

## A TWO-STAGE SIMP-ACO METHOD FOR RELIABILITY-BASED TOPOLOGY OPTIMIZATION OF DOUBLE LAYER GRIDS

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### ABSTRACT

For reliability-based topology optimization (RBTO) of double layer grids, a two-stage optimization method is presented by applying “Solid Isotropic Material with Penalization” and “Ant Colony Optimization” (SIMP-ACO method). To achieve this aim, first, the structural stiffness is maximized using SIMP. Then, the characteristics of the obtained topology are used to enhance ACO through six modifications. As numerical examples, reliability-based topology designs of typical double layer grids are obtained by ACO and SIMP-ACO methods. Their numerical results reveal the effectiveness of the proposed SIMP-ACO method for the RBTO of double layer grids.

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**KEY WORDS:** Double layer grids; reliability-based topology optimization; solid isotropic material with penalization; ant colony optimization; two-stage optimization method

### 1. INTRODUCTION

Topology optimization, which involves distributing a given amount of material in a domain subjected to certain loading and support conditions so as to optimize a given objective function, has been recognized as one of the most challenging tasks in structural design [1]. A topology optimization method enables designers to find an acceptable material formation for the required performances of systems [2]. This type of optimization is a relatively new but extremely rapidly expanding research field, which has interesting theoretical implications in mathematics, compliant mechanisms [3 and 4], multi-physics [5 and 6] and multi-material [7] and also important practical applications by the manufacturing car and aerospace industries in

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particular and is likely to have a significant role in nanotechnologies [8].

Many optimization methods and algorithms have been developed in the past decades for both linear and nonlinear structural problems in many disciplines such as solid mechanics, thermal, eigenvalue, and acoustic problems [2].

Nowadays, the most reasonable numerical finite element-based topology optimization method is the SIMP method which has been developed in the late eighties. It is sometimes called “material interpolation”, “artificial material”, “power law”, or “density” method, but it is currently known as “SIMP” [8]. The term “SIMP” stands for Solid Isotropic Microstructure (or Material) with Penalization for densities between 0 and 1. The basic idea of this method was proposed by Bendsoe [9], while the term “SIMP” was created later and first introduced in a paper by Rozvany et al. [10]. In recent years, SIMP has been popularly accepted in the topology optimization community for its conceptual and implementing simplicity, since in this method the only design variable of each element is its pseudo-density and the material property is assumed to be constant within each element [11]. It must be noted that, Sigmund’s educational article [12] with a 99-line SIMP-code and his web-based topology optimization program [13] played an important role in general acceptance of SIMP.

Topology optimization has become increasingly popular in a wide range of industries including automotive, aerospace, heavy industry, etc. partly because of development and promotion through commercial Finite Element Analysis (FEA) software (e.g., OptiStruct, Genesis, MSC/Nastran, Ansys, Tosca, etc.) [8]. All software have implemented the SIMP method, except Tosca that has used an Evolutionary Structural Optimization (ESO) type method [8]. Recent publications [14] indicate that Tosca has also started to adopt the SIMP approach combined with the Method of Moving Asymptotes (MMA) [8].

In skeletal structures, topology optimization methods using the ground structure approach are effective at the conceptual design stage. Real structural behavior are subject to uncertain conditions such as applied loads, material properties, and dimensional variation due to manufacturing factors. Because these uncertainties may affect desired performances, there is a great need for optimization methods which can effectively work despite having uncertain conditions. Reliability-based topology optimization is a useful strategy to consider such uncertainties [15 and 16].

The ACO is a relatively recent heuristic method for solving optimization problems through simulating the behavior of real ant colonies. Similar to genetic algorithm (GA), ACO is a good choice for structural topology optimization due to discrete characteristics of the structures. Recently some researchers have employed ant algorithms for topology optimization of continuous and discrete structures [17 and 18]. In skeletal structures topology optimization, the operation of these heuristic methods can be increased if they are combined with gradient-based methods. For example, in [19] a two-stage optimization method has been introduced for RBTO of double layer grids which has been performed by employing the methods of moving asymptotes (MMA) and ACO. Also, an ESO-ACO method has been presented in [20] which consists of the evolutionary structural optimization (ESO) and ant colony optimization (ACO), for minimizing the weight of double layer grid while artificial ground motion is used for calculating the structural dynamic responses.

This paper discusses a reliability-based topology optimization that uses ground structure approach for double layer grids. In this approach, a new combined method for RBTO of

double layer grids is presented that considers uncertainties in applied loads. The optimization is performed using “solid isotropic material with penalization” (SIMP) and “ant colony optimization” (ACO) method, which is called SIMP-ACO method. To execute SIMP-ACO, first, an introductory optimization is performed using SIMP in which the structural stiffness is maximized and the optimum cross-sectional areas are calculated from continuous quantities. Then, the obtained topology, optimum cross-sectional areas, and the internal forces of members (elements) are employed for improving the ACO approach through the following six modifications: (I) using the obtained topology as a new ground structure, (II) size optimization of the new ground structure to be considered as a good initial design, (III) finding the structural importance rate of elements and using it to assign an unequal amount of pheromone on the paths that associates with the presence or absence of the member (element) variables, (IV) limiting the lower limit of available cross-sectional areas of each element group, (V) determining the number of compressive and tensile element types, and (VI) modifying the generation of random stable structures.

Two criteria, i.e. stiffness and eigenvalue, are considered as two failure modes and the failure probability of these modes is evaluated by combining Monte Carlo Simulation (MCS) and third order approximation (TOA) [19].

In several numerical examples, the optimum topologies of double layer grids which obtained by SIMP-ACO and ACO are compared. The numerical results reveal the capability and robustness of SIMP-ACO for topology optimization of large scale skeletal structures with discrete cross-sectional areas and various constraints.

## 2. STRUCTURAL RELIABILITY ANALYSIS

Reliability analysis is a tool to compute the probability of failure corresponding to a given failure mode. A reliability analysis is normally formulated using a limit state function,  $g(\mathbf{Z})$ , where  $\mathbf{Z}$  is the vector of random variables. The condition  $g(\mathbf{Z}) \leq 0$  defines violation of the limit state, and so, the failure probability is expressed by the following expression [21]:

$$p_f = p[g(\mathbf{Z}) \leq 0] = \int_{g(\mathbf{Z}) \leq 0} f_{\mathbf{Z}}(\mathbf{Z}) d\mathbf{Z} \quad (1)$$

where  $f_{\mathbf{Z}}$  is the joint probability density function.

For complex and/or large-scale structures, the structural responses have to be calculated by a numerical procedure such as finite element analysis. In such cases, several computational approaches could be pursued to the solution of (1). These can be divided into three categories [21]: (a) Monte Carlo simulation, (b) semi-probabilistic methods, and (c) approximate probabilistic methods.

Monte Carlo simulation (MCS) is a simulation method that presents the following characteristics: it can be applied to many practical problems which have implicit limit state function; it is able to calculate the failure probability with the desired precision; it is easy to use.

To calculate  $p_f$  using MCS, for each random vector  $\mathbf{Z}$  a sufficient number of  $N$  independent random samples is produced using a specific probability density function. The value of the limit state function is calculated for each random sample  $Z_j$ , and the Monte Carlo

simulation estimate of  $p_f$  is given in terms of the sample mean by:

$$p_f = \frac{N_H}{N} \quad (2)$$

where  $N_H$  is the number of cases which are in failure domain ( $g(\mathbf{Z}) \leq 0$ ).

### 3. FAILURE CRITERIA IN RELIABILITY-BASED TOPOLOGY OPTIMIZATION

For the reliability-based topology optimization of double layer grids, two criteria are introduced. The first one is the compliance which is used for static criterion (SC); the other is the eigenvalue which is used for dynamic criterion (DC) [19].

The compliance is introduced as a quantity of the structural stiffness defined as follows:

$$SC = \mathbf{F}^T \mathbf{d}, \quad \mathbf{F} = \mathbf{K} \mathbf{d} \quad (3)$$

where  $\mathbf{K}$  is the global stiffness matrix, and  $\mathbf{F}$  and  $\mathbf{d}$  are the load and displacement vectors, respectively.

The  $j$ th eigenvalue can be evaluated by the Rayleigh quotient as [19]:

$$I_j(z^M) = \frac{\boldsymbol{\varphi}_j^T \mathbf{K} \boldsymbol{\varphi}_j}{\boldsymbol{\varphi}_j^T (\mathbf{M}_C + \mathbf{M}_L(z^M)) \boldsymbol{\varphi}_j} \quad (4)$$

in which

$$\mathbf{M}_L(z^M) = \left( \frac{z^M}{MM} \right) \mathbf{M}_L^{MM} \quad (5)$$

where  $\boldsymbol{\varphi}_j$  is the eigenvector corresponding to the  $j$ th eigenvalue ( $\lambda_j$ ),  $\mathbf{M}_C$  is the global mass matrix of structural elements,  $\mathbf{M}_L(z^M)$  is the lumped mass matrix of distributed non-structural mass  $z^M$ ,  $MM$  denotes the mean applied distributed non-structural mass, and  $\mathbf{M}_L^{MM}$  is the lumped mass matrix of  $MM$ . This distributed non-structural mass is assigned to the nodes of the top grid in the proportion of their load bearing area. In the present article the following dynamic criterion is adopted [19]:

$$DC = \frac{\sqrt{I_1}}{2p} \quad (6)$$

### 4. DOUBLE LAYER GRIDS RELIABILITY ANALYSIS

When the applied load or the non-structural mass are not certain values, the static criterion (3) and the dynamic criterion (6) also exhibit variations, which results in a deleterious effect upon the performance of double layer grids. The degree of degradation in performance is evaluated

based on structural reliability theory, and such variations are, therefore, modeled as probabilistic random variables. The double layer grid is failed when the static criterion  $SC$  is higher than the prescribed upper bound  $SC_U$ , or the dynamic criterion  $DC$  is lower than the prescribed lower limit  $DC_L$ . Therefore, the following two limit state functions are introduced [19]:

$$g_1(\mathbf{Z}^{AL}) = SC_U - SC(\mathbf{Z}^{AL}) \tag{7}$$

$$g_2(\mathbf{Z}^M) = DC(\mathbf{Z}^M) - DC_L \tag{8}$$

where  $\mathbf{Z}^{AL}$  is random vector consisting of the applied load.

The double layer grid is failed when either of the limit state functions in (7) and (8) has a negative value. That is, the system is modeled as a system consisting of two iterative failure modes [19].

### 5. TOPOLOGY OPTIMIZATION

In double layer grids topology optimization, geometry of the structure, support positions, and coordinates of nodes are kept constant while the presence or absence of the elements and also cross-sectional areas are selected as design variables. The symmetry properties of the structure are used for the tabulation of elements which lead to decrease in the design space. Therefore, the elements are deleted in groups of 8 or 4 [20]. Presence or absence of each element group is identified by a variable (topology variable), taking 1 and 0 for two cases, respectively. A zero amount of the  $i$ th topology variable indicates that the  $i$ th element group should be deleted from the ground structure. In this optimization problem, the number of design variables ( $NDV$ ) is the summation of the number of topology variables ( $NTV$ ), and the number of compressive and tensile member types [19 and 20].

Discrete variables are used for determining the suitable cross-sectional area of structural members. These variables are selected from pipe sections with specified thickness and outer diameter. Topology optimization is carried out in two cases which their details are presented as follows:

#### 5.1. Deterministic topology optimization (DTO)

In deterministic optimum topology design of the double layer grids, the optimum values of design variables are found from discrete values to minimize the weight of the structure ( $W$ ) under constraints on stress ( $g_\sigma$ ), slenderness ratio ( $g_\lambda$ ) and displacement ( $g_\delta$ ):

$$\begin{aligned} \text{Find: } \bar{\mathbf{A}} &= [J_1, J_2, \dots, J_{NTV}, a_1, a_2, \dots, a_{NMG}]^T \\ J_i &\in [0,1], \quad i = 1, 2, \dots, NTV \\ a_k &\in \tilde{\mathbf{A}}, \quad k = 1, 2, \dots, NMG \end{aligned} \tag{9}$$

$$\text{to minimize : } W = \rho^e \sum_{k=1}^{NMG} a_k \sum_{i=1}^{N_k} l_i$$

$$\text{subject to : } g_s, g_l, g_d \leq 0$$

where  $NMG$  is the number of member groups,  $N_k$  is the number of members in  $k$ th member

group,  $a_k$  is the discrete cross-sectional area of the  $k$ th member group selected from steel pipes in a given profile list ( $\tilde{\mathbf{A}}$ ),  $\rho^e$  is the material density, and  $l_i$  is the length of the  $i$ th element.

The stress and the slenderness ratio constraints are as follows:

$$g_s = \sum_k \max \left( \frac{|s_k|}{\bar{s}_k} - 1, 0 \right) \quad (10)$$

$$g_l = \sum_k \max \left( \frac{I_k}{\bar{I}_k} - 1, 0 \right) \quad (11)$$

where  $s_k$  is the member stress,  $\bar{s}_k$  is the allowable stress,  $I_k$  is the member slenderness ratio, and  $\bar{I}_k$  is its upper limit for the  $k$ th member of double layer grids. In this study, the AISC code provisions [22] are employed for the stress limits and local buckling criteria as follows:

$$\begin{aligned} \bar{s}_k &= 0.6F_y \quad (\text{for tension members}) \\ \bar{\sigma}_k &= \bar{\sigma}_c \quad (\text{for compression members}) \end{aligned} \quad (12)$$

in which

$$\bar{\sigma}_c = \begin{cases} \left( 1 - \frac{I_k^2}{2C_c^2} \right) F_y & \text{for } I_k < C_c, (\text{Plastic Buckling}) \\ \frac{5}{3} + \frac{3I_k}{8C_c} - \frac{I_k^3}{8C_c^3} & \\ \frac{12p^2 E}{23I_k^2} & \text{for } I_k \geq C_c, (\text{Elastic Buckling}) \end{cases} \quad (13)$$

where  $F_y$  is the yield stress,  $E$  is the modulus of elasticity, and  $C_c$  is taken as  $\sqrt{2p^2 E/F_y}$ .

The maximum slenderness ratio is limited to 300 and 240 for tension and compression members, respectively. Hence, the slenderness related design constraints can be formulated as follows:

$$\begin{aligned} I_k &= \frac{K_k l_k}{r_k} \leq 300 \quad (\text{for tension members}) \\ \lambda_k &= \frac{K_k l_k}{r_k} \leq 240 \quad (\text{for compression members}) \end{aligned} \quad (14)$$

where  $K_k$  is the effective length factor of the  $k$ th member ( $K_k=1$  for all truss members), and  $r_k$  is its radii of gyration.

In the optimization process, the allowable vertical displacement ( $\delta_v$ ) is adopted as the width of double layer grids divided by 360 [23]. So, the displacement constraint is expressed as follows:

$$g_d = \sum_j \max \left( \frac{|d_j|}{d_v} - 1, 0 \right) \tag{15}$$

where  $\delta_j$  is the displacement of the  $j$ th node.

5.2. Reliability-based topology optimization (RBTO)

In RBTO of double layer grids, the optimum value of  $\bar{\mathbf{A}}$  is found from discrete values to minimize  $W$  under the constraints on system failure probability ( $g_f$ ), stress, displacement, and slenderness ratio:

$$\begin{aligned} & \text{Find: } \bar{\mathbf{A}} \\ & \text{to minimize : } W \\ & \text{subject to : } g_s, g_l, g_d, g_f \leq 0 \end{aligned} \tag{16}$$

in which

$$g_f = \sum_{i=1}^2 \max \left( \frac{P_f^i}{P_U^i} - 1, 0 \right) \tag{17}$$

where  $P_U^i$  is the failure probability upper limit in the  $i$ th failure mode. The system failure probability in each mode ( $P_f^1, P_f^2$ ) is obtained from (2).

In each case of topology optimization, the optimization problems (9) and (16) will be solved using ACO and SIMP-ACO. To solve a constrained optimization problem, its objective function ( $W$ ) should be modified in such a way that the constrained problem should be converted to an unconstrained one, with a modified objective function ( $\Psi$ ), where  $\Psi$  is defined as:

$$\Psi = W(1 + C)^2 \tag{18}$$

in which

$$C = \begin{cases} \sum_{i=1}^{ne} (g_{s,i} + g_{l,i}) + \sum_{j=1}^{nj} g_{d,j}, & \text{for (9)} \\ \sum_{i=1}^{ne} (g_{s,i} + g_{l,i}) + \sum_{j=1}^{nj} g_{d,j} + \sum_{i=1}^2 g_{f,i}, & \text{for (16)} \end{cases} \tag{19}$$

where  $C$  is the penalty function,  $ne$  is the number of elements, and  $nj$  is the number of joints.

The optimum topology design of double layer grids is a minimization problem and hence, the fitness function must be chosen such that the higher the weight of the structure, the lower its fitness is and vice-versa. The following relation is selected as the measure of fitness function [24]:

$$F_i = \Psi_{\max} + \Psi_{\min} - \Psi_i \tag{20}$$

where  $\Psi_{\max}$ ,  $\Psi_{\min}$ , and  $\Psi_i$  are the maximum and minimum modified objective function values in a cycle and the modified objective function value of the  $i$ th structure, respectively.

## 6. TWO-STAGE OPTIMIZATION METHOD (SIMP-ACO)

In this paper, in order to achieve better topology using ACO, the location of members with high structural importance is identified by SIMP. To achieve this aim, the structural stiffness is maximized using SIMP approach, where the objective is to minimize the compliance. The optimization problem can be written as follows [12]:

$$\begin{aligned} \text{Minimize : } C(\mathbf{a}) &= \mathbf{d}^T \mathbf{K} \mathbf{d} = \sum_{i=1}^{ne} (a_i)^p d_i^T k_0 d_i \\ \text{Subject to : } \mathbf{K}(\mathbf{a}) \mathbf{d} &= \mathbf{F} \\ V(\mathbf{a}) &= f \sum_{i=1}^{ne} l_i \\ \mathbf{a} \in \hat{\mathbf{A}} &= \{ \mathbf{a} \in R^{ne} : 0 \leq a_i \leq 1, i = 1, \dots, ne \} \end{aligned} \quad (21)$$

where  $V(\mathbf{a})$  is the material volume,  $\mathbf{a}$  is a real-valued vector in a bounded space  $\hat{\mathbf{A}}$ , and  $f$  (volfrac) is the prescribe volume fraction. It must be noted that in [12], to avoid the structural instability, the nonzero amounts have been assigned to the cross-sectional areas, but in the present work, the zero value can be allocated to them, too.

After solving the optimization problem (21), the optimum topology, the obtained optimum cross-sectional areas, and the internal forces of members are used to accomplish the following six modifications (Sections 6.1-6.6). With these adaptations, ACO effectively results in the optimum topology such that all of the stress, displacement, slenderness ratio, and reliability constraints are satisfied, and the cross-sectional areas are selected from discrete quantities (i.e. solving Eqs. (9) and (16)).

### 6.1. Using the obtained topology as a new ground structure

The optimization problem (21) is solved to achieve a better solution for (9) and (16). In this paper, during the optimization procedure of (21) using SIMP, some of the redundant elements are removed. Therefore, if the number of removed elements are controlled, the obtained topology can be a good initial configuration for solving (9) and (16). To achieve this aim, the obtained topology of (21) is used as a new ground structure. In the new ground structure there are lower elements than the initial one, therefore, the search space is reduced. It is noted that since the optimum topology problem (9) is different from (16), and because of the existing of different deterministic and reliability constraints during the solution of (21), the dissimilar termination conditions must be chosen.



6.2. *Size optimization of the new ground structure to be considered as a good initial design for ACO*

In initial optimization stages of large scale skeletal structures, there is not any relation between topology and cross-sectional area variables which is a challenging issue in the topology optimization process. To improve this deficiency, the optimum cross-sectional areas are determined for the new ground structure obtained in the previous section, while the constraints of (9) or (16) are satisfied using ACO. Then, the optimum structure is considered as an initial design when the optimization problem (9) or (16) is solved.

6.3. *Calculating the pheromones of the topology variables [20]*

The rate importance of the  $i$ th group elements ( $IR_i$ ) is calculated as follows:

$$IR_i = \frac{MA_i}{MA_{max}}, MA_{max} = \max(MA_i), i = 1, \dots, nbwe \quad (22)$$

where  $MA_i$  is the cross-sectional area of a member of the  $i$ th group and  $nbwe$  is the number of bottom and web grids elements.

Thereafter, the pheromone of the  $i$ th topology variable is identified as follows:

$$\begin{aligned} \tau(i, 1) &= \max[\tau_{min}, \tau_0(1 - IR_i)] \\ \tau(i, 2) &= \max[\tau_{min}, \tau_0(IR_i)] \end{aligned} \quad (23)$$

where  $\tau(i,1)$  and  $\tau(i,2)$  are the pheromone of the absence and presence of  $i$ th group elements, respectively. Therefore, more  $IR_i$  means more presence pheromone of the  $i$ th group elements and the probability of existing of such elements in the structure increases.  $\tau_{min}$  is the minimum pheromone which is determined as follows [25]:

$$\tau_{min} = \tau_0 \left( \frac{\tau_1}{\tau_2} \right) \quad (24)$$

where

$$\begin{aligned} \tau_1 &= (1 - 0.05^{(1/NDV)}) \\ \tau_2 &= \left[ (-1 + \frac{NDV}{2}) \times 0.05^{(\frac{1}{NDV})} \right] \end{aligned} \quad (25)$$

6.4. *Enhancing the generation of new random stable structures [20]*

Numerous unstable structures are produced in topology optimization procedure of double layer grids using ACO and SIMP-ACO. In this article, after finding each unstable structure, a new structure is randomly constructed and then its stability is checked. This process is continued until a stable structure is obtained. To increase the efficiency of SIMP-ACO for producing a new structure,  $i$ th group elements will be deleted randomly from the new ground structure if the following relationship is satisfied:

$$IR_i \leq IR_{min} \quad (26)$$

where  $IR_{min}$  is the minimum importance rate. Selecting a small amount of  $IR_{min}$  removes a few number of elements. Also, if  $IR_{min}$  is selected a value near to one, large number of elements are subjected to be deleted randomly. Therefore, the removing chance of elements which have large and small amount of  $IR$ , will be equal. So, the amount of  $IR_{min}$  should be chosen such that the proper number of elements be deleted by chance [19].

#### 6.5. Identifying the number of compressive and tensile member types [20]

After solving the optimization problem (21), the member forces are found. With consideration of the stress and slenderness ratio constraints in design stage, the proper cross-sectional area is chosen from the available profiles for any member with tensile and compressive internal force. After designing all of the members, those that have the same cross-sectional area are located in the same type with respect to the compressive and tensile internal force. Then, computing the various cross-sectional areas determines the number of compressive and tensile element types.

#### 6.6. Shortening the available list profile for elements of each type [20]

The assigned profile to each element in Section 6.5 is based on the optimization problem (21) which has only two constraints. When in the optimization problem there are several deterministic and reliability constraints, greater cross-sectional area should be probably assigned to the elements. Therefore, the allocated cross-sectional area to the elements in Section 6.5 can be considered approximately as the lowest limit of the available profile. To have more security, the following relationship is suggested to the lowest limit of available profile for  $i$ th element types ( $lb_i$ ):

$$lb_i = dp_i - 0.1(NP), \quad lb_i \geq 1, \quad i = 1, 2, \dots, NMG \quad (27)$$

where  $dp_i$  is the digit of the profile for the  $i$ th element types which is calculated in section 6.5.

The process of SIMP-ACO method is schematically shown in Figure 1.

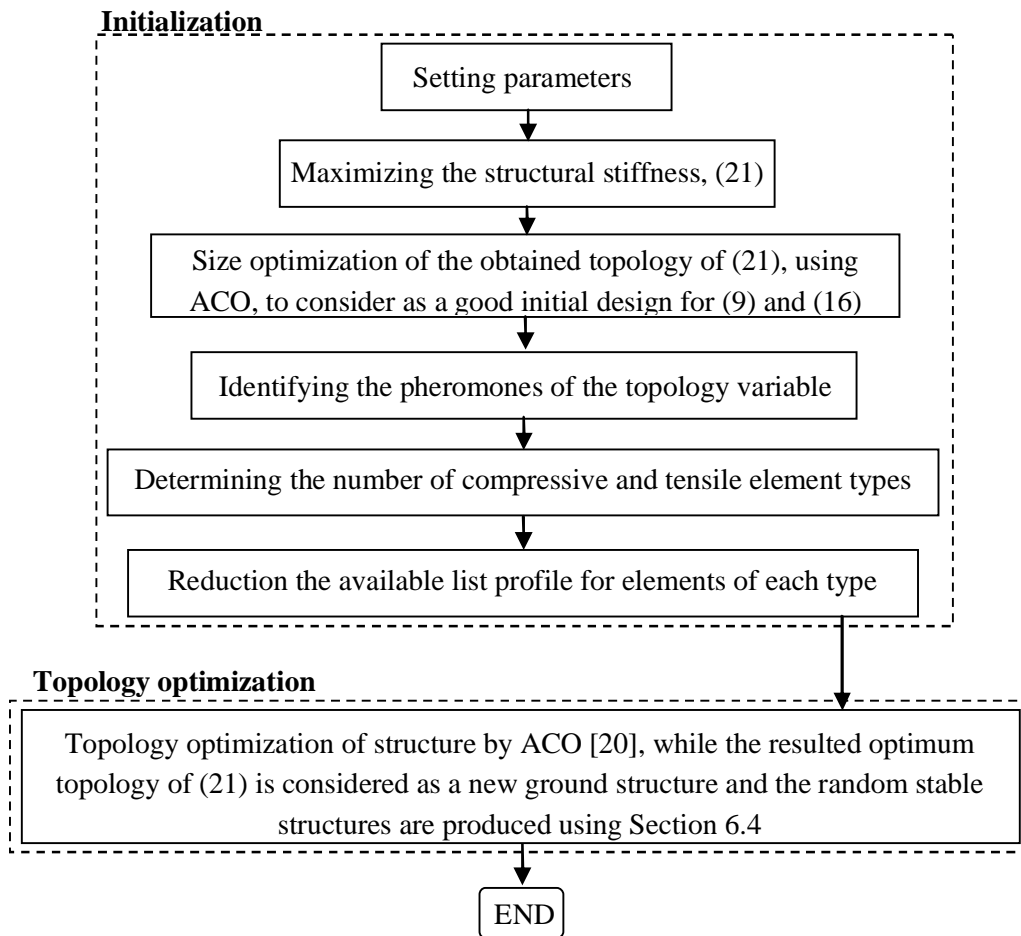


Figure 1. The process of SIMP-ACO method

It is noted that the amendment described in Section 6.3 is employed specially for ACO. While the optimization problem of (9) or (16) is solved using other heuristic optimization methods such as GA and PSO, the other general improvements which are described in Sections 6 (except Section 6.3), can be used to enhance their solutions.

### 7. EXAMPLE: 20×20 DOUBLE LAYER GRID

A square-on-square double layer grid with 841 nodes (joints) and 3200 members (elements) is presented to examine and verify the proposed optimization method. In [19], the presence or absence of the bottom joints is considered as topology variables, but in this article, the presence or absence of the elements is considered as topology variables. Therefore, the bottom and web elements are tabulated in 310 different groups ( $NTV=310$ ) instead of  $NTV=55$  in [19]. The depth of the double layer grid is 450 cm and the node spacing in the top and bottom chord is 300 cm. The ground structure is assumed to be supported at perimeter nodes of bottom grid (Figure 2) [19].

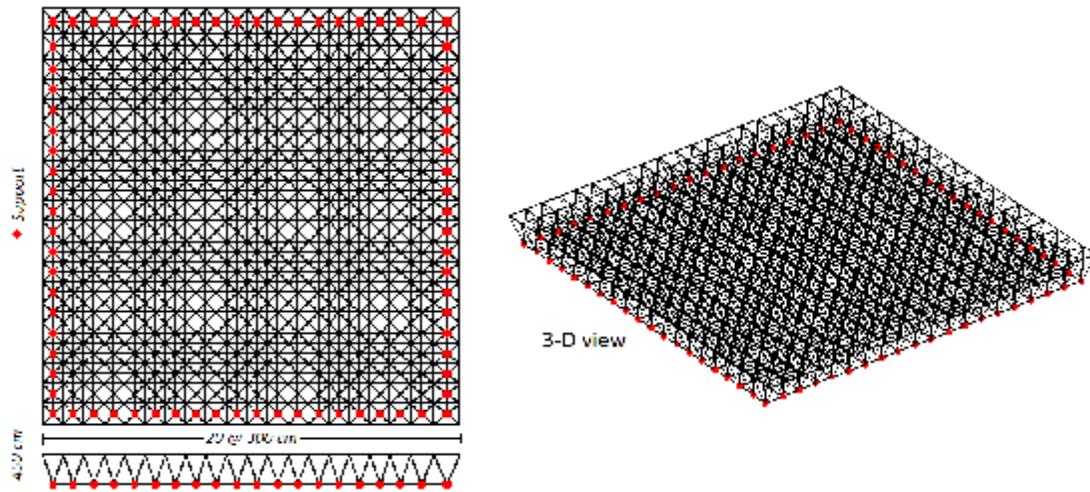


Figure 2. A 20×20 Double layer grid [19]

The assumed material is steel with a Young's modulus and material density as  $2.1 \times 10^6 \text{ kg/cm}^2$  and  $7850 \text{ kg/m}^3$ , respectively. These material constants are assumed to have deterministic values. Gravity acceleration is considered as  $981 \text{ cm/s}^2$ . The random variable is assumed to be normally distributed. The mean and the standard deviation of distributed applied load on double layer grid are  $1765.8 \text{ N/m}^2$  and  $176.58 \text{ N/m}^2$ , respectively.

The cross-sectional area of members is selected from the pipe profiles available in Table 1, where  $OD$  is the outer diameter and  $T$  is the thickness in centimeter [19]. The number of ants,  $\gamma$ ,  $\rho$ , and  $\theta$  are selected as 100, 10, 0.5, and 0.67, respectively [26]. The computational results show that the other specifications of ACO and SIMP-ACO which are shown in Table 2 are good choices.

Table 1. Available pipe profiles [19]

No.	OD	T	No.	OD	T	No.	OD	T	No.	OD	T
1	4.83	0.26	6	10.80	0.36	11	16.86	0.45	16	32.39	0.71
2	6.03	0.29	7	11.43	0.36	12	19.37	0.45	17	35.56	0.80
3	7.61	0.29	8	13.30	0.40	13	21.91	0.45	18	40.64	0.88
4	8.89	0.32	9	13.97	0.40	14	24.45	0.63	19	45.72	1.00
5	10.16	0.36	10	15.90	0.45	15	27.30	0.63			

Table 2. Specifications of ACO and SIMP-ACO methods

Parameter	$\alpha$	$\beta$	$IR_{min}$
Value	1	0.2	0.18

Bendsoe and Sigmund [27] have proved that the physical microstructures of SIMP scheme would exist with the satisfaction of some simple conditions (e.g. penalty exponent  $p \geq 3$  for Poisson's ratio  $1/3$ ). A reasonable penalty exponent should be necessarily appointed [11].

In [12], the proper amounts of  $f$  and  $p$  are determined as 0.5 and 3, respectively; for topology optimization of continuous structures the Poisson's ratio is considered as 0.3. In the present paper, the proper amounts of these parameters are chosen so that before the structural instability, the most number of elements are removed from the structure. To achieve this aim, the optimization problem (21) is solved considering 3 and 41 different values for  $f$  and  $p$ , respectively, with the same increasing steps, where the results are shown in Figure 3.

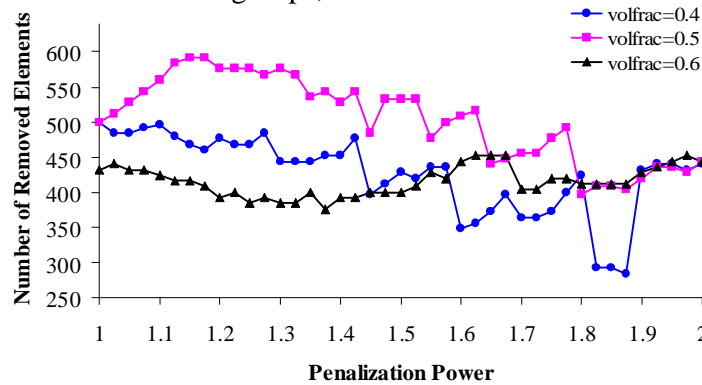


Figure 3. The effect of  $f$  and  $p$  on the number of removed elements

Figure 3 indicates that the most number of elements (592 elements) are removed when  $f$  is equal to 0.5 and  $p$  is considered as 1.150 or 1.175. It must be noted that when  $p$  is selected as 1.150 or 1.175, the optimization problem (21) is terminated in 19 or 20 iterations, respectively, using SIMP. Because of faster convergence in the second case, in this paper, these two values ( $f=0.5$  and  $p=1.175$ ) are selected to solve (21) using SIMP. The number of removed elements in the optimization procedure, the convergence history of the objective function, and the optimum resulted topology are shown in Figures (4) to (6), respectively.

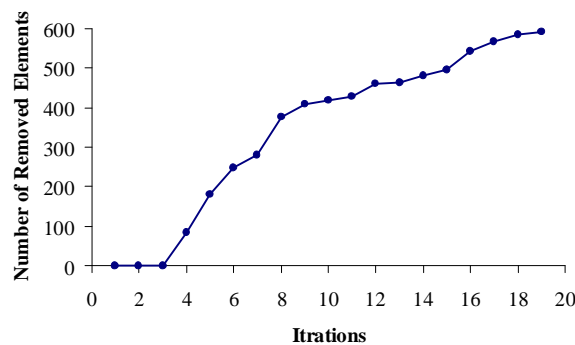


Figure 4. The number of removed elements in each iteration

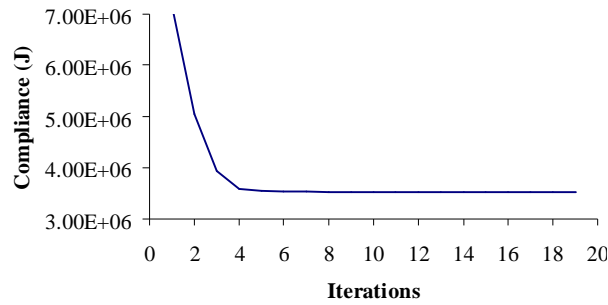


Figure 5. The convergence history of the objective function

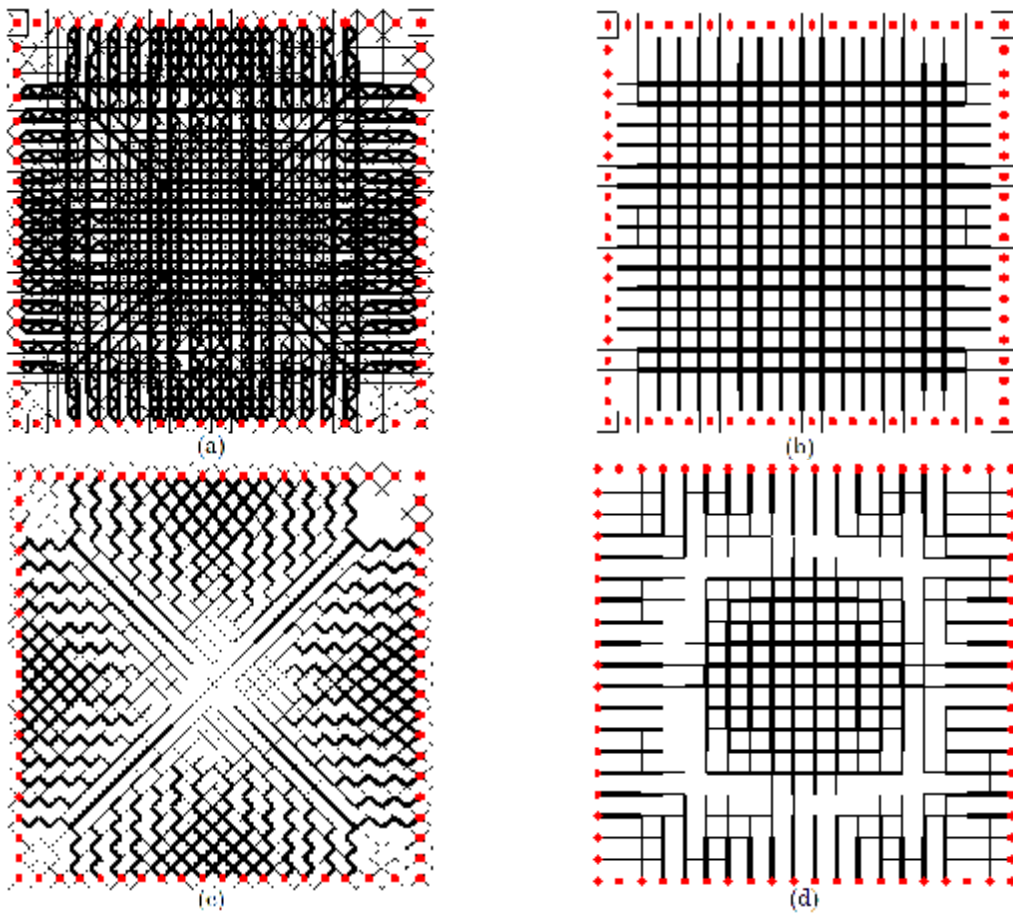


Figure 6. The optimal solution of SIMP: (a) double layer grid, (b) top layer, (c) diagonal layer, and (d) bottom layer

In this paper, the optimum topology shown in Figure 6 is used as the ground structure to solve (9) which has only deterministic constraints. But in this structure, the removed elements are so many that in final optimization steps, despite the increase of the structural stiffness, the dynamic criterion (6) is decreased (see Figure 7).

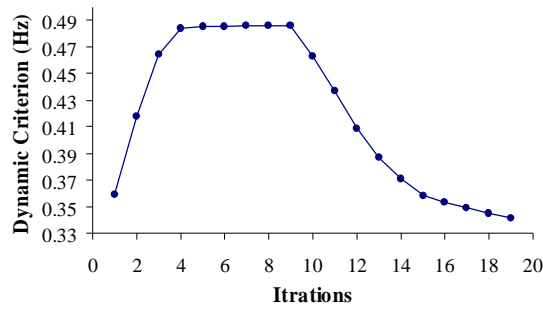


Figure 7. The dynamic criterion in each iteration

Therefore, for solving (16) which has frequency constraint, too, the optimum topology obtained in the 9<sup>th</sup> iteration, that is the structure with the most frequency, is used as the ground structure from which 408 elements are removed (Figure 8) and is convenient for using instead of the previous one (Figure 6).

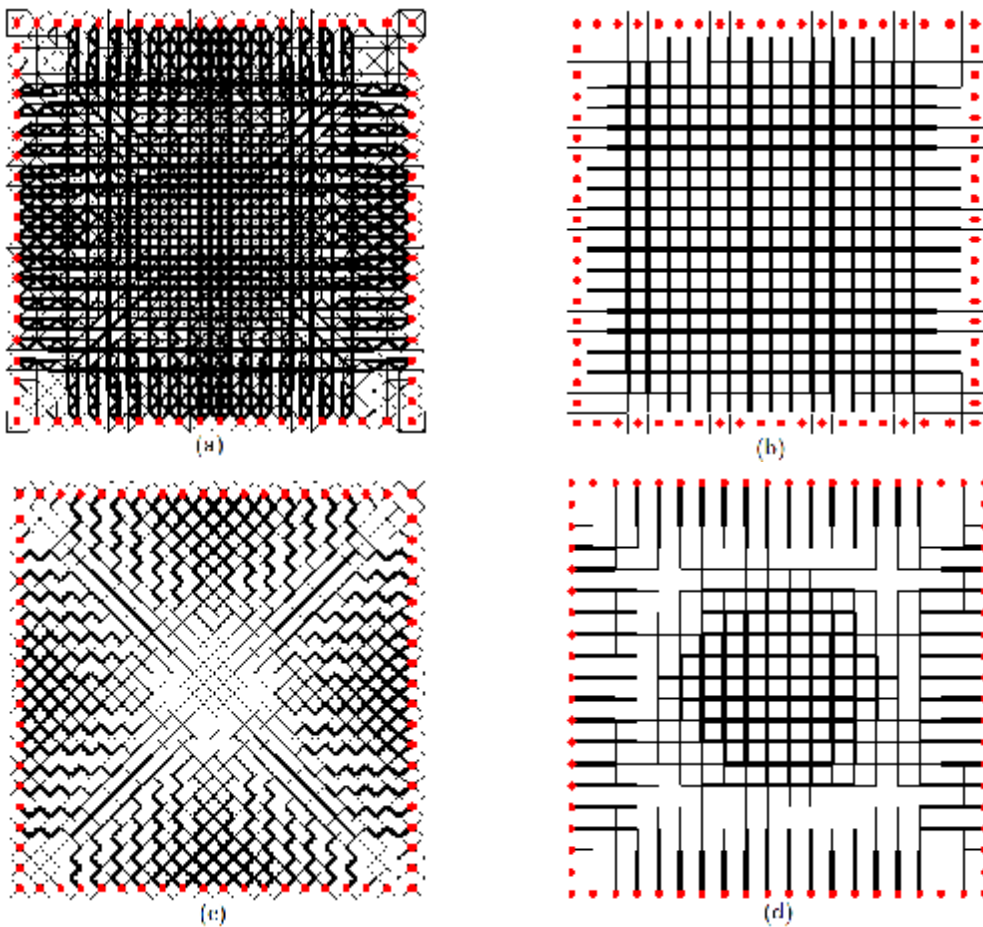


Figure 8. The optimal solution of SIMP: (a) double layer grid, (b) top layer, (c) diagonal layer, and (d) bottom layer

After solving the optimization problem (21), the number of member types is obtained equal to 4 for tensile members and 12 for compressive members (Section 6.5).

For better comparison of the two obtained topologies shown in Figures 6 and 8, the optimum cross-sectional areas are found using ACO and satisfying the constraints (9) and (16). The optimum weights are shown in Table 3 where the minimum weights are highlighted.

Table 3. The optimum weights of the two obtained topologies shown in Figures 6 and 8

Optimization Problem	Weight (kg)		
	Figure 6	Figure 8	Optimum Ground Structure (Figure 2) [19]
Deterministic Topology Optimization	84332	89734	97335
Reliability-based Topology Optimization	134971	119781	125698

The noticeable point is that the optimum weights of SIMP's topologies, which are obtained with a few numbers of structural analyses, are much lower than that of the optimum ground structures in [19]. The other point is that when the optimization problem (16) is solved, the weight of the structure shown in Figure 6 is more than that of the ground structure in [19]. This issue shows that in this structure some necessary elements are removed, too. Therefore, in SIMP-ACO method, the shown topologies in Figures 6 and 8 are used to solve the optimization problems (9) and (16), respectively.

The double layer grid shown in Figure 7 is optimized in two cases as follows:

*Case 1:* Topology optimization using SIMP-ACO.

*Case 2:* Topology optimization using ACO.

The members are grouped for reduction of the search space. To achieve this aim, a preliminary static analysis is first carried out in which all of the members have the same cross-sectional area. Then, the entire range of axial forces is divided into several equal ranges for both the tension and compression members. Each member of the double layer grid is placed into different groups according to its quantity of axial force. It must be noted that the number of these equal ranges for tension and compression members are determined in Section 6.5.

To consider the stochastic nature of the ACO and SIMP-ACO approaches, five sample optimization runs are performed for each design case and the achieved optimal solutions for Cases 1 and 2 are presented. In Figures (9) to (12), (a) is double layer grid, (b) is top layer, (c) is diagonal layer, and (d) is bottom layer. Furthermore, in these figures the thickness of each element is proportional to its cross-sectional area.

### 7.1. Deterministic topology optimization

In Cases 1 and 2 the optimum structures are shown in Figures (9) and (10). The optimum weights of these structures are obtained as 79580 kg and 84327 kg, respectively.



7.2. Reliability-based topology optimization

The optimum structures in Cases 1 and 2 are shown in Figures (11) and (12). The optimum weights of these structures are obtained as 113398 kg and 117920 kg, respectively.

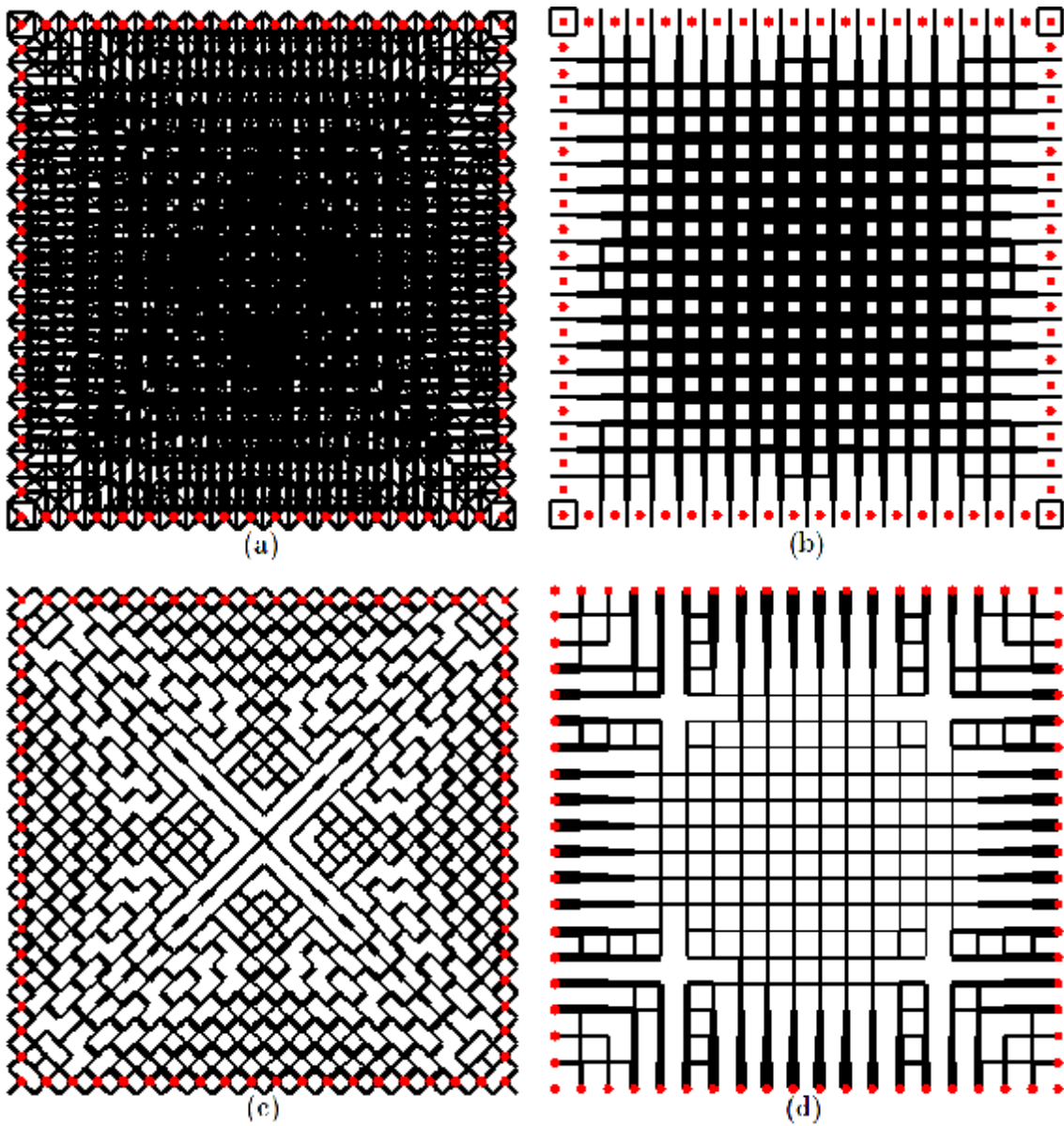


Figure 9. Optimum topology in Case 1

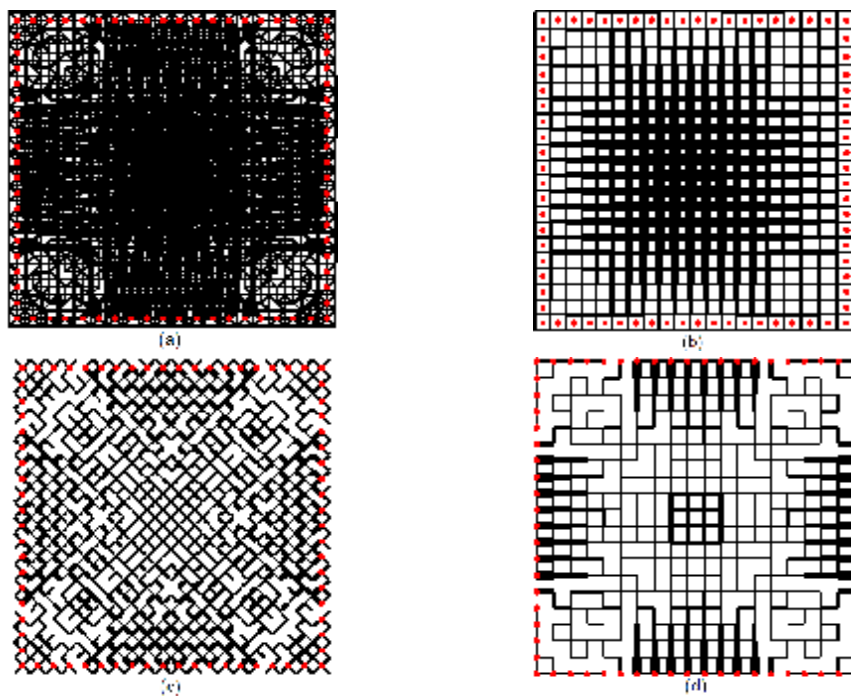


Figure 10. Optimum topology in Case 2

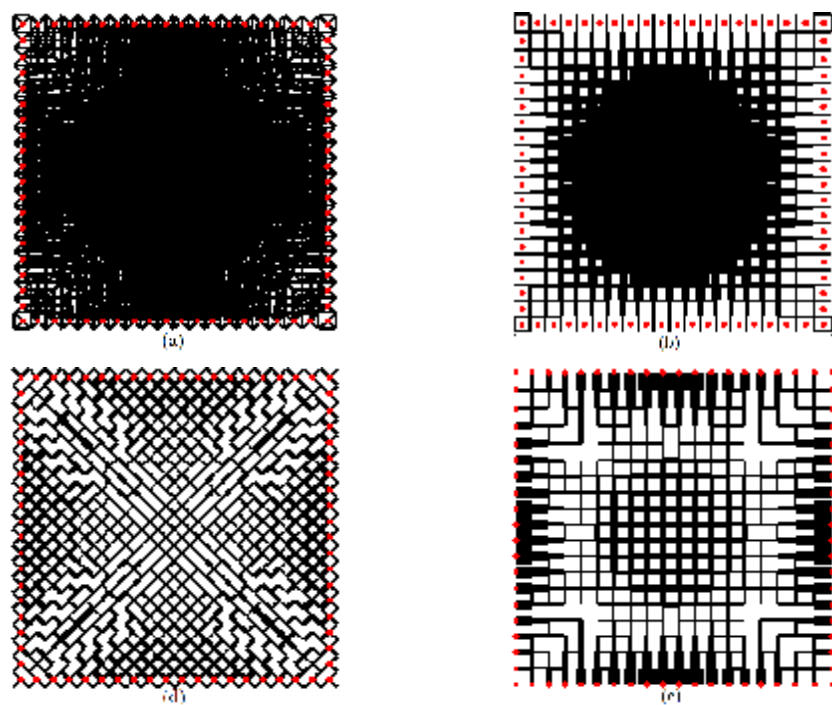


Figure 11. Optimum topology in Case 1

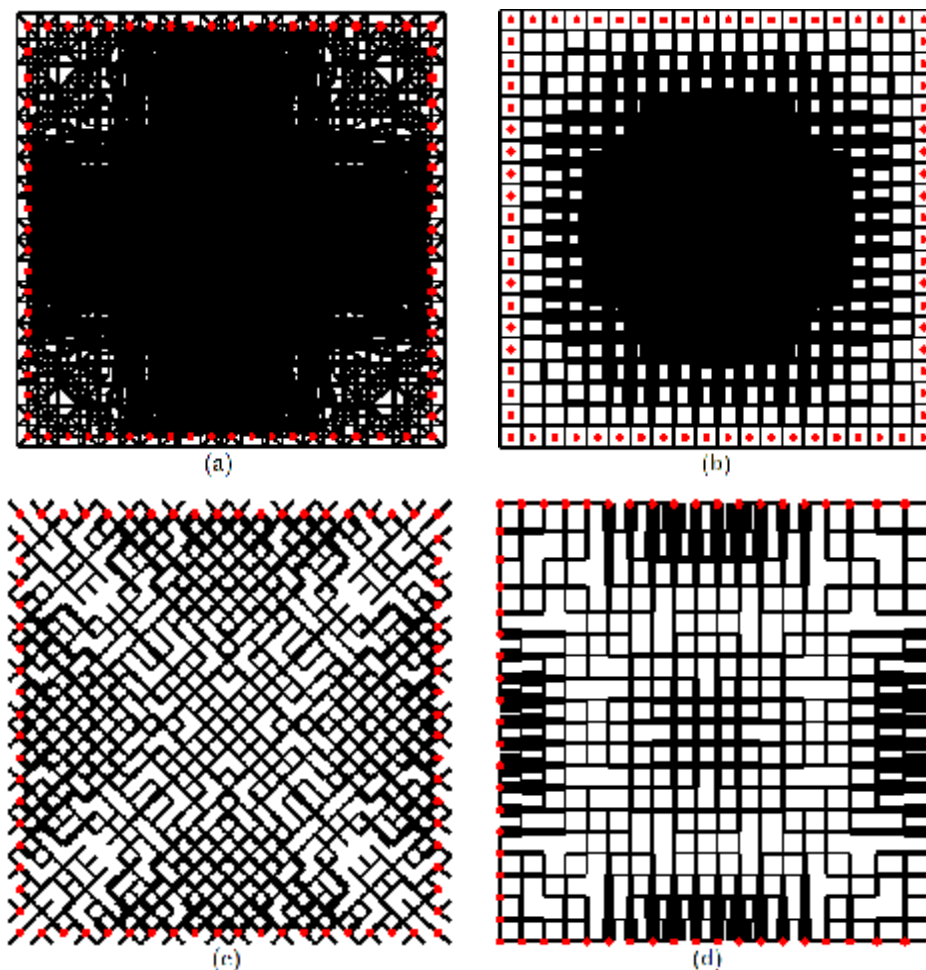


Figure 12. Optimum topology in Case 2

The minimum weights of all cases are listed in Table 4 to be compared with each other.

Table 4. Comparison of the optimum weights of the structures of Cases 1 and 2

Optimization Problem	Optimum Weight (kg)		
	Case 1	Case 2	The Optimum Topology [19]
DTO	79580	84327	81927
RBTO	113398	117920	115581

For a better comparison of ACO and SIMP-ACO, and to achieve the optimum topologies, optimum weights of the topologies attained for Cases 1 and 2 are listed in Table 5 for five sample runs where the weight of topologies shown in Figures (9) to (12) is highlighted.

Table 5. Optimum weights of the topologies attained for Cases 1 and 2 for five sample runs

Case No.	Optimization Problem	Optimum Weight (kg)				
		Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
1	DTO	79580	80653	80126	80286	80541
	RBTO	115802	114933	113398	116324	114010
2	DTO	84327	90270	87403	89213	93772
	RBTO	123050	119300	120509	119200	117920

The mean ( $\bar{W}$ ) and the standard deviation ( $SD$ ) of these optimum weights are listed in Table 6.

Table 6. The mean and the standard deviation of the optimum weights

Optimization Problem	Case No.	(kg) $\bar{W}$	(kg) $SD$
DTO	1	80237	422
	2	88997	3493
RBTO	1	114893	1214
	2	119996	1938

The statistical values of Table 6 demonstrate that the SIMP-ACO in topology optimization of double layer grids performs better than the ACO.

## 8. CONCLUSIONS

In this paper, a two-stage optimization method (SIMP-ACO) has been proposed for RBTO of double layer grids that uses ground structure approach. In optimization process of SIMP-ACO, the weight of the structure is minimized under deterministic and indeterministic constraints.

For implementation of SIMP-ACO, the location of members with high structural significance can be first recognized. To achieve this aim, the solution of the topology optimization problem was found using SIMP. Then, the outcomes of SIMP (optimum topology, optimum cross-sectional areas, and the internal forces of members) were used to improve ACO through six modifications.

In this paper, it is shown that the optimum weights of SIMP's topologies, which are obtained with a few numbers of structural analyses, are much lower than that of the optimum ground structures. To use this interesting issue, the SIMP's obtained topology is used to attain a better topology with considering various constraints using a heuristic method (ACO). To achieve this aim, a strategy is proposed to control the number of removed elements in SIMP's

optimization process.

The proposed method was applied for topology optimization of a double layer grid and the optimization was implemented in two cases which their results are as follows: (a) SIMP-ACO method obtains the optimum topologies with lower weight than those of optimum topologies attained by ACO with consideration of various constraints, (b) with respect to the ACO method, SIMP-ACO has better solutions and minimum standard deviations. Therefore, SIMP-ACO approach is more reliable than ACO, and (c) considering the presence or absence of the members causes that double layer grids satisfy the mentioned deterministic and reliability constraints with better weight than that of the presence or absence of the joints.

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