructing a Lagrangian function as:

$$L(\boldsymbol{w},b,\boldsymbol{e};\boldsymbol{x}) = J(\boldsymbol{w},\boldsymbol{e}) - \sum_{i=1}^{N}\alpha_i(\boldsymbol{w}^T\phi(\boldsymbol{x}_i) + b + e_i - y_i) \tag{4}$$

where α_i is the Lagrangian multipliers. The conditions for optimality are given by:

$$\frac{\partial L}{\partial \boldsymbol{w}} = 0\,,\ \frac{\partial L}{\partial b} = 0\,,\ \frac{\partial L}{\partial e_i} = 0\,, \ldots,\ \frac{\partial L}{\partial \alpha_i} = 0 \tag{5}$$

which together with the elimination of $\boldsymbol{w}$ and $\boldsymbol{e}$ result in the following Karush–Kuhn–Tucker (KKT) system:

$$\begin{bmatrix} \Omega + \boldsymbol{V}_\gamma & \mathbf{1}_N^T \\ \mathbf{1}_n & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ b \end{bmatrix} = \begin{bmatrix} \boldsymbol{y} \\ 0 \end{bmatrix} \tag{6}$$

in which

$$\begin{gathered} \boldsymbol{V}_\gamma = diag\{1/\gamma \bar{v}_1, \ldots, 1/\gamma \bar{v}_N\}\ ;\ \Omega_{i,j} = \left\langle \phi(\boldsymbol{x}_i), \phi(\boldsymbol{x}_j) \right\rangle_H \quad i, j = 1,2,\ldots,N \\ \boldsymbol{y} = [y_1, \ldots, y_N]^T ;\ \mathbf{1}_N^T = [1,\ldots,1]\ ;\ \boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_N] \end{gathered} \tag{7}$$

where the weight factors $\bar{v}_k$ are given by Widodo and Yang [28]:

$$\bar{v}_k = \begin{cases} 1 & \text{if } \left| e_i / \hat{s} \right| \le c_1 \\ \dfrac{c_2 - |e_i / \hat{s}|}{c_2 - c_1} & if\ \ c_1 < \left| e_i / \hat{s} \right| \le c_2 \\ 10^{-4} & \text{otherwise} \end{cases} \tag{8}$$

where $\hat{s}$ is a robust estimation of the standard deviation of the error variables ($e_i = a_i / D_{ii}^{-1}$); the constants c_1 and c_2 are typically chosen as $c_1 = 2.5$ and $c_2 = 3$. Here D_{ii}^{-1} denotes the i^{th} primal diagonal element of the inverse of matrix $\boldsymbol{D}$, which is the matrix on the left–hand side of Eq. (6).

Based on the Mercer's Theorem, a kernel $K(.,.)$ is selected such that:

$$K(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_j) = \left\langle \phi(\boldsymbol{x}_i), \phi(\bar{\boldsymbol{x}}_j) \right\rangle_H \ \ ;\ \ i, j = 1,2,\ldots,N \tag{9}$$

The WLS–SVM model is then obtained as:

$$y(\boldsymbol{x}) = \sum_{i=1}^{N} \alpha_i K(\boldsymbol{x}_i, \boldsymbol{x}) + b \tag{10}$$

In the WLS–SVM approach, the Gaussian radial basis function (RBF) is commonly employed as the kernel function, and it is expressed as [17]:

$$K_{\mathrm{RBF}}(\boldsymbol{x},\bar{\boldsymbol{x}}) = \exp\left(-\frac{\|\boldsymbol{x}-\bar{\boldsymbol{x}}\|^2}{\sigma^2}\right) \tag{11}$$

where σ^2 is a positive real constant, and it is usually called the kernel width. The structure of the WLS–SVM approach is shown in Fig. 1.

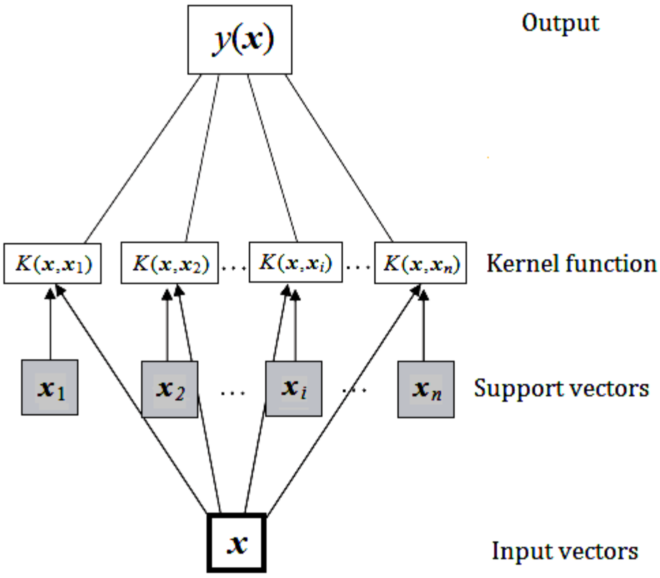


Figure 1. The structure of the WLS–SVM approaches [27, 28]

5. WAVELET KERNEL FUNCTION

Among different SVM approaches proposed in the literature, the approaches with wavelet kernel function have successfully been used in a variety of engineering problems [28, 18, 31, 32]. The following section provides an introduction for the wavelet kernel function.

Based on the wavelet concept, a signal or function can be expressed or approximated by a family of functions generated by the dilation (scaling) and translation of a function $\psi(x)$ called the mother wavelet as [33]:

$$\psi_{a,c}(x) = \frac{1}{\sqrt{a}}\psi\left(\frac{x-c}{a}\right) \tag{12}$$

where a and c are the dilation and translation factors, respectively. The wavelet transform of a function $f(x) \in L_2(\boldsymbol{R})$ can then be expressed as:

$$W_{a,c}(f) = \left\langle f(x), \psi_{a,c}(x) \right\rangle \tag{13}$$

where $\langle .,. \rangle$ denotes the dot product in $L_2(\boldsymbol{R})$.

The continuous wavelet transform (CWT) of a function $f(x) \in L^2(\boldsymbol{R})$ given an admissible mother wavelet $\psi(x)$ is defined as:

$$CWT(a,c) = \int_{-\infty}^{+\infty} f(x)\psi(x)\,dx = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} f(x)\psi^*(\frac{x-c}{a})\,dx \tag{14}$$

where $\psi^*(\cdot)$ denotes the complex conjugation of $\psi(x)$.

In the work of Khatibinia et al. [18], the cosine–Gaussian Morlet wavelet was used as kernel function in the WLS–SVM approach. This wavelet is the real part of the Morlet wavelet, which is expressed as [34]:

$$\psi(x) = [\cos(\omega_0 x) + j\sin(\omega_0 x)]\exp(-0.5x^2) \tag{15}$$

where ω_0 is the central frequency of the wavelet function. Using Eqs. (12) and (15), the cosine–Gaussian Morlet wavelet expression of a function is expressed as:

$$\psi(x) = \frac{1}{\sqrt{a}}\cos\left(\omega_0\left(\frac{x-c}{a}\right)\right)\exp\left(-0.5\left(\frac{x-c}{a}\right)^2\right) \tag{16}$$

The translation–invariant wavelet kernels can be given by [18]:

$$K(\boldsymbol{x},\bar{\boldsymbol{x}}) = \prod_{k=1}^{N}\psi\left(\frac{\boldsymbol{x}_i - \bar{\boldsymbol{x}}_i}{a}\right) \tag{17}$$

which gives the following expression for the kernel function of the cosine–Gaussian Morlet wavelet [18]:

$$K_{\text{Wavelat}}(\boldsymbol{x},\bar{\boldsymbol{x}}) = \prod_{k=1}^{N}\frac{1}{\sqrt{a}}\cos\left(\omega_o\,\frac{\boldsymbol{x}_i - \bar{\boldsymbol{x}}_i}{a}\right)\exp\left(-0.5\frac{\|\boldsymbol{x}_i - \bar{\boldsymbol{x}}_i\|}{a^2}\right) \tag{18}$$

It was revealed that the kernel function in the WLS–SVM approach had the highest accuracy [18].

6. MULTIPLE–KERNEL SVM APPROACH

In the prediction process of functions, the selection of a kernel function is very important for SVMs and a time–consuming task. In other words, the performance and accuracy of SVMs depend on choosing the appropriate kernel function and their parameters [35]. It was demonstrated that the kernel machines with a single kernel function such as SVMs could not utilized solving some complicated problems [35]. Hence, the combination of kernel functions as an interesting area of research has received great deal of attentions by numerous researchers. The results reveal that using multiple–kernel functions instead of a single one can enhance the capability of the SVM approaches and improve their performances [35–38].

In order to improve the performance and generalization ability of the WLS–SVM approach, a multiple–kernel WLS–SVM called MK–SVM is proposed based on the wavelet and RBF kernel functions in this study, as follows:

$$K_{\text{mix}} = \beta K_{\text{Wavelet}} + (1-\beta) K_{\text{RBF}} \tag{19}$$

where β is the controlled parameter. Since the RBF and wavelet kernel functions satisfy the Mercer's Theorem, the linear convex combination presented in Eq. (19) satisfies the Mercer's Theorem. Khatibinia et al. [18] demonstrated that the wavelet kernel function offered higher performance than the RBF kernel function. Hence, in this study the controlled parameter β is considered in the interval [0.5, 1) in order to consider the high portion of the wavelet kernel function in the MK–SVM approach.

7. NUMERICAL RESULTS

7.1 Scaling and dividing database

To evaluate the effectiveness and accuracy of the proposed MK–SVM approach, the flow number of asphalt–aggregate mixtures is estimated using the proposed MK–SVM approach. In order to achieve this purpose, the database of laboratory testing results for 118 samples outlined in Table 1 is selected. In the database, the input variables consist of the percentages of coarse aggregate (*CP*), filler (*FP*), bitumen (*BP*), air voids (*Va*), voids in mineral aggregate (*VMA*) and Marshall stability to flow ratio (*M/F*), while the flow number (F_n) is considered as the output variable. Before dividing database into training and testing sets, the values of the input variables are normalized between 0.2 and 0.8 as follows:

$$\overline{x}_i = b_1 \frac{x_i - x_{\min}}{x_{\max} - x_{\min}} + b_2 \tag{20}$$

where $\overline{x}_i$, $x_{\max}$ and $x_{\min}$ are the normalized, maximum and minimum values of the input variables, respectively. In this study, b_1 and b_2 are assumed to be equal to 0.6 and 0.2, respectively. Then, the database is randomly divided into training and testing sets including 89 (75%) and 29 (25%) samples, respectively.

7.2 Model construction using the proposed MK–SVM

The performance and the accuracy of the proposed MK–SVM depend on the controlled parameter β. In order to investigate the effect of the controlled parameter, the values 0.5, 0.6, 0.7, 0.8 and 0.9 are selected for the parameter. Furthermore, the values $\beta = 0$ and $\beta = 1$ actually correspond to the RBF and wavelet kernel functions, respectively. In the proposed

MK–SVM, the grid search method is used for finding the optimal values of kernel functions, and the 10–fold cross–validation prevents the model from over–fitting.

7.3 Estimating the accuracy evaluation of MK–SVM

In order to model the flow number of asphalt–aggregate mixtures, the proposed MK–SVM is trained by using the aforementioned database. For evaluating the performance of the MK–SVM approach, Mean Absolute Error (*MAE*), Mean Absolute Percentage Error (*MAPE*), Root Mean Squared Error (*RMSE*) and Coefficient of Determination (R^2) are considered as the statistical criteria. The *MAE* and *MAPE* is a quantity adopted to measure how closely a prediction matches the outcome. The *MAE* and *MAPE* are given as follows:

$$MAE = \sum_{i=1}^{n} \frac{|y_i - \bar{y}_i|}{n} \tag{21}$$

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \frac{|y_i - \bar{y}_i|}{y_i} \times 100 \tag{22}$$

The *RMSE* between actual output and desired output is considered as the objective function, which can be expressed as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2}{n}} \tag{23}$$

The R^2 represents the degree to which two variables are linearly related and is expressed as:

$$R^2 = \frac{\left(\sum_{i=1}^{n} (y_i - y_{ave})(\bar{y}_i - \bar{y}_{ave}) \right)^2}{\sum_{i=1}^{n} (y_i - y_{ave})^2 \sum_{i=1}^{n} (\bar{y}_i - \bar{y}_{ave})^2} \tag{24}$$

where y_{ave} and $\bar{y}_{ave}$ are the mean of the measurement and predicted values in the data samples.

According to the aforementioned statistical criteria, the statistic results from the proposed MK–SVM based on the different values of the controlled parameter β are summarized in Table 2 for the training and testing processes. In other to compare the performance of MK–SVM with that of the WLS–SVM approach, the statistical criteria for the WLS–SVM approach based on the RBF and wavelet kernel functions are also shown in Table 2.

Table 2: The statistical results of the proposed MK–SVM and WLS–SVM models

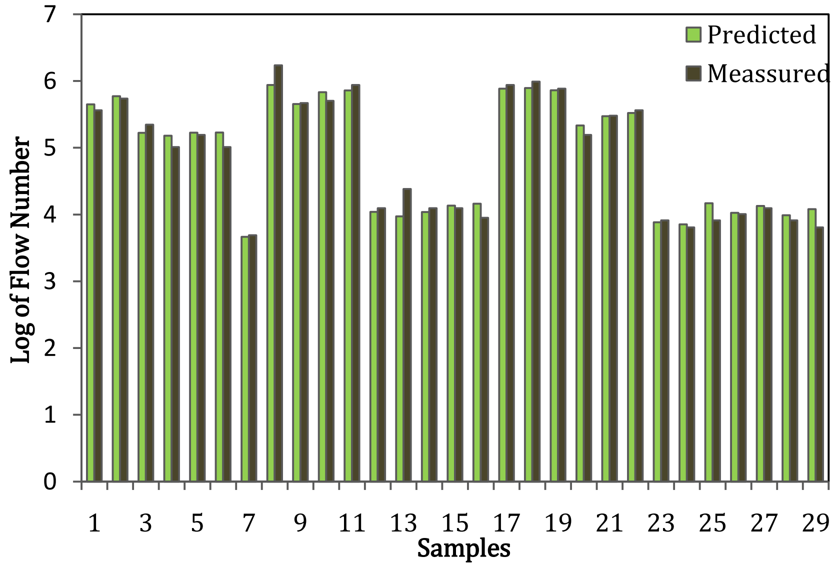
Model	β	Training process			Testing process		
		MAPE	*RMSE*	R^2	*MAPE*	*RMSE*	R^2
MK–SVM	0.5	1.6349	0.0121	0.9996	2.4120	0.0219	0.9991
	0.6	1.6138	0.0190	0.9996	2.3858	0.0217	0.9991
	0.7	1.4311	0.0098	0.9996	2.2755	0.0214	0.9991
	0.8	1.5927	0.0117	0.9996	2.3330	0.0216	0.9991
	0.9	1.5801	0.0116	0.9996	2.3226	0.0215	0.9991
WLS–SVM (RBF)	1	3.0611	0.0446	0.9975	5.8035	0.2786	0.9904
WLS–SVM (Wavelet)	0	2.0899	0.0372	0.9986	4.2530	0.1111	0.9961

The comparative results of Table 2 show that the proposed MK–SVM exhibits the best performance than the WLS–SVM approach with single kernel function. In fact, the linear combination of the kernel functions can integrate their merits and can prevent the decision of choosing the appropriate kernel function and its parameters. Hence, the combination of the RBF and wavelet kernel functions can improve the learning and generalization ability of the WLS–SVM approach. Furthermore, it can be observed that the value $\beta = 0.7$ has the slightly best performance in comparison with the other values for the controlled parameter.

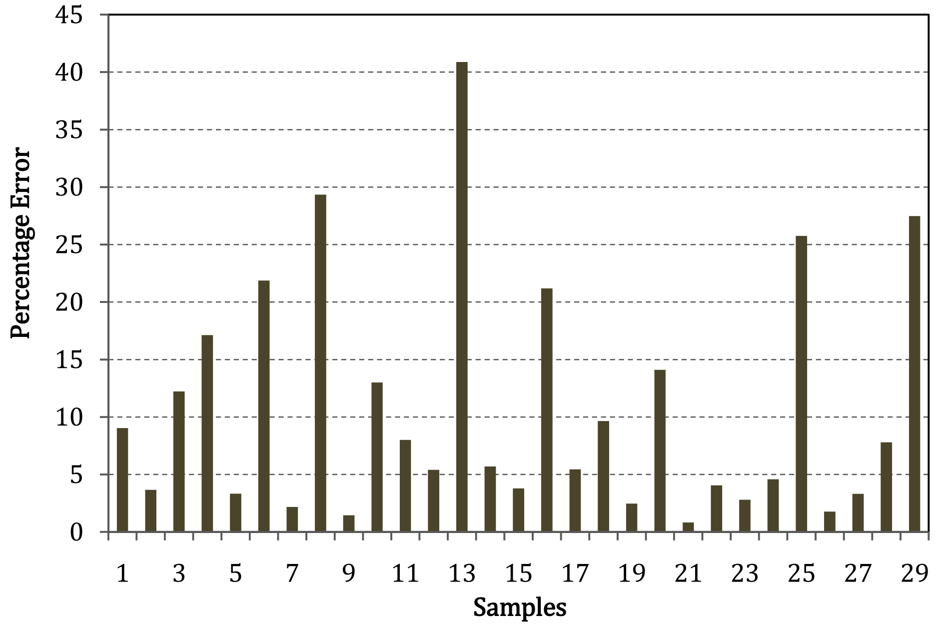
It can be seen from Figs. 2 through 4, the proposed MK–SVM approach in comparison with the WLS–SVM approach with RBF and wavelet kernel functions predicts the flow number at high accuracy rate.

Furthermore, Figs. 5 to 7 show the scatter diagrams for the actual values and the predicted values of the testing data of the proposed MK–SVM approach and the WLS–SVM approach with the RBF and wavelet kernel functions, respectively.

As obvious from Figs. 5 to 7, the flow number estimates of the models are closer to the corresponding measured values although the proposed MK–SVM model performs better than the WLS–SVM model.

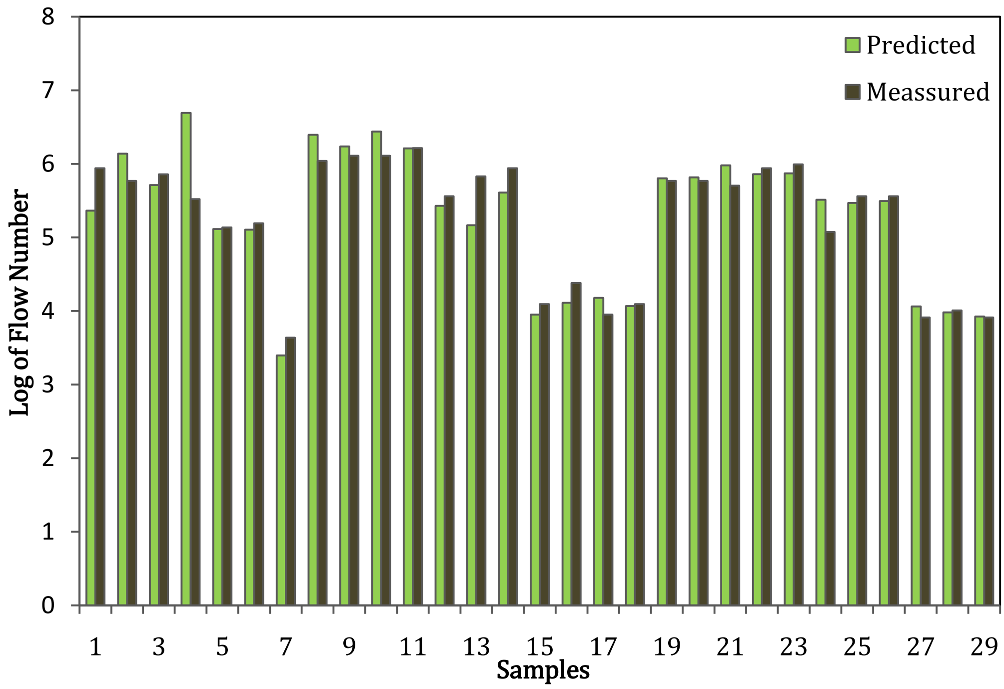


(a)

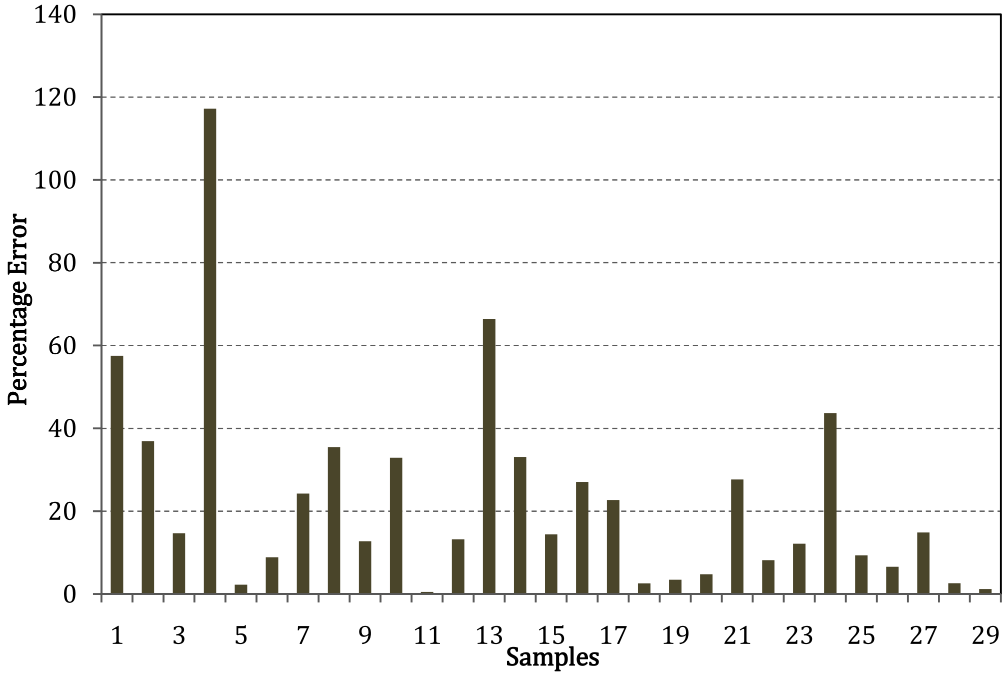


(b)

Figure 2. Results of MK–SVM in the testing process: (a) Comparison between the measurement and predicted flow number, (b) Error value between the measurement and predicted flow number

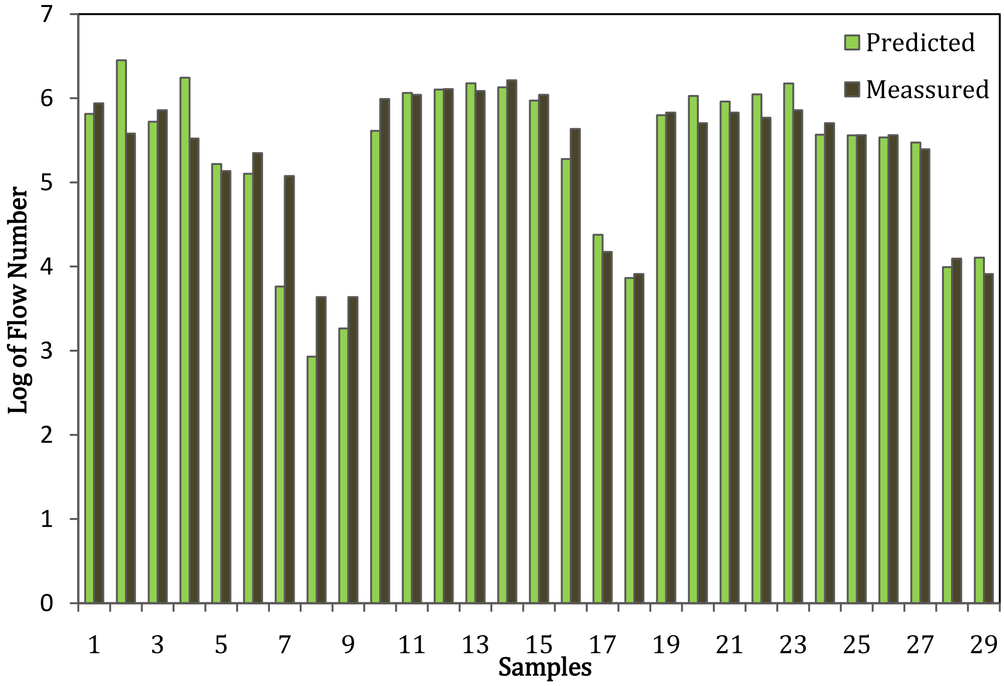


(a)

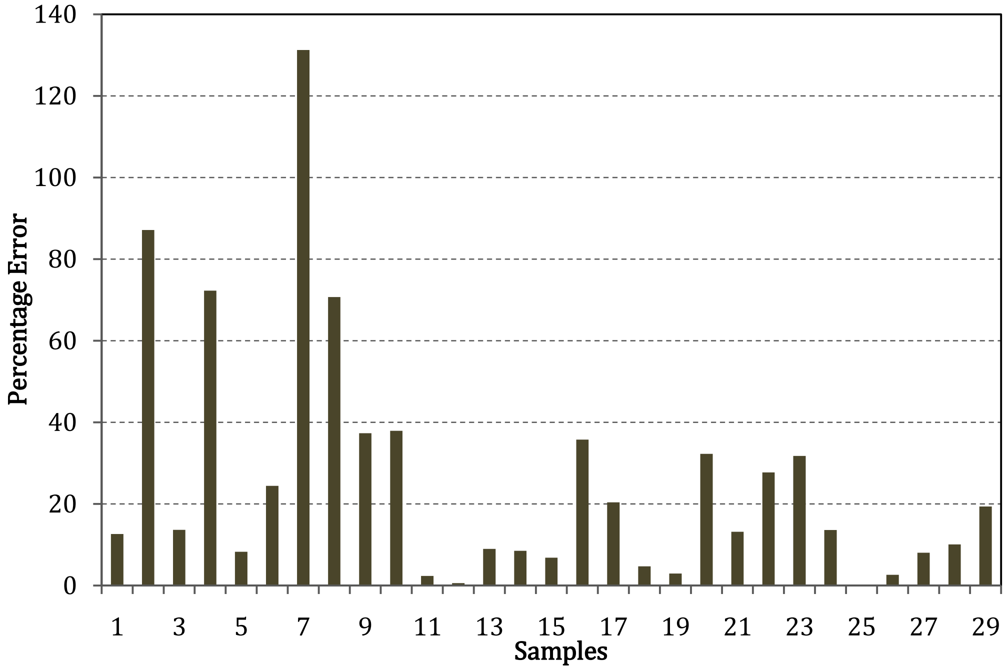


(b)

Figure 3. Results of WLS–SVM with wavelet kernel function in the testing process: (a) Comparison between the measurement and predicted flow number, (b) Error value between the measurement and predicted flow number



(a)



(b)

Figure 4. Results of WLS–SVM with RBF kernel function in the testing process: (a) Comparison between the measurement and predicted flow number, (b) Error value between the measurement and predicted flow number

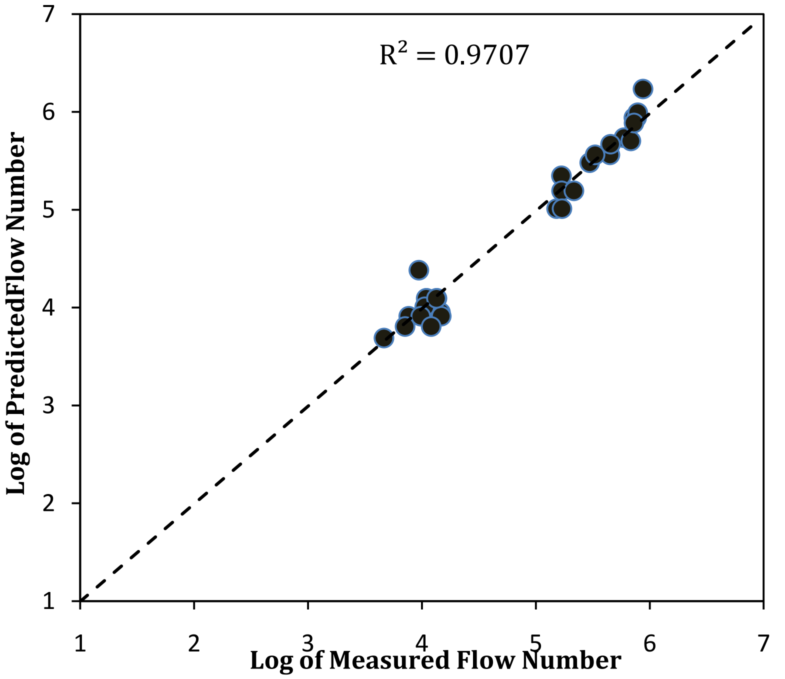


Figure 5. The scatter plots of the measured and estimated flow number values in the testing process for the KM–SVM approach

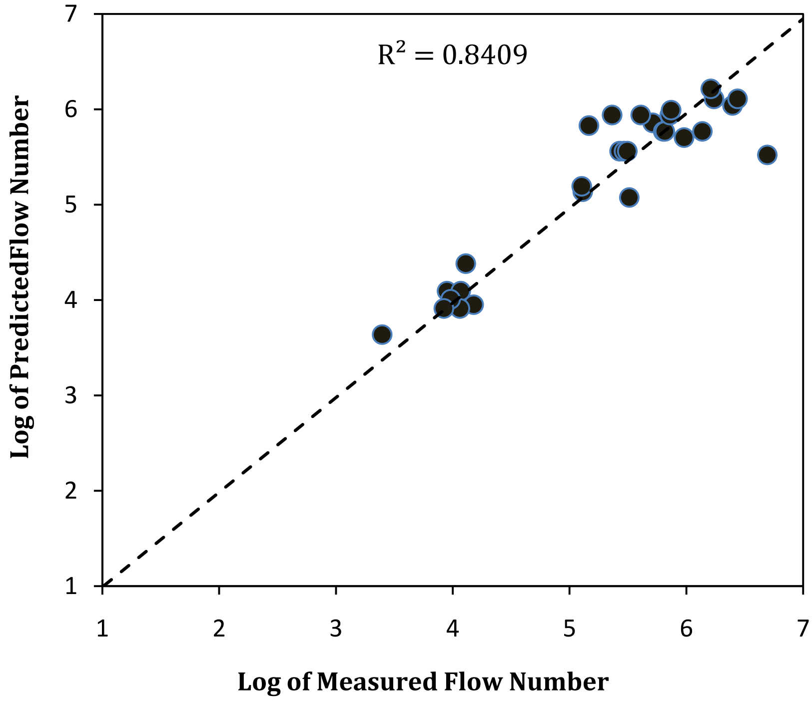


Figure 6. The scatter plots of the measured and estimated flow number values in the testing process for the WLS–SVM approach with wavelet kernel function

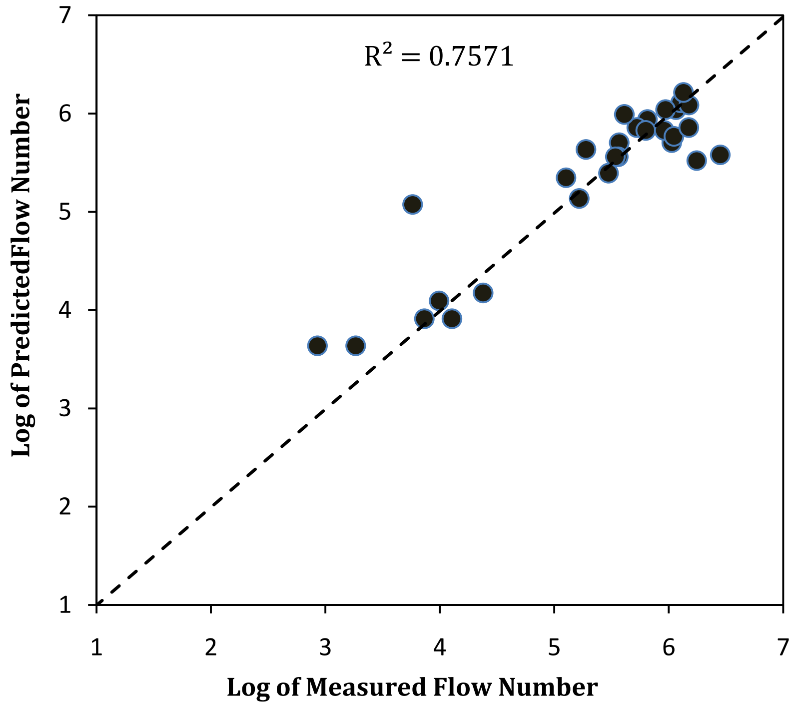


Figure 7. The scatter plots of the measured and estimated flow number values in the testing process for the WLS–SVM approach with RBF kernel function

7.4 Performance comparison of results with other techniques

This section presents the comparison of the proposed MK–SVM approach with other prediction models including the gene expression programming (GEP) [4], multi expression programming (MEP) [16], hybrid of genetic programming and simulated annealing (GP/SA) [3], generalized regression neural network (GRNN) [3] and a multivariable least squares regression (MLSR) [3]. Table 3 presents the input parameters used in each of the models.

Table 3: Input parameters of different F_n prediction models

Case	Input parameters	Model
1	*CP/SP*, V_a, *VMA*, *M/F*	GEP [4]
2	*FP*, *BP*, *VMA*, *M/F*	MEP [16], GP/SA [3],GRNN [3], MLSR [3]

The results for modeling the flow number of asphalt–aggregate mixtures are obtained by the proposed MK–SVM based on the controlled parameter β and also compared with other existing models which were shown in Table 3. The comparison of results for the testing procedure is summarized in Table 4.

It is evident from Table 4 that the proposed MK–SVM has the highest accuracy for modeling the flow number of asphalt–aggregate mixtures by comparing with the other methods. Thus, the proposed MK–SVM is more robust and is reliably utilized for prediction of the flow number of asphalt–aggregate mixtures.

Table 4: Comparison of the performance results of various methods

Case	Method	*MAE*	*RMSE*	R^2
1	MK–SVM	0.0540	0.0048	0.9991
	GEP [4]	0.0990	0.1265	0.8949
2	MK–SVM	0.0467	0.0036	0.9992
	MEP [16]	0.071	NR*	0.9506
	GP/SA [3]	0.094	0.1140	0.9370
	GRNN [3]	0.0620	0.0949	0.9584
	MLSR [3]	0.1250	0.1612	0.8593

* No reported

9. CONCLUSION

In this study, a multiple–kernel based support vector machine (MK–SVM) approach was proposed to accurately model the flow number of asphalt mixtures. The main aim of this approach was to prevent the decision of choosing the appropriate kernel function and its parameters. For achieve this purpose, the weighted least squares–support vector machine (WLS–SVM) integrated the radial basis function (RBF) and Morlet wavelet kernel functions. In the proposed MK–SVM, the linear convex combination of these kernel functions was adopted.

In order to validate the efficiency of the proposed MK–SVM, a database including 118 uniaxial dynamic creep test results was firstly selected. Then, the results of MK–SVM were obtained and compared with the WLS–SVM and other models. Therefore, the following conclusions can be derived from the results presented in this study:

- The results demonstrate that the combination of the kernel function accurately predict the flow number of asphalt mixtures and improve the performance of WLS–SVM. In fact, this approach can integrate the merit of these kernel function and their advantages are simultaneously used in the training procedure of WLS–SVM.
- The results demonstrate that the proposed MK–SVM approach has more superior performance than the single kernel based WLS–SVM and other methods found in the literature.

The proposed MK–SVM can reliably utilize for estimating the flow number and can be replaced instead of carrying out the expensively laboratory tests.

REFERENCES

1. Alavi AH, Hasni H, Zaabar I, Lajnef N. A new approach for modeling of flow number of asphalt mixtures, *Arch Civ Mech Eng* 2017; **17**: 326-35
2. Sousa JB, Craus J, Monismith CL. Summary report on permanent deformation in asphalt concrete. Strategic Highway Research Program (SHRP), National Research Council, Institute of Transportation Studies, University of California, 1991.
3. Alavi AH, Ameri M, Gandomi AH, Mirzahosseini MR. Formulation of flow number of asphalt mixes using a hybrid computational method, *Constr Buil Mater* 2011; **25**(3): 1338–55
4. Gandomi AH, Alavi AH, Mirzahosseini MR, Moqhadas Nejad F. Nonlinear genetic–based models for prediction of flow number of asphalt mixtures, *J Mater Civil Eng–ASCE* 2011; **23**(3): 248–63
5. Witczak MW, Kaloush K, Pellinen T, El–Basyouny M, Von Quintus H. Simple performance test for Superpave mix design, NCHRP Rep. 465, National Research Council, Transportation Research Board, Washington, 2002.
6. Bahuguna S. Permanent deformation and rate effects in asphalt concrete: Constitutive modeling and numerical implementation, Ph.D. Thesis, Case Western Reserve University, 2003.
7. Zhou F, Scullion T, Sun L. Verification and modeling of three–stage permanent deformation behavior of asphalt mixes, *J Transp Eng–ASCE* 2004; **130**(4): 486–94.
8. Salajegheh E, Gholizadeh E, Khatibinia M. Optimal Design of Structures for Earthquake Loads by a Hybrid RBF–BPSO Method, *Earth Eng Eng Vib* 2008; **7**: 13–24.
9. Kim YR. *Modeling of Asphalt Concrete*, 1st Ed., 11, McGraw–Hill, New York, 2008.
10. Inthata S, Kowtanapanich W, Cheerarot R. Prediction of chloride permeability of concretes containing ground pozzolans by artificial neural networks, *Mater Struct* 2013; **46**(10): 1707–21.
11. Gandomi AH, Yun GL, Alavi AH. An evolutionary approach for modeling of shear strength of RC deep beams, *Mater Struct* 2013; **46**(12): 2109-19.
12. Golafshani EM, Rahai A, Sebt MH. Artificial neural network and genetic programming for predicting the bond strength of GFRP bars in concrete, *Mater Struct* 2015: **48**(5): 1581-602.

13. Güneyisi EM, Mermerdaş K, Güneyisi E, Gesoğlu M. Numerical modeling of time to corrosion induced cover cracking in reinforced concrete using soft–computing based methods, *Mater Struct* 2015; **48**(6): 1739–56.
14. Güneyisi EM, Gesoğlu M, Güneyisi E, Mermerdaş K. Assessment of shear capacity of adhesive anchors for structures using neural network based model, *Mater Struct* 2016; **49**(3): 1065–77.
15. Khatibinia M, Mohammadizadeh MR. Intelligent fuzzy inference system approach for modeling of debonding strength in FRP retrofitted masonry elements, *Struct Eng Mech* 2017; **6**(2): 283-93.
16. Mirzahosseini MR, Aghaeifar A, Alavi AH, Gandomi AH, Seyednour R. Permanent deformation analysis of asphalt mixtures using soft computing techniques, *Expert Syst Appl* 2011: **38**(5): 6081-100.
17. Suykens JAK, De Brabanter J, Lukas L, Vandewalle J. Weighted least squares support vector machines: robustness and sparse approximation. *Neurocomput* 2009; **48**: 85–105.
18. Khatibinia M, MFadaee MJ, Salajegheh J, Salajegheh E. Seismic reliability assessment of RC structures including soil–structure interaction using wavelet weighted least squares support vector machine, *Reliab Eng Syst Safe* 2013; **110**: 22–33.
19. Khatibinia M, Khosravi Sh. A hybrid approach based on an improved gravitational search algorithm and orthogonal crossover for optimal shape design of concrete gravity dams, *Appl Soft Comput* 2014; **16**: 223–33.
20. Gharehbaghi S, Khatibinia M. Optimal seismic design of reinforced concrete structures under time–history earthquake loads using an intelligent hybrid algorithm, *Earth Eng Eng Vib* 2015; **14**: 97–109.
21. Saber Mahani A, Shojaee S, Salajegheh E, Khatibinia M. Hybridizing two stage meta–heuristic optimization model with weighted least squares support vector machine for optimal shape of double–arch dams, *Appl Soft Comput* 2015; **27**: 205–18.
22. Khatibinia M, Gharehbagh S, Moustafa A. Seismic reliability–based design optimization of reinforced concrete structures including soil–structure interaction effects, IN: Earthquake Engineering–from Engineering Seismology to Optimal Seismic Design of Engineering Structure, 2015; 267–304.
23. Yazdani H, Khatibinia M, Gharehbaghi S, Hatami K. Probabilistic performance–based optimum seismic design of RC structures considering soil–structure interaction effects, *ASCE–ASME J Risk Uncertain Eng Syst A Civil Eng* 2017; 3(2).
24. Nikafshan Rad H, Hasanipanah Rezaei M, Lotfi Eghlim. Developing a least squares support vector machine for estimating the blast–induced flyrock, *Eng Copmut* 2017, (In Press).
25. Sheykhi H, Bagherpour R, Ghasemi E, Kalhori H. Forecasting ground vibration due to rock blasting: a hybrid intelligent approach using support vector regression and fuzzy C–means clustering, *Eng Copmut* 2018; **34**(2): 357–65.
26. Xulei Y, Qing S, Cao A. Weighted support vector machine for data classification, *Proceedings of the 2005 IEEE International Joint Conference on Neural Networks* 2005; pp. 859-864.
27. Li HS, Lü ZZ, Yue ZF. Support vector machine for structural reliability analysis, *Appl Math Mech* 2006; **27**: 1295-303.

28. Widodo A, Yang BS. Wavelet support vector machine for induction machine fault diagnosis based on transient current signal, *Expert Syst Appl* 2008; **35**: 307-16.
29. Mirzaei Z, Akbarpour A, Khatibinia M, Khashei Siuki A. Optimal design of homogeneous earth dams by particle swarm optimization incorporating support vector machine approach, *Geomech Eng* 2015; **9**(6): 709–27.
30. Khatibinia M, Feizbakhsh A, Mohseni E, Ranjbar MM. Modeling mechanical strength of self–compacting mortar containing nanoparticles using wavelet– based support vector machine, *Comput Concr* 2016; **18**(6): 1065–82.
31. Wu Q. Product demand forecasts using wavelet kernel support vector machine and particle swarm optimization in manufacture system, *J Comput Appl* 2010; **233**: 2481–2491.
32. Wu Q. Hybrid model based on wavelet support vector machine and modified genetic algorithm penalizing Gaussian noises for power load forecasts, *Expert Syst Appl* 2001; **38**: 379-85
33. Zhang L, Zhou W, Jiao L. Wavelet support vector machine, *IEEE Transact Syst, Man, Cybernet, Part B (Cybernetics)* 2004; **34**: 34–9.
34. Kronland–Martinet R, Morlet J, Grossman A. Analysis of sound patterns through wavelet transforms, *Int J Pattern Recogn* 1987; **1**: 273–302
35. Bach FR, Lanckriet GRG, Jordan MI. Multiple kernel learning, conic duality, and the SMO algorithm, In: *Proceedings of the 21st International Conference on Machine Learning* 2004; pp. 26–13.
36. Sonnenburg S, Rätsch G, Schäfer C, Schölkopf B. Large scale multiple kernel learning, *J Mach Learn Res* 2006; **7**: 1531–65.
37. Wang Z, Chen S, Sun T. MultiK–MHKS: a novel multiple kernel learning algorithm, *IEEE Trans Patter Anal Mach Intell* 2008; **30**(2): 348–53.
38. Zhenyu C, Jianping L, Liwei W. A multiple kernel support vector machine scheme for feature selection and rule extraction from gene expression data of cancer tissue, *Artif Intell Med* 2007; **4**: 161–75.