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A BINARY LEVEL SET METHOD FOR STRUCTURAL TOPOLOGY OPTIMIZATION

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ABSTRACT

This paper proposes an effective algorithm based on the level set method (LSM) to solve shape and topology optimization problems. Since the conventional LSM has several limitations, a binary level set method (BLSM) is used instead. In the BLSM, the level set function can only take 1 and -1 values at convergence. Thus, it is related to phase-field methods. We don't need to solve the Hamilton-Jacobi equation, so it is free of the CFL condition and the reinitialization scheme. This favorable properties lead to a great time advantage in this method. In this paper, the BLSM is implemented with the additive operator splitting (AOS) scheme and several numerical issues of the implementation are discussed. The proposed scheme is much more efficient than the conventional level set method. Several 2D examples are presented which demonstrate the effectiveness and robustness of the proposed method.

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KEY WORDS: topology, level set method, binary, structural optimization.

1. INTRODUCTION

Nowadays structural topology optimization problems are very important and challenging in many engineering fields. This branch of engineering science has made notable progress in recent three decades. Up to now, considerable researches and various topology optimization methods such as homogenization methods [1-4], Solid Isotropic Material with Penalization

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(SIMP) methods [5,6] and Evolutionary Structural Optimization (ESO) methods [7] have been proposed. Recently, the level set methods [8-10] which were originally proposed for tracking the propagation of fluid interfaces, have been applied into structural topology optimization effectively [11,12]. In this method the boundaries of design domain are implicitly represented by the zero level set of a higher dimensional function. Thus, level set method (LSM) can easily handle different shape and topology changes such as merging, splitting and developing sharp corners. In this method, one has to solve Hamilton-Jacobi equation and this causes several limitations such as reinitialization process, CFL condition and dependency of final design to initial guess. To overcome these difficulties several alternative LSMs have been proposed [13-23]. One of the recent variants of the traditional LSM is the binary level set method (BLSM) [21-23]. In this method, the level set functions are discontinuous functions at convergence. In contrast to the classical level set method which the level set function should be chosen continuous and smooth function, for the BLSM, the level set function is discontinuous and we require the its value to be 1 and –1 at convergence. This method is in fact very similar to the phase filed model, which has been applied to image processing [24] and optimal shape design [25,26]. Recently, the BLSM is used for image segmentation and elliptic inverse problems [22,27].

In this paper, the BLSM is employed to solve constrained minimization problems in structural topology optimization problems, using the augmented Lagrangian method. Since the BLSM is closely related to the phase-field model, an operator splitting scheme, often used for phase-field models, is combined with the binary level set model for an efficient implementation of the numerical computation of the BLSM.

The remainder of this paper is organized as following. First the conventional level set method is introduced in section 2. In section 3, the statement of optimization problem is planned. After that, the binary level set method is presented in section 4. Then, this method in the structural topology optimization problem is applied in section 5. Some implementation issues are given in section 6, and to show the advantages of this method, we present several numerical examples in section 7. Finally, the conclusion is given in the last section.

2. LEVEL SET METHOD

The level set method is an implicit method for describing the evolution of an interface between two domains. It makes use of a function ϕ, referred to as the level set function, which represents the boundary as the zero level set and nonzero in the domain [9, 10]. According to the value of the level set function,

$$\begin{cases} \phi(x(t)) > 0 : \forall x(t) \in D \setminus \Omega \\ \phi(x(t)) = 0 : \forall x(t) \in \partial\Omega \\ \phi(x(t)) < 0 : \forall x(t) \in \Omega \setminus \partial\Omega \end{cases} \tag{1}$$

where $D \subset \Re^d$ denotes the design domain, which all admissible form of Ω a smooth boundary open set place in.(i.e. $\Omega \subset D$) and $t \in \Re^{+}$ is time. In the above context, the level set function is used as a switch to distinguish between the two domains present in the computational space. The boundary is embedded as the zero level of the level set function. During the optimization process, the level set surface may move up and down, and this causes the embedded boundary to undergo drastic shape or topological changes. From beginning to end, the value of the level set function on the boundary is constantly kept to be zero,

$$\phi(x) = 0.0\,,\ \forall x \in \partial\Omega \tag{2}$$

If we differentiate the above equation with respect to time t, we can get:

$$\frac{\partial\phi}{\partial t} + \nabla\phi.v(x) = 0.0 \tag{3}$$

where $v(x) = \dfrac{dx}{dt}$ is the velocity vector field, provided based on sensitivity analysis. Considering $n = \dfrac{\nabla\phi}{|\nabla\phi|}$ and $\nabla\phi.v = (v_n)|\nabla\phi|$, we can write equation (3) as

$$\frac{\partial\phi}{\partial t} + (v_n)|\nabla\phi| = 0.0 \tag{4}$$

3. SHAPE AND TOPOLOGY OPTIMIZATION PROBLEM

3.1. Statement of optimization problem

The optimization goal of the procedure presented in this paper is to minimize the compliance (global strain energy) over the structural domain for general loading conditions with a constraint on total material volume resource. There exist numerous equivalent formulations of the minimum compliance problem that we use which was given in Allaire [11].

Let Ω be a bounded open set, all admissible shapes in working domain D, occupied by a linear isotropic elastic material with Hook's law ***A*** in design domain. The objective function (compliance) is denoted by $J(\Omega)$ is then formulated as follows:

$$J(\Omega) = \int_{\Omega} f.udv + \int_{\Gamma_N} g.uds = \int_{\Omega} Ae(u).e(u)dv \tag{5}$$

where Γ_N is Neumann boundary condition, *f, g* are body force and surface load respectively, u is the displacement field based on the following linear elasticity equations:

$$\begin{cases} -\operatorname{div}(Ae(u)) = f & \text{in } \Omega \\ u = 0 & \text{on } \Gamma_{\mathrm{D}} \\ (Ae(u))n = g & \text{on } \Gamma_{\mathrm{N}} \end{cases} \tag{6}$$

where Γ_D is Dirichlet boundary condition. The standard notion for minimum compliance design problems can be mathematically defined as follows:

$$\begin{cases} \text{Minimize } J(\Omega) = \int_\Omega f.udv + \int_{\Gamma_N} g.uds = \int_\Omega Ae(u).e(u)dv \\ \text{Subject to}: \int_\Omega dv - V_{\max} \le 0 \end{cases} \tag{7}$$

3.2. Shape Derivative

In this section, we use a based gradient method, shape derivative, to guarantee minimization of Eq. (7). In [27] Simon and Murat introduce a technique for constructing shape derivative by parameterization of domains. We use their approach as follows:

$$\Omega_\theta = (\Im + \theta)\Omega \tag{8}$$

where Ω is a smooth open set domain, $\Im$ is identity mapping in $\Re^N, \theta \in W^{1,\infty}(\Re^N, \Re^N)$. The shape derivative of objective function $J(\Omega): \Re^N \to \Re$, is defined as the Frechet derivative in $\boldsymbol{W}^{1,\infty}(\Re^N, \Re^N)$. For θ small enough,

$$J((\Im + \theta)\Omega) = J(\Omega) + J'(\Omega)\theta + O(\theta) \tag{9}$$

where $\boldsymbol{J'(\Omega)}$ is a continuous linear form on $\boldsymbol{W}^{1,\infty}(\Re^N, \Re^N)$ given as the unique solution to Eq. (7). Above equation is called Frechet derivative-based sensitivity and the sensitivity of the mean compliance (7) is given as follows on the discussion in [11, 13].

$$J'(\Omega)\theta = \int_{\Gamma_N} (2[\frac{\partial(g.u)}{\partial n} + Hg.u + f.u] - Ae(u).e(u))\theta.nds + \int_{\Gamma_D} Ae(u).e(u))\theta.nds \tag{10}$$

In this formulation H is mean curvature defined by $H = \operatorname{div} n$, $\partial\Omega$ is the boundary of material domain Ω decomposed in three parts $\Gamma_D, \partial D_N, \Gamma_O$. Γ_D is admissible Dirichlet boundary conditions such that $\Gamma_D \subset \partial D_D, \Gamma_N = \partial D_N \cup \Gamma_O$ is Neumann boundary conditions where ∂D_N supports a non-homogeneous one and Γ_O supports homogeneous one. Let us suppose that there is no body force then in (5) $f = 0.0$, thus the objective function is defined as,

$$J(\Omega) = \int_{\partial D_N} g.uds \tag{11}$$

Therefore, the Frechet derivative of the mean compliance and the volume constrain are of the forms,

$$J'(\Omega)\theta = \int_{\Gamma_N} (-Ae(u).e(u))\theta.nds \tag{12}$$

$$V'(\Omega)\theta = \int_{\partial\Omega} \theta(x).n(x)ds \tag{13}$$

In this paper, for solving the optimization problem, the augmented Lagrangian method is used. The following augmented Lagrangian $\bar{J}(\Omega)$ is defined using the Lagrange multiplier (λ^K) and penalization parameter (Λ^K).

$$\bar{J}(\Omega) = J(\Omega) + \lambda^K \left[\int_\Omega dv - V_{\max}\right] + \frac{1}{2\Lambda^K}\left[\int_\Omega dv - V_{\max}\right]^2 \tag{14}$$

The Lagrange multiplier and penalization parameter are updated as follows at each iteration of the optimization process,

$$\lambda^{K+1} = \lambda^K + \frac{1}{\Lambda^K}\left[\int_\Omega dv - V_{\max}\right] \tag{15}$$

$$\Lambda^{K+1} = \alpha\Lambda^K \tag{16}$$

where $\alpha \in (0,1)$ is a constant parameter. The shape derivative of the augmented Lagrangian which there is no body force, is obtained,

$$\bar{J}'(\Omega)\theta = \int_{\Gamma_0} v\theta.nds \tag{17}$$

$$v = \left(\lambda + \frac{1}{2\Lambda}\left[\int_\Omega dv - V_{\max}\right]^2 - Ae(u).e(u)\right) \tag{18}$$

To ensure the decrease of the objective function in level set method, the normal velocity field must be chosen appropriately. The fast descent or the steepest descent method is used as it was proposed in [11, 12] where $\theta = -vn$. The normal velocity field in H-J equation is substituted with normal component of this direction $\theta.n = -v$.

$$\frac{\partial\phi}{\partial t} - v|\nabla\phi| = 0.0 \tag{19}$$

The solving of the above equation needs appropriate choice of the upwind difference schemes, reinitialization algorithm and extension velocity method, which may require excessive amount of computational efforts. Thus, these limit the utility of the level set methods [9, 10]. For example, in using of upwind scheme, the time step should satisfy the CFL condition which requires the front to cross no more than one grid cell each time step. Moreover, in optimization process, the level set surface may become too steep or too flat, this may numerical instability. So it is necessary to regularize the level set surface (reinitialization). In the next section, the binary level set method is used to overcome these drawbacks of the conventional level set method.

4. A BINARY LEVEL SET METHOD

The binary level set method (BLSM) was originally proposed by Lie et al [21]. In the BLSM, the sub regions are defined by the discontinuous level set functions which take the values 1 and –1 at convergence. The representation of two regions (Ω_1, Ω_2) can be given by

$$\phi(x) = \begin{cases} 1 & x \in \Omega_1 \\ -1 & x \in \Omega_2 \end{cases} \tag{20}$$

where Γ can be given implicitly by $\Gamma = \{x \in \Omega \mid \phi(x) = k\}$ for arbitrary $k \in (-1,1)$. Now, let $\rho(x) = c_1$ in Ω_1, and $\rho(x) = c_2$ in Ω_2 then $\rho(x)$ can be defined as

$$\rho(x) = \frac{1}{2}\left[c_1(\phi+1) + c_2(1-\phi)\right] \tag{21}$$

Generally, we can use N binary level set functions $\{\phi_i\}_{i=1}^{N}$ to represent 2^N sub regions $\{\Omega_j\}_{j=1}^{2N}$. Let us introduce the vectors $\phi = \{\phi_1, \phi_2, \cdots, \phi_N\}$ and $c = \{c_1, c_2, \cdots, c_N\}$. For $i = 1,2,\cdots,2^N$ let $(b_1^{i-1}, b_2^{i-1}, \ldots, b_N^{i-1})$ be the binary representation of *i*–1, i.e., $b_j^{i-1} = 0$ or 1. If we set,

$$s(i) = \sum_{j=1}^{N} b_j^{i-1} \tag{22}$$

and

$$\psi_i = \frac{(-1)^{s(i)}}{2^N} \prod_{j=1}^{N} (\phi_j + 1 - 2b_j^{i-1}) \tag{23}$$

The piecewise constant function r can be represented as

$$\rho = \sum_{i=1}^{2N} c_i \psi_i \tag{24}$$

To ensure the BLSFs converges to values 1 and −1 at every point in Ω, these functions are required to satisfy,

$$K(\phi_i) = \phi_i^2 - 1 = 0\,,\ i = 1,2,\cdots,N \tag{25}$$

We can also calculate the volume and the perimeter of each sub domain with the following formulation,

$$|\Omega_i| = \int_\Omega \psi_i dx \quad , \quad |\partial\Omega_i| = \int_\Omega |\nabla \psi_i| dx \tag{26}$$

4.1. Binary level set method for topology optimization problem

We want to employ the BLSM for solving structural topology optimization problems. In this paper, we implement it in two phases. Thus, we just need one BLSF to satisfy the Eq. (25). Now, we define the piecewise constant density function in two phases by,

$$\rho(x) = \frac{1}{2}\left[c_1(1-\phi) + c_2(1+\phi)\right] \tag{27}$$

Where c_1 and c_2 are the specified characteristic values of the void material, with $c_1 = 0$, and the solid material, with $c_2 = 1$. To ensure that the level set function ϕ converges to a unique value in each sub domain, the piecewise constant constraint is defined as:

$$K(\phi) = 0 \quad , \qquad K(\phi) = \phi_i^2 - 1 \tag{28}$$

This indicates that every point in the design domain must belong to one and only one phase and there is no vacuum and overlap between different phases. In this paper, the optimization goal is to minimize the compliance over the structural domain for general loading condition and several constraints. It can be defined as

$$\begin{aligned}
&\min\ J(u,\phi) = \int_\Omega \rho(\phi)F(u)d\Omega + \beta\int_\Omega |\nabla\phi| d\Omega \\
&\text{S.t.}\quad \mathrm{H}_1 = \int_\Omega \rho(\phi)dx - V_0 \le 0 \\
&\qquad\ \ \mathrm{H}_2 = K(\phi) = 0 \\
&\qquad\ \ a(u,v,\phi) = l(v,\phi) \\
&\qquad\ \ a(u,v,\phi) = \int_\Omega \rho(\phi)E_{ijkl}\varepsilon_{ij}(u)\varepsilon_{kl}(v)d\Omega \\
&\qquad\ \ l(v,\phi) = \int_\Omega f.vd\Omega + \int_{\Gamma_N} g.vd\Gamma \\
&\text{for all } v \in U\ ,\ u|_{\Gamma_D} = u_0
\end{aligned} \tag{29}$$

where Ω is the structural domain and its boundary is represented by $\Gamma = \partial\Omega$. Also in the linearly elastic equilibrium equation, u denotes the displacement field, u_0 is the prescribed displacement on Γ_D, E_{ijkl} is the elasticity tensor, ε_{ij} is the strain tensor and *f*, *g* are body force and surface load respectively.

In the objective function $J(\Omega)$, the first term, is the mean compliance where functional $F(u) = 1/2\, E_{ijkl}\varepsilon_{ij}(u)\varepsilon_{kl}(u)$ is the strain energy density and ρ is the material density ratio. The second term in the objective function is the regularization term and β is a nonnegative value to control the effect of this term. Indeed, this term controls both the length of interfaces and the jump of ϕ, since the value of ϕ may not be continuous in the BLSM. H_1 defines the material fraction for different phases and V_0 is the maximum admissible volume of the design domain. H_2 as mentioned before, is the piecewise constant constraint to guarantee the level set function belongs to only one phase. If we use the augmented Lagrangian method to convert Eq. (29) into an unconstraint one, we have the following form:

$$L(\phi,\lambda) = J(\phi) - a(u,v,\phi) + l(\phi,\lambda) + \lambda_1 H_1 + \frac{1}{2\mu_1} H_1^{\,2} + \lambda_2 \int_\Omega H_2 d\Omega + \frac{1}{2\mu_2}\int_\Omega H_2^{\,2} d\Omega \qquad (30)$$

where $\lambda_1 \in R$ and $\lambda_2 \in L^2(\Omega)$ are Lagrange multiplier and $\mu_1, \mu_2 > 0$ are penalty parameters. Now, we need to find a saddle point of the augmented Lagrangian functional L. To find the saddle point of this function when there is no body force, f we have the following equation as suggested by Wei and Wang [20, 29]:

$$\int_\Omega \Psi(u,\phi,\tilde{\lambda}_1,\tilde{\lambda}_2)\partial\phi d\Omega = 0 \qquad (31)$$

$$\Psi(u,\phi,\tilde{\lambda}_1,\tilde{\lambda}_2) = -\frac{1}{2}\rho'(\phi)E_{ijkl}\varepsilon_{ij}(u)\varepsilon_{kl}(u) + \beta\nabla.\left(\frac{\nabla\phi}{|\nabla\phi|}\right) + \tilde{\lambda}_1\rho'(\phi) + \tilde{\lambda}_2 K'(\phi) \qquad (32)$$

where

$$\rho'(\phi) = \frac{\partial\rho(\phi)}{\partial\phi} = \frac{1}{2}(c_2 - c_1) \qquad (33)$$

$$K'(\phi) = \frac{\partial K(\phi)}{\partial\phi} = 2\phi \qquad (34)$$

and

$$\tilde{\lambda}_1 = \lambda_1 + \frac{1}{\mu_1}\left(\int_\Omega \rho(\phi)d\Omega - V_0\right) \qquad (35)$$

$$\tilde{\lambda}_2 = \lambda_2 + \frac{1}{\mu_2}K(\phi) \qquad (36)$$

In the proposed approach to satisfy the Eq. (31) we use the steepest descent method.

$$\frac{d\phi}{dt} = -\psi \ , \phi^0 = \phi_0 \tag{37}$$

Thus, the optimization problem is transformed into an ordinary differential problem with initial value ϕ_0. The simplest approach for solving the Eq. (37) is to use an explicit scheme. But, in this paper, due to the existence of the diffusion term in Eq. (32), we employ a semi-implicit method with the additive operator splitting (AOS) scheme [30, 31], which effectiveness and success have been proved. For updating Lagrange multipliers λ_i and penalty parameters μ_i, we have the following equations,

$$\begin{aligned} \lambda_1^{k+1} &= \lambda_1^{k} + \frac{1}{\mu_1^{k}}(\int_{\Omega} \rho(\phi) d\Omega - V_0) \\ \lambda_2^{k+1} &= \lambda_2^{k} + \frac{1}{\mu_2^{k}} K(\phi) \\ \mu_i^{k+1} &= \alpha \mu_i^{k}, \ i = 1,2 \end{aligned} \tag{38}$$

In next section, some important numerical implementations are presented and then several 2D numerical examples are investigated to verify the robustness and the efficiency of the proposed method.

5. NUMERICAL IMPLEMENTATION

In this section some important issues for implementing of the proposed BLSM are discussed. These implementations are developed in order to improve the performance of the proposed method. In the BLSM, one can represent the design domain in an implicit manner by a binary level set model that is embedded in a scalar function of a higher dimension.

$$S = \{ \ x : \phi(x) = K \ \} \tag{39}$$

Where x is a point in space on the iso-surface ϕ, and K is the iso-value, determined by the values of level set functions of the adjoining phases, usually the average. For example, in two phase problems, the values of the level set function are set 1 or -1 to determine the different areas of the two phases, and then the iso-surface value K is chosen as 0.

The second term in the objective function in Eq. (29) is the regularization term, which plays a very important role. Because of ill-posedness of the original problem, the regularization term is necessary. In this work, we use Total-Variation (TV) regularization. The TV regularization controls both the perimeter of the level set curves and the jumps of the binary level set surfaces. We use the parameter β in this term to control the influence of the regularization term. This parameter plays a vital role not only for the convergence rate of the method, but also for the computed solution.

6. NUMERICAL EXAMPLE

In this section, two widely studied examples in structural topology optimization are used to illustrate the potentials of the present binary level set method. The finite element analysis is based on "ersatz material'' scheme [11], which fills the void areas with one weak material. All numerical examples have the following data; Young's modulus of real material is assumed 1 and ersatz material 10^{-3}. This also means $c_2 = 1$ and $c_1 = 0.001$, Poisson' ratio for two materials is assumed 0.3 and the thickness $t = 1$. The termination criterion of iteration is the relative difference of the objective function values between five successive iterations is less than 0.001.

6.1. Cantilever beam

We consider the well-known cantilever beam structure, as the first numerical example. Figure 1 shows the design domain of this kind of structure. The boundary of the left side is fixed, and a vertical concentrated force F=1N is loaded at the middle of its right free side. The size of the design domain is 80×40 with a squared mesh of size 1×1 and the volume fraction is 50%. First, to benchmark the present binary level set method, we apply the conventional level set method to solve this example. For this method, the size of time step is $\tau = 0.5$ and the level set function is reinitialized every three steps.

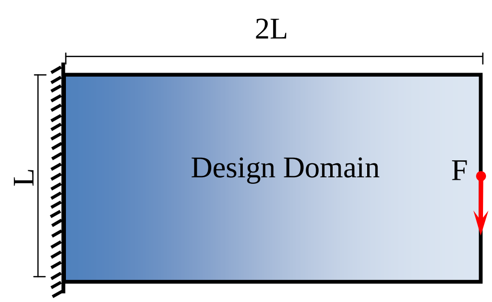

Figure 1. A cantilever beam

The evolution process of the optimal topology and the level set surface is shown in Figure 2. Also figure 3 shows the convergence speed of the objective function and the volume ratio for the cantilever beam. To ensure the stability of the explicit scheme, the time step size has to be set 0.5 in order to meet the CFL condition. This condition was limited the conventional level set method and causing this example is converged after 200 iterations. Moreover, the reinitialization procedure is usually computational expensive and also prevent the nucleation of new holes inside the design domain. Therefore, the final design will become strongly dependent on the initial guess. It is a trouble work to find the appropriate location of these holes at the initial design. These drawbacks limit the first advantages of the implicit representation of the design domain. If we use the binary level set method, we will overcome the disadvantages of the classical LSM. The evolution process of the optimal topology and the binary level set surface is shown in Figure 4. For this method, the size of time step is $\tau = 10$ and the other parameters are considered as $\beta = 1e-4, \mu_1 = 60, \mu_2 = 400$ and $\alpha = 0.9$.

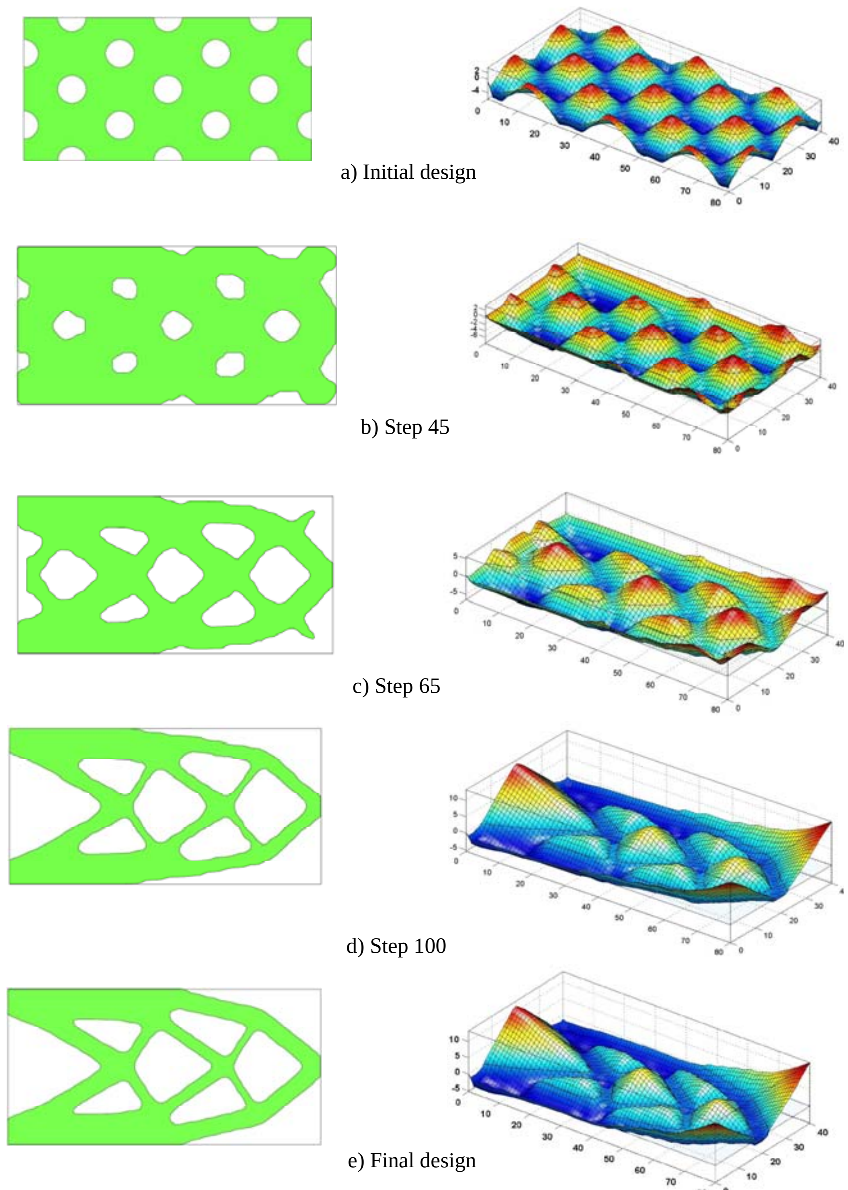

Figure 2. The evolution process of optimal design and the level set function with the conventional LSM

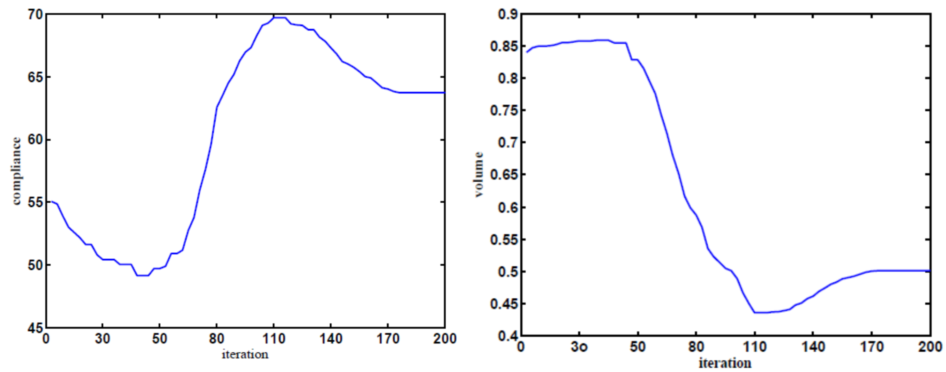

Figure 3. The evolution process of the compliance and the volume ratio

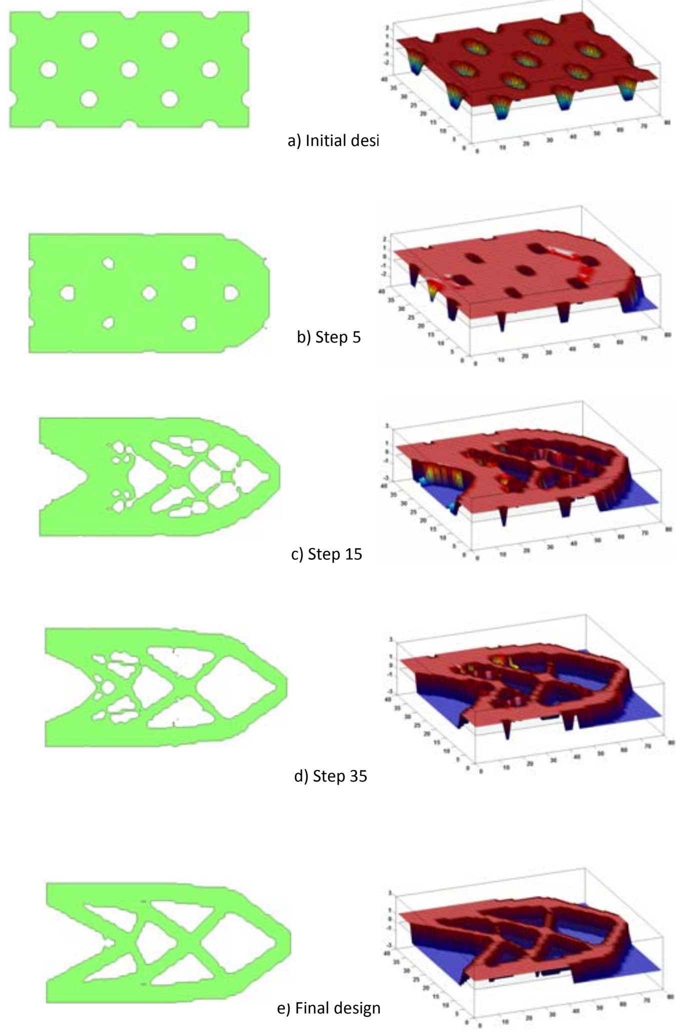

Figure 4. The evolution process of optimal design and the level set function with the BLSM.

The difficult part is to find these parameters which can be chosen after testing different values for these parameters. Therefore, different values may lead to different optimal topology. Figure 5 shows the strain energy and the volume fraction in different iterations. It can be seen that the compliance converges in a fast and stable way due to the present BLSM. Compared with the conventional level set method with the upwind numerical scheme, the proposed BLSM does not involve any reinitialization for the level set function. Also, for this method, we don't need to solve the Hamilton-Jacobi equation, thus, the level set evolution has no apparent restriction by the CFL condition to limit the step size.

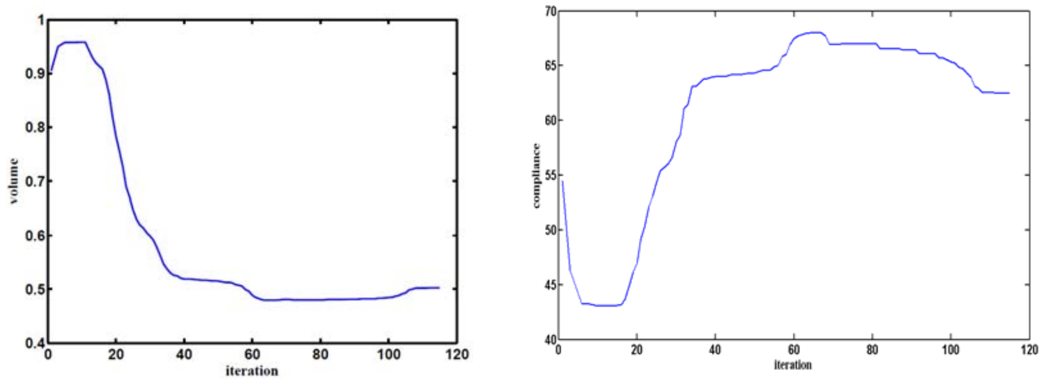

Figure 5. The evolution process of the compliance and the volume ratio

One of the other advantages of the proposed method is the nucleation property. In the BLSM, the nucleation of new holes occurs automatically without any artificial intervention. Different material phases can exchange in the entire design domain, and not just on the interface, thus it causes creating new holes in the process of level set evolution. To represent the aforementioned advantage, we also solve the optimal topology of the cantilever beam example when the initial guess of the design domain is solid. The values we have used for this scheme are β=1e-5, μ_1=50, μ_2=450 and τ=15. The evolution process of the optimal topology and the level set surface is shown in figure 6. Also figure 7 shows the convergence speed of the objective function and the volume ratio for the cantilever beam. The table 1 shows the results obtained with classical LSM and the BLSM with holes and without holes respectively, where N is the total number of steps, T denotes the total time of the optimization and $J(\Omega)$ is the total strain energy.

Table 1. Comparison of the conventional LSM and the BLSM

Schemes	***J(Ω)* (objective)**	***T(s)* (total time)**	***N* (iterations)**
Conventional LSM	63.88	763.72	200
BLSM with holes	62.73	291.14	115
BLSM without holes	64.18	219.87	100

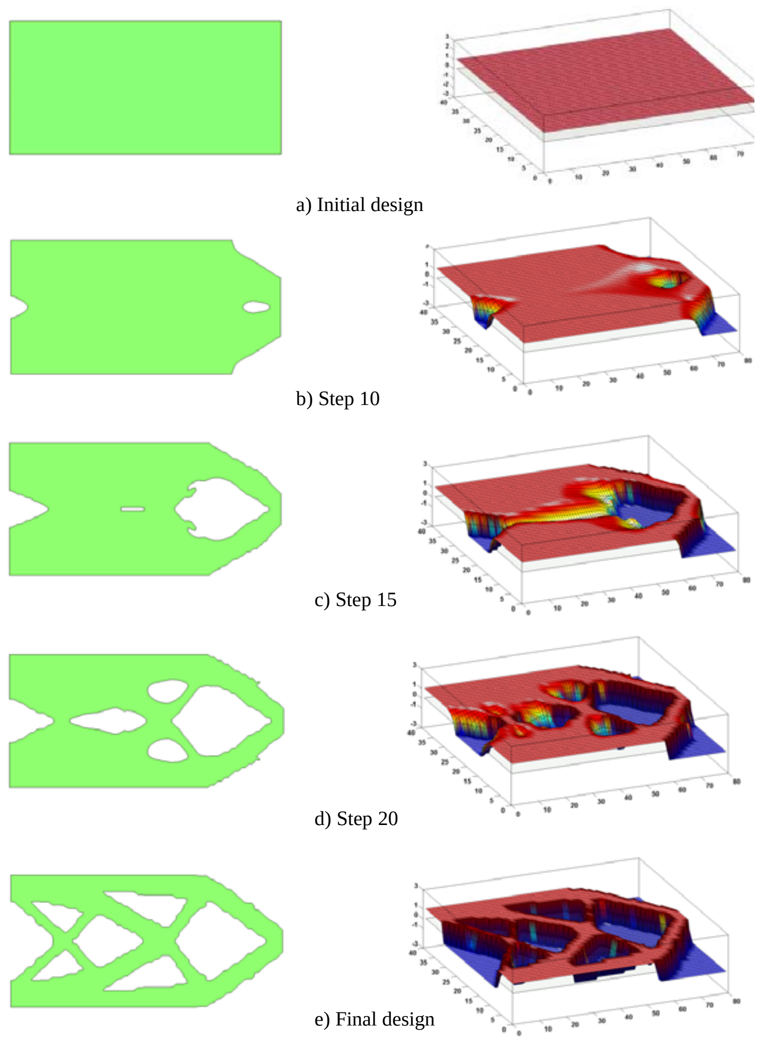

Figure 6. The evolution process of optimal design and the level set function with the BLSM.

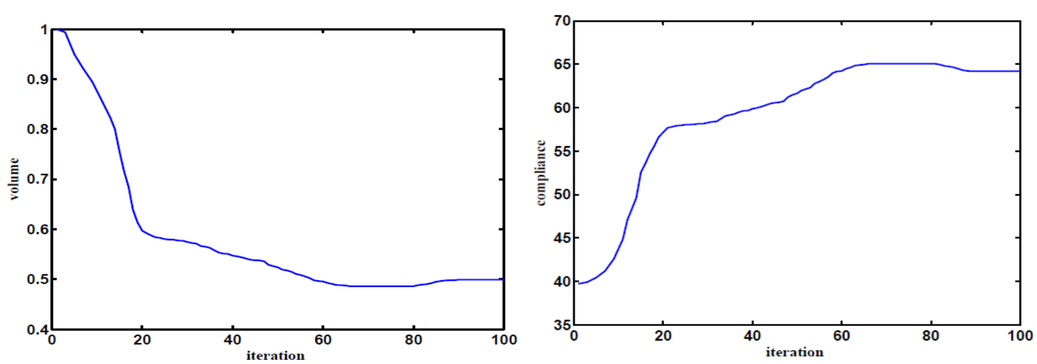

Figure 7. The evolution process of the compliance and the volume ratio

6.2. Michell type structure with multiple loads

As the final example, we want to consider the Michell type structure with multiple loads. Figure 8 shows the boundary condition of this kind of structure. The left corner of the bottom of the design domain is fixed and its right corner is simply supported. Three forces are applied at the equal spaced point at the bottom boundary with $F_1 = 30N$ and $F_2 = 15N$ .The design domain is 80×40 which is discretized with 3200, 1×1 squared elements. The volume fraction is chosen 40%. The BLSM is used for solving this problem without any holes in the initial design domain. The time step size is $\tau = 8$ and other parameters are β=1e–5, μ_1=45, μ_2=450, α=0.95. In Figure 9, the evolution of optimal topology is displayed by using the present BLSM. Also figure 10 shows the convergence speed of the objective function and the volume ratio for the Michell type structure.

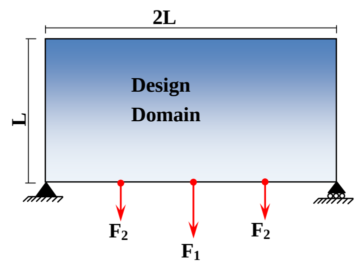

Figure 8. A Michell type structure

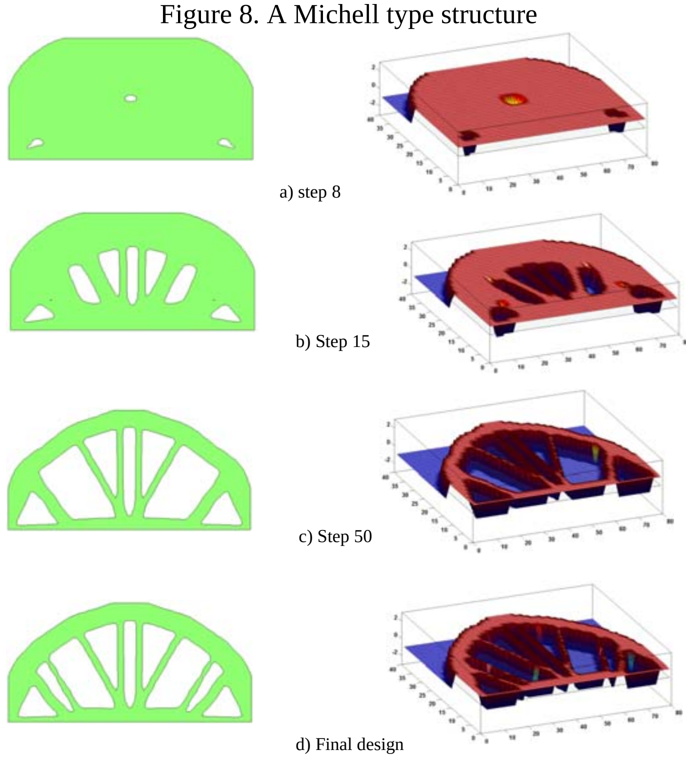

Figure 9. The evolution process of optimal design and the level set function with the BLSM

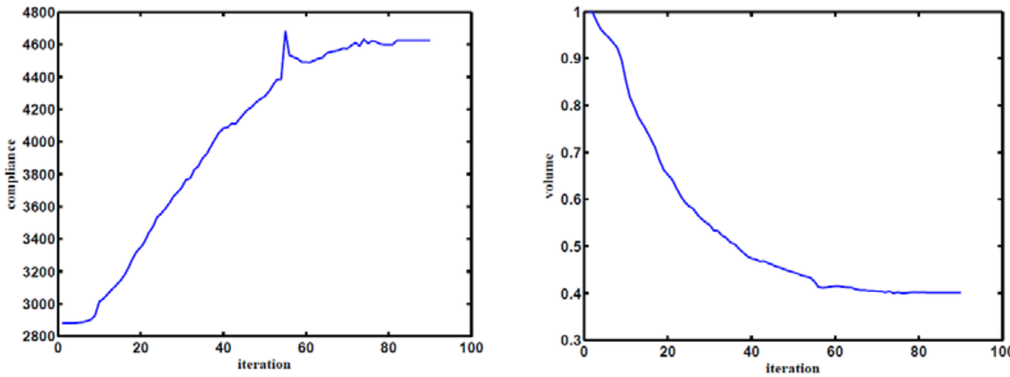

Figure 10. The evolution process of the compliance and the volume ratio

7. CONCLUSION

In this paper, we proposed the binary level set method for the structural topology optimization problems. AS the conventional LSM has several drawbacks, this method can be an interesting alternative for it. In the BLSM, we don't need to solve the Hamilton-Jacobi equation, causing we don't have to meet the CFL condition and the reinitialization procedure. Another great advantage is the nucleation property with which the nucleation of new holes occurs automatically without any artificial intervention. Therefore, in contrary to the Conventional LSM, in the BLSM, the final design is not relevant to the initial guess. Moreover, we also solved the minimization problem with the additive operator splitting (AOS) scheme for an efficient implementation of the numerical computation of the BLSM. Finally, the proposed BLSM is implemented for minimum compliance design of 2D structures, and the numerical examples indicate that the proposed method gives as good results as the conventional level set methods do.

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