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EIGENVECTORS OF COVARIANCE MATRIX FOR OPTIMAL DESIGN OF STEEL FRAMES

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ABSTRACT

In this paper, the discrete method of eigenvectors of covariance matrix has been used to weight minimization of steel frame structures. Eigenvectors of Covariance Matrix (ECM) algorithm is a robust and iterative method for solving optimization problems and is inspired by the CMA-ES method. Both of these methods use covariance matrix in the optimization process, but the covariance matrix calculation and new population generation in these two methods are completely different. At each stage of the ECM algorithm, successful distributions are identified and the covariance matrix of the successful distributions is formed. Subsequently, by the help of the principal component analysis (PCA), the scattering directions of these distributions will be achieved. The new population is generated by the combination of weighted directions that have a successful distribution and using random normal distribution. In the discrete ECM method, in case of succeeding in a certain number of cycles the step size is increased, otherwise the step size is reduced. In order to determine the efficiency of this method, three benchmark steel frames were optimized due to the resistance and displacement criteria specifications of the AISC-LRFD, and the results were compared to other optimization methods. Considerable outputs of this algorithm show that this method can handle the complex problems of optimizing discrete steel frames.

Keywords: Frame Design Optimization, Discrete Optimization, Meta-Heuristic Algorithms, Eigenvectors Of Covariance Matrix.

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1. INTRODUCTION

Optimal design of structures is a difficult and challenging task for designers and engineers. In practical cases, the complexity of the problems and related constraints makes

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classical methods less efficient in the field of engineering issues. The meta-heuristic methods have great potential for solving engineering problems and the efficiency and practicality of these methods in optimal design of structures is undeniable. In the past decade, many metaheuristic techniques have been used for optimal design of structures, Kaveh [1,2]. In the following, some of the techniques which have been utilized to optimize steel frame structures are mentioned. Kaveh and Ilchi Ghazaan used CBO and ECBO [3], Azad and Hasancebi employed GSS [4], Kaveh and Farhoudi used Dolphin Echolocation [5], and Kaveh and Talatahari utilized BB-BC [6], HPSACO [7], CSS [8], ICA [9] and IACO [10] for optimal design of steel frames. Carraro et al with the help of SGA [11]. Fernandez-Caban and Masters used particle swarm and big bang-big crunch [12], Kaveh et al. employed MDM [13], and Talatahari et al. employed the technique of ES–DE [14] to optimize steel frames.

ECM is one of the powerful meta-heuristic optimization methods [15] derived from the CMA-ES technique [16]. The ECM initially generates a population at random. It then evaluates the population using a penalty function, and selects and stores top responses among them. Afterwards, the covariance matrix is calculated based on the superior solutions. The covariance matrix defines the dependency between the two variables, this algorithm then obtains the eigenvectors and eigenvalues of the covariance matrix. After that, with the help of the PCA, ECM identifies the directions that contain the largest number of superior responses, and then generates a new population using these directions and random normal distribution. This increases the chance of finding better solutions. In this paper, to evaluate the capability of ECM algorithm in optimum design of frames, three benchmark steel frameworks have been evaluated through this method with AISC-LRFD specifications.

In Section two of this paper, the ECM algorithm is briefly described and the relationships required for optimal design of steel frames are presented. Section three introduces the AISC-LRFD requirements and specifications for steel frame design. In Section four, the three benchmark steel frames are optimized by using the ECM method and the results are compared to other studies. The final section gives an overview of the ECM optimization technique and its results in the optimal frame design.

2. ECM OPTIMIZATION ALGORITHM

The main idea behind the ECM method is to use principal component analysis to increase the chance of finding successful steps. The ECM algorithm is an iterative method that generates and evaluates a number of populations at each stage. It then selects a number of good results and, determines how they are distributed through PCA and obtains their distribution directions. By the weighted combination of the larger dispersed directions, a new step or direction is achieved. By adding the step of the new stage to the best response to that stage, and using the random normal distribution, a new population is generated. To make the ECM algorithm more consistent to solve discrete problems, minor modifications have been made to this method. The details of the ECM discrete optimization method used for optimal design of steel frames are summarized below.

At first, the ECM algorithm generates the initial population randomly with respect to the upper band and lower band. A penalty function is used to evaluate the produced responses and transform the unconstrained optimization problems into constrained problems. The following penalty function is used for this purpose.

$$f_{penalty}(x) = W(\boldsymbol{X})(1 + \varepsilon_1 C)^{\varepsilon_2} \tag{1}$$

$f_{penalty}(x)$ is the penalty function and $\boldsymbol{X}$ is design variables vector. ε_1 is a constant coefficient and is considered to be 50. ε_2 is 1.5 at the beginning, and 4 eventually. C is the sum of the absolute value of the constraint violations, which will be discussed how to be calculated in the next section. $W(\boldsymbol{X})$ is the weight of the frame obtained as following.

$$W(\boldsymbol{X}) = \sum_{j=1}^{Ne} L_j W_{n_j} \tag{2}$$

In this regard, L_j is the length of the member j, W_{n_j} is the nominal weight of the member j and Ne is the number of frame members. To determine the distribution of good results from all the population produced, some will be chosen as good results. Different criteria can be chosen for good results, but the criteria used by this method are as follows; Good results include two superior solutions and the solutions which allocated value from equation 1 is at most three times greater than the value assigned to the best response. By this criterion those responses that have no or minor violations will be chosen. Then by duplicating the vectors of the design variables the good results are columned into a square matrix $N \times N$. For example, if the number of design variables is 10 and the number of good responses is 4, we will yield a matrix of 10×8 dimensions by repeating of good solutions twice. Now two more vectors from the top answers are randomly selected and added to the previous matrix to obtain a 10×10 square matrices. The covariance matrix of this matrix is then calculated. After that, the eigenvectors and eigenvalues of the covariance matrix are obtained. The eigenvector corresponding to the largest eigenvalue, represents the largest direction of dispersion of good solutions. The covariance matrix eigenvectors are arranged in ascending order according to their eigenvalues. Covariance matrix eigenvectors with larger eigenvalues are used to generate the new population. The number of eigenvectors used to generate the new population is suggested as following.

$$\mu = 3 + \text{floor}(3\ \text{Ln}(N)) \tag{3}$$

Here μ is the number of eigenvectors used to generate the new population and N is the number of design variables. Floor is an operator which its output is the smallest integer, less than or equal to the input. The following weight vector is used for further contribution of the directions on which the more successful results are distributed.

$$\boldsymbol{WV} = \begin{pmatrix} \text{Ln}(3 + \text{floor}(3\ \text{Ln}(N))/2 + 1/3) - \text{Ln}(1) \\ \text{Ln}(3 + \text{floor}(3\ \text{Ln}(N))/2 + 1/3) - \text{Ln}(2) \\ \vdots \\ \text{Ln}(3 + \text{floor}(3\ \text{Ln}(N))/2 + 1/3) - \text{Ln}(\mu) \end{pmatrix}_{\mu \times 1} \tag{4}$$

$\boldsymbol{WV}$ is the weight vector. All the components of the weight vector are subdivided by the sum of the values of this vector to normalize the weight vector. Thus, the sum of the weight vector components is equal to one.

$$\boldsymbol{WV^n} = \boldsymbol{WV}/\text{sum}(\boldsymbol{WV}) \tag{5}$$

$\boldsymbol{WV^n}$ is the normal weight vector. It should be noted that as the response vectors approach each other, the eigenvector corresponding to the largest eigenvalue converges to a fixed vector, so it is not used to define the next step. Now we can define the direction of displacement to generate a new population by using the normal weight vector and covariance matrix eigenvectors. This direction will be added to the best response produced until step i and is defined as follows.

$$\boldsymbol{S_i} = \sum_{j=1}^{\mu} WV_j^n \; \boldsymbol{V_{j+1}^i} \tag{6}$$

In this equation, $\boldsymbol{S_i}$ is the step or direction created in the i th stage and WV_j^n is the j th normalized weight vector component. $\boldsymbol{V_{j+1}^i}$ represents the $j+1$ th eigenvector of covariance matrix in the i th stage. It is mentioned that, the covariance matrix eigenvectors have been ascending downward according to their respective eigenvalues. The step size in the first step of the implementation of the algorithm is given as following.

$$\sigma_1 = (ub - lb)/20 \tag{7}$$

σ_1 is the initial step size and ub and lb are upper bound and lower bound, respectively. If the calculated best response from the beginning of the optimization process improves at a certain number of iterations, the step size will be increased by 15%, otherwise the step size will be reduced by 5%. The number of iterations after which the step size change is suggested between 3 and 7 steps. In order to avoid random search, the maximum step size is limited to the following value.

$$\sigma_1 = (ub - lb)/3 \tag{8}$$

At the end, the new population is produced by the following equation:

$$(\boldsymbol{X_{i+1}})_{N\times 1} = (\boldsymbol{X_b})_{N\times 1} + (\,\boldsymbol{h_i})_{N\times N}\,(\sigma_i \boldsymbol{S_i})_{\,N\times 1} \tag{9}$$

In this equation, $\boldsymbol{X_b}$ is the best answer to the i th stage, σ_i and $\boldsymbol{S_i}$ are the step size and the direction produced in the i th stage, respectively. $\boldsymbol{h_i}$ is a diagonal matrix in the i th stage where the elements have a standard normal distribution. As previously mentioned, the ECM algorithm is iterative and the criterion chosen to stop duplicates in the examples in this article is as follows. If the step size falls below 1.5, the optimization process is terminated.

3. STEEL FRAME OPTIMIZATION CONSTRAINTS

In this paper, frame members are selected from 267 W-shape profile list of AISC database, as shown in Table 1. These sections are arranged in ascending order according to the cross section.

Table 1. 267 AISC steel W-shape database.

	AISC Label	w_n (lb/ft)	A (in^2)	I_x (in^4)	I_y (in^4)	r_x (in)	r_y (in)	J (in^4)	S_x (in^3)	Z_x (in^3)	C_w (in^6)
1	W6×8.5	8.5	2.51	14.8	1.99	2.43	0.889	0.033	5.08	5.71	15.8
2	W6×9	9	2.68	16.4	2.2	2.47	0.905	0.0405	5.56	6.23	17.7
3	W80×10	10	2.96	30.8	2.09	3.22	0.841	0.0426	7.81	8.87	30.9
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
266	W36×798	798	235	62600	4200	16.3	4.23	1050	2980	3580	1490000
267	W14×808	808	237	16000	5550	8.2	4.82	1840	1400	1830	434000

According to the AISC-LRFD specifications [17], inter-story drift and strength constraints are applied. C is constraint violation function and expressed as following.

$$C = \left(\sum_{j=1}^{N_s} C_j^d \ + \sum_{j=1}^{N_{bc}} C_j^s \right) \tag{10}$$

where C_j^d and C_j^s are the constraint violation for inter-story drift, and strength. N_s and N_{bc} are the number of stories, and the number of beam columns. C_j^d is defined as follows.

$$C_j^d = max\left(\left(|d_j| - h_j/300 \right); 0 \right) \quad . \quad j = 1; 2; \ \dots; \ N_s \tag{11}$$

d and h are respectively inter-story drift of story, and story height. The LRFD strength constrains is calculated as [17]

$$\begin{cases} C_j^s = \dfrac{P_u}{2\ \emptyset_c P_n} + \left(\dfrac{M_{ux}}{\emptyset_b M_{nx}} + \dfrac{M_{uy}}{\emptyset_b M_{nx}} \right) - 1 & \text{for} \quad \dfrac{P_u}{\emptyset_c P_n} < 0.2 \\ C_j^s = \dfrac{P_u}{\emptyset_c P_n} + \dfrac{8}{9} \left(\dfrac{M_{ux}}{\emptyset_b M_{nx}} + \dfrac{M_{uy}}{\emptyset_b M_{nx}} \right) - 1 & \text{for} \quad \dfrac{P_u}{\emptyset_c P_n} \geq 0.2 \end{cases} \tag{12}$$

where P_u is the required tensile or compressive strength; P_n is the nominal tensile or compressive strength; $\emptyset_c$ is the resistance factor ($\emptyset_c = 0.9$ for tension, $\emptyset_c = 0.85$ for compression); M_{ux} and M_{uy} are respectively required flexural strengths in x and y directions; M_{nx} and M_{ny} are respectively the nominal flexural strengths in x and y directions; and $\emptyset_b$ is the flexural resistance reduction factor ($\emptyset_b = 0.9$). The nominal strengths P_n is calculated by [17]:

$$\begin{cases} P_n = A_g\, F_y & \text{for tensile strength} \\ P_n = A_g\, F_{cr} & \text{for compressive strength} \end{cases} \tag{13}$$

where A_g is the gross section area of member; F_y is the yield stress of steel; and F_{cr} is calculated as [17]

$$\begin{cases} F_{cr} = \left(0.658^{\lambda_c^2}\right) F_y & \text{for } \lambda_c \le 1.5 \\ F_{cr} = \left(\dfrac{0.877}{\lambda_c^2}\right) F_y & \text{for } \lambda_c > 1.5 \end{cases} \tag{14}$$

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} \tag{15}$$

where L is the member length; r is the radius of gyration; E is the modulus of elasticity; and K is the effective length factor which is calculated as [18]

$$K = \sqrt{\frac{1.6G_A G_B + 4(G_A + G_B) + 7.5}{G_A + G_B + 7.5}} \tag{16}$$

where G_A and G_B are stiffness ratios of columns and girders at two end joints. The G ratio at each end is computed as following.

$$G = \frac{\sum(I/L)_{column}}{\sum(I/L)_{beam}} \tag{17}$$

where I is the moment of inertia. M_{nx} and M_{ny} in Eq.(12) are calculated as follows [17].

$$\begin{cases} M_P & L_b \le L_p \\ C_b\left[M_P - (M_P - M_r)\left(\dfrac{L_b - L_p}{L_r - L_p}\right)\right] & L_p < L_b \le L_r \\ M_{cr} \le M_P & L_b > L_r \end{cases} \tag{18}$$

where L_b is the laterally unbraced length of the member; L_p is the limiting laterally unbraced length for full plastic bending capacity; L_r is the limiting laterally unbraced length for inelastic lateral–torsional buckling; M_r is the limiting buckling moment; M_{cr} is the critical elastic moment for the lateral-torsional buckling. These parameters are defined in reference [17]. M_P is defined as follows.

$$M_P = \mathrm{Z}\, F_y \le 1.5\ \mathrm{S}\, F_y \tag{19}$$

where Z and S are respectively the plastic section modulus and the elastic section modulus.

4. TEST PROBLEMES

In this section, the ECM algorithm is applied to optimize three benchmark steel frames with AISC-LRFD specifications. These include: 1- Two-bay three-story frame 2- Three-bay fifteen-story frame 3- Three-bay twenty four-story frame. These frames before this, have already been optimized by a number of researchers. The results of applying this algorithm to optimal design of these frames have been compared to those of a number of studies. According to the number of design variables, in the first problem, the population size equals 6, in the second problem equals 15, and the last is 20. As previously mentioned, the criterion for terminating the program is when the step length is less than 1.5. To collect the statistical results, each problem was designed 30 times independently with the ECM algorithm. Due to the discretization of the design variables, the continuous design variables is replaced with one of the sections in Table 1 which has the largest cross-sectional area of less than or equal to the continuous design variable. This algorithm is coded in Matlab and the frames are analyzed using direct stiffness method.

4.1 Two-bay three-story frame

The first example is a two-bay three-story frame that comprises 9 columns and 6 beams. The topology and loading conditions of this frame are shown in Figure 1. The cross sections of all columns are the same and are selected among the eighteen W10 sections. The cross sections of all beams are the same and are selected among all 267 W-shaped sections. This problem involves two design variables, the modulus of elasticity of the material is $E = 29000\ ksi$ and the yield stress is $F_y = 36\ ksi$. The effective length factor of frame members is calculated by the sway-permitted frame from the simplified transcendental equations of reference [18] and $K_x \geq 1.0$. The out of plane effective length factor of members is defined as $K_y = 1.0$. All columns are unbraced throughout their length and unbraced length of beams are 1/6 of span length. According to the LRFD specification, the only constraint to design this frame is the strength constrain.

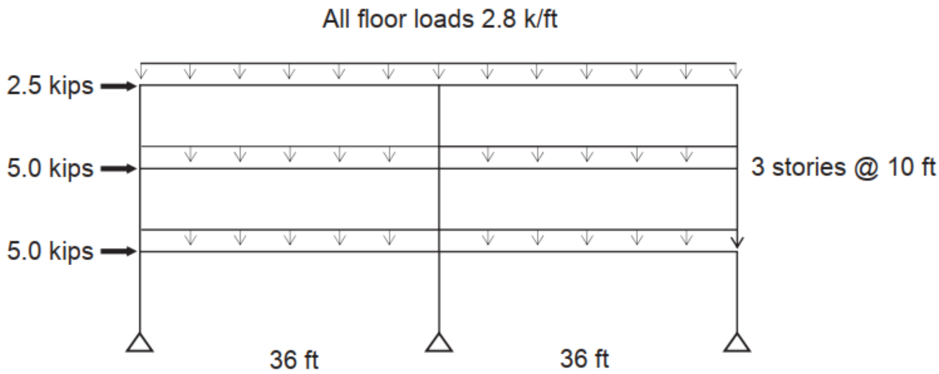

Figure 1. Schematic of the two-bay three-story frame.

Table 2 contains the optimal design variables, the best weight, mean, standard deviation of the responses, and the number of analysis for the two-story three-bay frame. The ECM algorithm resulted in two responses. The first result is similar to other studies in Table 2 and the second result is in Table 2. As shown in the table, the best weight is the same in all methods. Although the ECM method has been stopped after 3120 analyzes, it achieved an optimal response after 264 analyses. The reason for non-stop algorithm after finding the optimal solution is that the stopping criterion is not met. All of the optimal solutions in Table 2 have no violation, and in Fig. 2 the convergence history has been shown for this frame. Since all iterations of the algorithm result in the same response, the standard deviation for this frame is zero. Figure 3 shows the stress ratio of all members of the two-story three-bay frame along with the maximum allowable stress ratio.

Table 2. Optimal design for the two-bay three-story frame.

Element group	SBO [19]	ACO [20]	DDHS [21]	GA [22]	Present study
Beam	W24×62	W24×62	W24×62	W24×62	W21×62
Column	W10×60	W10×60	W10×60	W10×60	W10×60
Best weight (Ib)	18,792	18,792	18,792	18,792	18,792
Mean (Ib)	18,792	19,163	18,792	22,080	18,792
Standard deviation (Ib)	0	1,693	0	5,818	0
Number of analyses	502	880	N/A	900	3,120

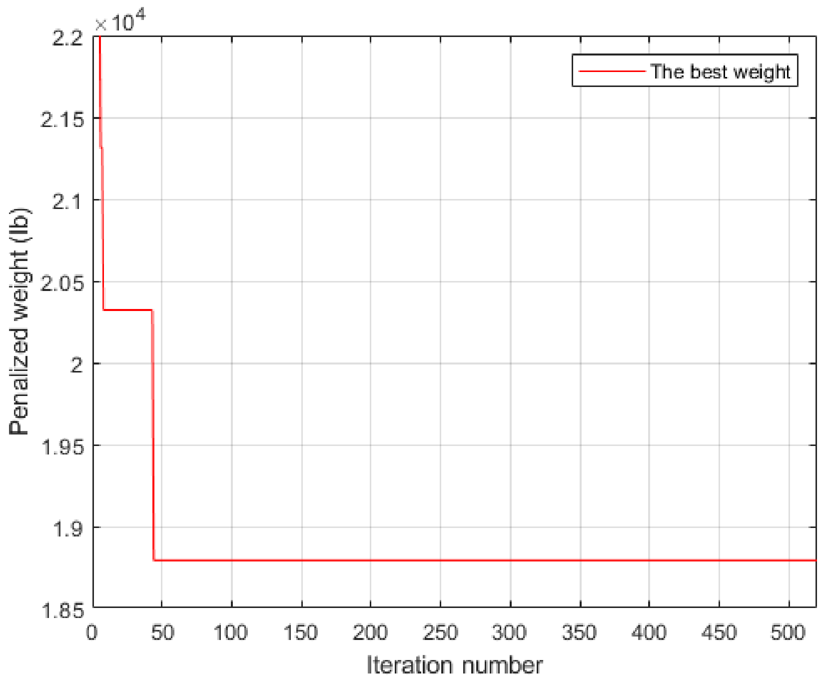

Figure 2. Convergence history of the two-bay three-story frame.

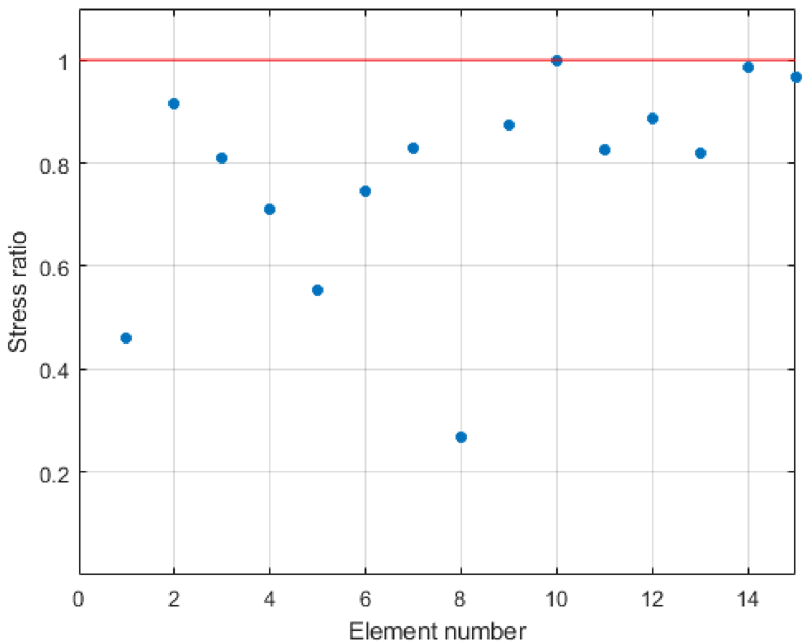

Figure 3. Stress ratios for the best designs of the two-bay three-story frame.

4.2 Three-bay fifteen-story frame

The second example is a three-bay fifteen-story frame, comprising 60 columns and 45 beams. The topology and loading conditions of this frame are shown in Figure 4. The columns of this frame are divided into 10 groups with the same cross-sectional area as shown in Fig. 4 and all the beams of this frame have the same cross-sectional area. The beams and columns are selected from all 267 W-shape sections. This issue involves eleven design variables and the material has a modulus of elasticity $E = 29000\ ksi$ and yield stress $F_y = 36\ ksi$. The effective length factor of members for the frame with the sway-permitted frame is calculated from simplified transcendental equations of reference [18] and $K_x \geq 0$. The out of plane effective length factor members is defined as $K_y = 1.0$. All columns are unbraced throughout their length, and unbraced length for all beams are 1/5 of span length. Based on the AISC-LRFD specification, the design constraints used to design this frame are strength constraint and inter-story drift.

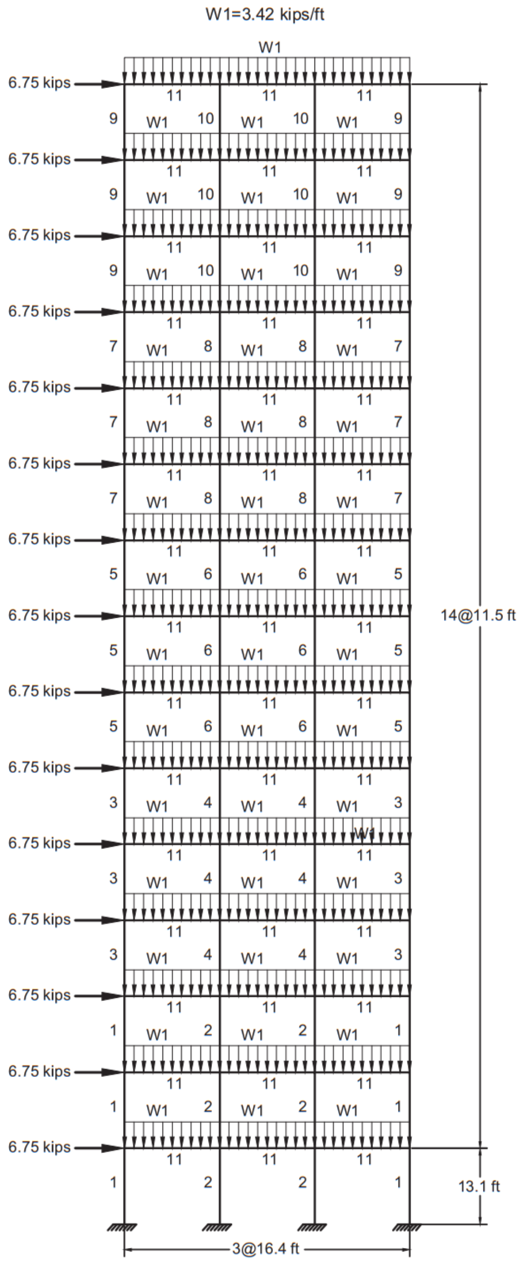

Figure 4. Schematic of the three-bay fifteen-story frame.

Table 3 contains the optimal design values for 11 design variables of the three-bay fifteen-story frame, the best weight and the mean, standard deviation of responses, and the number of analyses. As shown in Table 3, although the number of analyses of the ECM method is greater than most of the methods, this algorithm has led to the lightest design. Figure 5 shows the convergence history of the best and average designs for this frame. Figure 6 shows the stress ratio of all members of three-bay fifteen-story frame along with the maximum allowable stress ratio. Figure 7 shows the inter-story drift of all the stories with the maximum permitted drift.

Table 3. Optimal design for the three-bay fifteen-story frame.

Element group	ECBO [3]	EWOA [23]	CSS [24]	ICA [9]	HPSACO [7]	AWEO [25]	Present study
1	W14×99	W14×99	W21×147	W24×117	W21×111	W14×99	W14×90
2	W27×161	W27×161	W18×143	W21×147	W18×158	W27×161	W36×170
3	W27×84	W27×84	W12×87	W27×84	W10×88	W27×84	W14×82
4	W24×104	W24×104	W30×108	W27×114	W30×116	W24×104	W24×104
5	W14×61	W21×68	W18×76	W14×74	W21×83	W14×61	W16×67
6	W30×90	W18×86	W24×103	W18×86	W24×103	W30×90	W18×86
7	W14×48	W21×48	W21×68	W12×96	W21×55	W16×50	W21×48
8	W14×61	W14×68	W14×61	W24×68	W27×114	W21×68	W14×61
9	W14×30	W8×31	W18×35	W10×39	W10×33	W14×34	W12×30
10	W12×40	W10×45	W10×33	W12×40	W18×46	W8×35	W10×39
11	W21×44	W21×44	W21×44	W21×44	W21×44	W21×44	W21×44
Best weight (Ib)	86,986	88,090	92,761	93,850	95,850	87,538	86,917
Mean (Ib)	88,410	90,784	N/A	N/A	N/A	88,893	91385
Standard deviation (Ib)	N/A	N/A	N/A	N/A	N/A	N/A	2041
Number of analyses	9,000	19,940	5,000	6,000	6,800	10,670	15,585

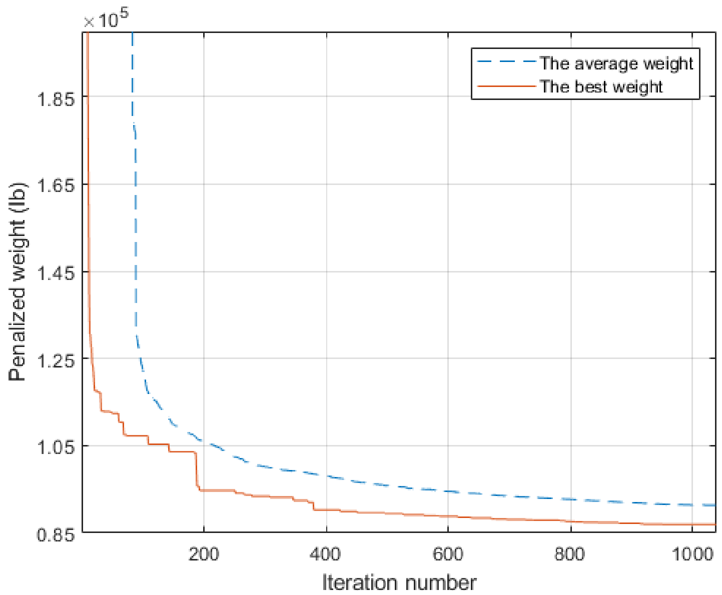

Figure 5. Convergence history of the three-bay fifteen-story frame.

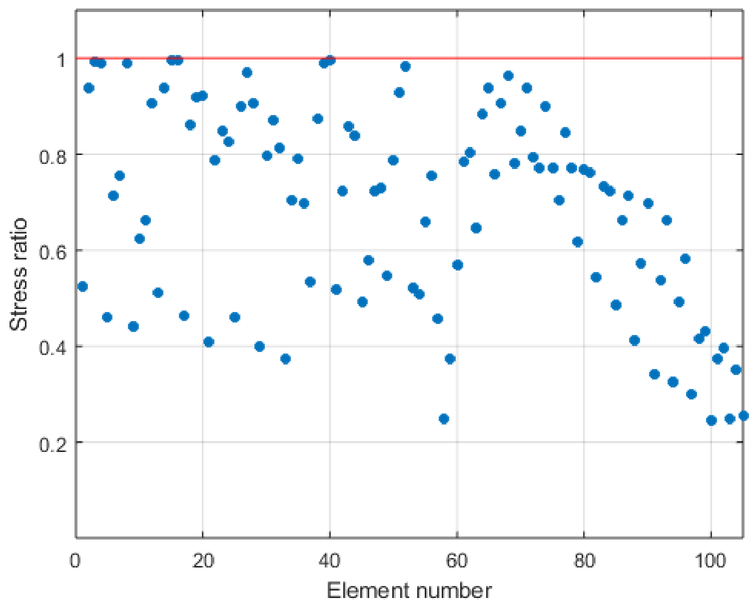

Figure 6. Stress ratios for the best designs of the three-bay fifteen-story frame.

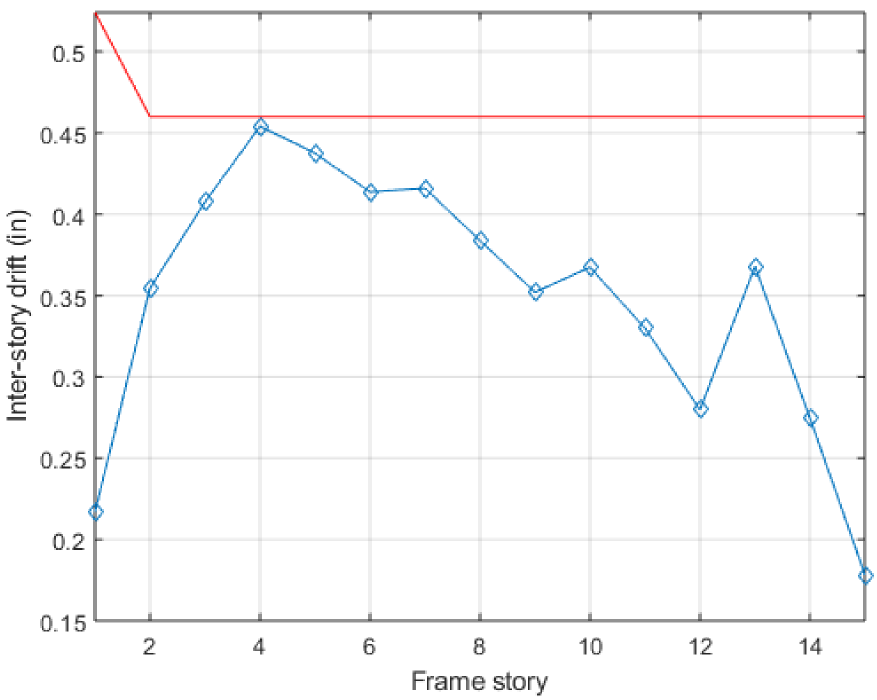

Fig. 7. The inter-story drift for the best designs of the three-bay fifteen-story frame.

4.3. Three-bay twenty four-story frame

The third design example is a three-bay twenty four-story frame comprising 96 columns and 72 beams. The topology and loading conditions of this frame are shown in Figure 8. The columns and beams of this frame, according to Fig. 8, are divided into 16 groups with the same cross section and 4 groups with the same cross section respectively. The beams are selected from all 267 W-shape sections and columns from the thirty-seven W14 sections. This issue includes twenty design variables and has a modulus of elasticity of material $E = 29732\, ksi$ and yield stress $F_y = 33.4\, ksi$. The effective length factor of members for the frame with the sway-permitted frame is calculated from simplified transcendental equations of reference [18] and $K_x \geq 0$. The out-of-plane effective length factor of member is defined as $K_y = 1.0$. All columns and beams are unbraced throughout their length. Based on the AISC-LRFD specification, the constraints used for designing this frame are strength constraint and inter-story drift.

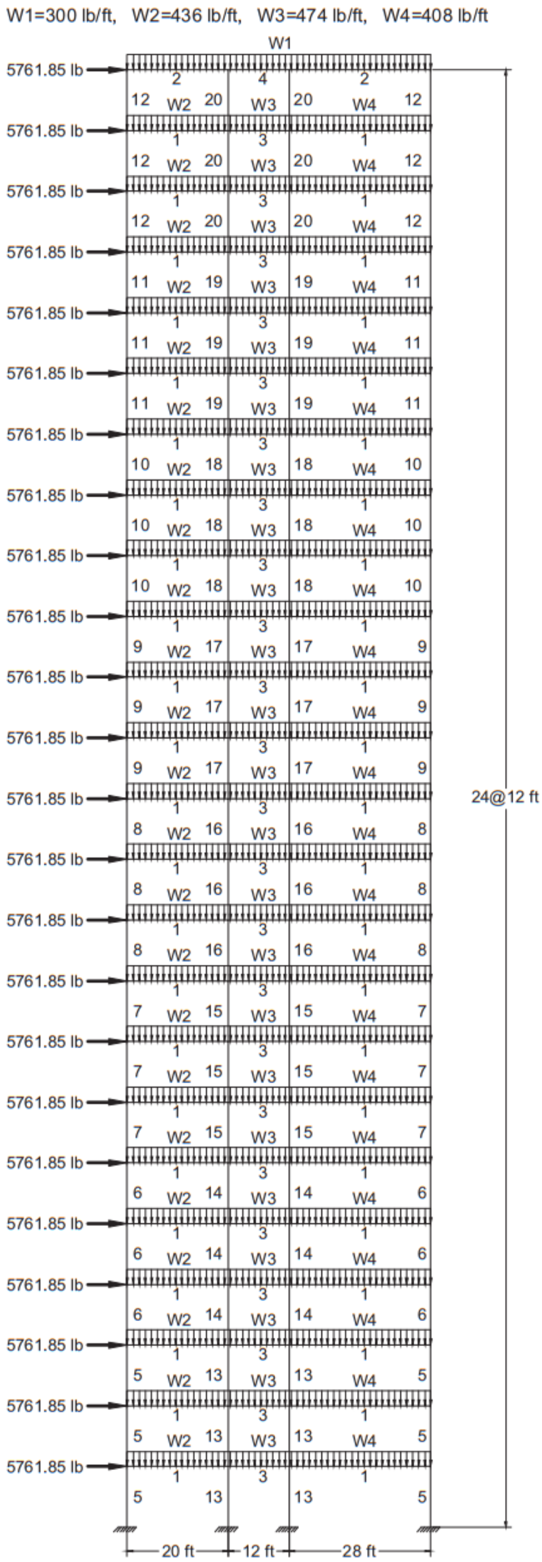

Figure 8. Schematic of the three-bay twenty four-story frame.

Table 4 contains the optimal design values for twenty design variables of the three-bay twenty four-story frame, the best weight and the mean, standard deviation of responses and the number of analyses. Although the number of analyses of the ECM method is higher than that of the other methods, it has led to the lightest weight. Figure 9 shows the convergence history of the best design and average designs for this frame. Figure 10 shows the stress ratio of all members of the three-bay twenty four-story with the maximum allowable stress ratio. Figure 11 shows the inter-story drift of all the stories with the maximum permitted drift.

Table 4. Optimal design for the three-bay twenty four-story frame.

Element group	SBO [19]	ECBO [3]	CSS [24]	ICA [9]	TLBO [26]	HS [27]	EWOA [23]	Present study
1	W30×90	W30×90	W30×90	W30×90	W30×90	W30×90	W30×90	W30×90
2	W8×18	W6×15	W21×50	W21×50	W8×18	W10×22	W10×30	W6×15
3	W21×48	W24×55	W21×48	W24×55	W24×62	W18×40	W24×55	W24×55
4	W6×8.5	W6×8.5	W12×19	W8×28	W6×9	W12×16	W6×8.5	W6×8.5
5	W14×152	W14×145	W14×176	W14×109	W14×132	W14×176	W14×159	W14×159
6	W14×120	W14×132	W14×145	W14×159	W14×120	W14×176	W14×99	W14×132
7	W14×109	W14×99	W14×109	W14×120	W14×99	W14×132	W14×120	W14×109
8	W14×74	W14×90	W14×90	W14×90	W14×82	W14×109	W14×74	W14×74
9	W14×82	W14×74	W14×74	W14×74	W14×74	W14×82	W14×74	W14×53
10	W14×43	W14×38	W14×61	W14×68	W14×53	W14×74	W14×43	W14×43
11	W14×34	W14×38	W14×34	W14×30	W14×34	W14×34	W14×30	W14×38
12	W12×19	W14×22	W14×34	W14×38	W14×22	W14×22	W14×22	W14×22
13	W14×109	W14×99	W14×145	W14×159	W14×109	W14×145	W14×90	W14×90
14	W14×109	W14×99	W14×132	W14×132	W14×99	W14×132	W14×120	W14×99
15	W14×99	W14×99	W14×109	W14×99	W14×99	W14×109	W14×90	W14×90
16	W14×99	W14×82	W14×82	W14×82	W14×90	W14×82	W14×99	W14×90
17	W14×68	W14×68	W14×68	W14×68	W14×68	W14×61	W14×68	W14×82
18	W14×61	W14×61	W14×43	W14×48	W14×53	W14×48	W14×61	W14×61
19	W14×34	W14×30	W14×34	W14×34	W14×34	W14×30	W14×43	W14×30
20	W14×22	W14×22	W14×22	W14×22	W14×22	W14×22	W14×22	W14×22
Best weight (Ib)	202,422	201,618	212,449	212,725	203,008	214,860	203,490	201,330
Mean (Ib)	209,560	209,644	215,313	N/A	N/A	222,620	208,648	211,115
Standard deviation (Ib)	7,052	N/A	2,448	N/A	N/A	5,800	N/A	6,774
Number of analyses	14,572	15,360	5,500	7,500	12,000	14,651	18,820	21,400

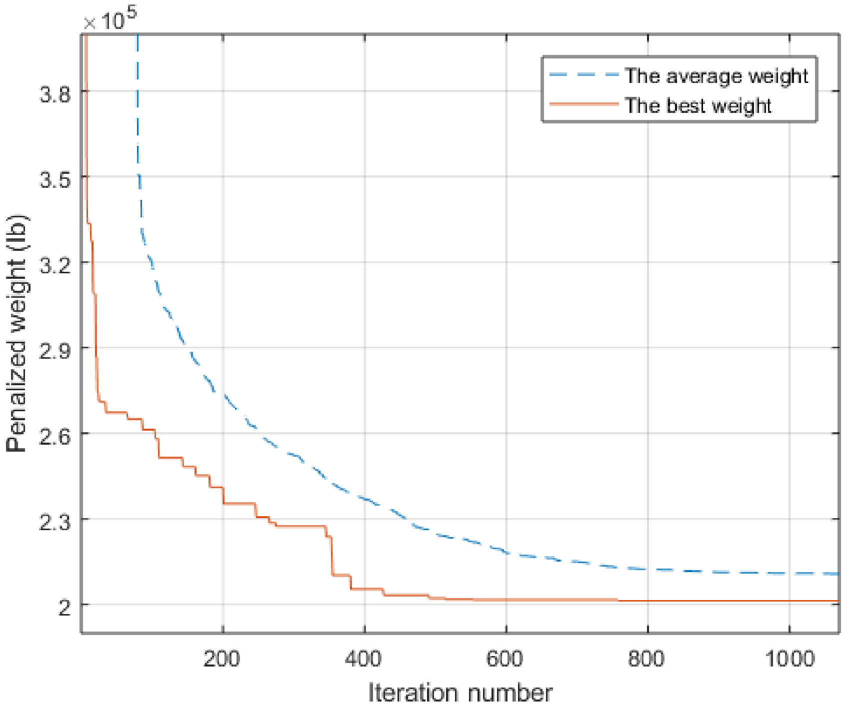

Figure 9. Convergence history of the three-bay twenty four-story frame.

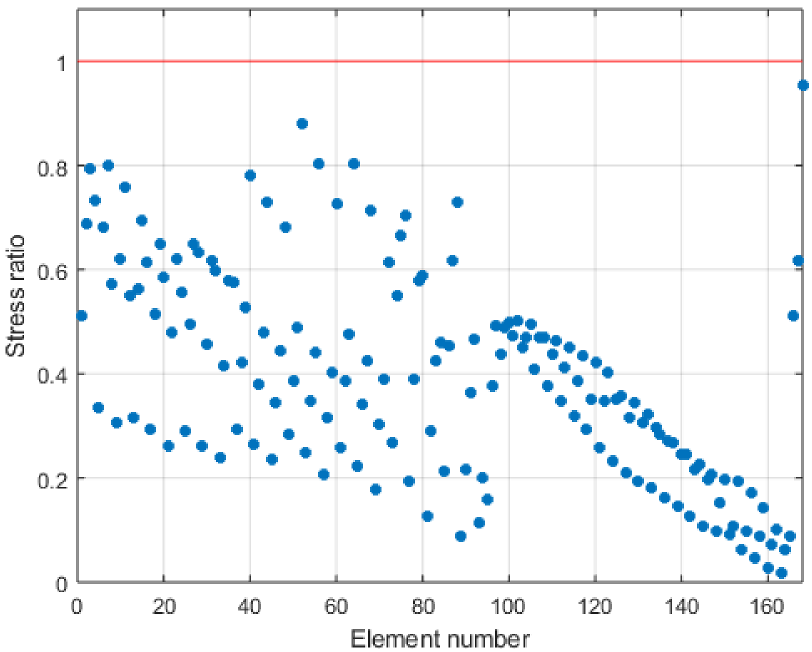

Figure 10. Stress ratios for the best designs of the three-bay twenty four-story frame.

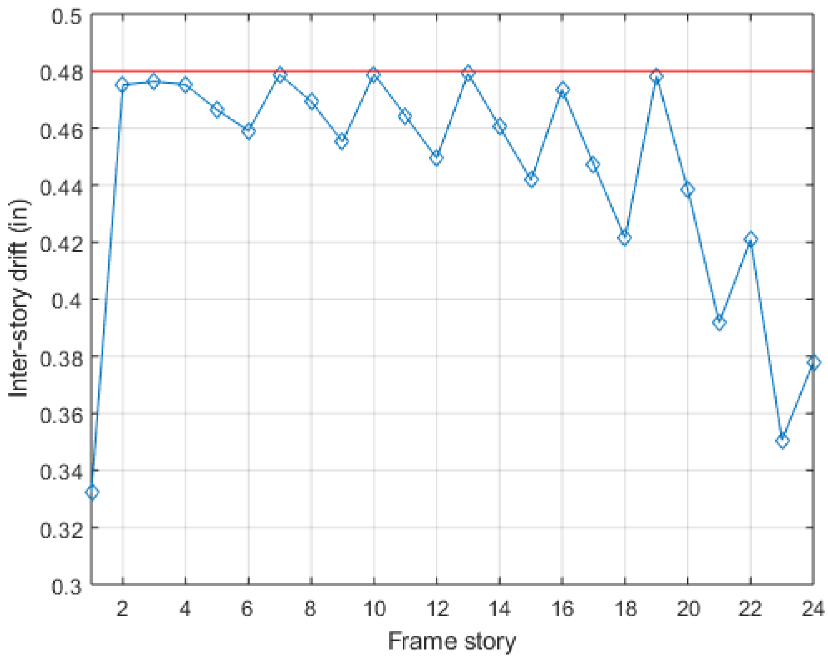

Figure 11. The inter-story drift for the best designs of the three-bay twenty four-story frame.

5. CONCLUSION

In this study, the discrete ECM technique was used to optimize discrete steel frames. The objective function is to minimize the weight of the steel frames under constraints, the strength and the inter-story drift of the AISC-LRFD specification. The ECM algorithm identifies the successful results at each stage and, by the help of the PCA, determines how they are distributed and utilizes the successful distributions to create a new population. Using this method of statistical concepts increases the chance of finding successful steps. Three benchmark steel frame were examined to evaluate the efficiency and usefulness of this technique. In the two-bay three-story frame, the best response obtained with the ECM, was the same as the best of the other methods. In the three-bay fifteen-story frame and the three-bay twenty four-story frame, the results of this method were lighter than other algorithms. It may be possible to consider the number of further analyses of this technique as a weak point of this algorithm in comparison with other methods, but the good results of this algorithm partially offsets this shortcoming. According to the received statistical results, ECM is a robust and suitable method and it can be utilized for optimal design of steel frame problems.

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