INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING
Int. J. Optim. Civil Eng., 2011; 1:141-153

MATHEMATICAL AND TECHNICAL OPTIMA IN THE DESIGN OF WELDED STEEL SHELL STRUCTURES

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ABSTRACT

In some cases the optimum is the minimum of the objective function (mathematical optimum), but in other cases the optimum is given by a technical constraint (technical optimum). The present paper shows the both types in two problems. The first problem is to find the optimum dimensions of a ring-stiffened circular cylindrical shell subject to external pressure, which minimize the structural cost. The calculation shows that the cost decreases when the shell diameter decreases. The decrease of diameter is limited by a fabrication constraint that the diameter should be minimum 2 m to make it possible the welding and painting inside of the shell. The second problem is to find the optimum dimensions of a cantilever column loaded by compression and bending. The column is constructed as circular or conical unstiffened shell. The cost comparison of both structural versions shows the most economic one.

Received: February 2011; Accepted: June 2011

KEY WORDS: structural optimization, circular and conical cylindrical shells, cost calculation, buckling of plates and shells, economy of welded structures

1. INTRODUCTION

Cylindrical shells are used in various engineering structures, e.g. in pipelines, offshore structures, columns and towers, bridges, silos etc. The shells can be stiffened against buckling by ring-stiffeners or stringers or orthogonally. The effectiveness of stiffening depends on the kind of load. Many cases of loads and stiffening have been investigated by realistic numerical structural models and design aspects have been concluded by cost comparisons of optimized structural versions [1-3].

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Since in Eurocodes design method for stiffened shell buckling is not given, the design rules of Det Norske Veritas (DNV) are used. In this new investigation newer DNV shell buckling formulae are applied [4].

Optimum design of ring-stiffened cylindrical shells has been treated in [5,6]. Results of model experiments for cylindrical shells used in offshore oil platforms have been published in [7]. In [8] the proposed strength formulation is compared with DNV rules, British Standard BS 5500 and experimental results.

The tripping of open section ring-stiffeners is treated in [9]. Buckling solutions for shells with various end conditions, stiffener distributions and under various pressure distributions have been presented in [10,11].

In [12] the adopted approach aims at simultaneously minimizing the shell vibration, associated sound radiation, weight of the stiffening rings as well as the cost of the stiffened shell. The production, life cycle and maintenance costs are computed using the Parametric Review of Information for Costing and Evaluation (PRICE) model (Price Systems, N.J. Mount Laurel, 1999) without any detailed cost data.

In the optimization process the optimum values of shell diameter and thickness as well as the number and dimensions of ring-stiffeners are sought to minimize the structural volume or cost. In order to avoid tripping welded square box section stiffeners are used, their side length and thickness of plate elements should be optimized.

Besides the constraints on shell and stiffener buckling the fabrication constraints can be active. To make it possible the welding of stiffeners inside the shell the minimum shell diameter should be fixed (2000 mm). The calculations show that the volume and cost decreases when the shell diameter is decreased. Thus, the shell diameter can be the fixed minimum value. Another fabrication constraint is the limitation of shell and plate thickness (4mm).

The remaining unknown variables can be calculated using the two buckling constraints and the condition of volume or cost minimization. The relation between the side length and plate thickness of ring-stiffeners is determined be the local buckling constraint. To obtain the optimum values of variables a relative simple systematic search method is used.

The cost function contains the cost of material, assembly, welding and painting and is formulated according to the fabrication sequence.

Columns or towers are used in many engineering structures, e.g. in buildings, wind turbine towers, piers of motorways, etc. They can be constructed as rectangular boxes or shells. Walls of boxes can be designed from stiffened plates or cellular plates. Shells can be unstiffened or stiffened circular or conical. A ring-stiffened conical shell is treated for external pressure in the case of equidistant and non-equidistant stiffening in [3, 13].

Previous studies have shown that, when the constraint on horizontal displacement of the column top is not active, the unstiffened circular shell can be cheaper than that of stringer stiffened one. In the present study the unstiffened circular shell is compared to the slightly conical one to show the economy of conical shells over the circular ones.

In previous studies the fabrication has been realized by using 3 m long plate elements to form unstiffened shell elements. In the present study 1.5 m wide plate elements are used. Therefore, the shell thicknesses can be varied in more shell parts. With equidistant shell elements of the same thickness the fabrication can be realized more easily.

The optimal thickness for each shell element is calculated from the shell buckling constraint according to the Det Norske Veritas [4] design rules.

In the previous studies the fabrication sequence is designed so that the circumferential welds have been realized for the completely assembled shell. In order to ease the welding inside the shell the fabrication is changed and it is supposed that these welds are welded successively. Thus the next 1.5 m wide shell part is welded to the previous longer structure and so the number of assembled parts is always 2.

Firstly, the conical shell is optimized by using different radii with a constant inclination angle. Secondly, this angle is changed to show its effect. Thirdly, the optimal circular shell radius is sought to minimize the cost.

2. RING-STIFFENED CYLINDRICAL SHELL LOADED BY EXTERNAL PRESSURE

2.1. Characteristics of the optimization problem

Given data: external pressure intensity $p = 0.5$ N/mm^2, safety factor $\gamma = 1.5$, shell length $L = 6000$ mm, steel yield stress $f_y = 355$ MPa, elastic modulus $E = 2.1\times10^5$ MPa, Poisson ratio $\nu = 0.3$, density $\rho = 7.85\times10^{-6}$ kg/mm^3, the cost constants are given separately.

Unknown variables: shell radius R, shell thickness t, number of spacings between ring-stiffeners n, thus, the spacing between stiffeners is $L_r = L/n$, the side length of the square box section stiffener h_r, the thickness of stiffener plate parts t_r.

2.2. Constraint on shell buckling

According to the DNV rules [4]

$$\sigma = \frac{\gamma p R}{t} \le \frac{f_y}{\sqrt{1+\lambda^4}}, \lambda = \sqrt{\frac{f_y}{\sigma_E}} \tag{1}$$

$$\sigma_E = \frac{C\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{L_r}\right)^2 \tag{2}$$

$$C = \psi\sqrt{1+\left(\frac{\rho_1 \xi}{\psi}\right)^2}, \psi = 4, \rho_1 = 0.6 \tag{3}$$

$$\xi = 1.04\sqrt{Z}, Z = \frac{L_r^2}{Rt}\sqrt{1-\nu^2} \tag{4}$$

2.3. Constraint on ring-stiffener buckling

The moment of inertia of the effective stiffener cross-section should be larger than the required one

$$I_x \geq I_{req} \tag{5}$$

The effective shell length between ring-stiffeners is the smaller of

$$L_e = \frac{1.56\sqrt{Rt}}{1+12\frac{t}{R}} \text{ or } L_r \tag{6}$$

The distance of the gravity centre of the effective ring-stiffener cross-section (Figure 1)

$$y_E = \frac{L_e t\left(h_r + \frac{t+t_r}{2}\right) + h_r t_r \left(h_r + t_r\right)}{3t_r h_r + L_e t} \tag{7}$$

The moment of inertia of the effective stiffener cross-section

$$I_x = \frac{t_r h_r^3}{6} + 2t_r h_r \left(\frac{h_r + t_r}{2} - y_E\right)^2 + h_r t_r y_E^2 + \frac{L_e t^3}{12} + L_e t\left(h_r + \frac{t+t_r}{2} - y_E\right)^2 \tag{8}$$

The relation between h_r and t_r is determined by the local buckling constraint

$$t_r \geq \delta h_r, \delta = \frac{1}{42\varepsilon}, \varepsilon = \sqrt{\frac{235}{f_y}} \tag{9}$$

For $f_y = 355$ $\delta = 1/34$, the required t_r is rounded to the larger integer, but $t_{r\min} = 4$ mm. The required moment of inertia

$$I_{req} = \frac{\gamma p R R_0^2 L_r}{3E}\left[1.5 + \frac{3E y_E 0.005R}{R_0^2\left(\frac{f_y}{2} - \sigma\right)}\right] \tag{10}$$

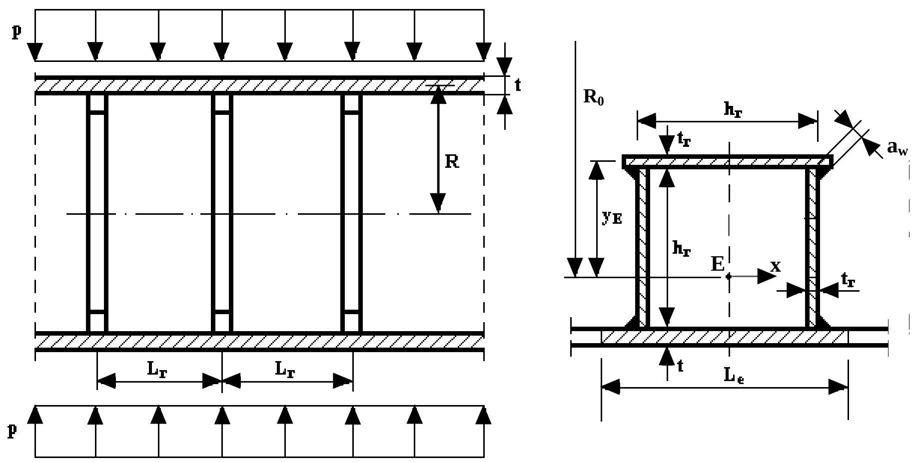

Figure 1. Ring-stiffened cylindrical shell loaded by external pressure

2.4. The cost function

The cost function contents the cost of material, assembly, welding and painting and is formulated according to the fabrication sequence.

The cost of assembly and welding is calculated using the following formula [1-3]

$$K_w = k_w \left(C_1 \Theta \sqrt{\kappa \rho V} + 1.3 \sum_i C_{wi} a_{wi}^n C_{pi} L_{wi} \right) \quad (11)$$

where k_w [\$/min] is the welding cost factor, C_1 is the factor for the assembly usually taken as C_1 = 1 min/kg$^{0.5}$, Θ is the factor expressing the complexity of assembly, the first member calculates the time of the assembly, κ is the number of structural parts to be assembled, ρV is the mass of the assembled structure, the second member estimates the time of welding, C_w and n are the constants given for the specified welding technology and weld type, C_p is the factor of welding position (for downhand 1, for vertical 2, for overhead 3), L_w is the weld length, the multiplier 1.3 takes into account the additional welding times (deslagging, chipping, changing the electrode).

The fabrication sequence is as follows:

(a) Welding the unstiffened shell from curved plate parts of dimensions 6000×1500 mm and of number

$$n_p = \frac{2R\pi}{1500},$$

which should be rounded to the larger integer. Use butt welds of length

$$L_{w1} = n_p L, \;\; \Theta = 3, \kappa_1 = n_p, V_1 = 2R\pi L t, k_W = 1, \quad (12)$$

welding technology SAW (submerged arc welding)

$$\text{For } t = 4\text{−}15 \text{ mm} \quad C_{W1} = 0.1346\times10^{-3} \text{ and } n_1 = 2, \tag{13a}$$

$$\text{for } t > 15 \text{ mm} \quad C_{W1} = 0.1033\times10^{-3} \text{ and } n_1 = 1.9, \tag{13b}$$

$$K_{W1} = k_W\left(\Theta\sqrt{\kappa_1 \rho V_1} + 1.3 C_{W1} t^{n_1} L_{W1}\right) \tag{14}$$

(b) Welding the ring-stiffeners separately from 3 plate parts with 2 fillet welds (GMAW-C – gas metal arc welding with CO_2):

$$K_{W2} = k_W\left(\Theta\sqrt{3\rho V_2} + 1.3x0.3394x10^{-3} a_W^2 L_{W2}\right) \tag{15}$$

where

$$V_2 = 4\pi h_r t_r\left(R - \frac{h_r}{2}\right) + 2\pi h_r t_r (R - h_r) \tag{16}$$

$$L_{W2} = 4\pi(R - h_r), a_W = 0.7 t_r \tag{17}$$

(c) Welding the (n+1) ring-stiffeners into the shell with 2 circumferential fillet welds (GMAW-C)

$$K_{W3} = k_W\left(\Theta\sqrt{(n+2)\rho V_3} + 1.3x0.3394x10^{-3} a_W^2 L_{W3}\right) \tag{18}$$

where

$$V_3 = V_1 + (n+1)V_2, L_{W3} = 4R\pi(n+1) \tag{19}$$

Material cost

$$K_M = k_M \rho V_3, k_M = 1 \text{ \$/kg} \tag{20}$$

Painting cost

$$K_P = k_P S_P, k_P = 28.8x10^{-6} \text{ \$/mm}^2, \tag{21}$$

$$S_P = 2R\pi L + 2R\pi[L - (n+1)h_r] + 2\pi(R - h_r)h_r(n+1) + 4\pi\left(R - \frac{h_r}{2}\right)h_r(n+1) \tag{22}$$

The total cost

$$K = K_M + K_{W1} + (n+1)K_{W2} + K_{W3} + K_P \tag{23}$$

2.5. Results of the optimization

In the following the minimum cost design is obtained by a systematic search using a MathCAD algorithm.

For a shell thickness t the number of stiffeners n is determined by the shell buckling constraint (Eq. (1)) and the stiffener dimensions (h_r and t_r) are determined by the stiffener buckling constraint (Eq. (5)).

The search results for R = 1851 and 1500 (Tables 1 and 2) show that the volume and cost decreases when the radius is decreased. Thus, the realistic optimum can be obtained by taking the radius as small as possible. This minimum radius is determined by the requirement that the internal stiffeners should easily be welded inside of shell, i.e. $R_{\min}$ = 1000 mm. Therefore the more detailed search is performed for this radius (Table 3).

Table 1. Systematic search for R = 1850 mm. Dimensions are in mm. The minimum cost is marked by bold letters

t	n	$\sigma<\sigma_{adm}$ MPa	h_r	t_r	$I_x>I_{req}$ $\times10^{-4}$ mm^4	$V\times10^{-5}$ mm^3	K $
11	7	126<152	180	6	3352>3341	10490	18770
12	6	115<143	180	6	3530>3502	10830	18640
13	5	106<124	190	6	4245>4014	11290	18650
14	4	99<109	200	6	5050>4888	11710	**18620**
15	4	92<121	200	6	5252>4718	12400	19390

Table 2. Systematic search for R = 1500 mm. Dimensions are in mm. The minimum cost is marked by bold letters

t	n	$\sigma<\sigma_{adm}$ MPa	h_r	t_r	$I_x>I_{req}$ $\times10^{-4}$ mm^4	$V\times10^{-5}$ mm^3	K $
8	10	140<157	160	5	1745>1616	6830	13890
9	8	125<140	160	5	1590>1550	6870	13250
10	6	112<115	160	5	1995>1885	7130	**12900**
11	5	102<106	150	5	2109>2102	7480	12950
12	5	93<120	160	5	2217>2003	8050	13570

Table 3. Systematic search for R = 1000 mm. Dimensions are in mm. The optima are marked by bold letters

t	n	$\sigma<\sigma_{adm}$ MPa	h_r	t_r	$I_x>I_{req}$ $\times10^{-4}$ mm^4	$V\times10^{-5}$ mm^3	K $
5	16	150<156	110	4	402>364	3192	8338
6	12	125<141	100	4	353>296	**3177**	7631
7	9	107<123	100	4	387>336	3343	7321
8	7	94<111	100	4	419>400	3579	7244
9	5	83<90	110	4	572>557	3854	**7221**
10	4	75<82	120	4	759>703	4186	7419
11	3	68<69	130	4	982>953	4505	7598

It can be seen from Table 3. that the optima for minimum volume and minimum cost are different. It is caused by the larger value of fabrication (welding and painting) cost. The details of the cost for K = 7221 $ are given in Table 4. (The sum of the welding and painting costs is 4196 $).

Table 4. Details of the minimum cost in $.

K_M	K_{W1}	$(n+1)K_{W2}$	K_{W3}	K_P	K
3025	673	474	665	2384	7221

3. CIRCULAR AND CONICAL SHELLS FOR A CANTILEVER COLUMN LOADED BY AXIAL COMPRESSION AND BENDING

3.1. Constraint on conical shell buckling

According to the DNV rules [4] the buckling of conical shells is treated like buckling of an equivalent circular cylindrical shell.

The thickness, the average radius and the length of the i^{th} equivalent shell part are

$$t_{ei} = t_i \cos\alpha, R_{eai} = \frac{R_i + R_{i+1}}{2\cos\alpha}, \quad L_{ei} = \frac{L_i}{\cos\alpha} , \tag{24}$$

The inclination angle is defined by

$$\tan\alpha = \frac{R_{\max} - R_0}{L_0} \tag{25}$$

The sum of the axial and bending stresses should be smaller than the critical buckling stress

$$\sigma_{ai} + \sigma_{bi} = \frac{N_F}{2R_i \pi t_{ei}} + \frac{H_F \left(\sum_{j=0}^{i-1} L_j + \frac{L_i}{2} \right)}{R_i^2 \pi t_{ei}} \le \sigma_{cri} = \frac{f_y}{\sqrt{1+\lambda_i^4}} \tag{26}$$

where the reduced slenderness

$$\lambda_i^2 = \frac{f_y}{\sigma_{ai} + \sigma_{bi}} \left(\frac{\sigma_{ai}}{\sigma_{Eai}} + \frac{\sigma_{bi}}{\sigma_{Ebi}} \right) \tag{27}$$

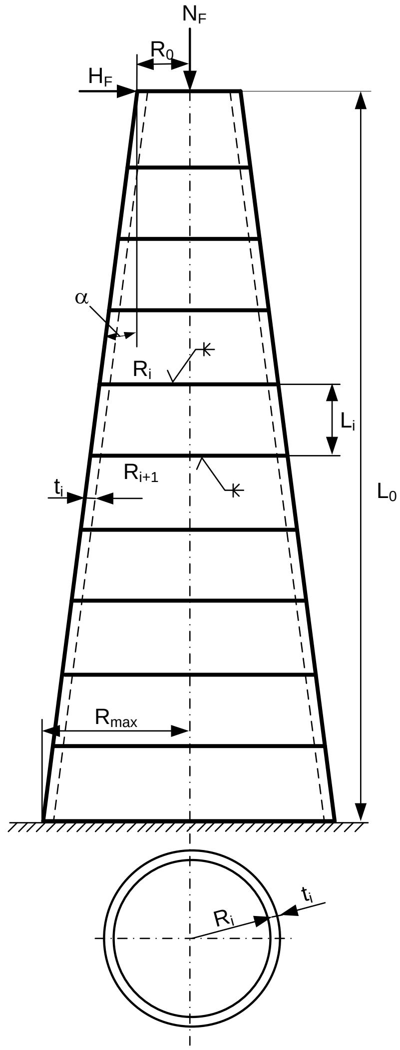

Figure 2. Conical shell cantilever column loaded by axial compression and bending

The elastic buckling stress for the axial compression is

$$\sigma_{Eai} = C_{ai}\left(1.5 - 50\beta\right)\frac{\pi^2 E}{10.92}\left(\frac{t_{ei}}{L_{ei}}\right)^2 \tag{28}$$

$$C_{ai} = \sqrt{1 + \left(\rho_{ai}\xi_i\right)^2}, \rho_{ai} = 0.5\left(1 + \frac{R_{eai}}{150 t_{ei}}\right)^{-0.5} \tag{29}$$

$$\xi_i = 0.702 Z_i, Z_i = \frac{L_{ei}^2\sqrt{1-\nu^2}}{R_{eai} t_{ei}}, \nu = 0.3 \tag{30}$$

The elastic buckling stress for bending is

$$\sigma_{Ebi} = C_{bi}\left(1.5 - 50\beta\right)\frac{\pi^2 E}{10.92}\left(\frac{t_{ei}}{L_{ei}}\right)^2 \tag{31}$$

$$C_{bi} = \sqrt{1 + \left(\rho_{bi}\xi_i\right)^2},\ \rho_{bi} = 0.5\left(1 + \frac{R_{eai}}{300 t_{ei}}\right)^{-0.5} \tag{32}$$

Note that the residual welding distortion factor is $1.5 - 50\beta = 1$ when t > 9 mm. The detailed derivation of it is treated in [2].

3.2. The cost function

The cost function contains the cost of material, forming of plate parts into conical or circular shell elements, welding and painting and is formulated according to the fabrication sequence.

The material cost is given by

$$K_M = k_M \rho V, k_M = 1.0 \text{ \$/kg},\ \rho = 7.85\times10^{-6} \text{ kg/mm}^3 \tag{33}$$

$$V = 2\pi \sum_{i=1}^{10} R_{eai} L_{ei} t_i \tag{34}$$

The cost of forming of plate parts into conical or circular shell elements

$$K_F = k_F \Theta \sum_{i=1}^{10} e^{\mu_i},\ \mu_i = 6.8582513 - 4.52721 t_i^{-0.5} + 0.009531996\left(2R_{eai}\right)^{0.5} \tag{35}$$

The coefficient for the complexity of assembly is $\Theta = 3$. The specific fabrication cost factor is taken as k_F = 1.0\$/min.

For a shell element 3 axial butt welds are needed (GMAW-C – Gas Metal Arc Welding with CO_2)

$$K_{W0i} = k_F\left(\Theta\sqrt{\kappa\rho V_i} + 1.3x0.152x10^{-3} t_i^{1.94} 3L_{ei}\right) \tag{36}$$

The number of assembled elements is $\kappa = 3$.

Cost of welding of circumferential welds between shell elements. The welding is performed successively, so one weld is connecting only two parts in each fabrication step.

$$K_{Wi} = k_F \left(\Theta \sqrt{2\rho \left(\sum_{j=1}^{i-1} V_j + V_i \right)} + 1.3x0.152x10^{-3} t_i^{1.94} 2\pi R_i \right) \tag{37}$$

Cost of painting

$$K_P = k_P 4\pi \frac{R_{\max} + R_0}{2} L_0, k_P = 28.8x10^{-6} \text{ \$/mm}^2. \tag{38}$$

The total cost

$$K = K_M + K_F + \sum_{i=1}^{10} K_{W0i} + \sum_{i=1}^{10} K_{Wi} + K_P \tag{39}$$

3.3. Numerical data and results

L_0 = 15 m, this height is divided in 10 shell parts, each length of L_i = 1500 mm. This uniform length is selected for easy fabrication. N_F = 3400 kN, $H_F = 0.1N_F$, f_y = 355 MPa, E = 2.1×10^5 MPa.

The calculation is performed by using a MathCAD algorithm. Results are given in Tables 5, 6 and 7.

Table 5. Cost parts ($) of conical shells of inclination angle 2.86^0 for different radii (mm)

R_0	R_{max}	K_M	K_{F0}	K_{W0}	K_W	K_P	K
750	1500	26300	19895	9702	14750	6107	76754
850	1600	25660	19360	8300	13753	6650	73723
1050	1800	**24750**	18492	6536	12300	7736	69814
1250	2000	24790	17974	5664	11796	8822	**69046**
1450	2200	25320	17709	5191	11640	9907	69767
1650	2400	26090	17565	4881	11754	10990	71280

In Table 5 the minimum material cost (volume) and total cost are marked by bold letters. It can be seen that the minimum volume and minimum cost correspond to different radii. This difference is caused by high fabrication costs. The optimum is found, since the decrease of radii causes increase of thicknesses, which increases the material and welding cost, on the other hand the increase of radii causes increase of material and painting cost.

Table 6. Cost parts ($) of conical shells of different inclination angles (the average radius is 1625 mm)

Angle	R_0	$R_{\max}$	K_M	K_{F0}	K_{W0}	K_W	K_P	K
4.38^0	1050	2200	24870	17961	5676	11582	8822	**68911**
6.65^0	750	2500	25160	18246	5920	11424	8822	69572

The thicknesses for the optimal conical shell of inclination angle 4.38^0 are from above as follows: 18, 19, 20 and all others 21 mm.

Table 7. Cost parts ($) of circular shells for different radii. The minimum cost is marked by bold letters

$R_0 = R_{\max}$	K_M	K_{F0}	K_{W0}	K_W	K_P	K
1450	25750	18661	7070	13640	7872	72993
1650	25500	17960	5825	12393	8957	**70635**
1750	25500	17920	5596	12385	9500	70901
1850	25730	17809	5333	12250	10040	71162

The thicknesses for the optimal circular shell of radius 1650 mm are as follows: 14,15,17,18,20,21,23,24,26,27 mm.

4. CONCLUSIONS

In the first problem, the structural volume and the cost decrease when the shell radius is decreased. Thus, the shell radius should be taken as small as possible. The minimum radius is determined by the fact that the internal ring-stiffeners should welded into the shell ($R_{\min}$ = 1000 mm).

The shell thickness and the number of ring-stiffeners can be calculated using the constraint on shell buckling. In order to avoid ring-stiffener tripping, welded square box section rings are used. The dimensions of the rings can be determined from the constraint on ring-stiffener buckling. The constraints on buckling are formulated according to the newer DNV design rules.

In the cost function the costs of material, assembly, welding and painting are formulated. The welding cost parts are calculated according to the fabrication sequence. The optima for minimum volume and minimum cost are different, since the fabrication cost parts are relative high as compared to the whole cost.

The ring-stiffening is very effective, since in the case of n = 1 (only 2 end stiffeners) the required shell thickness is t = 18 mm, the volume is $V = 7144\times10^{-3}$ mm^3 and the cost is K = 10450$, i.e. the cost savings achieved by ring-stiffeners is (10450−7221)/10450×100 = 31%.

In the second problem, the following fabrication aspects are considered: the change of shell thickness is designed in equal distances, the circumferential welds are welded successively to ease the welding inside of the shell, only integer numbers are used for shell thicknesses.

The structural volume or components of cost vary with radii in such manner that for both circular or conical unstiffened shells optimum radius can be found.

Three inclination angles of conical shell have been investigated and one of them was optimal.

The comparison of conical and circular shells shows that the cost of optimal conical shell is lower than that of circular one, but the difference is not very large (70635-68911)/ 70635x100= 2.8%.

Acknowledgements: The research was supported by the Hungarian Scientific Research Fund OTKA T-75678 and by the TÁMOP 4.2.1.B-10/2/KONV entitled “Increasing the quality of higher education through the development of research and innovation program” at the University of Miskolc.

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