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A THEORETICALLY CORRECT RESOURCE USAGE VISUALIZATION FOR THE RESOURCE-CONSTRAINED PROJECT SCHEDULING PROBLEM

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ABSTRACT

The cumulative resource constraints of the resource-constrained project scheduling problem (RCPSP) do not treat the resource demands as geometric rectangles, that is, activities are not necessarily assigned to the same resource units over their processing times. In spite of this fact, most papers on resource-constrained project scheduling mainly in the motivation phase use a strip packing of rectangles (SPR) like visualization to illustrate the resource allocation. A novice researcher inspired by the "artistic" SPR visualization may think that the "rectangles" are essential elements of the RCPSP, and that the RCPSP is a special counter-intuitive strip packing problem (SPP) which can be solved without explicitly defined strip packing constraints. In this context "artistic" means, that we have to use a "drawing tool" to produce a SPR like visualization, because the standard model of the RCPSP knows nothing about the rectangles. In the RCPSP, the rectangles can be torn vertically and horizontally, which is absurd in the SPP, and the existence of a cumulative solution is only a necessary but not sufficient condition of the existence of the SPR like visualization, as proven by several researchers. Therefore the popular SPR visualization is theoretically wrong and misleading, and hides a real problem, which is connected to the dedicated resource assignment. In this paper, we prove that replacing the rectangles with a set of strips with unit height we can always generate a theoretically correct strip packing of strips (SPS) like dedicated assignment, where dedicated means that each demand unit is served by exactly one resource unit over its duration without "hidden" transfer time and cost.

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KEY WORDS: resource-constrained project scheduling; resource-profile visualization; strip-

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packing of strips; spatial resource constraint; commulated resource constraint

1. INTRODUCTION

In this paper a very popular NP-hard optimization problem namely the resource-constrained project scheduling problem (RCPSP) is considered. Howevere, the RCPSP is very challenging but sometimes very frustrating problem family which is in the centre of interest for several decades in the heuristic community driven by the "good quality solution within reasonable time" slogan.

In order to tackle the RCPSP A single project problem is considered which consists of A real activities $a \in \{1, 2, \mathbf{K}, A\}$ with nonpreemptable duration of D_a periods. The activities are interrelated by precedence and resource constraints. Precedence constraints force an activity not to be started before all its predecessors are finished. These are given by relations $a \to b$, where $a \to b$ means that activity b cannot start before activity a is completed. Furthermore, activity $a = 0$ $(a = A+1)$ is defined to be the unique dummy source (sink).

In order to be processed, activity a, $a \in \{1, 2, \mathbf{K}, A\}$ requires R_{ar} units of resource type r, $r \in \{1, \mathbf{K}, R\}$ during every period of its duration. Since resource r, $r \in \{1, \mathbf{K}, R\}$ is only available with the constant period availability of R_r units for each period, activities might not be scheduled at their earliest (precedence-feasible) start time but later.

Our objective is to schedule the activities such that precedence and resource constraints are satisfied and the makespan of the project is minimized. All parameters are assumed to be non-negative integer valued.

Let $\overline{T}$ denote an upper bound on the precedence and resource feasible project's makespan. The time periods are labeled by consecutive numbers: $1 \le t \le \overline{T}$.

Let S_a denote the start time of activity a, $a \in \{1, 2, \mathbf{K}, A+1\}$. Because preemption is not allowed, the ordered set $S = \{S_1, \mathbf{K}, S_{A+1}\}$ defines a schedule of the project. Let $\underline{s}_a$ $(\bar{s}_a)$ denote the earliest (latest) precedence feasible starting time of activity a, $a \in \{1, 2, \mathbf{K}, A+1\}$ according to upper bound $\overline{T}$.

Let S_{as}, where $\underline{s}_a \le s \le \bar{s}_a$, denotes a zero-one decision variable:

$$S_{as} = \begin{cases} 1 & if\ activity\ a\ is\ started\ in\ period\ s \\ 0 & otherwise \end{cases},\ a \in \{1, 2, \mathbf{K}, A+1\}. \tag{1}$$

According to the applied notation:

$$S_a = \sum_{s=\underline{s}_a}^{\bar{s}_a} s * S_{as},\ \sum_{s=\underline{s}_a}^{\bar{s}_a} S_{as} = 1,\ a \in \{1, 2, \mathbf{K}, A+1\}. \tag{2}$$

Let $PS = \{a \to b \mid a \ne b,\ a \in \{0, \mathbf{K}, A\},\ b \in \{1, \mathbf{K}, A+1\}\}$ denote the set of network (predecessor-successor) relations.

A schedule is network (precedence) feasible if it satisfies the predecessor-successor relations:

$$S_a + D_a \le S_b,\ a \to b \in PS\ . \tag{3}$$

Let denote the set of active activities in period t

$$A_t = \left\{ a \mid S_a \le t < S_a + D_a \right\},\ t \in \left\{ 1, \ldots, \overline{T} \right\}, \tag{4}$$

and let denote the amount of resource r used in period t

$$U_{tr} = \sum_{a \in A_t} R_{ar}\ ,\ t \in \left\{ 1, \ldots, \overline{T} \right\},\ r \in \left\{ 1, \ldots, R \right\}. \tag{5}$$

A network feasible schedule is resource feasible if it satisfies the following relations:

$$U_{tr} \le R_r,\ t \in \left\{ 1, \ldots, \overline{T} \right\},\ r \in \left\{ 1, \ldots, R \right\}. \tag{6}$$

where U_{tr} is the height of the bar in period t in the resource usage histogram (resource profile) of r.

Let $\boldsymbol{S}$ denote the set of all resource feasible schedules according to upper bound $\overline{T}$. The RCPSP can be described by the following 0-1 linear programming model (see e.g. Pritsker et al. [1]):

$$\left\{ \boldsymbol{min}\, S_{A+1} \mid S \in \boldsymbol{S} \right\}. \tag{7}$$

In the model of the RCPSP, the cumulative resource constraints do not treat the resource demands as geometric rectangles, that is, activities are not necessarily assigned to the same resource units over their processing times. In spite of this fact, most papers on resource-constrained project scheduling mainly in the motivation phase use a strip packing of rectangles (SPR) like visualization to illustrate the resource allocation. In the RCPSP, the rectangles can be torn vertically and horizontally, which is absurd in the SPP, and the existence of a cumulative solution is only a necessary but not sufficient condition of the existence of the SPR like visualization, as proven by several researchers [2-4]. Therefore the popular SPR visualization is theoretically wrong and misleading, and hides a real problem, which is connected to the dedicated resource assignment. Unfortunately, the published counter examples were unable to change the presentation practice of the project scheduling community, it remained essentially the same (see e.g. Alcaraz and Maroto [5], Hartmann [6], Valls and Ballestín [7], Palpant et al. [8], Lambrechts et al. [9]). A rigorous paper (Herroelen and Leus [10]) trying to identify and illuminate the popular misconceptions about project scheduling, illustrates the possible misconceptions with totally misleading SPR visualizations. The similar thing is true for two "bibles": In the books of Demeulemeester and Herroelen [11] and Neumann, Schwindt, and Zimmerman [12] all of the resource profile visualizations are wrong and misleading.

The remainder parts of the paper is organized as follows. In Section 2 we present a motivating example. In Section 3 we propose a new theoretically correct strip packing of strips (SPS) like resource usage visualization for the RCPSP. Finally, Section 4 presents the conclusions and future work plans.

2. A MOTIVATING EXAMPLE

The visualization problem will be illustrated by example schedules generated for a project shown in Table 1 and Figure 1. In Table 1 the set of immediate predecessors of activity a, $a \in \{1, 2, \mathbf{K}, A\}$ is denoted by IP_a. The project is based on a counter-example introduced by Hujter [2] and investigated by Poder et al. [13] in a different context. In this paper we redefined the original counter example as a standard RCPSP without essential modifications. The project consists of only eight real activities.

Table 1. Project description

a	D_a	$\underline{s}_a$	$\bar{s}_a$	R_{a1}	IP_a
0	1	0	0		
1	5	1	9	2	{0}
2	8	1	9	2	{0}
3	6	6	15	1	{1}
4	5	6	14	1	{1}
5	2	9	17	1	{2}
6	3	9	17	1	{2}
7	2	11	19	2	{4, 5}
8	1	12	20	2	{6}
9	1	21	21		{3, 7, 8}

The activities are represented by bars, the predecessor-successor relations by lines. There are only one resource type and four units are available from this resource type in each period. The activities, periods and resource units are labeled by consecutive integer numbers. In Figure 1, the "random" schedule is resource feasible.

The resource usage histogram as a presentation tool is a theoretically correct visualization of the resource usage in the standard RCPSP model. Any other visualization may result in a more complicated model. We have implemented the applied program in Visual Basic® 6.0, to solve the mixed integer linear programming (MILP) problems a state-of-the-art callable solver (ILOG® CPLEX 12.2) was used. The visualizations presented in this paper are resizable Windows® meta-files. The input of the solver and the visual representation of the output have been generated by the program according to the selected model type.

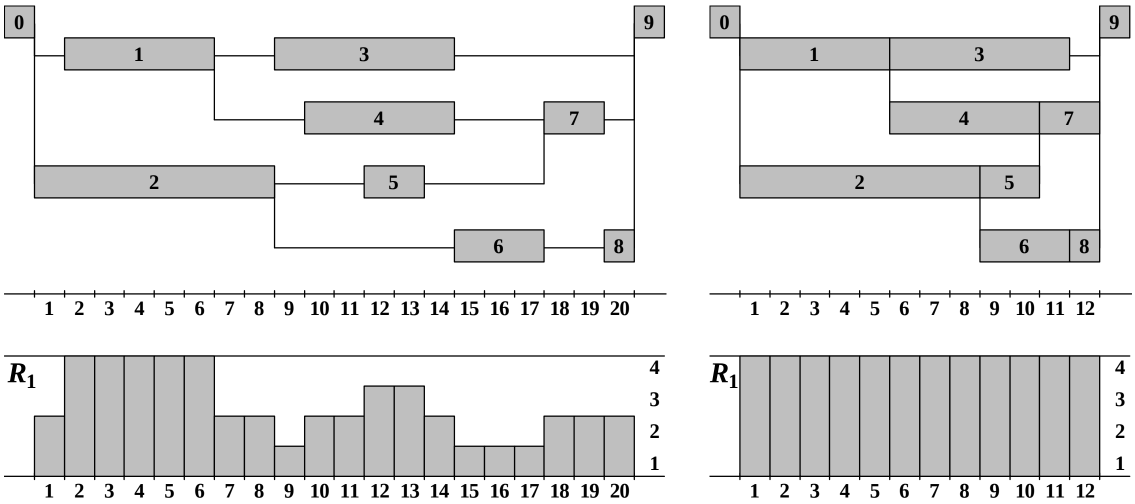

Figure 1. A random and the makespan minimal resource feasible schedules

Using the standard cumulative resource constraints the optimal makespan is 12 and the resource profile is a dream (every resource unit is working continuously from the first period up to the last one). When we reformulate the model according to the standard SPR visualization and solve the makespan minimization problem again we get a different solution with longer makespan as shown in the left side of Figure 2. The result well illustrates the fact, that the cumulative resource constraint set is a relaxation of the SPR constraint set. In this case the additional non-overlapping constraints enforce each pair of "rectangles" not to overlap. According to the additional constraints, the optimal makespan will be 13. When we replace the rectangles with a set of strips with unit height and prescribe that the strips always move together we receive the original solution again (see the right side of Figure 2), as an illustration of the next section, where we prove that the standard resource usage histogram always can be replaced by the SPS visualization (in other words, the resource usage always can be visualized as a set of strips which not necessarily forms a rectangle). The dedicated assignment constraints enforce to serve each demand unit with the same server (resource) unit. This modification is a description of a natural requirement, namely, we have to manage the execution of the project's activities without "hidden" transfer time and cost.

We have to emphasize, that in the standard RCPSP the resource dimensions have no physical meaning therefore the "closeness" is meaningless. But there are several interesting problems where the adjacency is an essential requirement of the scheduling process (see e.g. Hartmann [14], Kok [15], Duin and Van der Sluis [16]).

According to nature of the investigated problem, the adjacency requirement may be strong (check-in desks for a flight have to be adjacent) or weak (hotel room reservation for a group, when the adjacency is not necessary but desirable). To illustrate this, in Figure 3 we present three resource usage visualizations, which differ only in the resource assignment for activity 2 (the demand units of this activity are indicated by light grey color).

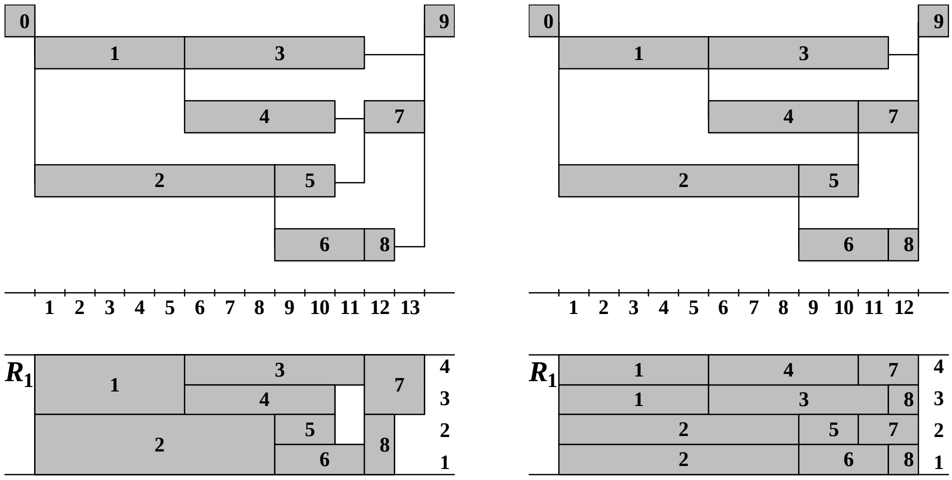

Figure 2. The optimal solutions obtained by SPR and SPS

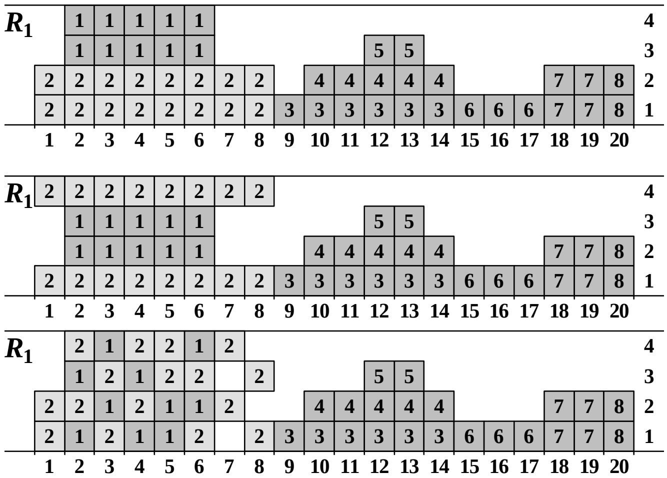

Figure 3. Three resource usage visualizations: the demand units are indicated by light grey color

We have to note, that we can not discriminate among the assignments, when we treat the problem as a standard RCPSP. In other words, these are equally good solutions. But, when we assume that, the visualizations illustrate the booking practice of a small hotel, which has only four rooms, the situation will be totally different. Assuming that activity 2 is a friendly company of two couples which try to spend an eight day long holiday together, then the first assignment is excellent, the second one is a more or less acceptable (the hotel is small and the assignments are dedicated), but the third one is a total failure of the management and a nightmare for the guests according to the permanent move.

3. SPS VISUALIZATION

In this section we propose theoretically correct resource usage visualization for the RCPSP. We will prove that in the RCPSP the existence of a cumulative solution is a necessary and sufficient condition of the existence of the SPS visualization. In this assignment each resource unit is working in dedicated mode without hidden transfer time and cost, but the strips correspond to demands of a given activity not necessarily form a rectangle. Let T , $T \leq \overline{T}$ define the last working period of a resource-feasible schedule.

The constructive proof is very simple:

Firstly, we split the columns of the resource usage histograms into unit squares and distribute the unit squares among the active activities according to their resource demands. Secondly, we arrange the labeled unit squares arbitrarily (naturally, without destroying the resource feasibility).

After that, for each $r \in \{1,2,\mathbf{K},R\}$ histogram and for each $t \in \{2,3,\mathbf{K},T\}$ period from left to right we apply the following two rules in the given order:

(1) We select the active activities which were active in the previous period and connect the corresponding demand units to the previously allocated ones;

(2) We assign the resource demands of the starting activities to the free resource units arbitrarily.

In order to illustrate the essence of the constructive proof, in Figure 4 we show the main steps of the transformation process for the makespan minimal resource constrained solution of our project example. In Figure 3 the already scheduled demand units are indicated by light grey color.

R_1													
	2	2	2	2	2	4	4	4	6	6	7	8	4
	2	2	2	2	2	3	3	3	5	5	7	8	3
	1	1	1	1	1	2	2	2	4	4	6	7	2
	1	1	1	1	1	2	2	2	3	3	3	7	1
	1	2	3	4	5	6	7	8	9	10	11	12	

R_1													
	2	2	2	2	2	4	4	4	6	6	7	8	4
	2	2	2	2	2	3	3	3	5	5	7	8	3
	1	1	1	1	1	2	2	2	4	4	6	7	2
	1	1	1	1	1	2	2	2	3	3	3	7	1
	1	2	3	4	5	6	7	8	9	10	11	12	

R_1													
	2	2	2	2	2	2	2	2	6	6	7	8	4
	2	2	2	2	2	2	2	2	5	5	7	8	3
	1	1	1	1	1	4	4	4	4	4	6	7	2
	1	1	1	1	1	3	3	3	3	3	3	7	1
	1	2	3	4	5	6	7	8	9	10	11	12	

R_1													
	2	2	2	2	2	2	2	2	6	6	7	8	4
	2	2	2	2	2	2	2	2	5	5	7	8	3
	1	1	1	1	1	4	4	4	4	4	6	7	2
	1	1	1	1	1	3	3	3	3	3	3	7	1
	1	2	3	4	5	6	7	8	9	10	11	12	

R_1													
	2	2	2	2	2	2	2	2	6	6	6	8	4
	2	2	2	2	2	2	2	2	5	5	7	8	3
	1	1	1	1	1	4	4	4	4	4	7	7	2
	1	1	1	1	1	3	3	3	3	3	3	7	1
	1	2	3	4	5	6	7	8	9	10	11	12	

R_1													
	2	2	2	2	2	2	2	2	6	6	6	8	4
	2	2	2	2	2	2	2	2	5	5	7	7	3
	1	1	1	1	1	4	4	4	4	4	7	7	2
	1	1	1	1	1	3	3	3	3	3	3	8	1
	1	2	3	4	5	6	7	8	9	10	11	12	

Figure 4. The main steps of the transformation process

The starting assignment is in lexicographical order in the activity index from left to right in the time scale. From period 2 to period 5 there is nothing to do, because the columns satisfy the requirements without rearrangements. From period 6 to period 8, according to the already allocated activity 2, we have to apply rule 1 and rule 2. From period 9 to 10 also there is nothing to do, because in the starting arrangement the demand units of activity 3 and 4 connect to their previously allocated demand units. In period 11 we have to apply rule 1 and rule 2 again. The result reveals the fact, that a makespan minimal dedicated assignment not necessarily means "rectangles".

We have to note, that the resource allocation algorithm, which follows the logic of the constructive proof, can be replaced by a simple MILP. In the MILP formulation, we can exploit the fact, that in the starting schedule, according to essence of the time-oriented approach, the activity starting times are fixed. Our preliminary results show, that this MILP may be a very useful tool in the "best" SPS visualization searching process.

4. CONCLUSION

In this paper, we presented a theoretically correct SPS visualization for the RCPSP. It is known that the cumulative resource constraints of the RCPSP do not treat the resource demands as geometric rectangles, that is, activities are not necessarily assigned to the same resource units over their processing times. In spite of this fact, most papers on resource-constrained project scheduling mainly in the motivation phase use a SPR like visualization to illustrate the resource allocation. As proven by several researchers, the cumulative solution is only a necessary but not sufficient condition of the existence of the SPR visualization. Therefore the popular SPR visualization is theoretically wrong and misleading, and hides a real problem, which is connected to the dedicated resource assignment. In this paper, we proved that replacing the rectangles with a set of strips with unit height we always can generate a theoretically correct SPS like dedicated assignment, where dedicated means that each demand unit is served by exactly one resource unit over its duration without "hidden" transfer time and cost. An open and challenging question is that what would be the "best" SPS visualization from a practical point of view. This question will be investigated in a forthcoming article.

REFERENCES

1. Pritsker A, Watters L, Wolfe P. Multiproject scheduling with limited resources: A zero–one programming approach. *Manag Sci,* 1969; **16**: 93–107.
2. Hujter M. On the dynamic storage allocation problem. Manuscript, Computer and Automation Institute, Hungarian Academy of Sciences, 1990.
3. Gimadi E, Zalyubovskii V, Sharygin P. The Problem of Strip Packing: An asymptotically exact approach. *Russian Math*, 1997; **41**(12): 32-42.
4. Rykov IA, Approaches to solving RCPSP using relaxed problem with consumable resources. In: Waldmann, K.-H., Stocker, U.M. (eds.): *Oper Res Proc,* Springer, Berlin, 2006.
5. Alcaraz J, Maroto C. A robust genetic algorithm for resource allocation in project scheduling. *Ann Oper Res,* 2001; **102**: 83–109.
6. Hartmann S. Project scheduling with multiple modes: A genetic algorithm. *Ann Oper Res,* 2001; **102**: 111–35.
7. Valls V, Ballestín F. A population-based approach to the resource-constrained project scheduling problem. *Ann Oper Res,* 2004; **131**: 305–24.

8. Palpant M, Artigues C, Michelon P. LSSPER: Solving the resource-constrained project scheduling problem with large neighborhood search. *Ann Oper Res*, 2004; **131**: 237–57.
9. Lambrechts O, Demeulemeester E, Herroelen W. Time slack-based techniques for robust project scheduling subject to resource uncertainty. *Ann Oper Res*, 2011; **186**: 443–64.
10. Herroelen W, and Leus R, Identification and illumination of popular misconceptions about project scheduling and time buffering in a resource-constrained environment. *J Oper Res Soc*, 2005; **56**: 102-9.
11. Demeulemeester E, Herroelen W. *Project Scheduling: A Research Handbook*. Kluwer Academic Publishers, Boston, 2002.
12. Neumann K, Schwindt C, and Zimmerman J. *Project Scheduling with Time Windows and Scarce Resources*, second ed. Lecture Notes in Economics and Mathematical Systems second ed., vol. 508 Springer, Berlin, 2003.
13. Poder E, Beldiceanu N, and Sanlaville E. Computing a lower approximation of the compulsory part of a task with varying duration and varying resource consumption. *Eur J Oper Res*, 2004; **153**: 239–54.
14. Hartmann S. Packing problems and project scheduling models: an integrating perspective. *J Oper Res Soc*, 2000; **51**: 1083-92.
15. Kok AL. Spatial Resource Constrained Project Scheduling. Master thesis, University of Twente, 2006.
16. Duin CW, Van der Sluis E. On the complexity of adjacent resource scheduling. *J Scheduling*, 2006; **9**: 49-62.