INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING
Int. J. Optim. Civil Eng., 2012; 2(1):137-152

KOLMOGOROV-SMIRNOV TEST TO TACKLE FAIR COMPARISON OF HEURISTIC APPROACHES IN STRUCTURAL OPTIMIZATION

A. Csébfalvi*, †
Department of Structural Engineering, University of Pécs, H-7624 Pécs, Boszorkány u. 2. Hungary

ABSTRACT

This paper provides a test method to make a fair comparison between different heuristics in structure optimization. When statistical methods are applied to the structural optimization (namely heuristics or meta-heuristics with several tunable parameters and starting seeds), the "one problem - one result" is extremely far from the fair comparison. From statistical point of view, the minimal requirement is a so-called "small-sample" according to the fundamental elements of the theory of the experimental design and evaluation and the protocol used in the drug development processes. The viability and efficiency of the proposed statistically correct methodology is demonstrated using the well-known ten-bar truss on a set of the heuristics from the brutal-force-search up to the most sophisticated hybrid approaches.

Received: 1 February 2012; Accepted: 30 March 2012

KEY WORDS: statistical comparison; Kolmogorov-Smirnov test; heuristics; meta-heuristics; structural optimization

1. INTRODUCTION

In this paper we present a statistically correct methodology for the fair comparison of optimization results given by different stochastic approaches in structural optimization. According to our opinion, we have to adapt the appropriate elements of the very rigorous protocol used to test new drugs, or compare the effects of different drugs or treatments [1].

---

*Corresponding author: A. Csébfalvi, Department of Structural Engineering, University of Pécs, H-7624 Pécs, Boszorkány u. 2. Hungary
†E-mail address: csebfalvi@pmmik.pte.hu (A. Csébfalvi)

Naturally, the problem of fair comparison, as a fundamental requirement of the real progress is not connected only to the structural optimization. It is a general problem of the heuristic community without detectable results [2-7]. When we use statistical methods in the structural optimization (namely heuristics meta-heuristics, hybrid methods with several tunable parameters and starting seeds) then the usual presentation practice: "one problem - one result" is extremely far from the fair comparison.

From statistical point of view, the least requirement of the fair comparison is a "small-sample" for each investigated approach and an appropriate "nonparametric-small-sample-test" according to the experimental design theory and the very slowly changing structural optimization presentation standard. By definition "small-sample" means 10-30 independent runs, and as a "nonparametric-small-sample-test" the well-known nonparametric Kolmogorov-Smirnov test (KST) can be applied.

In the next section, we briefly describe a hybrid metaheuristic [8, 9] which will be used as a solution generator to illustrate the methodology of the fair statistical comparison. In section 3 we present a very popular structural optimization problem with several solutions given by a wide spectrum of approaches and summarize our impressions about current presentation standard in the structural optimization. Section 4 illustrates the scenario of the statistically fair comparison using a very simple motivating example. The paper closes with some concluding remarks.

2. THE STRUCTURAL OPTIMIZATION PROBLEM

In this paper, we use a highly simplified but very efficient hybrid metaheuristic for structural weight minimization with continuous size variables and displacement and stress constraints to illustrate the statistical problems connected to the fair comparison. The "supernatural" ANGEL method combines ant colony optimization (AN), genetic algorithm (GE) and local search (L) strategy. In the algorithm, AN and GE search alternately and cooperatively in the solution space. The powerful L algorithm, which is based on the local linearization of the constraint set, is applied to yield a better feasible or less unfeasible solution from the solution generated by AN or GE. The highly nonlinear and non-convex large-span and large-scale shallow truss examples show that ANGEL can be more efficient and robust than the conventional gradient based deterministic or the traditional population based heuristic (meta-heuristic) methods in solving structural optimization problems. ANGEL produces highly competitive results in significantly shorter run-times than the previously developed approaches. The benefit of synergy can be demonstrated by standard statistical tests.

Generally, a weight minimizing continuous structural engineering optimization problem can be written as follows:

$$\mathbf{W}(\mathbf{X}) \rightarrow \min \tag{1}$$

$$\mathrm{G}_j(\mathrm{X}) \in \left[\underline{G}_j, \overline{G}_j\right] \quad j \in \{1,2,\mathbf{K},M\} \tag{2}$$

$$X_i \in \left[\, \underline{X}_i, \overline{X}_i \right], \; i \in \{1,2,\ldots,N\} \tag{3}$$

Where $\boldsymbol{X} = (X_1, X_2, \ldots, X_N)$ is the vector of the design variables, $\boldsymbol{W}(\boldsymbol{X})$ is the weight of the structure, $\boldsymbol{G}_j(\boldsymbol{X}),\; j \in \{1,2,\ldots,M\}$ are the implicit response variables (nodal displacements and element stresses).

In ANGEL, a design is represented by set of $\{W, l, \mathbf{X}, \Phi\}$, where W is the weight of the structure $W = \mathbf{W}(\mathbf{X})$, l is the penalty factor $(0 \le l \le 1)$, $\boldsymbol{X}$ is the current set of size variables (cross-sectional areas) for member groups. The fitness function (the pheromone intensity) $\Phi = \boldsymbol{\Phi}(\mathbf{X})$ $(0 \le \Phi \le 2)$ is defined as following:

$$F = \begin{cases} 2 - \dfrac{W - \underline{W}}{\overline{W} - \underline{W}} & l = 1 \\ & \text{if} \\ l & l < 1 \end{cases} \tag{4}$$

where $\overline{W}(\underline{W})$ is the minimal (maximal) weight of the structure, according to the given design space and l is non-smooth function of the "normalized" constraint violation terms:

$$l = \mathbf{max}\left\{ \left\{ \overset{\mathbf{s}}{G}_j, \overset{\mathbf{r}}{G}_j \right\} \mid j \in \{1,2,\ldots,M\} \right\} \tag{5}$$

where

$$\left\{ \overset{\mathbf{s}}{G}_j, \overset{\mathbf{r}}{G}_j \right\} = \left\{ \boldsymbol{max}\left(0, 100 * \frac{\underline{G}_j - G_j}{\underline{G}_j} \right), \boldsymbol{max}\left(0, 100 * \frac{G_j - \overline{G}_j}{\overline{G}_j} \right) \right\} \tag{6}$$

The applied structural model was a large deflection truss model with analytical derivatives. According to the systematic simplification, ANGEL is based only three operators (see Figures 1-3): random selection (AN+GE), random perturbation (AN) and random combination (GE). In the algorithm the traditional mutation operator is replaced by the local search procedure as an "optimal" mutation. That is, rather than introducing small random perturbations into an offspring solution, a gradient based local search is applied to improve the solution until a local optimum or the maximal number of iterations is reached. The main procedure of the proposed meta-heuristic method follows the repetition of these two steps: (1) AN with L and (2) GE with L. In other words, firstly generates an initial population, after that, in an iterative process AN and GE search alternately and cooperatively on the current solution set. The initial population is a totally random set. The random perturbation and random combination procedures which are based on the normal distribution, call the random selection function, to select a “more or less good” solution from the current population. The higher the fitness values of a solution, the higher the chance that it will be selected by the function. The random perturbation procedure uses the continuous inverse method to generate a new solution from the old one. The random combination procedure generates an offspring solution from the selected mother and father solutions. The offspring solution is generated from the combined distribution, where the combined distribution is the weighted sum of the parent's distributions. The two procedures are controlled by the standard deviation, which is decreasing exponentially step by step. In our algorithm in the GA phase, an offspring not necessarily will be the member of the current population, and a parent not necessarily will die after mating. The reason is straightforward, because our algorithm uses very simple rule without explicit pheromone evaporation handling: If the current design is better than the worst solution of the current population than the worst one will be replaced by the better one.

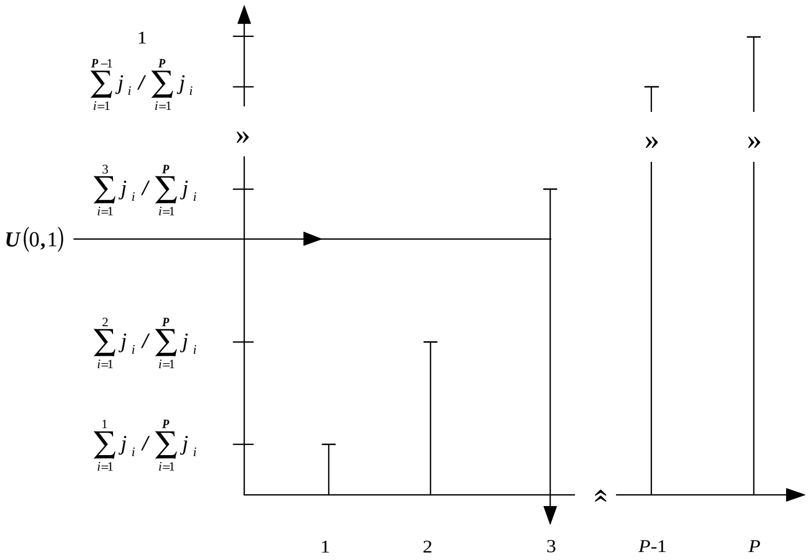

Figure 1. Random selection

ANGEL has only three "tunable" parameters $\{P, G, \bar{I}\}$, where P is the size of the population, G is the number of generations, $\bar{I}$ is the maximal number of local search iterations $(0 \le \bar{I} \le 100)$, and an additional parameter pair $\{\bar{S}, \underline{S}\}$ which defines a exponentially decreasing multiplier in the function of generation g, $g \in \{1, 2, \ldots, G\}$:

$$\boldsymbol{S}(g) = \bar{S} * \boldsymbol{exp}\left(\boldsymbol{log}\left(\frac{\underline{S}}{\bar{S}}\right) * \frac{g-1}{G-1}\right) \tag{7}$$

The parameter pair $\{\bar{S}, \underline{S}\}$ can be kept "frozen" in the algorithm:

$$\{\bar{S}, \underline{S}\} = \{1.0, 0.01\} \tag{8}$$

which means, that ANGEL is practically a "tuning-free" algorithm.

The monotonically decreasing standard deviation function for each design variable can be defined in the following way:

$$S_i^g = S(g) * \left(\overline{X}_i - \underline{X}_i\right), \; g \in \{1,2,\mathbf{K},G\} \; i \in \{1,2,\mathbf{K},N\} \tag{9}$$

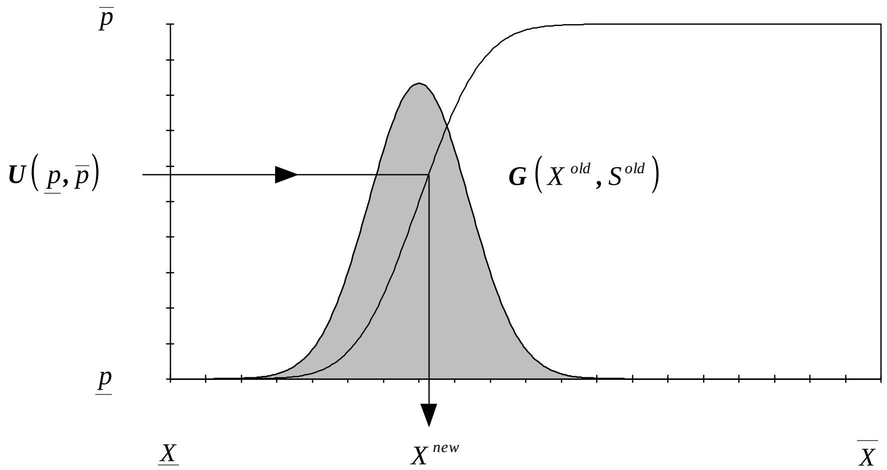

Figure 2. Random perturbation

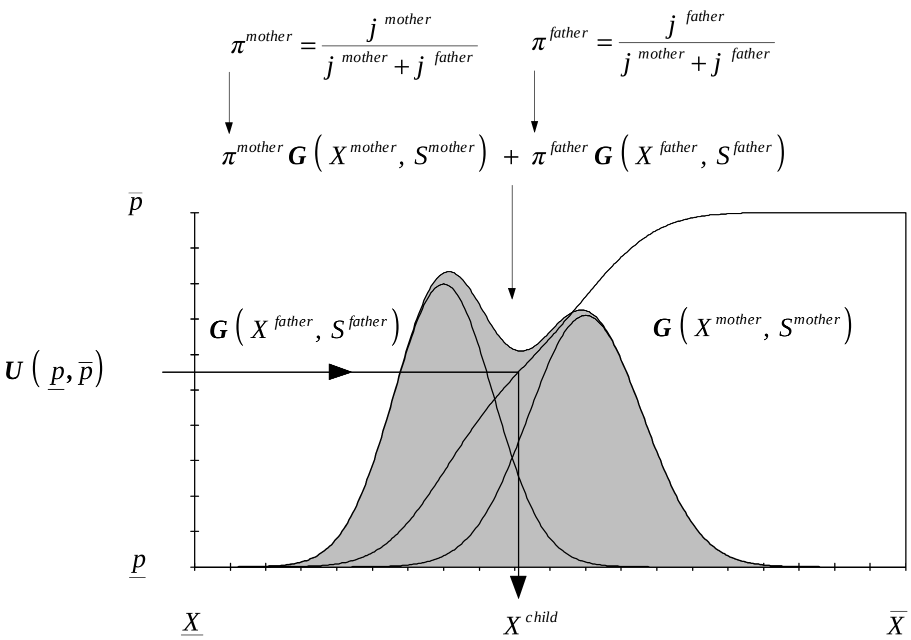

Figure 3. Random combination

3. THE TEST EXAMPLE

In this section, we present computational results for the very popular ten-bar truss weight minimization problem with size variables and displacement and stress constraints for the first load case (see Figure 4). Table 1 presents the input parameters, and Tables 2-4 satisfy the usual solution presentation requirements of structural engineering, but from statistical point of view the presentation is meaningless, therefore unable to characterize the real progress in this area.

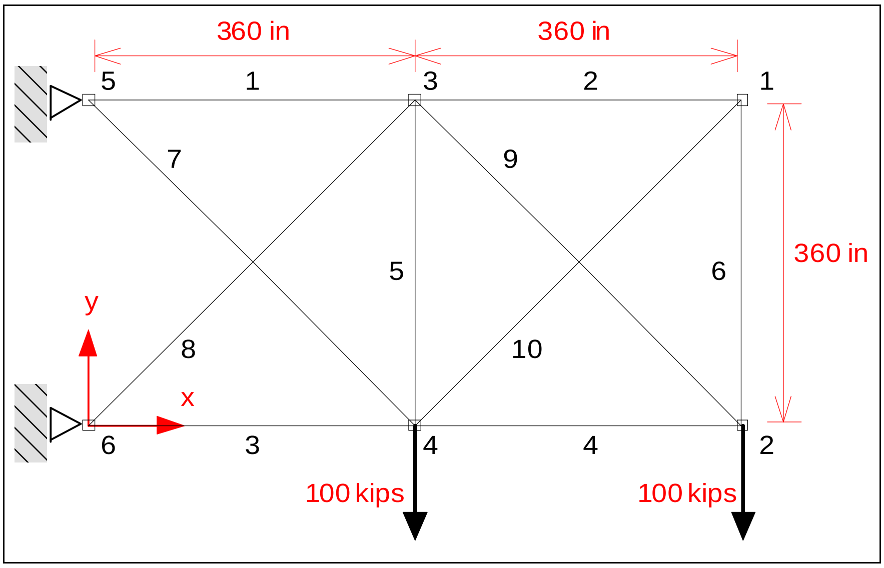

Figure 4. The benchmark-example

Table 1. The initial data of the ten-bar truss benchmark-example

Design variables		$A_i \in [0.1,\ 35.0]\ (in^2)$; $i \in \{1,2,...,10\}$	
Design constraints	Stress constraints	$s_i = \pm 172.4\ MPa\ (\pm 25\ ksi)$; $i \in \{1,2,\mathbf{K},10\}$	
	Deflection constraints	$u_j = \pm 5.08\ cm\ (\pm 2\ in)$; $j \in \{1,2,\mathbf{K},8\}$	
Load	Nodes	X	Y
	2	–	$-445.37kN$ (-100 kips)
	4	–	$-445.37kN$ (-100 kips)
Material properties	Elasticity modulus	$E = 6.895 \times 10^4\ MPa\ (1.0 \times 10^4\ ksi)$	
	Density	$r = 0.0272\ N/cm^3\ (0.1\ lb/in^3)$	

The following important impressions can be concluded of the formerly obtained results presented Tables 2-5. Independently from the totally different solution strategies, the results are more or less the same.

Table 2. Comparison results of the ten-bar truss benchmark-example

Design variables	**Reference [10]**	**Reference [11]**	**Reference [12]**	**Reference [13]**	**Reference [14]**	**Reference [15]**
A_1	30.420	33.432	30.500	30.670	30.730	29,226
A_2	0.128	0.100	0.100	0.100	0.100	0,100
A_3	23.410	24.260	23.290	23.760	23.930	24,182
A_4	14.910	14.260	15.430	14.590	14.733	14,947
A_5	0.101	0.100	0.100	0.100	0.100	0,100
A_6	0.101	0.100	0.210	0.100	0.100	0,395
A_7	8.696	8.388	7.649	8.578	8.542	7,496
A_8	21.084	20.740	20.980	21.070	20.950	21,925
A_9	21.077	19.690	21.820	20.960	21.840	21,291
A_{10}	0.186	0.100	0.100	0.100	0.100	0,100
$W[lb]$	**5084.90**	**5089.00**	**5080.00**	**5076.85**	**5076.70**	**5069.09**

Table 3. Comparison results of the ten-bar truss benchmark-example

Design variables	**Reference [16]**	**Reference [17]**	**Reference [18]**	**Reference [19]**	**Reference [20]**	**Reference [21]**
A_1	30.980	30.031	30.561	30.704	30.598	30.520
A_2	0.100	0.100	0.100	0.100	0.100	0.100
A_3	24.170	23.274	23.170	23.167	23.171	23.200
A_4	14.810	15.286	15.112	15.183	15.196	15.220
A_5	0.100	0.100	0.100	0.100	0.100	0.100
A_6	0.406	0.557	0.549	0.551	0.541	0.551
A_7	7.547	7.468	7.470	7.460	7.463	7.457
A_8	21.050	21.198	21.099	20.978	21.035	21.040
A_9	20.940	21.618	21.527	21.508	21.518	21.530
A_{10}	0.100	0.100	0.100	0.100	0.100	0.100
$W[lb]$	**5066.98**	**5061.60**	**5060.92**	**5060.92**	**5060.90**	**5060.80**

According to the stochastic nature of the solution searching algorithms, the presented solutions are random variable values. In other words, probably we see the first (best) elements from a set of ordered samples, but apart from this we know nothing about the samples (sample size, sample elements, appropriate statistics etc.).

Table 4. Comparison results of the ten-bar truss benchmark-example

Design variables	Reference [22]	Reference [23]	Reference [24]	Reference [25]	Reference [26]	Reference [27]
A_1	30.150	30.307	30.314	31.280	32.966	30,440
A_2	0.102	0.100	0.100	0.100	0.100	0,100
A_3	22.710	23.434	23.261	24.650	22.799	21,790
A_4	15.270	15.505	15.225	15.390	14.146	14,260
A_5	0.102	0.100	0.100	0.100	0.100	0,100
A_6	0.544	0.524	0.550	0.100	0.739	0,451
A_7	7.541	7.437	7.484	7.900	6381	7,628
A_8	21.560	21.079	20.920	21.530	20,912	21,630
A_9	21.450	21.229	21.612	19.070	20,978	21,360
A_{10}	0.100	0.100	0.100	0.100	0,100	0,100
$W[lb]$	**5057.88**	**5056.56**	**5055.30**	**5052.00**	**5013.24**	**4987.00**

According to the presented information, the "which is the best?" question meaningless, so to announce the "winner" when the selection process is based on the order of the first random elements of unknown ordered random sets is unfair and baseless (for example imagine a best drug selection procedure with similar "methodology").

Very interesting to see, that sometimes a pure stochastic method without gradient-based local search is able to rich a size border. There is only one acceptable reason for this statistical nonsense: there is a "hidden" rounding algorithm in the approach activated when the solution is near to a border.

There another problem which is connected to the requirement of fair comparison, and it is the quality of the solutions. In the function of the applied structural model (for example: linear or nonlinear) and the feasibility handling process (rigorous or compliant), a solution which is feasible in one approach, may be more or less unfeasible in another one (see Table 5).

When we use stochastic methods to get "good" but naturally random solutions, we have to use statistical methods to compare the efficiency of the different approaches, analyze a given method, or prove the success of an improvement or a "golden number" setting within a given approach. The key term here is the following: significant difference!

The selection of the appropriate methodology for the statistical comparison is a challenging but sometimes frustrating "problems in the problem".

Table 5. Comparison results of the ten-bar truss benchmark-example

Reference	$W_{App}[lb]$	$W_{Eff}[lb]$	Non-linear model	Linear model

			$u_{\max}[in]$	$m_{\max}[\%]$	$u_{\max}[in]$	$m_{\max}[\%]$
[10]	5089.00	5091.57	0	-1.99844	0	0
[11]	5084.90	5084.81	0	-1.99860	0	0
[12]	5080.00	5080.04	0	-1.99997	0	0
[13]	5076.85	5077.15	0	-1.99989	0	0
[14]	5076.66	5127.62	0	-1.98233	0	0
[15]	5069.09	5069.09	0.03100	-2.00000	0	0.03100
[16]	5066.98	5058.34	1.00889	-1.99982	0	1.00889
[17]	5061.60	5061.66	0.72248	-1.99999	0	0.72248
[18]	5060.92	5060.92	1.02775	-2.00000	0.02917	1.02775
[19]	5060.92	5060.91	1.04748	-2.00000	0.04417	1.04748
[20]	5060.90	5060.90	1.04763	-2.00000	0.04552	1.04763
[21]	5060.80	5060.93	1.06041	-1.99996	0.05660	1.06041
[22]	5057.88	5058.34	1.00889	-2.00181	0.09072	1.00889
[23]	5056.56	5056.59	1.05256	-2.00198	0.09922	1.05256
[24]	5055.30	5055.29	0.00961	-2.00284	0.14222	0.00961
[25]	5052.00	5052.63	0.92375	-2.01949	0.97431	0.92375
[26]	5013.24	5013.24	26.65840	-2.01312	25.20260	26.65840
[27]	4987.00	4999.22	1.33365	-2.02798	1.39896	1.33365

The first problem connected to the statistical methodology is well-known: we have to decide, whether a parametric or nonparametric approach would be the most appropriate in the given case. A "normal" parametric test, for example, may give a totally misleading "winner list" when the normality assumption is invalid.

The second very important methodological problem is connected to the sample size, because in the structural optimization the implicit function evaluation is time consuming (we have to solve the equilibrium equation system in every investigated point): therefore, we have to choose a small-sample or large-sample oriented approach according to the sample size. It is well-known that when the sample size is small, an extremely good but hardly reproducible solution may be statistically meaningless (the play of nature), but when the sample size is large, a small but statistically significant difference between solutions may be a good indicator of the real difference between methods.

4. THE COMPARISON STUDY

In this section, according to our fundamentally methodological point of view we present a simple statistical comparison example for the ten-bar truss problem which can help to understand the fundamental statistical problems connected to the fair comparison. The searching history and a "spider net" like visualization for BF, ANGE, and ANGEL are shown in Figures 5-10, and the ordered samples are presented in Table 6.

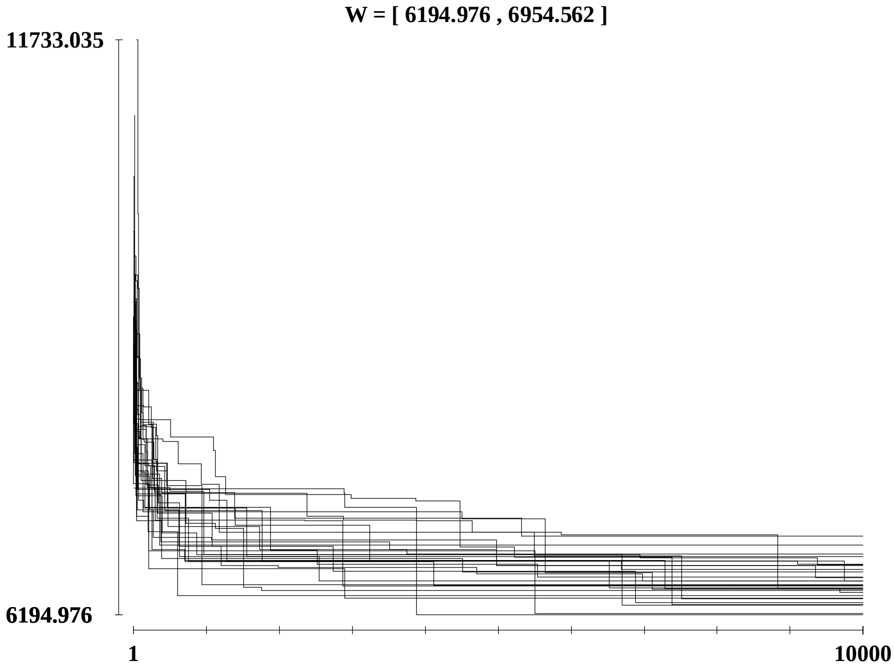

Figure 5. BF searching history

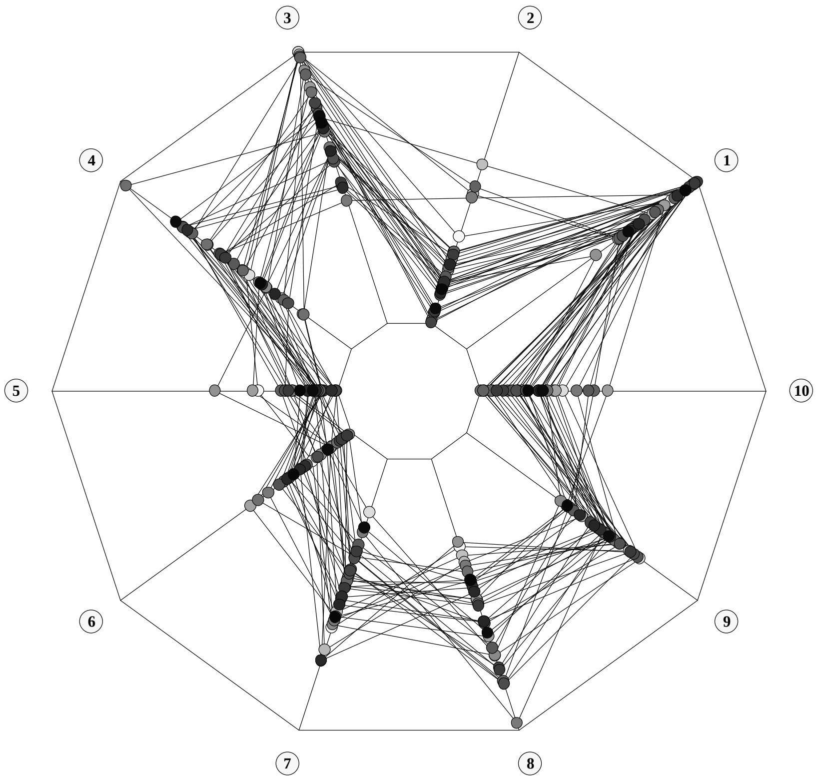

Figure 6. BF spider net

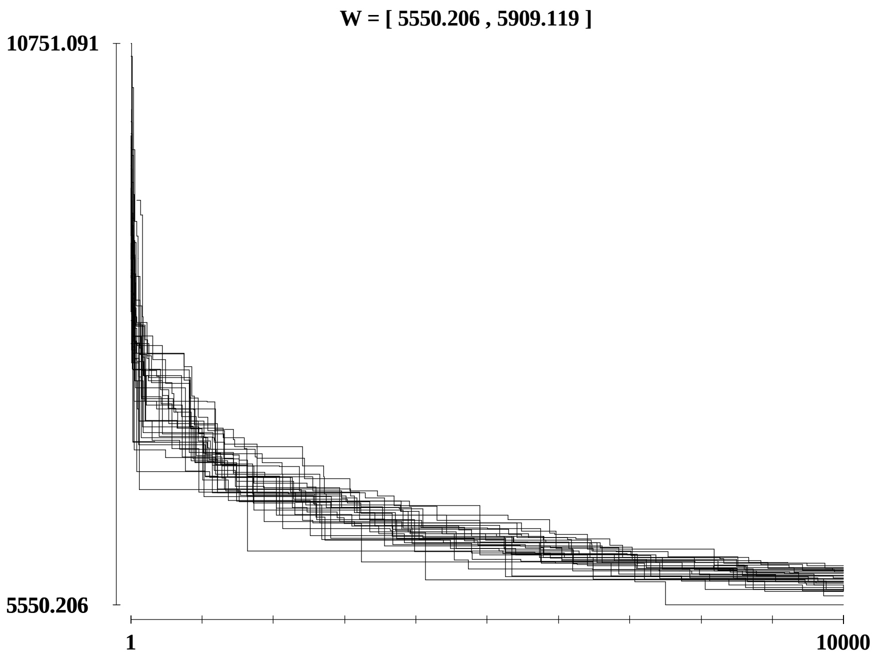

Figure 7. ANGE searching history

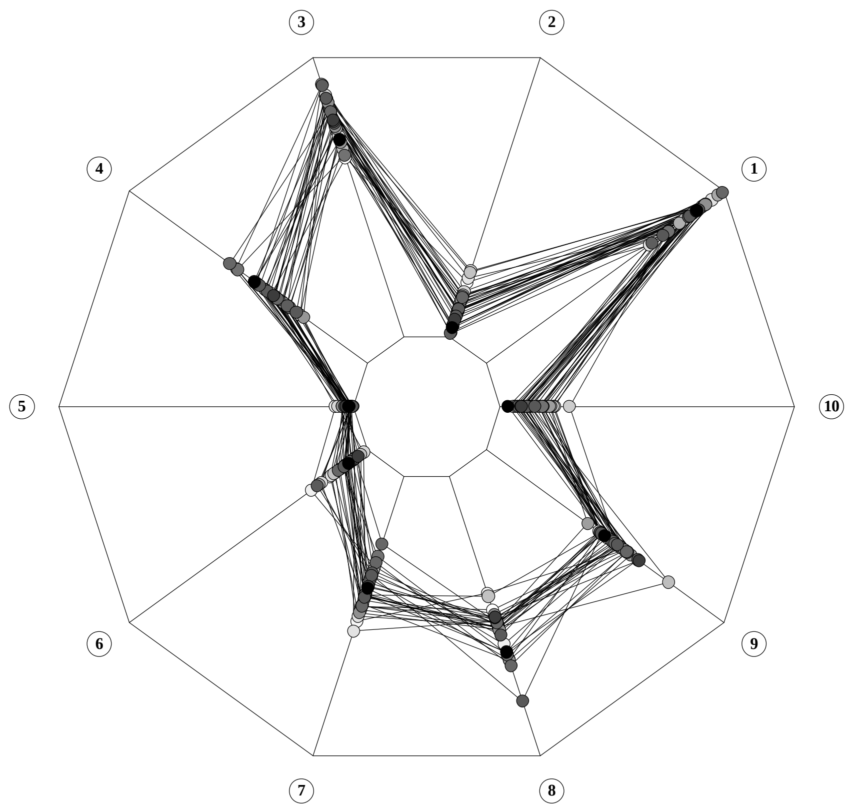

Figure 8. ANGE spider net

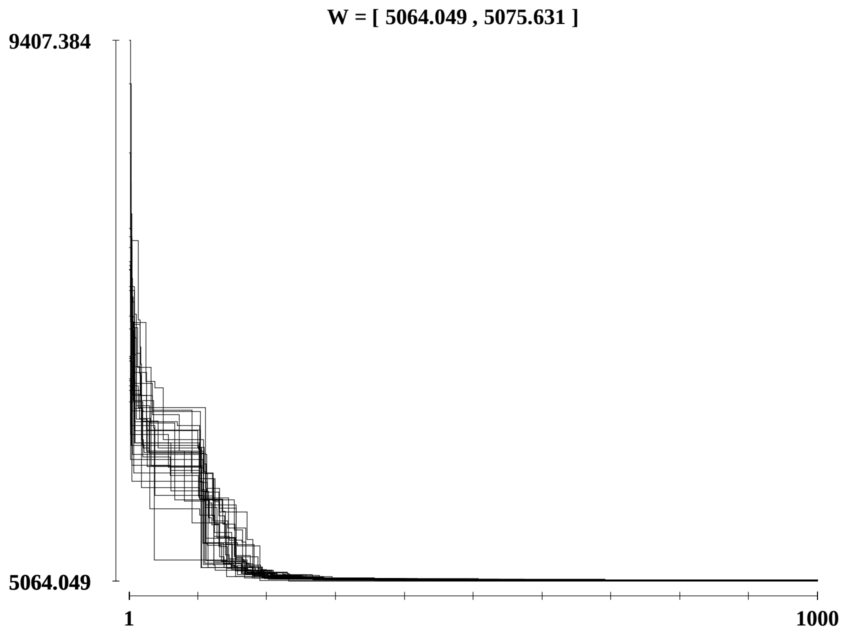

Figure 9. ANGEL searching history

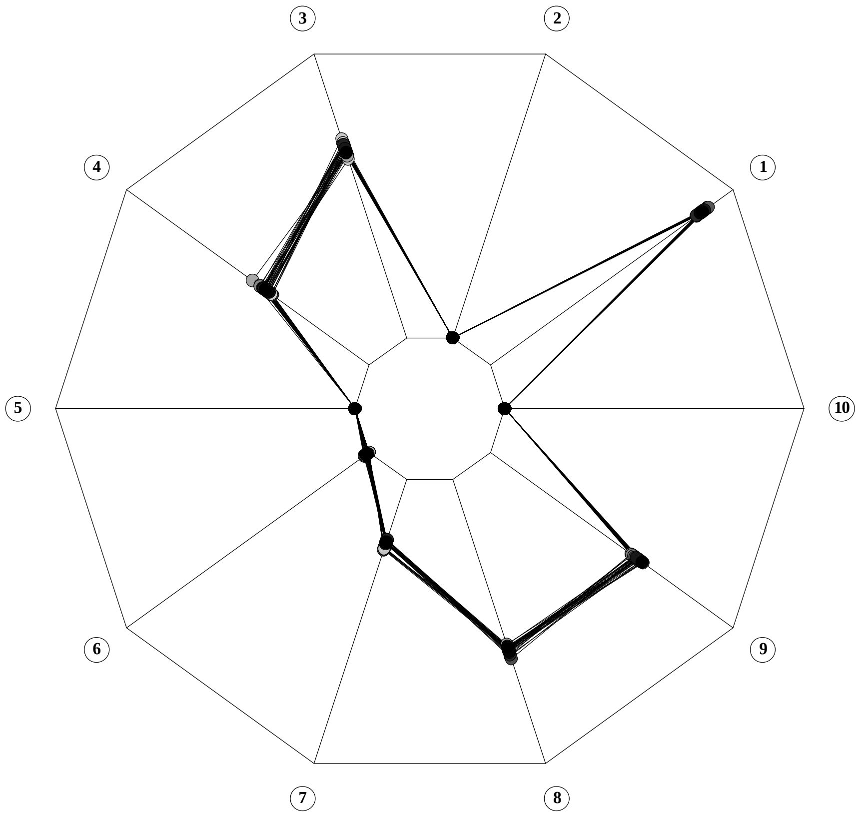

Figure 10. ANGEL spider net

Table 6. Ordered samples of ten-bar truss

	$W[lb]$		
Order	**BFS**	**ANGE**	**ANGEL**
1	6194.976	5550.206	5064.049
2	6210.553	5635.219	5064.265
3	6289.081	5673.552	5065.144
4	6295.906	5675.658	5065.644
5	6310.756	5678.372	5065.884
6	6351.857	5682.852	5066.257
7	6358.005	5694.187	5067.502
8	6383.326	5694.375	5067.983
9	6413.217	5695.173	5069.063
10	6428.944	5710.769	5069.901
11	6440.013	5750.338	5070.327
12	6447.865	5753.376	5070.590
13	6454.067	5763.015	5071.511
14	6474.522	5764.65	5071.636
15	6480.684	5772.142	5071.678
16	6486.377	5774.454	5071.844
17	6520.239	5788.412	5072.885
18	6521.964	5794.04	5072.910
19	6523.422	5797.794	5073.138
20	6552.367	5816.947	5073.252
21	6556.865	5823.144	5073.417
22	6608.343	5839.429	5073.468
23	6633.204	5853.423	5073.545
24	6667.815	5854.036	5073.807
25	6679.523	5864.765	5074.106
26	6683.527	5869.855	5074.503
27	6754.265	5870.178	5074.613
28	6780.186	5874.165	5075.152
29	6869.330	5888.304	5075.290
30	6954.562	5896.135	5075.631
$\min W[lb]$	6194.976	5550.206	5064.049
$\max W[lb]$	6954.562	5909.119	5075.631
$range W[lb]$	759.586	358.913	11.582

From statistical point of view, the minimal requirement of the fair comparison is a "small-sample" for each investigated approach and a "nonparametric-small-sample-test" according to the fundamental elements of the experimental design theory and the very slowly changing structural optimization presentation standard. By definition "small-sample" means 10-30 independent runs, and as a "nonparametric-small-sample-test" the nonparametric Kolmogorov-Smirnov test (NKST) can be applied to avoid the additional methodological problems.

We assume, that we would like to test the effect of the local search (L) in ANGE(L) with the following two settings:

1. $\{P, G, \bar{I}\} = \{1000, 10, \ 0\}$,
2. $\{P, G, \bar{I}\} = \{\ 100, 10, 10\}$.

We ran every model 30 times with "frozen" $\{\bar{S}, \underline{S}\} = \{1.0, 0.01\}$ and $\bar{I} \le 0.001\%$ values. According to the experimental settings in each case 10.000 evaluations is allowed but in different distributions.

We have to note, that setting $\bar{I} = 10$ not necessarily means always 10 iterations. Firstly, we present the "best" solutions when we tried to solve the problem with brutal-force-search (BF) algorithm generating $100 * 10 * 10 = 10.000$ random designs using a uniform random number generator to generate the size variables. NKST for two independent samples from a continuous field tests: $H_0 : F_1(x) = F_2(x)$, that is, the two samples are from populations with the same distribution function.

The alternative hypothesis is the following: $H_A : F_1(x) \neq F_2(x)$ for some x. In our example the result of NKST is trivial: the linearized local search procedure (L) significantly decreases the weight of the structure: $Z = 3.873\,(SIG = 0.000)$.

5. CONCLUSION

In this paper we presented a statistically correct methodology for to compare the efficiency of the different stochastic approaches developed to generate good quality solutions within reasonable time in structural optimization. When we use statistical methods in the structural optimization (namely heuristics or meta-heuristics with several tunable parameters and starting seeds), then the usual presentation practice: "one problem - one result" is extremely far from the fair comparison. From statistical point of view, the minimal requirement is a so-called "nonparametric small-sample test" according to the fundamental elements of the theory of the experimental design and evaluation and the protocol used in the drug development processes. The viability and efficiency of the proposed statistically correct methodology is demonstrated using the well-known ten-bar example on a set of the heuristics from the brutal-force-search up to the most sophisticated hybrid approaches.

REFERENCES

1. Tashkova1 K, Korošec P, Šilc J, Todorovski L, and Džeroski S. Parameter estimation with bio-inspired metaheuristic optimization: modeling the dynamics of endocytosis, *BioMed Central: Systems Biology*, 2011; **5**(159): 1–26.
2. Hooker JN. Testing Heuristics: We Have It All Wrong, *J Heuristics,* 1995; **1**: 33–42.
3. Barr RS, Golden BL, Kelly JP, Resende MGC, Stewart WR, Design and Reporting on Computational Experiments with Heuristic Methods, *J Heuristics,* 1995; **1**: 9–32.
4. Kaveh A, Talatahari S. An improved ant colony optimization for constrained engineering design problems, *Eng Comput,* 2010; **27**(1) 155–82,
5. Kaveh A, Talatahari S. A novel heuristic optimization method: Charged system search, *Acta Mech*, 2010; **213**(3-4): 267–89.
6. Kaveh A, Talatahari S. Charged system search for optimum grillage system design using the LRFD-AISC code, *J Constr Steel Res*, 2010; **66**: 767–71.
7. Kaveh A, Zolghadr A. Shape and size optimization of truss structures with frequency constraints using enhanced charged system search algorithm, *Asian J Civil Eng*, 2011; **12**(4) 487–509.
8. Csébfalvi A. Angel method for discrete optimization problems, *Periodica Polytechnica Civil Eng*, 2007; 51/2: 37–46.
9. Csébfalvi A. Multiple constrained sizing-shaping truss-optimization using ANGEL method, *Periodica Polytechnica Civil Eng,* 2011; **55**(1): 81–6.
10. Schmit LA Jr, Farshi B. Some approximation concepts for structural synthesis, *AIAA J*, 1974; **12**(5): 692–9.
11. Venkayya VB. Design of optimum structures, *Comput Struct*, 1971; **1**(1–2): 265–309.
12. Dobbs MW, Nelson RB. Application of optimality criteria to automated structural design, *AIAA J*, 1976; **14**(10): 1436–43.
13. Schmit LA Jr, Miura H. Approximation concepts for efficient structural synthesis, NASA CR-2552, Washington, DC: NASA, 1976.
14. Rizzi P. Optimization of multiconstrained structures based on optimality criteria, *AIAA/ASME/SAE 17th Structures*, *Structural Dynamics*, *and Materials Conference*, 1976.
15. Lemonge ACC, Barbosa HJC. An adaptive penalty scheme for genetic algorithms in structural optimization, *Int J Numer Meth Eng*, 2004; (59): 703–36.
16. Khan MR, Willmert KD, Thornton WA. An optimality criterion method for large-scale structures, *AIAA J*, 1979; **17**(7): 753–61.
17. Haug EJ, and Arora JS, Applied Optimal Design, Wiley: New York, 1979.
18. Lemonge ACC. Application of Genetic algorithms in Structural Optimization Problems. Ph.D. Thesis, Program of Civil Engineering–COPPE, Federal University of Rio de Janeiro, Brazil, 1999, in Portuguese.
19. Li LJ, Huang ZB, Liu F, Wu QH. A heuristic particle swarm optimizer for optimization of pin connected structures, *Comput Struct*, 2007; **85**(7-8) 340–9.
20. Koohestani K, Kazemzadeh Azad KS. An adaptive real-coded genetic algorithm for size and shape optimization of truss structures, in B.H.V. Topping, Y. Tsompanakis, (Editors), *Proceedings of the First International Conference on Soft Computing*

Technology in Civil, Structural and Environmental Engineering, Civil-Comp Press, Stirlingshire, UK, Paper 13, 2009, doi:10.4203/ccp.92.13.
21. Haftka RT, Kamat MP. Elements of Structural Optimization, Martinus Nighoff: Dordrecht, 1985.
22. Lee KS, Geem ZW. A new structural optimization method based on the harmony search algorithm, *Comput Struct*, 2004; **82**(9-10) 781–98.
23. Kaveh A, Talatahari S. Particle swarm optimizer, ant colony strategy and harmony search scheme hybridized for optimization of truss structures, *Comput Struct*, 2009; **87**(5-6) 267-83.
24. Csébfalvi A. A hybrid meta-heuristic method for continuous engineering optimization, *Periodica Polytechnica*, *Ser Civ Eng*, 2009; **53**(2): 93–100.
25. Adeli H, Kamal O. Efficient optimization of plane trusses, *Adv Eng Softw,* 1991; **13**(3): 116–22.
26. El-Sayed ME, and Jang TS. Structural optimization using unconstrained non-linear goal programming algorithm, *Comput Struct*, 1994; **52**(4): 723–7.
27. Galante M. *Structures Optimization by a Simple Genetic Algorithm, in Numerical Methods in Engineering and Applied Sciences*, CIMNE: Barcelona, Spain, 862–870, 1992.