

## DETERMINATION OF ATTENUATION RELATIONSHIPS USING AN OPTIMIZATION PROBLEM

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### ABSTRACT

The main objective of this study is to present new method on the basis of genetic algorithms for attenuation relationship determination of horizontal peak ground acceleration and spectral acceleration. The proposed method employs the optimization capabilities of genetic algorithm to determine the coefficients of attenuation relationships of peak ground and spectral accelerations. This method has been applied to 361 Iranian earthquake records with magnitudes between 4.5 and 7.4 obtained from two seismic zones, namely Zagros and Alborz-Central Iran. The obtained results indicated that the proposed method can be characterized as a powerful tool for prediction horizontal peak ground and spectral accelerations.

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### 1. INTRODUCTION

The estimation of strong ground motion parameters is vital for the purpose of seismic hazard analysis and seismic design of the structures in seismic zones. In order to derive the empirical attenuation relationships, the first step is to use a mathematical model relating independent and dependent variables of the ground motion parameters by applying regression analysis. In this

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method, selected function forms for attenuation equations are based on few theoretical considerations. The regression analysis was performed after selecting the function form either in one or two steps, in the one step case, regression against magnitude and in the two step case regression against distance was applied. Different methods for performing the regression of attenuation relationships have been presented by Joyner and Boore [1].

Recently, Ahmad et al. [2] presented a new methodology for the prediction of strong ground motion parameters using artificial neural networks. They suggested attenuation relationships for three peak ground motion parameters, i.e., peak ground acceleration, peak ground velocity, and peak ground displacement. Artificial neural network modeled important characteristics of peak ground motion attenuation including magnitude scaling, attenuation with distance, site amplification, saturation of peak ground parameters with distance and magnitude, and magnitude dependent attenuation.

In parallel with the progress of novel methods based on the traditional optimization techniques, liking has grown considerably in expanding inverse methods based on evolutionary algorithms. Evolutionary algorithms are powerful search algorithms based on the heuristic concepts of natural choice and genetic acts and they can be categorized into four classes: evolutionary programming, evolutionary strategies, genetic programming, and genetic algorithms. The study of genetic algorithms was originated in the mid 1970s [3] and has developed into a powerful optimization approach. Excellent introductions to genetic algorithms was given by Goldberg [4]. Genetic algorithm has been employed for solving a wide range of optimization problems such as structural optimization [5], and structural damage detection [6].

In this study, a new method presents to estimate empirical attenuation relationships of horizontal peak ground acceleration and spectral acceleration using genetic algorithm. The efficiency of the proposed method has been validated using Iranian earthquake records. Moreover, the performance of the method have been evaluated through the comparison of obtained results with those from other attenuation relationships.

## 2. THE PROPOSED METHODOLOGY

This paper is aimed at presenting a new method on the basis of genetic algorithm to determine horizontal peak ground and spectral accelerations. An attenuation relationship provides a functional relationship between earthquake properties or response quantities and several parameters such as magnitude, distance from seismic source, soil conditions, etc.

The functional form of an attenuation relationship depends on it's derived method [7]. One way to develop the attenuation relationship is to employ an empirical method. Empirical relationships are generally derived by applying a regression analysis approach to an ensemble of strong ground motion database.

It is assume that the form of function in the relationship:

$$\log(Y) = b_1 + b_2M + b_3M^2 + b_4 \log(R) + b_5M \log(R) + b_6P \quad (1)$$

where,  $b_1$  to  $b_6$  are constant coefficients of the attenuation relationship,  $Y$  is strong ground motion parameter,  $M$  and  $R$  are magnitude and distance, respectively, and  $P$  is the site

conations and source effects.

If it is assumed that the constant coefficients determination problem for an attenuation relationship is a regression analysis, then, it will be possible to consider the same problem as an optimization problem. Regression methods are not appropriate for solving these optimization problems. However, biologically inspired soft computing methods, such as genetic algorithms, can imitate the robust problem-solving strategies applied in nature by dealing with optimization problems.

The genetic algorithm is an optimization and search technique based on genetics and natural selection principles. A genetic algorithm allows a population composed of many individuals to evolve under specified selection rules to a state that maximizes the “fitness” (i.e., minimizes the objective function).

Some of the advantages of a genetic algorithm include that it [8]:

- Optimizes with continuous or discrete variables
- Derivative information isn't required
- Simultaneously searches from a wide sampling of the objective function
- Deals with a large number of variables
- Is well suited for parallel computers
- Optimizes variables with extremely complex objective function
- Provides a list of optimum variables and not only a single solution
- Works with numerically generated data, experimental data, or analytical functions.

These advantages are intriguing and produce stunning results while traditional optimization approaches fail miserably.

The genetic algorithm approach attempts to find the best solution to a given problem by minimizing an objective evaluation function. Therefore, the key point in a minimization problem is the objective function. This function is used to provide a measure of how individuals have performed in the problem domain. In the case of a minimization problem, the fit individuals will have the lowest numerical value of the associated objective function [4].

In this method, genetic algorithm has been used to determine the constant coefficients of the attenuation relationship by optimizing an objective function. Figure 1 shows the flowchart of the proposed method for attenuation relationship determination using the genetic algorithm.

In the proposed method, the general expression for the objective function is:

$$F = f(b_1, b_2, \dots, b_N) \quad (2)$$

where,  $b_1, b_2, \dots, b_N$  are constant coefficients of the attenuation relationship.

To construct the objective function  $F$ , it is necessary to use all kinds of terms of the attenuation relationship that are sufficiently sensitive to the constant coefficients.

If the strong ground motion parameters in Eq. (1) are replaced by a recorded database, a residual value can be defined over each strong ground motion recorded database:

$$R_i = \log(Y_i) - b_1 - b_2 M_i - b_3 M_i^2 - b_4 \log(R_i) - b_5 M_i \log(R_i) - b_6 P_i \quad i = 1, \dots, n \quad (3)$$

where,  $n$  is the total number of recorded databases from the earthquakes.

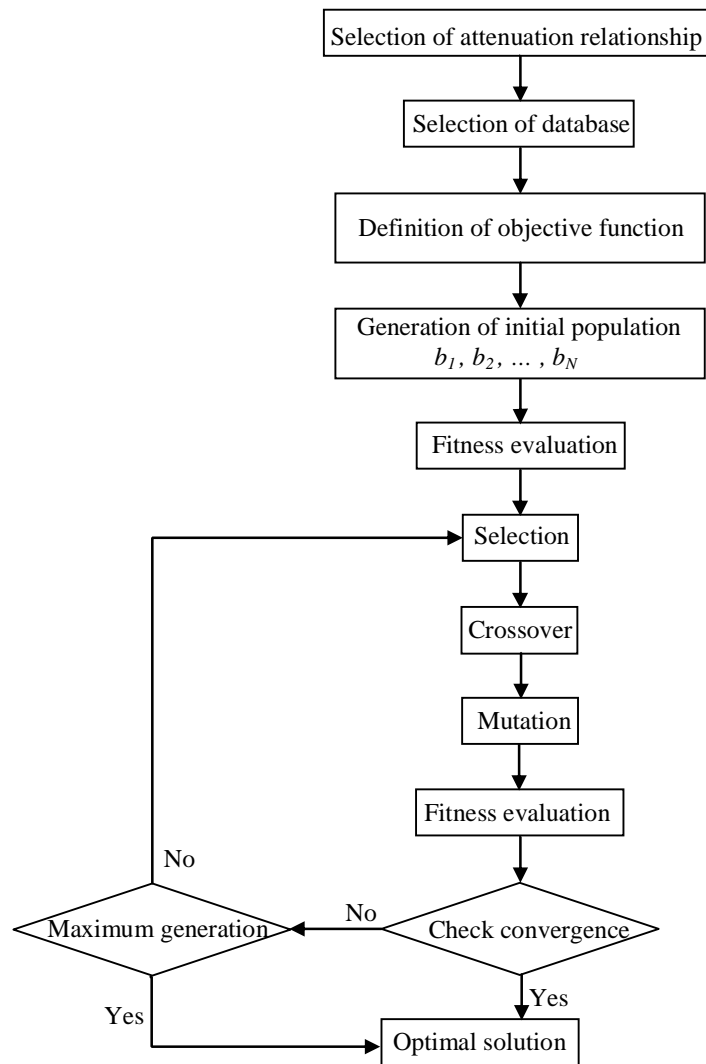


Figure 1. Flowchart of the proposed method using genetic algorithm

Therefore, the problem of the attenuation relationship determination for strong ground motion parameters can be formulated as an optimization problem. The objective is to minimize the objective function:

$$F = \sum_{i=1}^n (R_i(b_j))^2 \quad j = 1, \dots, N \quad (4)$$

In order to minimize this equation, an initial population of randomly generated candidate solutions, encoded as chromosomes, applying the principle of survival of the fittest to produce better and better approximations to a solution. At each generation, a new population is generated by numerical processes of selection, crossover, and mutation with the purpose of improving the best fitnesses.

At the selection operation, chromosomes are selected for future population reproduction based upon their fitness. Selection is a very important step within a genetic algorithm, since the quality of an individual is measured by its fitness value, if the selection involves only the most highly fit chromosomes the solution-space may be very limited due to the lack of diversity; on the other hand, random selection does not guarantee that future generations will increase its fitness.

Crossover operator takes the chromosomes of two parents, randomly selected, which then exchanges the part of their genes, resulting in the two new chromosomes for child generation. Therefore, the crossover does not create new material within the population; it simply intermix the existing population.

The mutation operator introduces a change in one or more of the chromosome's genes. Therefore, with this operator new material is introduced in the population and its main goal is to prevent the population from converging to a local minimum.

This process leads to the evolution of populations of individuals that are better suited to their environment than the individuals that were created, just as in the natural adaptation. Within the chromosome are separate genes that represent the independent variables for the problem at hand. The algorithm progresses with successive generations to reach an optimum solution for the studied problem.

### 3. AN ILLUSTRATIVE EXAMPLE

The presented method for the prediction of horizontal peak ground acceleration and spectral acceleration have been applied to a 361 earthquake record with magnitudes between 4.5 and 7.4 and hypocentral distances ranging from 5 to 291 km.

Since most of large-magnitude earthquakes in Iran have been reported in terms of surface wave magnitude ( $M_S$ ), therefore, the scale used to classify the size of the earthquake events is  $M_S$ . For cases in which magnitude has been reported in other terms such as wave magnitude ( $m_b$ ); it is required to convert them into  $M_S$ .

Distance is considered to be the second independent variables in the attenuation relationship. This parameter ( $R$ ) shows the distance between the earthquake focus (hypocenter) and its site. In the current study, in order to calculate the hypocentral distance, geometric methods have been utilized in conjunction with geographic coordinates of the epicenter, accelerograph and focal depth. Hypocentral distance ( $R$ ) is assumed to be the hypotenuse of a right-angled triangle in which focal depth ( $D$ ) and distance between epicenter and accelerograph ( $r$ ) are the other legs. Figure 3 shows the distribution of magnitude and distance of the records.

In accordance with the Iranian seismic code, 2800 Standard [9], the ground type is categorized into four main groups. The first two types are so firm called rock and the second two types are called soil in Iran with shear wave velocity 375  $m/s$  as a border.

According to the history of earthquakes in Iran, it has been realized that a large number of events occurred along the Zagros fold thrust belt. While in comparison, those earthquakes that have been occurred in Alborz, and central and eastern parts of Iran have higher magnitudes.

Stocklin [10] and Nowroozi [11] have suggested simplified divisions to classify seismotectonic structures of Iran. Therefore, Iran has been categorized into two zones of Zagros, and Alborz-Central Iran and the attenuation relationships for these two zones were derived separately.

### 3.1. Results

In the proposed method genetic algorithm has been used for identification of the functional form with minimum objective function, several models of attenuation relationships for horizontal peak ground acceleration in this study were examined. The final functional form selected is:

$$\log(PGA) = b_1 + b_2M + b_3M^2 + b_4 \log(\sqrt{r^2 + D^2}) \quad (5)$$

where,  $PGA$  is horizontal peak ground acceleration,  $M$  is surface wave magnitude,  $r$  is epicenter distance,  $D$  is focal depth, and  $b_1$  to  $b_4$  are constant coefficients of the attenuation relationship.

Using the proposed method with genetic algorithm, the constant coefficients of the attenuation relationship was determined by optimizing an objective function. The proposed method started with an initial generation of individuals randomly selected with no guess of the initial parameter values. The fitness of each solution was evaluated by computing the objective function using Eq. (4). Then the genetic operators of reproduction, namely crossover, and mutation were employed to obtain the fittest chromosome. Crossover operator has been performed using a single point scheme. The following parameters have been adopted in this paper: size population=100; probability of crossover=0.8; probability of mutation=0.01; and maximum number of generations=100.

The constant coefficients determined for the attenuation relationships are illustrated in Table 1. Figure 2 shows the peak ground acceleration model for the seismic zone of Alborz-Central Iran at focal deep of 15 km and magnitudes of 5, 6, 7, and 7.5. It is apparent that the values of peak ground acceleration decrease as the distance increases.

Table 1. Coefficients of attenuation relationships of the peak ground acceleration determined

Seismic zone	Site class	$b_1$	$b_2$	$b_3$	$b_4$	$\sigma$
Alborz-Central Iran	Rock	2.173	0.185	0.006	-0.938	0.351
	Soil	1.651	0.302	0.004	-1.082	0.261
Zagros	Rock	2.448	0.348	-0.020	-1.329	0.275
	Soil	2.639	-0.214	0.031	-0.579	0.305

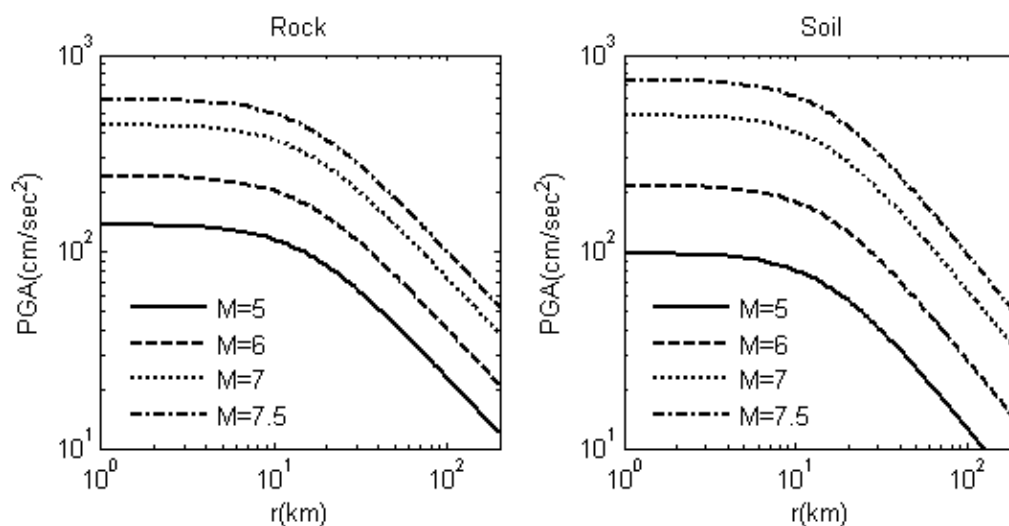


Figure 2. Results of peak ground acceleration model obtained for different magnitudes and distances (Alborz-Central Iran ,  $D=15$  km)

The functional form finally selected for the attenuation relationship of acceleration response spectra is:

$$\log[Sa(T, x)] = b_1(T) + b_2(T)M + b_3(T) \log(\sqrt{r^2 + D^2}) \quad (6)$$

where,  $Sa(T, \xi)$  is the acceleration response spectra within the period of  $T$  and damping ratio of  $\xi$  in  $\text{cm/s}^2$ .

The proposed method has been applied to attenuation relationships as follows. The constant coefficients determined for the attenuation relationships of acceleration response spectra with the damping ratio of 5% are cited in Tables 2 and 3. In Figure 3, the acceleration response spectra of seismic zones of Zagros and Alborz-Central Iran for focal deep 10km and magnitude 6 is shown. It should be noticed that the acceleration response spectra contains small peaks and valleys because the constant coefficients have not been smoothed.

### 3.2. Comparison of the results with other models

The proposed method using genetic algorithm has been employed to determine constant coefficients of the attenuation relationships. A comparison between the attenuation relationships for horizontal peak ground acceleration in this study and the relationships from previous studies such as Ambraseys et al. [12], Zare et al. [13] and Ghodrati et al. [14] was performed in this section. Since the data set of Zare et al. [13] and Ghodrati et al. [14] are on the basis of Iranian earthquakes; therefore they are appropriate for comparison. The attenuation relationship of Ambraseys et al. [12] was developed for Europe and the Middle East, and used from Iranian earthquake records. Figure 4 shows a comparison between the proposed method for seismic zone of Alborz-Central Iran and some previous studies.

The proposed model for spectral acceleration has been compared with other models such

as those developed by Ambraseys et al. [12], Sadiagh et al. [15], Khademi [16] and Ghodrati et al. [17]. The results for the seismic zone of Alborz-Central Iran and  $M_s=6.5$  are shown in Figure 5.

Table 2. Coefficients of attenuation relationships of the spectral acceleration determined seismic zone of Alborz-Central Iran

T(s)	Rock				Soil			
	$b_1$	$b_2$	$b_3$	$\sigma$	$b_1$	$b_2$	$b_3$	$\sigma$
0.1	2.716	0.187	-0.964	0.404	2.234	0.283	-1.131	0.303
0.2	2.273	0.225	-1.011	0.336	1.897	0.353	-1.089	0.304
0.3	2.208	0.220	-0.963	0.344	1.575	0.384	-1.064	0.287
0.4	2.238	0.242	-1.049	0.343	1.393	0.426	-1.132	0.296
0.5	1.068	0.392	-0.923	0.351	1.086	0.416	-0.990	0.304
0.6	1.414	0.319	-0.944	0.372	0.677	0.471	-0.980	0.323
0.8	0.767	0.406	-1.035	0.372	0.312	0.533	-1.049	0.323
1.0	0.620	0.391	-0.885	0.380	0.121	0.525	-0.982	0.367
1.2	0.332	0.465	-1.049	0.373	-0.042	0.555	-1.063	0.387
1.5	0.022	0.498	-1.060	0.391	-0.923	0.708	-1.120	0.401
2.0	0.091	0.454	-1.076	0.409	-1.127	0.712	-1.148	0.389
2.5	-0.679	0.583	-1.130	0.416	-1.547	0.787	-1.232	0.375
3.0	-0.905	0.549	-0.941	0.412	-1.467	0.723	-1.169	0.378
3.5	-1.019	0.637	-1.213	0.417	-1.783	0.811	-1.334	0.363
4.0	-1.090	0.628	-1.195	0.410	-2.052	0.805	-1.230	0.357

Table 3. Coefficients of attenuation relationships of the spectral acceleration determined for seismic zone of Zagros

T(s)	Rock				Soil			
	$b_1$	$b_2$	$b_3$	$\sigma$	$b_1$	$b_2$	$b_3$	$\sigma$
0.1	4.200	0.097	-1.713	0.327	2.454	0.113	-0.809	0.345
0.2	2.823	0.160	-1.092	0.360	1.852	0.230	-0.796	0.344
0.3	2.138	0.144	-0.728	0.369	1.898	0.141	-0.543	0.322
0.4	1.355	0.253	-0.688	0.380	0.801	0.331	-0.575	0.328
0.5	0.854	0.334	-0.716	0.364	0.630	0.300	-0.405	0.341
0.6	0.144	0.361	-0.452	0.368	0.301	0.321	-0.330	0.338
0.8	-0.190	0.482	-0.727	0.368	-0.207	0.398	-0.351	0.338
1.0	-0.797	0.507	-0.505	0.367	-0.929	0.436	-0.141	0.381
1.2	-1.313	0.652	-0.725	0.340	-0.921	0.383	-0.073	0.368
1.5	-2.091	0.585	-0.180	0.319	-1.240	0.426	-0.129	0.405
2.0	-2.467	0.742	-0.553	0.319	-1.709	0.424	0.000	0.361
2.5	-2.648	0.716	-0.462	0.321	-1.892	0.537	-0.322	0.360
3.0	-2.705	0.709	-0.495	0.305	-2.658	0.528	0.057	0.365
3.5	-2.134	0.474	-0.182	0.336	-2.448	0.581	-0.266	0.364
4.0	-3.378	0.683	-0.132	0.309	-2.770	0.641	-0.335	0.360



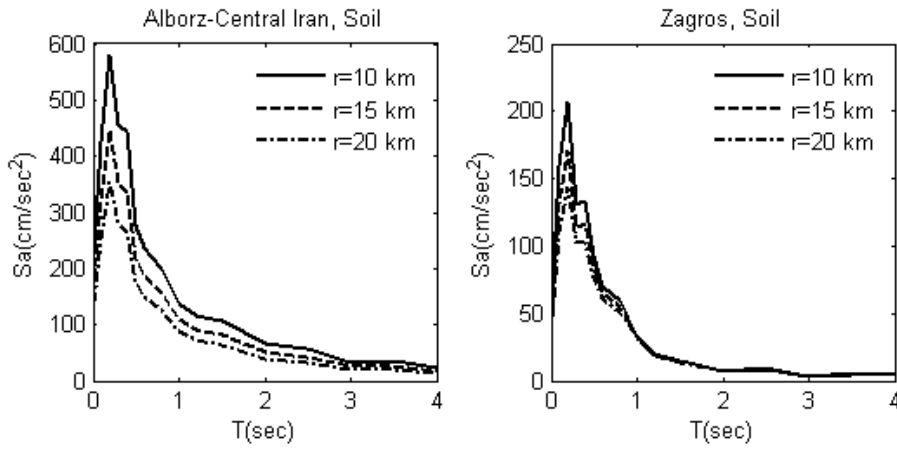


Figure 3. Results of spectral acceleration model obtained for different distances ( $M_s=6$ ,  $D=10$  km)

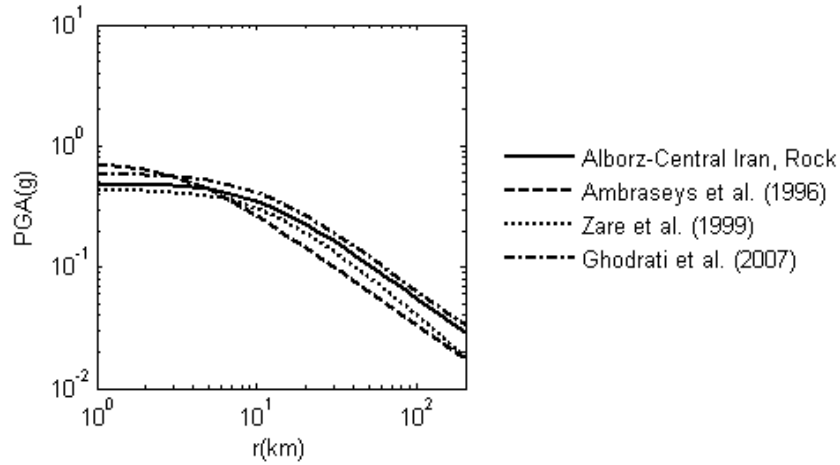


Figure 4. Comparison of the proposed model of peak ground acceleration for the seismic zone of Alborz-Central Iran and some previous models for  $M_s=6.5$

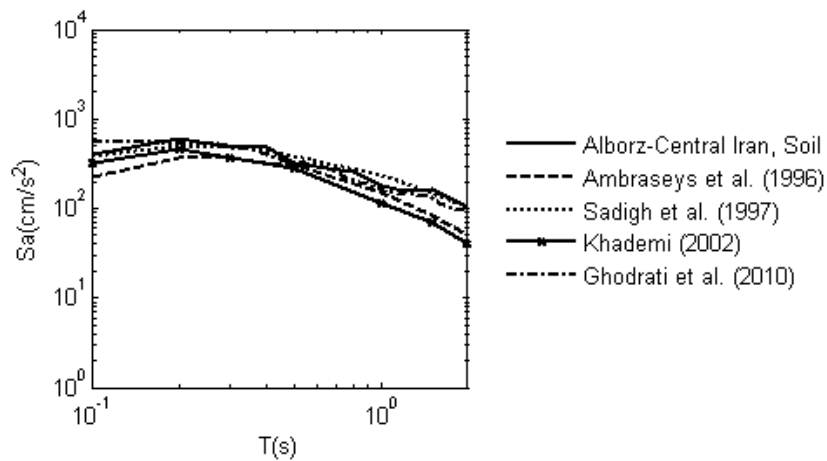


Figure 5. Comparison of the proposed model of spectral acceleration for the seismic zone of Alborz-Central Iran and other models for  $M_s=6.5$

#### 4. CONCLUSIONS

In this paper, a method has been suggested for the estimation of horizontal peak ground acceleration and spectral acceleration using genetic algorithm. The suggested method is based on determining the constant coefficients of attenuation relationship from an ensemble of strong ground motion database using an optimization problem. In this method for determining the constant coefficients in the attenuation relationship by optimizing an objective function proposed by applying the genetic algorithm.

In an interpretive example, the proposed method was applied to a sample of strong ground motion database of Iran. The obtained results clearly reveal that genetic algorithm can be viewed as a reliable tool for solving complex optimization problems such as attenuation relationship. Finally, the efficiency of the proposed method has been confirmed through the comparison of the obtained results and the other models result.

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