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AN EXTENSION TO STOCHASTIC TIME-COST TRADE-OFF PROBLEM OPTIMIZATION WITH DISCOUNTED CASH FLOW

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ABSTRACT

In this paper, an efficient multi-objective model is proposed to solve time-cost trade off problem considering cash flows. The proposed multi-objective meta-heuristic is based on Ant colony optimization and is called Non Dominated Archiving Ant Colony Optimization (NAACO). The significant feature of this work is consideration of uncertainties in time, cost and more importantly interest rate. A fuzzy approach is adopted to account for uncertainties. Mathematics of cash-flow analysis in a fuzzy environment is described. A case study is done using the proposed approach and the decision maker's options to handle the uncertainties are investigated and discussed.

KEYWORDS: Multi-objective optimization; time-cost trade off; cash-flow analysis; naaco; uncertainty; fuzzy sets theory

1. INTRODUCTION

One of the managerial issues for project scheduling is to decide upon the resources, processes, or technologies needed for operating tasks. In recent decades construction industry has witnessed drastic technological and operational improvements providing managers with more options to choose from. Inductively, when using advanced technology or allocating more resources to an activity, the duration of the activity could be shortened with possible increase in direct cost. As a result, there is a trade-off between time and cost of an activity with regard to different operational modes. This has led to the emergence of the Time-Cost Trade off Problem (TCTP) in the construction management domain. A detailed literature review on the TCTP may be found in Kalhor et al. [1]

In any construction project a realistic schedule should encompass the time value of money. The idea of considering net present value of money in scheduling was first introduced by

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Russell [2]. In his so-called Payment Scheduling Problem (PSP) he used Taylor expansion of net present value in connection with a linear programming (LP) model. Grinold [3] also used a linear programming approach to solve PSP using a weighted distribution problem. Erengus et al. [4] used a mixed integer nonlinear programming to solve PSP. Icmeli and Erenguc [5] suggested a heuristic procedure to maximize net present value of money.

While mathematical approaches are accurate and heuristic procedures are simple to follow, both methods are likely to show an exponential worst case complexity when the size of the problem increases (De et. al. [6]). Meta heuristic approaches, on the other hand, have shown a good capability to handle NP-hardness of problems like TCTP and PSP. Such algorithms search a vast solution space intelligently rather than completely, however they do not guarantee the optimal results. A variety of Meta heuristics are applied to address TCTP and PSP, including GAs ([7], [8], [9]), PSO ([10]), and ACO ([11], [1]).

So far, most of the researches carried out on the scheduling problems neglect the uncertainty of the project parameters such as activity duration, activity cost, and interest rate. However, a real construction project is governed by environmental, geotechnical, political, psychological and economical factors, which introduce uncertainty into the aforementioned projects' parameters. In order to address uncertainties Feng et al. [12] allocated a probability distributed function to activity cost and time and adopted the probability theory.

Bonnal et al. [13] argues that despite the indisputable power of probability theory in modeling statistical uncertainties, two basic assumptions of this theory make it less practical for modeling uncertainties in construction projects. First, one should have knowledge about all the possible discrete events associated with an uncertain phenomenon, and second, they should know the probability of occurance of each phenomenon so that their accumulated probability is 100%. On the other hand, uniqueness is one of the well perceived features of construction projects. It is safe to say that the [1] mission of studying and quantifying all the events associated with above mentioned governing factors dates back to centuries ago and no one know when it is the time to put an end to it.

Fuzzy sets theory, introduced by Zadeh [14] has provided researchers with a tool to approach their goal with the flickering light of information. Eshtehardian et al. [15] and Kalhor et al [1] successfully used fuzzy mathematics for time cost trade off optimization; Afshar et al. [16] also adopted fuzzy approach to address uncertainties in finance based scheduling problem.

In this study, as an extension to previous work done by Aladini et. al. [17], a Non-dominated Archiving Ant Colony Optimization (NAACO) approach is adopted to solve stochastic multi-objective TCTP with discounted cash flow. Uncertainties in activity duration and cost as well as uncertainties in interest rate are addressed by means of fuzzy mathematics. This study fully applies α cut approach to account for decision maker's attitude towards risks.

2. MODEL FORMULATION

2.1. Deterministic TCTP with disocunted cash flow

The deterministic TCTP with discounted cash flow can be defined as selecting an array of activity modes so as to minimize both time and discounted cash of the project. Mathematically, there is a set of activities denoted by i $\{i \in A\}$ and corresponding to each activity there is a set of implementation modes denoted by j {j$\in$ M (i)}. t_{ij} and c_{ij} are time and cost of activity i when it is implemented by mode j. Total project duration (T) and discounted cash flow (DC) shall be minimized as defined in equations (1) and (2):

$$\min(T) = \max(T_p) = \sum_{i\in A_p}\sum_{j=1}^{n_{M(i)}} x_{ij}t_{ij} \tag{1}$$

$$\min(DC) = \sum_{m=s}^{f}\frac{C_m}{(1+r)^m} = \sum_{m=s}^{f}\frac{\sum_{i\in A_m}\sum_{j=1}^{n_{M(j)}} x_{ij}c_{ij}\,pe_{i,m} + O_m}{(1+r)^m} \tag{2}$$

Where:

T_p : Duration of path p in the activity network

A_p : Set of activities on path p in the activity network

$n_{M(i)}$: Number of feasible operation modes of activity i

x_{ij} : Zero-one variable where $x_{ij} = \begin{cases} 1 & \text{if activity i is impelemented by mode j} \\ 0 & \text{otherwise} \end{cases}$

s and f : Start and finish months of project (usually s =1)

C_m : Total cost paid in m[th] month

r : interest rate

A_m : Set of activities which are entirely or partially pending in m[th] month

$pe_{i,m}$: Percent of duration of activity i pending in m[th] month

O_m : Overhead cost of month m

An answer for the bi-objective TCTP whith discounted cash flow yields a set of non-dominated solutions. A solution is dominated when there is another solution which yields lesser duration with the same discounted cash, or lesser discounted cash with same duration or both lesser duration and discounted cash. The detailed description of the problem can be found in Aladini et al.[17]

2.2. stochastic TCTP with disocunted cash flow

In this article, time and cost of activities as well as interest rate and their functions are dealt with as fuzzy numbers, the stochastic TCTP-DCF may be represented as (3):

$$\begin{gathered}\min(\tilde{T}) = \max(\tilde{T}_p) = \max\Big(\sum_{i\in A_p}\sum_{j=1}^{n_{M(i)}} x_{ij}\tilde{t}_{ij}\Big) \\ \min(\tilde{DC}) = \sum_{m=s}^{f}\frac{\tilde{C}_m}{(1+\tilde{r})^m} = \sum_{m=s}^{f}\frac{\sum_{i\in A_m}\sum_{j=1}^{n_{M(j)}} x_{ij}\tilde{c}_{ij}\,\tilde{pe}_{i,m} +}{(1+\tilde{r})^m}\end{gathered} \tag{3}$$

Where superscripted ~ symbolizes fuzzy numbers. Fuzzy numbers describe the degree to which a real number (or an object) belongs to a set. Correspondingly, a fuzzy number A is shown as (4)

$$\tilde{A} = \{(c, m_A(c)) \mid c \in C\} \tag{4}$$

In which c is a real number in the global set C and $m_A(c)\,(m \in [0,1])$ is the degree to which c belongs to set A. Graphically, most of the fuzzy numbers are triangular, rectangular, trapezoidal, Gaussians etc. For example, a triangular fuzzy number for cost of activity i is shown in Figure 1. As shown in Figure 1 L_2 belongs to low cost of activity i with the highest membership degree ($m = 1$) while it does not belong to medium cost of activity i. With no loss of generality, in this paper, only medium range of cost, duration and interest rate is considered and the numbers are graphically shown by a single triangle.

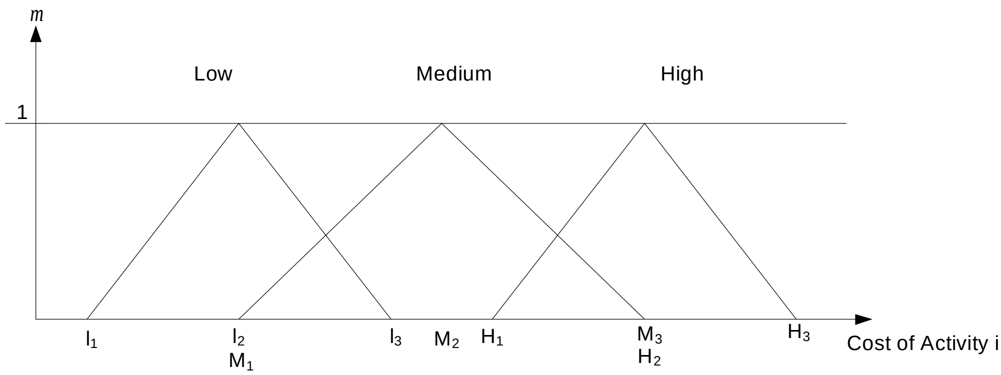

Figure 1. Fuzzy cost

In order to account for the decision maker's attitude towards risk, the concept of $a-cut$ is introduced. $a-cut$ set in a fuzzy set A in the universal set X is a crisp set denoted as A_a that consists of all elements x in X, whose membership degrees are greater than or equal to a . a it is mathematically shown as (5):

$$A_a = \{x \mid m_{\tilde{A}(x)} \geq a\,, a \in [0,1]\} \tag{5}$$

By choosing smaller $a-cut$, a decision maker shows that she is eager to consider a wider range of possibilities and vice versa. In other words, a value is a representation of decision maker's risk aversion/adoption. Guerra et. al. [18] used $a-cut$ setting to describe a fuzzy number mathematically and proposed fuzzy extensions of uni-variate functions. Basic fuzzy arithmatics are referred to [18] . The fuzzy Power functions applied in this model is presented as (6).

Assume that a continuous fuzzy number is a pair of functions u^- and u^+ ($u^{\pm} : [0,1] \rightarrow \Re$) which:

i. $u^- : a \rightarrow u_a^- \in \Re$ is bounded monotonic increasing (non decreasing) continuous function $\forall a \in [0,1]$;

ii. $u^+ : a \rightarrow u_a^+ \in \Re$ is bounded monotonic decreasing (non increasing) continuous function $\forall a \in [0,1]$;

iii. $u_a^- \leq u_a^+ \forall a \in [0,1]$ if $u_1^- \leq u_1^+$ we have a fuzzy interval and if $u_1^- = u_1^+$ we have a fuzzy number.

u^- and u^+ are lower and upper branches of $\tilde{u}$ respectively, and u_a is shown as $[u_a^-, u_a^+]$

When both u^- and u^+ are differentiable, one can represent fuzzy number $\tilde{u}$ as $(u_i^-, du_i^-, u_i^+, du_i^+)$ and simply $(u_i^-, d_i^-, u_i^+, d_i^+)$ if $u_0^- \leq u_1^- \leq ... \leq u_N^- \leq u_N^+ \leq u_{N-1}^+ \leq ... \leq u_0^+$ and the slopes $d_i^- \geq 0, d_i^+ \leq 0$ (as is true for triangular fuzzy numbers) then for three fuzzy numbers $\tilde{u}$ $(u_i^-, d_i^-, u_i^+, d_i^+)$, $\tilde{v}$ $(v_i^-, e_i^-, v_i^+, e_i^+)$ and $\tilde{z}$ $(z_i^-, f_i^-, z_i^+, f_i^+)$:

$$\tilde{v} = \tilde{u}^r \quad (r \in \Re)$$
$$\tilde{v} = \exp(r \ln(u))$$
$$\tilde{z} = \ln(\tilde{u}) \Rightarrow \begin{cases} z_i^- = \ln(u_i^-) \\ z_i^+ = \ln(u_i^+) \\ \mathrm{f}_i^- = \dfrac{d_i^-}{u_i^-} \\ \mathrm{f}_i^+ = \dfrac{d_i^+}{u_i^+} \end{cases} \tag{6}$$
$$\tilde{z} = \exp(\tilde{u}) \Rightarrow \begin{cases} z_i^- = \exp(u_i^-) \\ z_i^+ = \exp(u_i^+) \\ \mathrm{f}_i^- = (\exp(u_i^-))(d_i^-) \\ \mathrm{f}_i^+ = (\exp(u_i^+))(d_i^+) \end{cases}$$

In order to compare two fuzzy numbers various approaches are proposed. In this paper, left and right dominance approach is adopted. The left and right dominances of fuzzy number $\tilde{u}$ over $\tilde{v}$ are (7)

$$D_{\tilde{u},\tilde{v}}^L = \frac{1}{n+1} \sum_{i=0}^{n} (u_i^- - v_i^-)$$
$$D_{\tilde{u},\tilde{v}}^R = \frac{1}{n+1} \sum_{k=0}^{n} (u_i^+ - v_i^+) \tag{7}$$

In other words, left and right dominance are the average difference of the left and right spreads of two fuzzy numbers at some $a - cut$ s. overall dominance of fuzzy number u over v is denoted as $D_{u,v}(b)$ (8).

$$\begin{aligned}
&D_{\tilde{u},\tilde{v}}(b) = (1-b)D^{R}_{\tilde{u},\tilde{v}} + bD^{L}_{\tilde{u},\tilde{v}} \\
&if\ D_{\tilde{u},\tilde{v}}(b) > 0,\ then\ \tilde{u} > \tilde{v} \\
&f\ D_{\tilde{u},\tilde{v}}(b) = 0,\ then\ \tilde{u} = \tilde{v} \\
&f\ D_{\tilde{u},\tilde{v}}(b) < 0,\ then\ \tilde{u} < \tilde{v}
\end{aligned} \tag{8}$$

Where b is the index of optimism and belongs to [0, 1]. In a minimization problem, greater b values reflect that the decision maker is more optimistic and she pays more attention to possibilities of lesser values. In order to distinguish fuzzy parameters α and b from ACO similar parameters in the rest of this paper a_f and b_f are used for fuzzy parameters.

3. NON-DOMINATED ARCHIVING ANT COLONY OPTIMIZATION

Ant colony optimization (ACO) introduced by Dorigo [19] is inspired by the natural behavior of real ants. Artificial ants search a solution graph (including nodes and arcs) in a stepwise manner. At each step, a colony of ants select and travel on an arc from a set of feasible arcs and proceeds to the next step. When the final step is completed a solution is formed. Ants are attracted by the pheromone accumulated on paths, they are also reinforced by the heuristic information a planner or a resource other than ants gives to them.

Equations (9) to (11) represents the ACO rules (14-16).

$$t^{k}_{ij} \leftarrow t^{k-1}_{ij} \times (1-r) + \Delta t^{k}_{ij} \tag{9}$$

$$\Delta t^{k}_{ij} = \begin{cases} \dfrac{Q_k}{f^{k}(B)} & if\ \ path\, ij\ is\, traveled\, by\ k^{th}\ iteration's\ best\ ant \\ 0 & otherwise \end{cases} \tag{10}$$

$$p^{k}_{ij} = \begin{cases} \dfrac{t^{a}_{ij} \times h^{b}_{ij}}{\sum_{j} t^{a}_{ij} \times h^{b}_{ij}} & if\ \ j\ \ is\ an\ allowabl\ end\ node \\ 0 & otherwise \end{cases} \tag{11}$$

Where p_{ij} is the probability that an ant chooses node j when standing at point i. t^{k}_{ij} is the pheromone accumulated on arc ij at k^{th} generation, Δt^{k}_{ij} is the pheromone laid on path ij at k^{th} generation, and r is the evaporation rate which is set into the algorithm in order to prevent stagnation. Q_k is a constant and $f^{k}(B)$ is the best fitness value in the k^{th} generation (iteration best) or the global best fitness value found until the k^{th} generation(global best). h_{ij} is the heuristic information about path ij.

In order to optimize two objectives concurrently, NAACO approach uses two colonies of ants exchanging their pheromone information on a shared solution graph. The concept of NAACO is shown in Figure 2. Similar to single objective ACO, several generations of ants explore a solution graph to find optimal results. Pheromone reinitation and probability of selecting a path in NAACO similarly follows Eqs. (9) to (11). However, ants are devided to two colonies. Each colony seeks its own objective. For the case at hand, a colony corresponds to finding the minimum duration and the other colony aims to minimize the project's discounted cashflow. Ants of each colony try to find the path which result in the best solution according to their own objective. In order to find non-dominated solutions ants should share information about both objectives. In NAACO, this is furnished by using a common solution graph for both colonies. Colonies do not communicate with each other, rather they interact via a shared solution space.

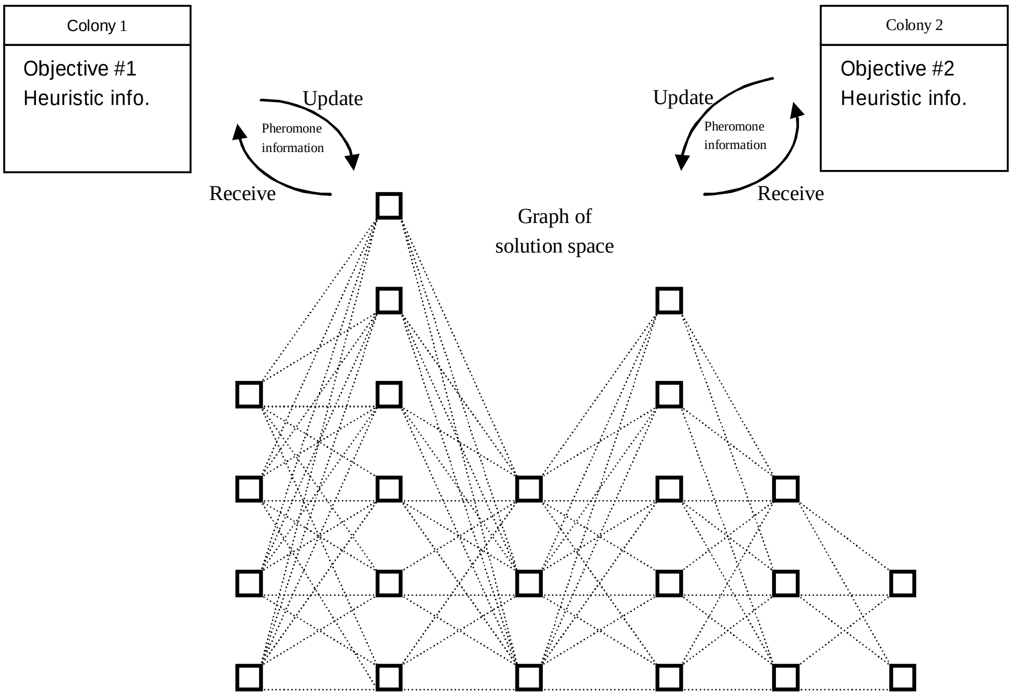

Figure 2. The concept of NAACO

Colonies explore the solution space separately. Receiving pheromone information on the solution graph, ants of a colony finds their solutions.Next, solutions are assessed based on the colony's objective. Corresponding to the best solution found, phermone information on paths are updated. The active colony stands by until the other colony go through a similar routine. By the end of the second colony's exploration, a cycle is completed.

All solutions resulted by both colonies are stored. Next, non-dominated solutions are extracted and archived while other solutions are discarded. The archive is updated each time a cycle is completed and any dominated solutions are deduced from the archive. In order to avoid stagnations, at some intervals pheromone on all paths are cleared and only paths regarding the non-dominated solutions in the archive receive pheromone; this is called

Pheromone Reinitiation. In addition, both iteration and global best solutions are used for phermone reinitiation as suggested by Dorigo and Stutzle [19] suggest. More detailed description of the proposed NAACO is refered to Kalhor et al. [1]

4. APPLICATION OF THE MODEL AND ANALYSIS OF THE RESULTS

An 18-activity project devised by Feng et al. [9] is solved using the proposed model. Network of the problem is shown in Figure 3. Table 1 shows 13 paths appearing on the network with corresponding activity set. The fuzzy input data regarding time and cost of activities are tabulated in Table 2. As noted earlier, the fuzzy numbers show only medium sets of cost and time for simplicity. The overhead cost and interest rate fuzzy numbers are also shown in Figure 4.

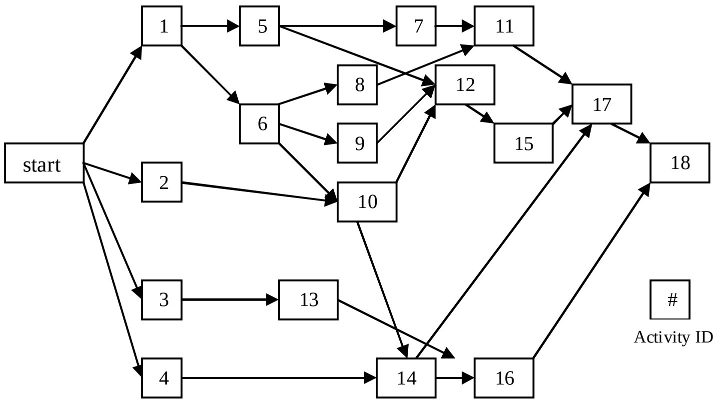

Figure 3. Example project's network

Table 1. Path list of case project's networks

path No.	Activities on Path						
1	1	5	12	15	17	18	
2	1	5	7	11	17	18	
3	1	6	10	12	15	17	18
4	1	6	9	12	15	17	18
5	1	6	10	14	17	18	
6	1	6	10	14	16	18	
7	1	6	8	11	17	18	
8	2	10	12	15	17	18	
9	2	10	14	17	18		
10	2	10	14	16	18		
11	3	13	16	18			
12	4	14	17	18			
13	4	14	16	18			

Table 2. Parameters of the example project

activity	option	time			cost			activity	option	time			cost		
1	1	10	14	19	2100	2400	2890	10	1	13	15	19	420	450	492
	2	11	15	20	1900	2150	2560		2	21	22	27	385	400	485
	3	13	16	20	1720	1900	2280		3	30	33	39	290	320	356
	4	16	21	24	1250	1500	2000	11	1	10	12	16	410	450	510
	5	19	24	31	985	1200	1750		2	14	16	20	313	350	395
2	1	12	15	21	2870	3000	3420		3	16	20	26	284	300	352
	2	16	18	23	2185	2400	2850	12	1	18	22	29	1850	2000	2450
	3	18	20	25	1650	1800	2255		2	19	24	30	1565	1750	2050
	4	20	23	28	1300	1500	1950		3	20	28	36	1325	1500	1880
	5	19	25	30	900	1000	1190		4	21	30	45	915	1000	1350
3	1	11	15	23	4250	4500	4990	13	1	12	14	18	3650	4000	4540
	2	17	22	29	3850	4000	4460		2	16	18	20	2970	3200	3385
	3	29	33	40	2985	3200	3560		3	23	24	26	1595	1800	2160
4	1	10	12	16	42050	45000	48800	14	1	7	9	11	2580	3000	3685
	2	13	16	21	38500	35000	39000		2	13	15	19	2200	2400	2880
	3	17	20	28	28500	30000	33550		3	16	18	23	2080	2200	2850
5	1	19	22	25	18500	20000	22550	15	1	10	12	15	4385	4500	4850
	2	22	24	27	16000	17500	19950		2	13	16	18	3200	3500	3750
	3	24	28	33	14150	15000	17050	16	1	18	20	23	2650	3000	3850
	4	29	30	34	8500	10000	12600		2	19	22	26	1850	2000	2480
6	1	12	14	17	38500	40000	42860		3	20	24	30	1340	1750	2240
	2	17	18	21	29800	32000	34550		4	22	28	34	1250	1500	1950
	3	21	24	29	16550	18000	21000		5	23	30	39	860	1000	1320
7	1	8	9	10	28500	30000	33670	17	1	12	14	18	3750	4000	4670
	2	11	15	19	21670	24000	28560		2	15	18	22	3000	3200	3530
	3	16	18	23	20000	22000	23560		3	22	24	29	1650	1800	2140
8	1	11	14	16	185	220	282	18	1	8	9	12	2850	3000	3575
	2	13	15	19	182	215	255		2	11	15	20	2030	2400	2950
	3	13	16	21	182	200	245		3	14	18	22	1950	2200	2660
	4	17	21	25	175	208	234								
	5	21	24	29	110	120	132								
9	1	11	15	20	290	300	313								
	2	16	18	23	212	240	288								
	3	18	20	22	165	180	225								
	4	20	23	28	125	150	196								
	5	21	25	28	85	100	124								

For implementation purposes, the detailed NA-ACO is completely coded using MATLAB and used to search for optimum Pareto front for the problem at hand. The size of solution space is 5.90E9 and the proposed NAACO surveyed to near optimal Pareto front in approximately 15 minutes. The parameters are tuned on a trial and error basis, and based on the authors' relative knowledge. Non-dominated solutions for α_f=0 and b_f=0.5 are shown in Table 3.

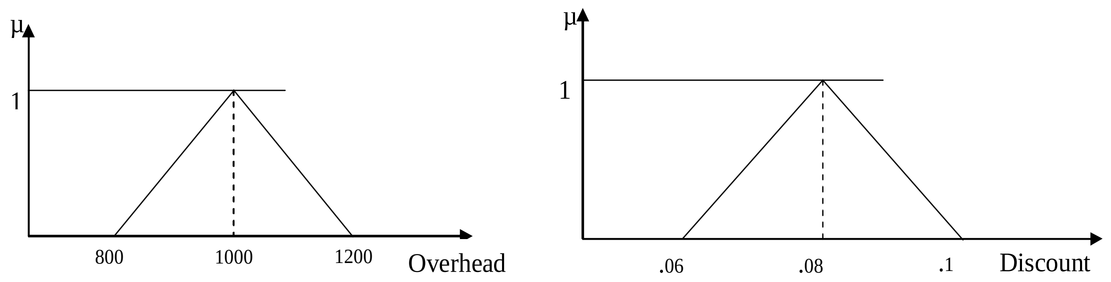

Figure 4. Graph of fuzzy overhead cost and discount rate

Table 3. Non-dominated solutions for α_f=0 and β_f =0.5

No.	Proeject Duration			Discounted cash-flow			Activities mode																	
							1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	83	100	130	245558.1	256578.6	268429.3	1	5	3	3	3	1	1	1	1	1	2	1	1	2	1	4	1	1
2	84	101	131	243412.2	254336.4	266083.6	2	5	3	3	3	1	1	1	1	1	3	1	1	3	1	4	1	1
3	87	101	130	239361.8	250104.1	261655.8	1	5	3	3	4	1	2	2	1	1	2	1	1	2	1	4	1	1
4	88	102	131	237744.9	248414.7	259888.4	2	5	3	3	4	1	2	1	1	1	1	1	1	3	1	4	1	1
5	88	104	134	167680.4	175205.8	183298.1	1	5	3	3	4	2	1	3	1	1	1	1	2	2	1	2	1	1
6	89	105	135	161755.8	169015.3	176821.7	2	5	3	3	4	2	2	2	1	1	2	1	1	2	1	4	1	1
7	89	107	135	158016.9	165108.6	172734.5	4	5	3	3	3	1	1	4	1	1	2	1	2	2	1	5	1	1
8	93	108	135	154616.5	161555.6	169017.4	4	5	3	3	4	1	2	4	1	1	2	1	1	3	1	4	1	1
9	92	110	142	96424.28	100751.7	105405.2	5	5	3	3	1	1	1	1	1	1	3	1	1	2	1	4	1	1

In Table 3, nine activities are higlighted. These are activities whose corresponding optimal modes are constant for all variations in α_f and β_f values. However, these activities do not share common characteristics in terms of budget, time, number of predecessors or successors in the network. Another finding is that for lesser α_f vlaues, fewer paths are appeared in the optimal solutions as critical (longest) paths (only paths 1 and 4 for α_f=0 , paths 1,4, and 6 for α_f=0.4 and paths 1,4,2, and 6 for α_f=1).

In some cases, for the same bi-objective values, various options are yielded. For example, for duration and discounted cash flow appeared on the sixth solution presented on Table 4, there are nine other alternatives. However, total cost of these simialr results are not the same. The longest path of all these solution is the same (4th path identified in table 1), and modes selected for activities on this path (activities 1,6,9,12,15,17, and 18) are similar. Differences are due to none-critical activities (solutions are deffuzifies for ease of comparison) .

Table 4. Solutions whith the same duration and discounted cash flow but different activity modes

Defuzzified Values			Activities mode																		Longest Path
T	DC	C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
109.67	169197.57	238193.33	2	5	3	3	4	2	2	4	1	1	2	1	3	3	1	4	1	1	4
109.67	169197.57	237995.67	2	5	3	3	4	2	3	4	1	1	1	1	2	1	1	2	1	1	4
109.67	169197.57	239702.67	2	5	3	3	4	2	2	2	1	1	3	1	2	1	1	5	1	1	4
109.67	169197.57	239872.33	2	5	3	3	4	2	2	5	1	1	1	1	2	2	1	3	1	1	4
109.67	169197.57	240533.33	2	5	3	3	4	2	2	2	1	1	2	1	1	2	1	4	1	1	4
109.67	169197.57	245066.00	2	5	3	3	4	2	1	2	1	1	3	1	3	1	1	3	1	1	4
109.67	169197.57	246237.33	2	5	3	3	4	2	1	5	1	1	1	1	2	1	1	4	1	1	4
109.67	169197.57	246356.00	2	5	3	3	4	2	1	2	1	1	3	1	1	3	1	4	1	1	4
109.67	169197.57	246951.67	2	5	3	3	4	2	1	1	1	1	2	1	1	3	1	2	1	1	4
109.67	169197.57	247007.67	2	5	3	3	4	2	1	3	1	1	3	1	1	2	1	2	1	1	4

T: Project Duration
DC: Discounted
Cash Flow C: Total Cost

The resulted project duration and net present value of project cost are both fuzzy numbers. The graph associated with project duration is a triangle because it is the sum of triangle-shaped fuzzy numbers. Nevertheless, the net present value of project cost is not necessarily triangle shaped. The fuzzy number associated with the net present value of project discounted cash for the solution represented on the first row of Table 3 is shown in Figure 5.

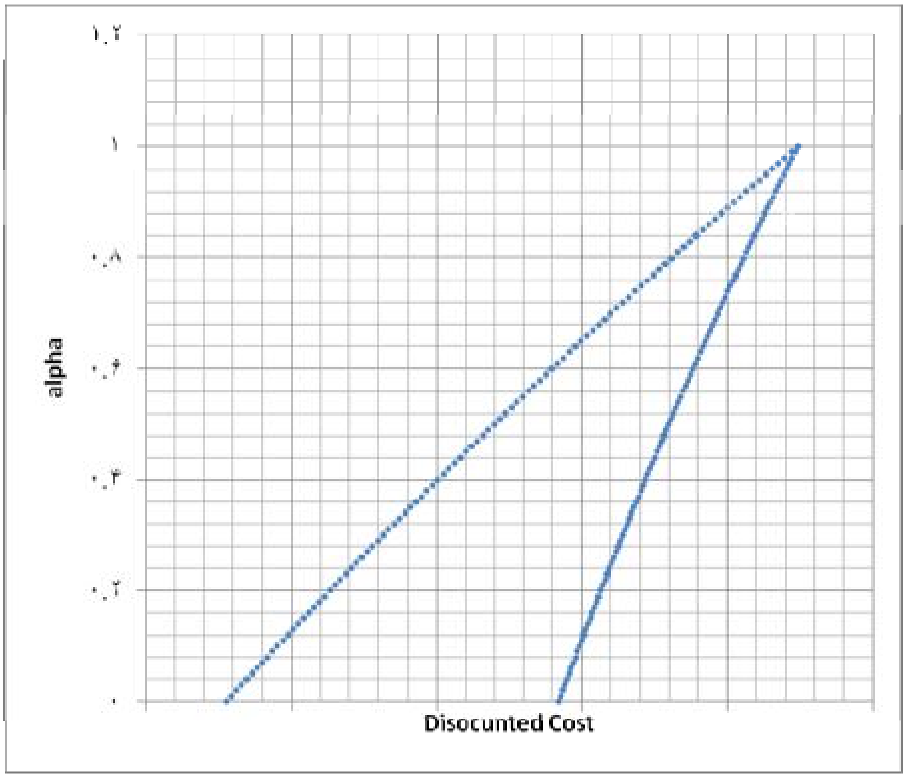

Figure 5. Graph of the fuzzy discounted cash flow regarding the solution presented in the first row of Table 2

As to verify the results, the problem input is set so that input data for α_f =1 matches the input data for deterministic problem. This provides a basis for comparison and verification of the fuzzy results with those of deterministic results. Compared to a NAACO model for deterministic problem with the same set of data, the proposed stochastic TCTP-DCF was verified.

The results of adopting different α_f values are compared in Figure 6. To make results visually understandable, they are de-fuzzified using the center of gravity approach. The Pareto fronts show a slight left shift as the α_f value increases. This is probably due to accounting for wider range of possibilities and consequently greater fuzzy numbers.

Results of various β_f values are also investigated, and results for different s (with f =0.5) are represented in Figure 7. As data suggest, the Pareto fronts are partly shifted from left to right as the β_f value increases, however the rest of the Pareto front overlap for all b_f values.

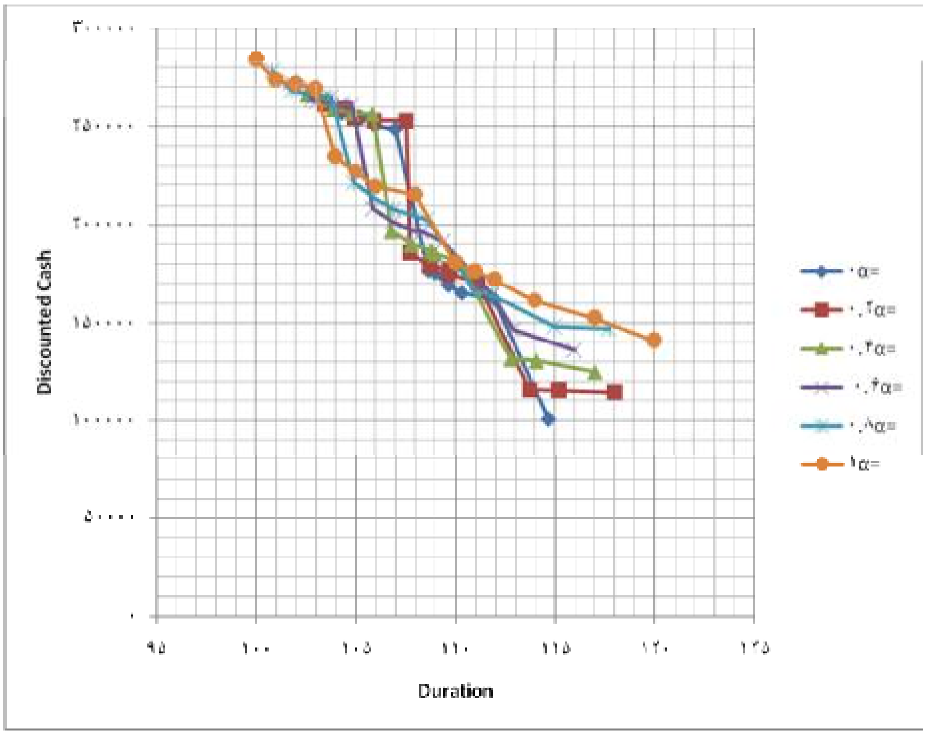

Figure 6. Non-dominated solutions regarding various αs (b_f =0.5)

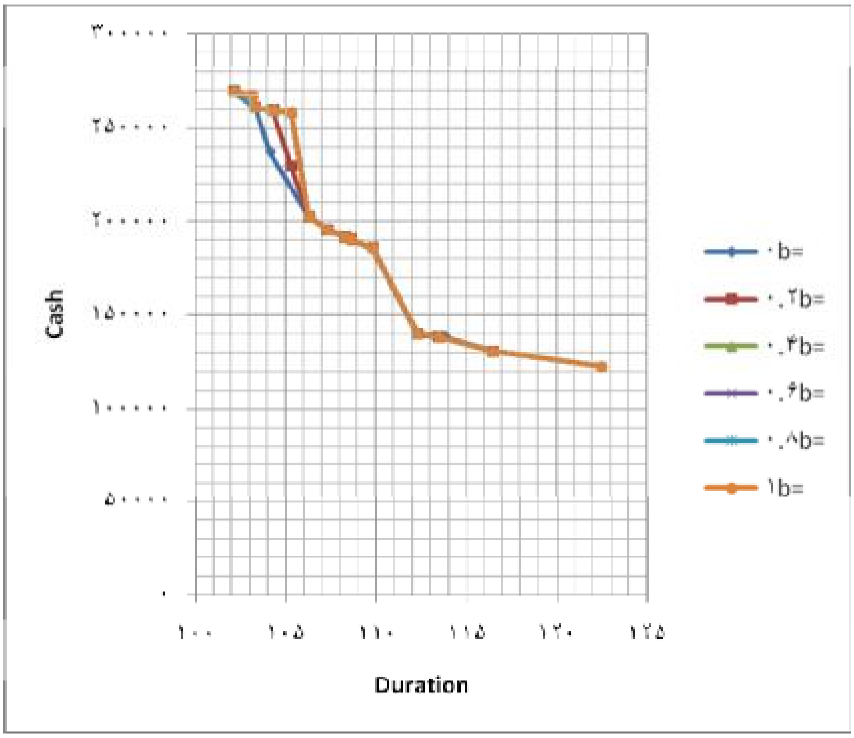

Figure 7. Results for various b values α_f =0.5

5. CONCLUSIONS

In this paper a non-dominate archiving ant colony approach is proposed to solve stochastic time cost trade-off problem with discounted cash flow. Fuzzy approach is adopted to answer for uncertainties in TCTP-DCF. Activities' duration and cost as well as interest rate are treated as fuzzy numbers.

Extension to Fuzzy exponential function within the model formulation is based on an almost recent work which uses differential rules to extend univariant fuzzy functions. The model comprehensively investigates decision maker's attitude towards risks. A case study is solved using the proposed model. The model shows capability of producing results in a reasonable time. In fact the NAACO model returned results after searching 0.033 percent of a 5904900000 sized solution space. Furthermore, the model resulted in various alternatives for similar objective values.

The solutions found by the model are resulted from fuzzy inputs and mapped by fuzzy arithmetic, and the results are returned as fuzzy numbers. The decision maker is provided with the Pareto set of results and she may select an answer from this set in a fuzzy format. However prior to run of the model she have the opportunity to specify how risk averse/seeker she is by means of a_f and b_f values. Effects of adopting various a_f and b_f values are discussed in the case study.

The future work of the authors is an in-depth assessment of parameters which affect the decision maker's choice of a_f and b_f for a stochastic scheduling problem in order to model expert judgment. In addition, there is a need to generate, test and verify fuzzy inputs for stochastic construction projects environment which will be considered by authors in a near future.

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