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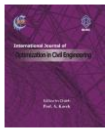

CONFIGURATION OPTIMIZATION OF TRUSSES USING A MULTI HEURISTIC BASED SEARCH METHOD

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ABSTRACT

Different methods are available for simultaneous optimization of cross-section, topology and geometry of truss structures. Since the search space for this problem is very large, the probability of falling in local optimum is considerably high. On the other hand, different types of design variables (continuous and discrete) lead to some difficulties in the process of optimization. In this article, simultaneous optimization of cross-section, topology and geometry of truss structures is performed by utilizing the Multi Heuristic based Search Method (MHSM) that overcome the above mentioned problem and obtains good results. The presented method performs the optimization by dividing the searching space into five subsections in which an MHSM is employed. These subsections are named procedure islands. Some examples are then presented to scrutinize the method more carefully. Results show the capabilities of the present algorithm for optimal design of truss structures.

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KEY WORDS: optimization; truss structures; multi heuristic based search method (MHSM)

1. INTRODUCTION

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Systems in structural engineering usually endure the burden and transfer the forces to supports. In addition to be safe and applicable, these systems should be designed with less cost and material. Therefore, optimal design of skeletal structures like trusses will be categorized to four main divisions: sizing optimization, topology optimization, shape or geometry optimization and configuration optimization.

In the case of sizing, optimization will be carried out for appropriate amounts of cross-sectional areas with fixed nodal coordinates (fixed geometry) and fixed connectivity of members (fixed topology). In application of engineering, standard sections which are available are usually used. In this regard, optimization of separated sections is developed by selecting the members from the available profiles [1-12].

Concerning the shape or geometry, nodal coordinates of truss are adjusted for optimal design. This issue is often considered conjugated with cross-sections in related articles. That is, the main goal in these type of optimization is to find the cross-sections and nodal coordinates [13-17].

In topology optimization, best connectivity of the members is determined. This optimization is often performed by simultaneous optimization of cross-section assuming the geometry of the structure to be fixed [18-25].

For configuration optimization of structures the best cross-section of members in the best configuration and positioning of nodes are determined. In this method, all three variables are included in variables vector of the design. This type of optimization leads corresponds to a large design space and increases the probability of getting trapped in local optimum. To avoid this problem, some researchers assume the design variables (geometry and topology) to be constant and then manage the optimization process. They then select the resulted optimized design as an initial design for total optimization and repeat the process again [24-26]. According to methods which are proposed by some other researchers, meta-heuristic based algorithms were employed to the simultaneous optimization of Sizing, topology and geometry (configuration) of truss structures [27-32]. Lack of cognition of constants and relations in such algorithms and also largeness of searching space, in some cases, result in local optimum.

Parallel global optimisation meta-heuristics using an asynchronous island-model is proposed by Izzo et al for parallel computing in optimization [33]. In this method, initial population is divided to smaller subsections which are called islands. Then, meta-heuristic based methods are assigned to each island and optimization is performed. Subsequently, after some specific repetitions, during migration process best design of each island is substituted to low quality design in other islands. This process is continued until completion of all repetitions based on determined amount for migration interval. Therefore, in each meta-heuristic based method, dependence of results on relations and parameters is decreased remarkably. On the other hand, using the parallel processing systems is accessible as a result of parallel search technique in design space. This ability result in an increase of the speed of optimization operation and more suitable results are obtained.

In order to select current methods in each island, meta-heuristic based algorithms are separately studied, their features are specified and the best ones are selected based on the quality of results and the speed of optimization operation. Then, the performance of the selected algorithms is improved by some additional alterations. Accordingly, the **M**ulti **H**euristic based **S**earch **M**ethod (MHSM) is designed for optimizing the configuration of truss structures.

2. FORMULATION OF TRUSSES CONFIGURATION PROBLEM

Formulation of trusses configuration problem is defined as the following:
Minimize

$$F(A,\xi)=\sum_{i=1}^{Ne}\left(\rho_i \ell_i(\xi) a_i\right) \tag{1}$$

Subject to

$$C1 : \text{Truss is acceptable to the user} \tag{2}$$
$$C2 : \text{Truss is kinematically stable} \tag{3}$$
$$C3 : \sigma_j \leq \sigma_{all\,(Ten)}\ ,\ |\sigma_j| \leq |\sigma_{all\,(Com)}| \quad j=1,2,\ldots,Ne \tag{4}$$
$$C4 : |\Delta_k| \leq |\Delta_k^{max}| \qquad k=1,2,\ldots,Ndof \tag{5}$$

In equation (1), design variables are the cross-sectional areas of the members [**A**], and design variables of geometry [ξ] are defined as following:

$$[\mathbf{A}]=[a_1,a_2,\ldots,a_{Nos}] \quad ; \quad a_i \in S \quad ; \quad i=1,\ldots,Nos \tag{6}$$
$$\xi_m^{min} \leq \xi_m (X_m, Y_m, Z_m) \leq \xi_m^{max} \quad , \quad m=1,\ldots,Nn \tag{7}$$

In equations (1) to (7)
ρ_i: Materials density of member *i*.
$l_i(\xi)$: Length of the *i*th member of truss which depends on geometric variables ξ.
a_i: Cross-section area of the member *i*.
Ne: The number of members of the optimal truss.
S: List of the available profiles whose sizing variables are selected.
Nos: Number of cross-section areas in each design.
Nn: Number of all the geometric variables in optimization problem.
σ_j: Stress in the *j*th element of structure.
σ_{all}: Allowable tensile and compressive stress.
Δ_k: Nodal displacement of *k*th degree of freedom.
Δ_k^{max}: Allowable displacement of *k*th degree of freedom.
ξ_m^{max}: Upper bounds of the design variable ξ_m.
ξ_m^{min}: Lower bounds of the design variable ξ_m.

Constraint C1. Some nodes like supports and points of applied load are remarkably important in optimization of trusses. In other words, in some cases it is required to find the optimal configuration as fixing some nodes. Therefore, design optimization should include free nodes and also nodes with fixed positions. On the other hand, to achieve a practical design is an important issue in truss optimization, and some time it is preferred to have symmetric nodal coordinates even if the design is not quite optimal. Star Graph is used as ground structure in this article [19-21]. This ground structure avoids producing long members by contacting each node to its neighbor nodes using additional members.

Constraint C2. Different designs with various configurations are created in optimization based on meta-heuristic methods, and some of them may be unstable. In this article, unstableness of each design is investigated before it is analyzed. If the design is unstable, it is fined. In this regard, firstly the static formulaes are utilized in order to control the geometric stability, and then if necessary, the stiffness matrix of structure is employed [21]. Kinematically stable structures have symmetric and positive define stiffness matrices [34]. Thus, a structure will be stable if all entries of main diagonal in the decomposed stiffness matrix (in the process of Cholesky method) are positive and non-zero.

Constraint C3. The stress, which is due to composition of loading, should be in allowed region for all the members of the truss with optimal configuration. This allowed amount is determined by code. Therefore, in optimization process, after controlling the stability of structure, stress of each member of truss will be calculated. The amount of stress constraint violation is determined by Eq. (8).

$$C3=\begin{cases} C_3^i=0 \quad if \quad \left|\dfrac{\sigma_i}{\sigma_{all}}\right|-1\le 0 \quad ; \quad i=1,\ldots,Ne \\ C_3^i=\left|\dfrac{\sigma_i}{\sigma_{all}}\right|-1 \quad if \quad \left|\dfrac{\sigma_i}{\sigma_{all}}\right|-1>0 \quad ; \quad i=1,\ldots,Ne \end{cases} \tag{8}$$

In this equation, the quantity of the constraint violation of the members will be summed together when the number of loading combinations is *nlc* [26].

Constraint C4. After analyzing the stable truss and computing the amounts of stress, the displacement of active nodes in each design will be calculated. If the displacement of the *i*th degree of freedom locates in the allowed range, the design will not be fined. Otherwise, the quantity of the constraint violation of displacement is obtained based on Eq. (9).

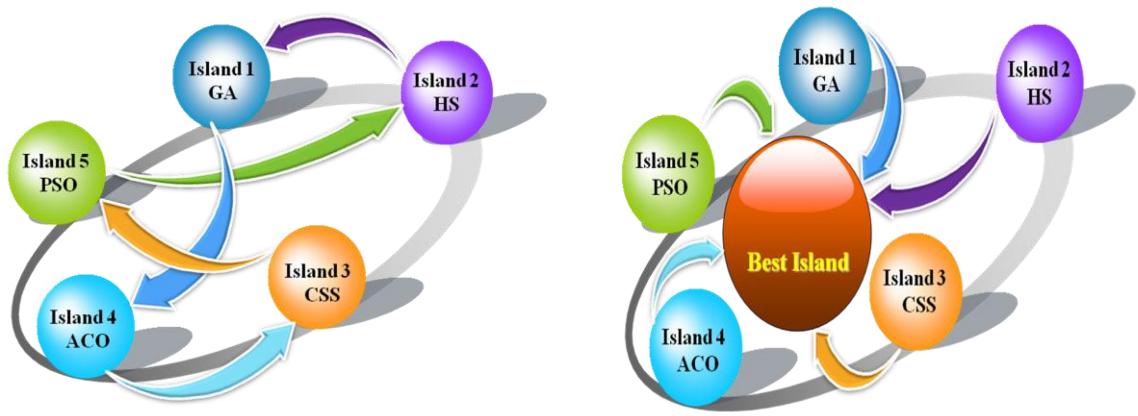
$$C4=\begin{cases} C_4^i=0 \quad if \quad \left|\dfrac{\Delta_i}{\Delta_i^{all}}\right|-1\le 0 \quad ; \quad i=1,\ldots,Ndof \\ C_4^i=\left|\dfrac{\Delta_i}{\Delta_i^{all}}\right|-1 \quad if \quad \left|\dfrac{\Delta_i}{\Delta_i^{all}}\right|-1>0 \quad ; \quad i=1,\ldots,Ndof \end{cases} \tag{9}$$

In these equations, the quantity of the constraint violation of nodal displacements will be summed together when the number of loading combinations is *nlc.*

3. PROPOSED OPTIMIZATION METHODS

Meta-heuristic or heuristic algorithms are intelligent random search methods which search the design space by different points (different design). The logic of these algorithms is in a way that they need to produce various enhanced designs during the optimization. Even though, various parameters in meta-heuristic methods and also the lack of information about the quantity of these parameters in each optimization problem results in local optimum in some cases. That is, finding correct amount of parameters and equations in each meta-heuristic method is a difficulty of optimization based on these algorithms. Various researchers tried to improve each method by offering different solutions and also strove to decrease the impact of parameters of related algorithms [36-40].

In this article, the optimization of configuration for the truss structures is performed by Multi meta-Heuristic based Search Method (MHSM). Reducing the effect of parameters of meta-heuristic algorithms and increasing the domain of searching of the design space is the special feature of this method. According to this method, initial population is divided to several islands. Each island has optimization method with distinctive structure based on a meta-heuristic algorithm. This arrangement of action leads to variation in answers. The proposed MHSM method is performed in two variants of MHSM.1 and MHSM.2 [21 and 41].



(a) MHSM.1 (b) MHSM.2

Figure 1 Two MHSMs of the **M**ulti **H**euristic based **S**earch **M**ethod

In MHSM.1, based on the migration rate, the best members of each island after several specific generations are alternatively transferred to other islands based on migration interval. Each subpopulation, in migration process, has a random destination which be identified in each period of migration. Migration operator, transfer a specific percent of subpopulation to another island and substitute a low quality member. Each existed meta-heuristic method, after migration process, combines migrated and residual members in order to obtain a population of higher quality. Due to the presence of migration mechanism in MHSM.1, answers have remarkable diversity during search process. Hence, each optimization problem is investigated by several methods and the design space is immediately searched. Then, the best results are shared among other islands and new members are allocated to each island. Figure 1 shows the optimization process based on MHSM.1.

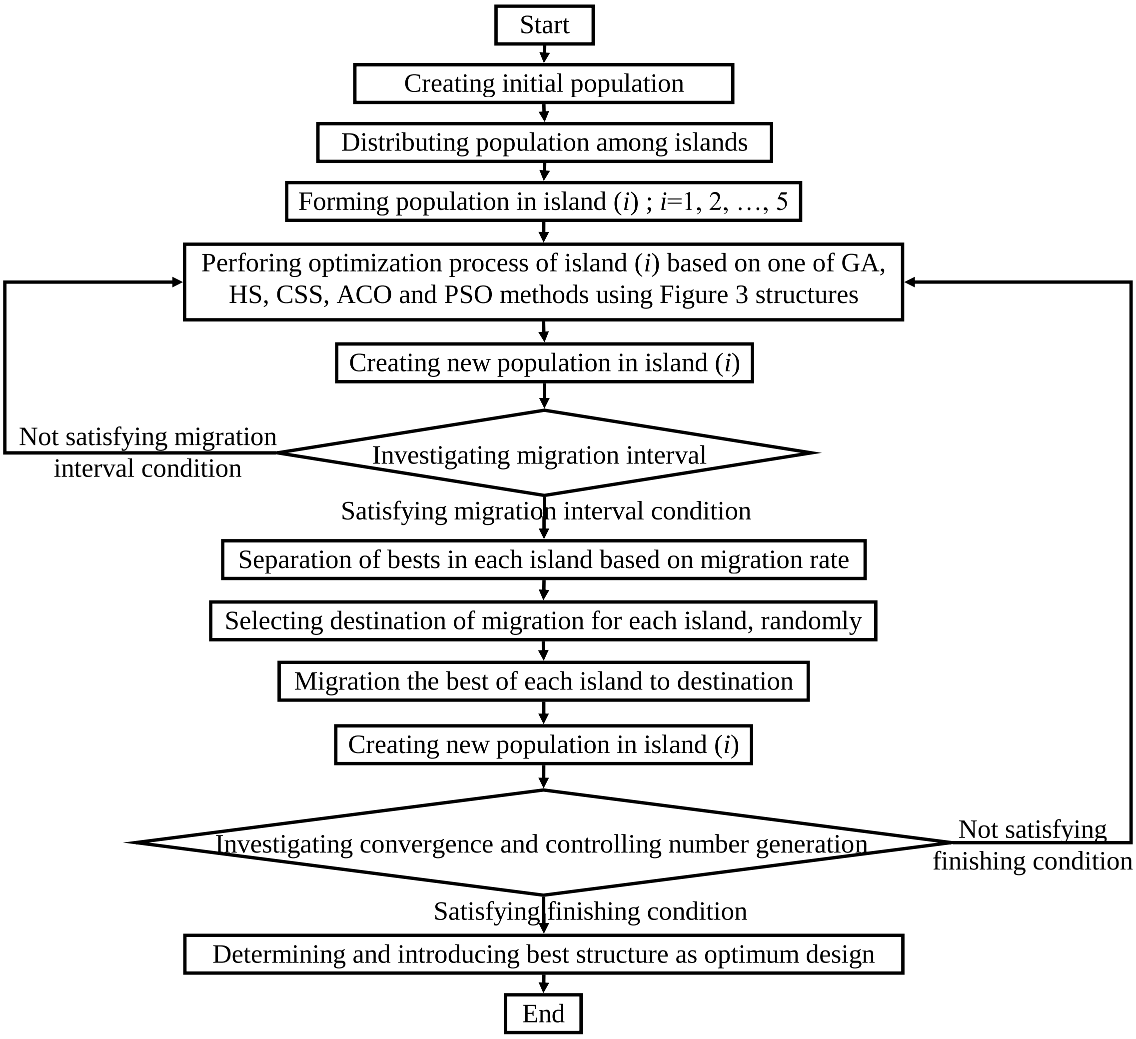


Figure 1. First variant of Multi Heuristic Searching Method (MHSM.1)

In MHSM.2 during the migration interval, most appropriate island is recognized in attaining to minimum. Then, the algorithm on the best island overcomes in the entire system. In other word, best results which are obtained in different islands, gather in the selected island and optimization process will be continued based on the algorithm of selected island. This feature results in increasing the speed of the optimization process.

Migration interval in this method is more than MHSM.1. Figure 2 illustrates the optimization process based on this method.

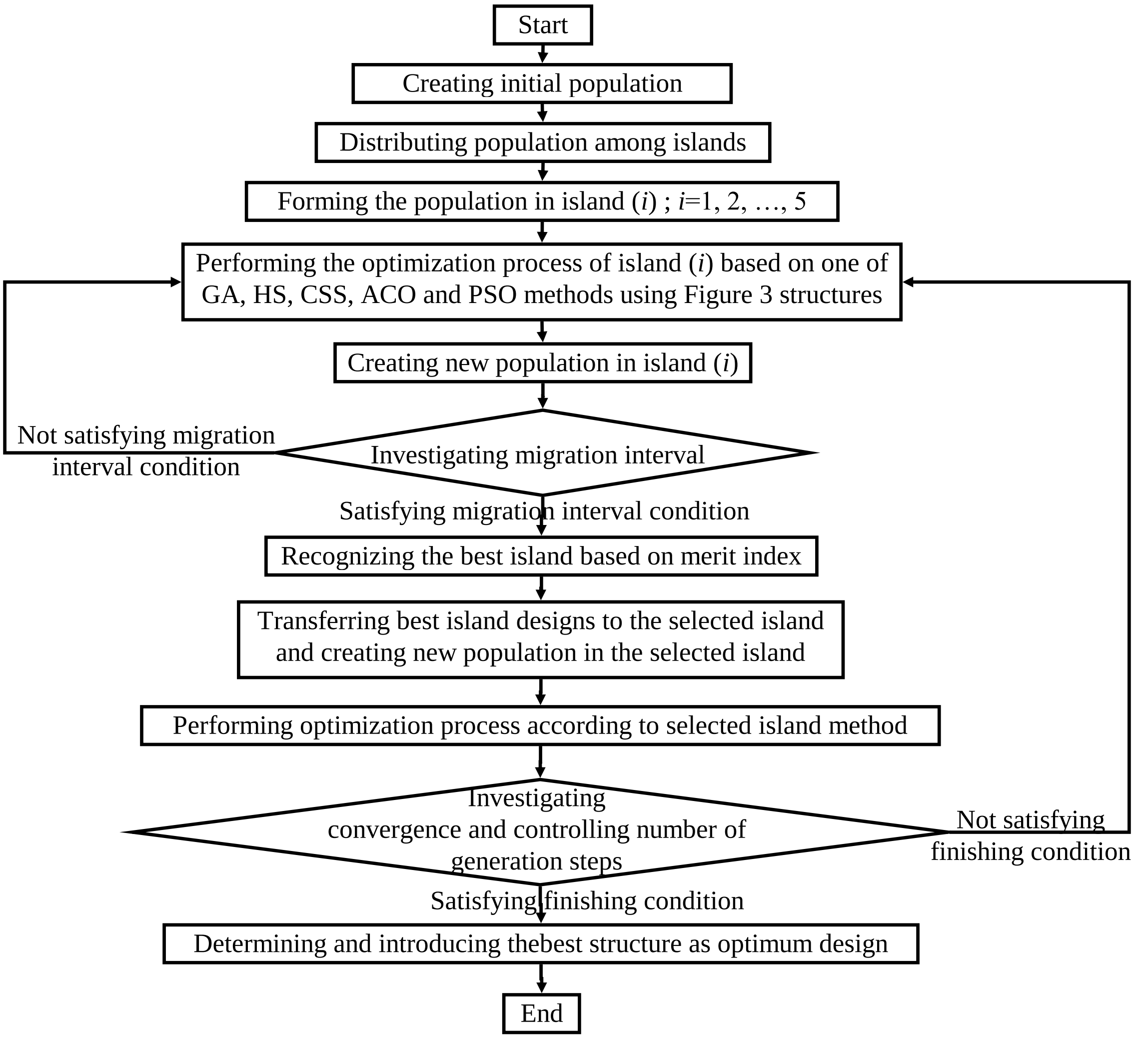


Figure 2. Second variant of the Multi Heuristic Searching Method (MHSM.2)

In issues such as optimization of sizing, topology and geometry that as a result of numerous design variables, searching space become large and the effect of parameters of each meta-heuristic method play important role in the optimization process, design space will be effectively investigated taking the advantages of the MHSM methods and parallel computers, and finally suitable answers will be obtained. Constant and stable state of MHSM methods results in the tendency of the optimization algorithm to global optimum answer. Flowchart of the structural section of the optimization process for truss structure configurations is presented in Figure 3.

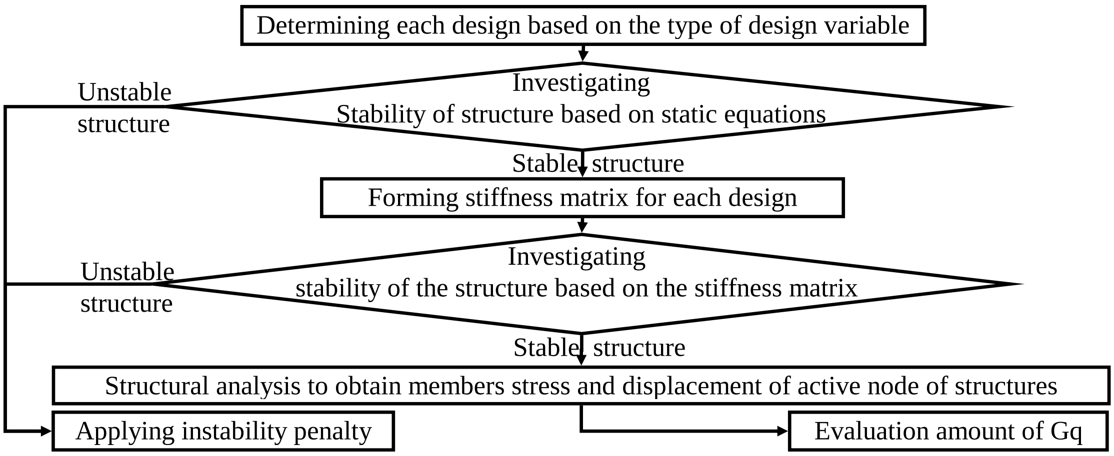


Figure 3. Part of the optimization process of truss configuration

3.1. Island (1)

In this article, optimization of island 1 is performed based on the Genetic Algorithm (GA) [42]. In this regard, optimization process is performed as the following [1, 13]:

Firstly, an initial population is randomly formed with binary characters. Then, the value of the object function and the constraint violations are determined. In this article, the proposed penalty function with dynamic features is used which has a good compatibility with the algorithms of the MHSM.

$$\begin{aligned} f_{penalty} &= F(A,\xi).K.C_g \\ C_g &= \sum^{nlc} \sum_{q=1}^{4} \max\left[0, g_{iq}(A,\xi)\right] \\ K &= k_j \times Ln(j+1) \quad ; \quad j = 1,..nk \end{aligned} \tag{10}$$

In this equation $g_{iq}(A,\xi)$ is the characteristic of the constraint violation, and C_g is the representative of the sum of all violations that has occurred by the structure in order to resist all the load combinations of the *nlc*. K is the constant of dynamic penalty, k_j is a constant quantity of each migration range for total number of *nk*, and j is the counter of each migration interval. Afterwards, the merit of each design is computed based on the object function and the proposed penalty function.

Then, the bests are tried to be selected using a replication process that is inspired by natural development rules. In this island, tournament method [43] is used for the selection process. Once the selection process is completed, the crossover operator is applied in order to produce a population of offsprings. For this purpose, uniform crossover is used with small changes [44]. Therefore, parent’s strings are selected based on the crossover rate. Then, a string which is called Mask, is randomly produced. This string consists of binary bits as length of each string. In the next step, a uniform random number is produced for each bit and is compared to the amounts resulted from Eq. (11). Offspring’s bits is selected based on the Mask pattern if the random number become more than the amount obtained by Eq. (11). That is, if the amount of the bit in the Mask is equal to one, the bit of the first offspring will be from the first parent, otherwise, it is selected from the second parent. Although, while the randomly produced number is less than the amount obtained from Eq. (11), the bit of the offspring's strings are selected from more meritorious parent.

$$P_{C2} = P_{C2}^{Min} + \left(P_{C2}^{Max} - P_{C2}^{Min}\right)\frac{t}{T} \tag{11}$$

where P_{C2} is the secondary rate of crossover in each generation for each bit, P_{C2}^{Max} and P_{C2}^{Min} are, respectively, maximum and minimum rate of the secondary crossover in the optimization process (based on the input of user), t is the number of current generation, and T is the total number of generations. In this article, a method is proposed which is used dynamically to apply the mutation operator. Thus, firstly, the total number of making generations is divided to the number of bits of each substring in design variable of sizing and several intervals are formed. Then, the common operator of the mutation is applied on all the bits in each substring of the cross-section area. After performing this process in the first interval, the first bit at the left-side of each substring of the cross-section area becomes stabilized, and the rate of the mutation probability for it will be equal to zero, and the optimization process will be also continued till the end of the second interval of the total number of making generations. Afterwards, the mutation rate of the two bits at the left-side becomes zero and this process is continued till the last bit in the substring of the cross-section area. It should be mentioned that the rate of the mutation probability for the residual bits in each interval is performed utilizing the following equation:

$$P_m = P_m^{Max} - \left(P_m^{Max} - P_m^{Min}\right)\frac{t}{T} \tag{12}$$

In which above equation, P_m is the mutation rate in each interval, P_m^{Max} and P_m^{Min} are, respectively, the maximum and minimum amount of mutation rate in optimization process (based on input of the user), t is the number of present interval and T is the number of all intervals.

It should be noted that bits of geometric planning variable is changed like mutation process of substring of cross-section, based on the numbers of bits in geometric substring. However, bit of topological design variable is changed during the entire process of optimization with equal probability.

Allocation of different mutation and crossover rate for each design variable are some other preparations in this article for island (1). Therefore, variables of geometry, sizing and topology are not merged with equal probability. This has considerable effect in escaping from the local optimum.

3.2. Island (2)

In island (2), the Harmony Search (HS) algorithm is used [45]. According to this algorithm in the process of optimization, each musician substitute with design variable and collection of musician make the vector of design variable. Quality of music is substituted by the value of the object function. Optimization process for this algorithm is performed as follows [4]:

First, HS parameters such as *HMCR*, *PAR*, *HMS*, etc. are initialized. Then, the initial population (*HM*) based on *HMS* (number of population members in island (2)) is randomly formed as a matrix.

Dimension of this matrix is determined by design variables (sizing, topology and geometry).

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_N^1 \\ x_1^2 & x_2^2 & \dots & x_N^2 \\ \vdots & \vdots & & \\ x_1^{HMS} & x_2^{HMS} & \dots & x_N^{HMS} \end{bmatrix}_{HMS\times N} \tag{13}$$

In the above matrix X_i refers to sizing, topology and geometry variables. In MHSM for the current island in spite of general process of HS, initial population without constraint violation is not required and fitness of each design is specified on the basis of constraint violation and objective function. In order to compute the amount of fitness for each design, proposed penalty function is used according to Eq. (10).

Optimization process is continued for *HM* by producing a new member based on the HS rules. Vectors of the new design variables $X' = [x_1', x_2', \ldots, x_N']$ are made by three possible variants of HS rules and *HMCR* and *PAR* parameters. Accordingly, each amount of x_1' proportionate to the type of design variable, can be randomly produced, again, or can be determined by the existed corresponding amounts in *HM*. This step is performed by producing a uniform random number between zero and one, and comparing it with the amount of *HMCR*. If the random number is more than *HMCR*, x_i' is determined randomly and based on the type and variable range; otherwise, the amount of x_i' is settled by *HM*. Determination of x_i' in *HM* is by *PAR* parameter. Therefore, a uniform random number between zero and one is produced and by comparing it with the value of the *PAR*, x_i' is defined. If the random number is less than *PAR*, x_i' will be selected from the existing corresponding value of the *HM*. Otherwise, x_i' is determined based on the value of the *bw* and from neighborhood of corresponding values with x_i' at *HM*. Finally, if the vector of design variable is better than the worst vector in HM, then the new vector will replace the worst vector. Otherwise, HM remains without change.

This article suggest that *PAR* and *bw* parameters should change based on the amount of migration the interval as follow:

$$PAR = PAR_{\min} + \frac{(PAR_{\max} - PAR_{\min})}{nk} \times j \quad j = 1,\ldots nk \tag{14}$$

$$bw = bw_{\max} \times \exp\left(\frac{\ln(bw_{\min}/bw_{\max})}{nk} \times j\right) \quad j = 1,...nk \tag{15}$$

where, the indices *max* and *min* refer to the maximum and minimum values of the related parameter. *j* and *nk* are the number of migration interval and total number of migration interval in the optimization process, respectively. Migration interval, based on amount of *PAR* and *bw* parameters, has different values that ascends for *PAR* and exponentially descends for *bw* during entire process of the optimization. Varying these parameters has valuable influence on the optimization process [46].

Allocating different rates of *HMCR*, *PAR* and *bw* for each design variables (geometry, sizing and topology) is another feature of the island (2) which is proposed in this article.

3.3. Island (3)

The Charged System Search (CSS) method is used to perform optimization process [47]. In the CSS method, optimization process is performed based on the charged particles laws and Newton laws of motion. Thus, each vector of design variables is considered as a charged particle which possesses electric field as a result of electric charge. Each particle is affected by electric field of other particles and proportional to the amount of electric force of other particles and Newton laws of motion, the particle move in design space and results in new position. The optimization process is performed as follows [12, 17 and 48]:

Firstly, like other heuristic methods, initial population is randomly produced and other parameters of the CSS method such as the number of particles, number of selected particles of *CMS* and so on are initialized. Then, the fitness of each particle is computed according to value of the object function and the proposed penalty function in Eq. (10). The magnitude of the charge of each particle (q_s) and motion probability of particle of *s* affected by the force of the particle *r*, P_{rs}, is obtained by the following equation:

$$q_s = \frac{fit_s - fit_{worst}}{fit_{best} - fit_{worst}} \qquad s = 1,\ldots,ChargeSize \tag{16}$$

$$P_{rs} = \begin{cases} 1 & \dfrac{fit_r - fit_{best}}{fit_s - fit_r} > ran \vee fit_s > fit_r \\ 0 & else \end{cases} \tag{17}$$

fit_{best} and fit_{worst} are, the fitness of best and worst existed design in current population, respectively. A small population which consists of the bests of the existed population is called *CMS* is produced after computing the p_{rs} and q_s. Then, the resultant electrical force acting on a particle is computed based on following equation:

$$F_s = q_s \sum_{r,r\neq s} \frac{q_r}{a^3} r_{rs} P_{rs} (X_r - X_s) \quad if \quad r_{rs} < a \tag{18}$$

$$F_s = q_s \sum_{r,r \neq s} \frac{q_r}{r_{rs}^2} P_{rs} \left(X_r - X_s \right) \quad if \quad r_{rs} \geq a \tag{19}$$

where a is the diameter of each particle, r_{rs} is the distance between two particles r and s that is defined according to position of the particles X_r and X_s. New position of each particle in the design space is determined by the following equation:

$$X_{s,new} = X_{s,old} + r_1 k_a F_s + r_2 k_v v_{s,old} \tag{20}$$

$$v_{s,new} = X_{s,new} - X_{s,old} \tag{21}$$

r_1 and r_2 are uniform random number between zero and one. v_s is also the velocity of the particle s, k_a and k_v are, respectively, velocity and acceleration coefficient which are computed to correspond with MHSM as:

$$k_a = 0.5(1 + j/nk) \quad j = 1,...nk \tag{22}$$

$$k_v = 0.5(1 - j/nk) \quad j = 1,...nk \tag{23}$$

New position of each particle is evaluated during the optimization process, providing the amount of exiting from the allowed range. Design variables will then modified based on the HS method and *CMS* population.

3.4. Island (4)

Ant Colony Optimization (ACO) is used in island 4 [49], that is performed by the following steps [25, 50, 51]:

Firstly, the amount of initial pheromone is specified based on the number of design variables and possible states of each variable. In order to calculate fit_0, the first cross-section area in the profile list is initialized for sizing variables. Additionally, topology and geometric variables are also calculated based on the ground structure, and then the initial pheromone is determined according to following equation:

$$\tau_0 = \frac{1}{fit_0} \tag{24}$$

Then, the probability of selection for each type of design variables is evaluated based on the following equation [6]:

$$p_{ij} = \frac{\tau_{ij}^{\alpha} \upsilon_i^{\beta}}{\sum_{k=1}^{N} \tau_{kj}^{\alpha} \upsilon_k^{\beta}} \tag{25}$$

where, τ_{ij} is the amount of existed pheromone in the *i*th path (state number *i* for the considered design variable) for the design variable number *j* and *N* is the number of possible states for the considered design variable. v_i is zero for the geometric and topological variables, and it is also calculated by Eq. (26) for sizing design variable.

$$\upsilon_i = \frac{1}{A_i} \tag{26}$$

A_i refers to the selected cross-section area of the *i*th path. Afterwards, the amount of the variable number *i* is determined by p_{ij} by the roulette wheel method in GA. After determining the amount of all design variables, the amount of the pheromone in the selected path is diminished as follows:

$$\tau_{ij}^{new} = \rho.\tau_{ij}^{old} \tag{27}$$

where ρ is the local update parameter assigned to a suitable value between zero and one.

Then, the amount of fitness is calculated and existed population designs are sorted. Pheromone evaporation process for all the possible paths is done based on the following equation:

$$\tau_{ij}^{new} = (1 - e_r).\tau_{ij}^{old} \tag{28}$$

In Eq. (28), e_r is a constant referred to as the evaporation rate. After evaporating of pheromone, the process of depositing pheromone in the selected paths is executed as follow:

$$\tau_{ij} = \tau_{ij} + e_r.\left[(\Delta T_{ij}) + \sum_{k=1}^{\lambda_r} (\lambda_r - k)(\Delta T_{ij})_k\right] \tag{29}$$

In the above equation, λ_r is number of the best existed population and *k* is the number of considered design in small population of the bests. $(\Delta\tau_{ij})_k$ in Eq. (29), is calculated for ant number *k* by the following equation:

$$(\Delta\tau_{ij})_k = \frac{1}{fit_k} \tag{30}$$

Finally, criterion of local search is investigated. In this article, if no variation is created in several continual populations for fitness of the best design, the local search process will be performed, based on the HS method, in the neighbourhood of the best design. Allocating evaporation rate and different local search parameters to each type of design variables (geometry, sizing and topology) are other facilities in this article.

3.5. Island (5)

In island (5), Particle Swarm Optimization (PSO) [52] proportional to type of design variable is used in two standard states according to references [53, 54] and binary state according to reference [24]. This algorithm is influenced by the social behaviour of the birds in searching food. The PSO algorithm begins by producing an initial random population. Particle number i (design) which introduces bird number i in the group of birds is defined by two variables $X_i = [x_{i1}, x_{i2}, ..., x_{iN}]$ and $V_i=[v_{i1}, v_{i2}, ..., v_{iN}]$. X_i is the position and V_i is velocity of the particle number i in the search space. In each step of group movement (repeat), particle position is changed by two amounts of $P_{best,i}$ and R_{best}. In order to this, position of each particle (design) is determined in the search space utilizing the following equations:

$$X_i^{k+1} = X_i^k + V_i^{k+1} \tag{31}$$

$$V_i^{k+1} = \omega V_i^k + c_1 r_1 \left(p_{best,i}^k - X_i^k\right) + c_2 r_2 \left(R_{best}^k - X_i^k\right) \tag{32}$$

In the above equation, X_i^k is the position of the ith particle in the iteration number k, V_i^k is velocity of the ith particle in the iteration number k, ω is the inertia weight in the previous step, r_1 and r_2 are the uniform random numbers between zero and one, C_1 and C_2 are the acceleration constants. $P^k_{best,i}$ is the best position of the particle i from first to iteration number k, R^k_{best} is the best position of a particle from the first to iteration number k among all the particles.

In this article, the amount of variable velocity of the particles is controlled by defining minimum and maximum velocity (v_{min} , v_{max}). In this regard, v_{min} and v_{max}, based on the coefficient of maximum and minimum amount of x is defined, proportional to the type of design variable [55].

In order to be compatible with the MHSM, the parameter ω is changed based on the number of migration interval as:

$$\omega = \omega_{\max} - \frac{(\omega_{\max} - \omega_{\min})}{nk} \times j \quad j = 1,...nk \tag{33}$$

ω_{max} and ω_{min} are the maximum and minimum values of ω, respectively. j and nk are the number and total number of migration interval in the optimization process, respectively. Therefore, the value of ω is linearly altered in each migration, with initial amount of ω_{max} and final amount of ω_{min}. This method in altering the ω resulting in a balance of the local and global search in the PSO algorithm [53].

In this article, acceleration constants (C_1 and C_2) and ω for each type of design variables (geometry, Sizing and topology) are different.

4. NUMERICAL EXAMPLES

Some common instances in optimization of truss structures configuration are subsequently evaluated in order to investigate the ability of the MHSM algorithm, and their results are compared to those of some reliable references. Results show that MHSM searches the space more accurately compared to other methods and leads to better results.

4.1. A ten-bars planar truss structure

A ten-bar planar truss is investigated as the first example. Figure 4 and Table 1 illustrate the necessary information for the considered truss.

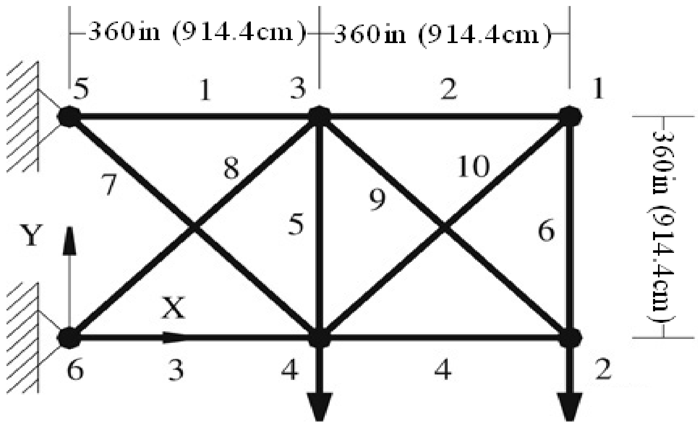


Figure 4. Initial configuration of a ten-bar planar truss

Table 1. Data for design of the ten-bar planar truss

Design variables
Size variables: A_i ; i = 1, 2, …, 10 ; Geometry variables: *Y1*, *Y3*, *Y5*
Constraint data
$\sigma_{all\ (Ten)} = \sigma_{all\ (Com)} = \pm 17240$ N/cm^2 = ±25 *Ksi* ; Δ^y_{all} = 5.04 *cm* = 2 *in* 180 *in* (457.2 *cm*) < *Y1* < 1000 *in* (2540 *cm*) ; 180 *in* (457.2 *cm*) < *Y3* < 1000 *in* (2540 *cm*) 180 *in* (457.2 *cm*) < *Y5* < 1000 *in* (2540 *cm*) ε = 10 *in* (25.4 *cm*)
List of available profiles
$a_i \in S$ = {1.62, 1.8, 2.38, 2.62, 2.88, 3.09, 3.13, 3.38, 3.63, 3.84, 3.87, 4.18, 4.49, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.5, 13.5, 13.9, 14.2, 15.5, 16.0, 18.8, 19.9, 22.0, 22.9, 26.5, 30.0, 33.5} (in^2) $a_i \in S$ = {10.45, 11.61, 15.35, 16.90, 18.58, 19.94, 20.19, 21.81, 23.42, 24.77, 24.97, 26.97, 28.97, 30.97, 32.06, 33.03, 37.03, 46.58, 51.42, 74.19, 87.1, 89.68,91.61, 100.0, 103.23, 121.29, 128.39, 141.94, 147.74, 170.97, 193.55, 216.13} (cm^2)
Loading data
P2Y = *P4Y* = –445.4 *kN* = –100 *Kips*
Material properties
E = 6895000 N/cm^2 =10^4 *ksi* ; ρ = 0.0271264 N/cm^3 =0.1 lb/in^3

Figure 5 is resulted as plan of an optimum six-node planar truss after executing the optimization process based on the MHSM.

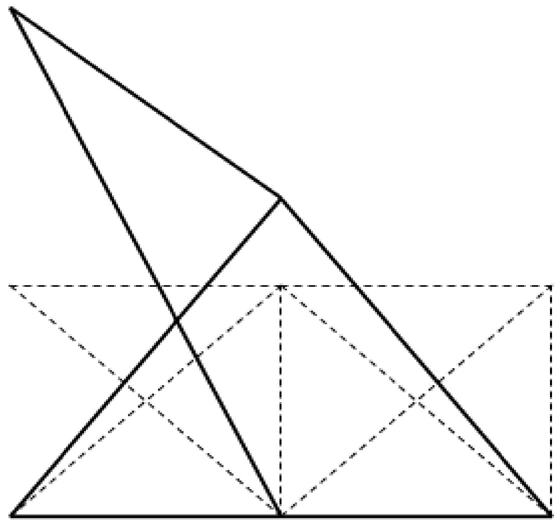


Figure 5. Optimal geometry and topology for the ten-bar truss, based on the MHSM

Results of the optimum design are compared to the related references in Table 2. This shows that the MHSM leads to more enhanced results than other references.

Table 2. Final design of sizing, shape and topology for the ten-bars truss

Design Variables	Rajan [26]	Tang [29]	Rahami [30]	MHSM
A1 - in^2 (*cm*^{2})	9.9 (63.871)	13.5 (87.1)	11.5 (74.19)	13.5 (87.1)
A2 - in^2 (*cm*^{2})	9.4 (60.645)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
A3 - in^2 (*cm*^{2})	11.5 (74.19)	7.97 (51.42)	11.5 (74.19)	11.5 (74.19)
A4 - in^2 (*cm*^{2})	1.5 (9.677)	7.22 (46.58)	5.74 (37.03)	7.22 (46.58)
A5 - in^2 (*cm*^{2})	0.0 (0.0)	1.62 (10.45)	0.0 (0.0)	0.0 (0.0)
A6 - in^2 (*cm*^{2})	12.0 (77.42)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
A7 - in^2 (*cm*^{2})	11.5 (74.19)	4.49 (28.97)	5.74 (37.03)	5.74 (37.03)
A8 - in^2 (*cm*^{2})	3.6 (23.226)	3.13 (20.19)	3.84 (24.77)	3.38 (21.81)
A9 - in^2 (*cm*^{2})	0.0 (0.0)	13.5 (87.1)	13.5 (87.1)	11.5 (74.19)
A10 - in^2 (*cm*^{2})	10.4 (67.097)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Y1 - in (*cm*)	186.5 (473.71)	--	--	--
Y3 - in (*cm*)	554.5 (1408.43)	527.9 (1340.866)	506.4203 (1286.308)	496.3779 (1260.8)
Y5 - in (*cm*)	786.9 (1998.726)	888.8 (2257.552)	789.7306 (2005.916)	793.3558 (2015.124)
Weight *lb* (*kN*)	**3254.0 (14.474)**	**2813.8 (12.516)**	**2723.05 (12.113)**	**2716.5 (12.083)**
$\sigma_{\max}$ *ksi* (*N/cm*^{2})	15.6 (10756)	18.5 (12755.3)	19.1463 (13201)	19.131 (13190.36)
$\Delta_{\max}$ *in* (*cm*)	1.99 (5.0546)	1.9998 (5.079492)	1.999996 (5.07998984)	1.997852 (5.07454408)

4.2. A 15-bar planar truss

In this example, a fifteen-bar planar truss is studied. The considered truss is shown in Figure 6 and the required information for the optimization process are accessible in Table 3.

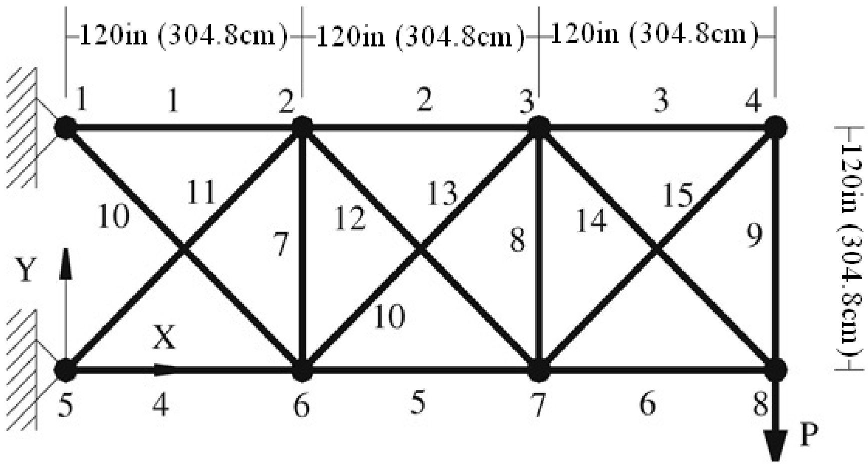


Figure 6. The primary geometry of the 15-bar truss

Table 3. Data for design of the 15-bar planar truss

Design variables
Size variables: A_i ; i = 1, 2, …, 15 ; Geometry variables: $Y2$, $Y3$, $Y4$, $Y6$, $Y7$, $Y8$, $X2=X6$, $X3=X7$
Constraint data
$\sigma_{all\ (Ten)} = \sigma_{all\ (Com)} = \pm 17240\ N/cm^2 = \pm 25\ Ksi$ 100 *in* (254 *cm*) < $X2$ < 140 *in* (355.6 *cm*); 220 *in* (558.8 *cm*) < $X3$ < 260 *in* (660.4 *cm*) 100 *in* (254 *cm*) < $Y2$ < 140 *in* (355.6 *cm*); 100 *in* (254 *cm*) < $Y3$ < 140 *in* (355.6 *cm*) 50 *in* (127 *cm*) < $Y4$ < 90 *in* (228.6 *cm*); –20 *in* (–50.8 *cm*) < $Y6$ < 20 *in* (50.8 *cm*) –20 *in* (–50.8 *cm*) < $Y7$ < 20 *in* (50.8 *cm*); 20 *in* (50.8 *cm*) < $Y8$ < 60 *in* (152.4 *cm*) ε = 0.01 *in* (0.0254 *cm*)
List of available profiles
$a_i \in S$ = {0.111, 0.141, 0.174, 0.22, 0.27, 0.287, 0.347, 0.44, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.8, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.3, 10.85, 13.33, 14.29, 17.17, 19.18} (in^2) $a_i \in S$ = {0.716, 0.91, 1.123, 1.419, 1.742, 1.852, 2.239, 2.839, 3.477, 6.155, 6.974, 7.574, 8.600, 9.600, 11.381, 13.819, 17.400, 18.064, 20.200, 23.00, 24.6, 31.0, 38.4, 42.4, 46.4, 55.0, 60.0, 70.0, 86.0, 92.193, 110.774, 123.742} (cm^2)
Loading data
$P8Y = -44.54\ kN = -10\ Kips$
Material properties
E = 6895000 N/cm^2 =10^4 *ksi* ; ρ = 0.0272 N/Cm^3 =0.1 lb/in^3

Figure 7 is resulted as optimum design by the MHSM after optimization.

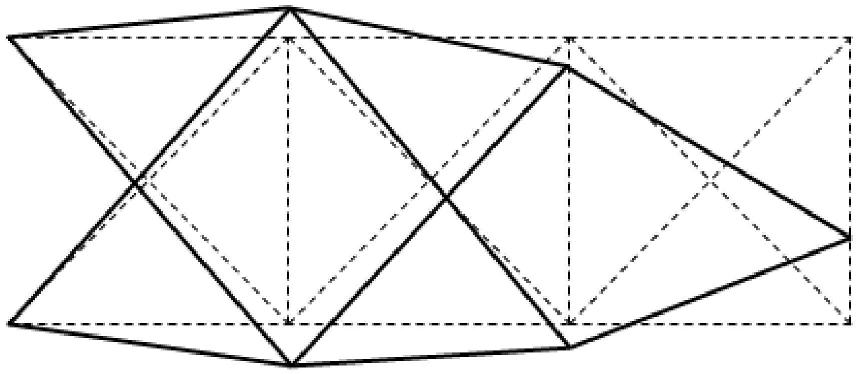


Figure 7. Optimal geometry and topology for the 15-bar truss based on the MHSM

Table 4 compares the results of the present method with those other references.

Table 4. Final design of sizing, shape and topology for the 15-bars truss

Design Variables	**Wu [27]**	**Tang [29]**	**Rahami [30]**	**MHSM**
A1 - in^2 (*cm*^{2})	1.174 (7.574)	1.081 (6.974)	0.954 (6.155)	0.954 (6.155)
A2 - in^2 (*cm*^{2})	0.954 (6.155)	0.539 (3.477)	0.954 (6.155)	0.539 (3.477)
A3 - in^2 (*cm*^{2})	0.440 ((2.839)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
A4 - in^2 (*cm*^{2})	1.333 (8.600)	1.081 (6.974)	1.081 (6.974)	0.954 (6.155)
A5 - in^2 (*cm*^{2})	0.954 (6.155)	0.954 (6.155)	0.539 (3.477)	0.539 (3.477)
A6 - in^2 (*cm*^{2})	0.174 (1.123)	0.440 (2.839)	0.539 (3.477)	0.440 (2.839)
A7 - in^2 (*cm*^{2})	0.440 (2.839)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
A8 - in^2 (*cm*^{2})	0.440 (2.839)	0.141 (0.91)	0.000 (0.000)	0.000 (0.000)
A9 - in^2 (*cm*^{2})	1.081 (6.974)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
A10 - in^2 (*cm*^{2})	1.333 (8.600)	0.270 (1.742)	0.440 (2.839)	0.440 (2.839)
A11 - in^2 (*cm*^{2})	1.174 (7.574)	0.270 (1.742)	0.220 (1.419)	0.539 (3.477)
A12 - in^2 (*cm*^{2})	1.174 (7.574)	0.539 (3.477)	0.111 (0.716)	0.174 (1.123)
A13 - in^2 (*cm*^{2})	0.347 (2.239)	0.141 (0.91)	0.347 (2.239)	0.174 (1.123)
A14 - in^2 (*cm*^{2})	0.347 (2.239)	0.440 (2.839)	0.539 (3.477)	0.539 (3.477)
A15 - in^2 (*cm*^{2})	0.440 (2.839)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
X2 - in (*cm*)	123.189 (312.9)	111.85 (284.099)	107.3869 (272.762)	120.0048 (304.812)
X3 - in (*cm*)	231.595 (588.251)	242.45 (615.823)	244.4534 (620.911)	240.2393 (610.209)
Y2 - in (*cm*)	107.189 (272.26)	104.02 (264.211)	125.4198 (318.566)	132.5079 (336.57)
Y3 - in (*cm*)	119.175 (302.705)	109.22 (277.419)	117.2854 (297.905)	107.5799 (273.253)
Y4 - in (*cm*)	60.462 (153.573)	--	--	--
Y6 - in (*cm*)	−16.728 (−42.489)	−10.82 (−27.483)	−1.6249 (−4.127)	−17.5873 (−44.671)
Y7 - in (*cm*)	15.565 (39.535)	−11.13 (28.27)	18.0828 (45.93)	−10.0463 (25.517)
Y8 - in (*cm*)	36.645 (93.078)	48.84 (124.054)	50.2040 (127.518)	38.5885 (98.014)
Weight *lb* (*N*)	**120.528 (536.135)**	**77.84 (346.116)**	**75.0966 (334.046)**	**73.263 (325.889)**
$\sigma_{\max}$ *ksi* (*N/cm*^{2})	24.762 (17072.8)	24.9552 (17206)	24.9930 (17232)	24.9896 (17230)

4.3. A 25-bar space truss

In this example, The twenty five-bar space truss with ten nodes shown in Figure 8 is optimized.

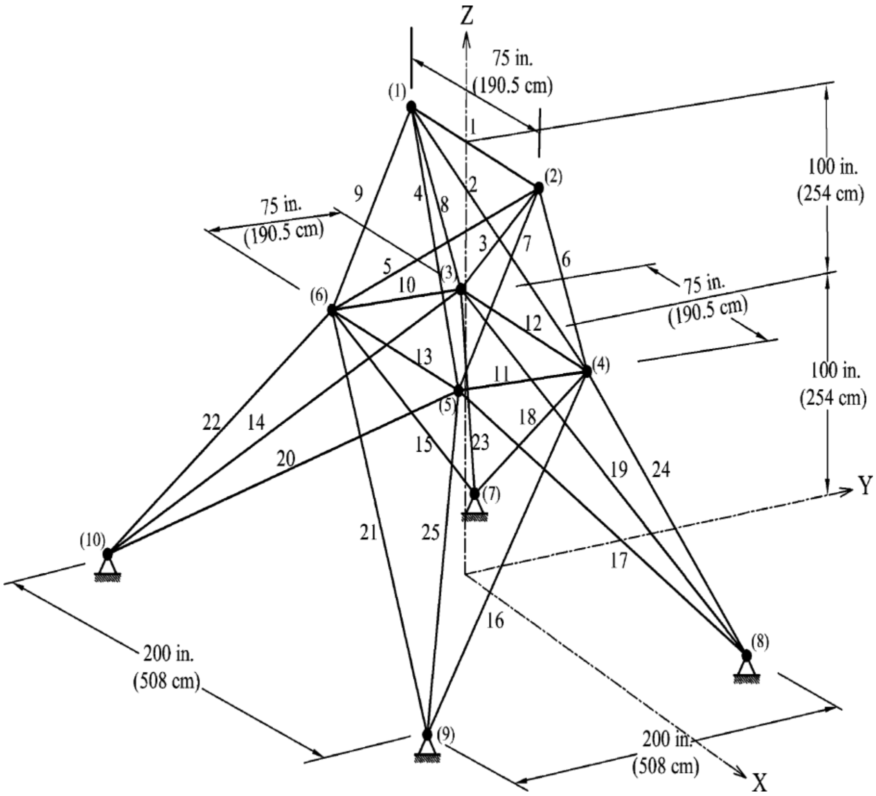


Figure 8. The primary geometry of the 25-bar truss

Table 5 shows the required information for the optimization of the 25-bar truss.

Table 5. Data for design of the 25-bar truss

Grouping members
A1 : 1 (1,2) ; A2 : 2(1,4) , 3 (2,3) , 4 (1,5) , 5 (2,6) A3 : 6 (2,5) , 7 (2,4) , 8 (1,3) , 9 (1,6) ; A4 : 10 (3,6) , 11 (4,5) A5 : 12 (3,4) , 13 (5,6) ; A6 : 14 (3,10) , 15 (6,7) , 16 (4,9) , 17 (5,8) A7 : 18 (3,8) , 19 (4,7) , 20 (6,9) , 21 (5,10) ; A8 : 22 (3,7) , 23 (4,8) , 24 (5,9) , 25 (6,10)
Design variables
Size variables: A_i ; $i = 1, 2, \dots, 8$ Geometry variables: $X4=X5=-X3=-X6$, $X8=X9=-X7=-X10$ $Y3=Y4=-Y5=-Y6$, $Y7=Y8=-Y9=-Y10$, $Z3=Z4=Z5=Z6$
Constraint data
$\sigma_{all\,(Ten)} = \sigma_{all\,(Com)} = \pm 27850\ N/cm^2 = \pm 40\ Ksi$; $\Delta^y_{all} = 0.889\ cm = 0.35\ in$ 20 *in* (50.8 *cm*) < $X4$ < 60 *in* (152.4 *cm*) ; 40 *in* (101.6 *cm*) < $X8$ < 80 *in* (203.2 *cm*) 40 *in* (101.6 *cm*) < $Y4$ < 80 *in* (203.2 *cm*) ; 100 *in* (254 *cm*) < $Y8$ < 140 *in* (355.6 *cm*)

90 *in* (228.6 *cm*) < $Z4$ < 130 *in* (330.2 *cm*) ; ε = 0.01 *in* (0.0254 *cm*)
List of available profiles
$a_i \in S = \{0.1j\ (j–1, …, 26), 2.8, 3.0, 3.2, 3.4\}\ (in^2)$
$a_i \in S = \{0.645j\ (j–1, …, 26), 18.064, 19.335, 20.645, 21.935\}\ (cm^2)$
Loading data
$P2Y = P4Y = –445.4\ kN = –100\ Kips\ ;\ P1X = 4.454\ kN = 1\ Kips$
$P1Y = P2Y = P1Z = P2Z = –44.537\ kN = –10\ Kips$
$P3X = 2.227\ kN = 0.5\ Kips\ ;\ P6X = 2.672\ kN = 0.6\ Kips$
Material properties
$E = 6895000\ N/cm^2 = 10^4\ ksi\ ;\ \rho = 0.0272\ N/Cm^3 = 0.1\ lb/in^3$

In this example, optimization process in performed in three states. In state (1), the possibility of elimination of each member in topological optimization process is feasible. Most of the articles consider the topological optimization process for each group that increases the possibility of unsteady design production due to deletion of each group. In state (2), the assumptions are considered exactly identical to other references, and possibility of deletion of each group in topological optimization process is feasible. This leads to the creation of optimum design with more precise shapes. In state (3), according to optimum points of state (2), nodal points interval for geometric variables is changed as shown in Table 6, and then design space with new intervals are again searched for geometric variables. Results of optimum design based on MHSM are shown in Figures 9, 10 and 11 and also in Table 7.

Table 6. New interval for geometry variable of the 25-bar truss

20 *in* (50.8 *cm*) < $X4$ < 60 *in* (152.4 *cm*) ; 40 *in* (101.6 *cm*) < $X8$ < 80 *in* (203.2 *cm*)
40 *in* (101.6 *cm*) < $Y4$ < 80 *in* (203.2 *cm*) ; 100 *in* (254 *cm*) < $Y8$ < 140 *in* (355.6 *cm*)
90 *in* (228.6 *cm*) < $Z4$ < 130 *in* (330.2 *cm*)

Table 7. Final design of sizing, shape and topology for the 25-bar truss

Design Variables	Wu [27]	Tang [29]	Rahami [30]	MHSM State 1	MHSM State 2	MHSM State 3
$A1$ - in^2 (cm^2)	0.1 (0.645)	--	--	0.1 (0.645)	--	--
$A2$ - in^2 (cm^2)	0.2 (1.29)	0.1 (0.645)	0.1 (0.645)	--	0.1 (0.645)	0.1 (0.645)
$A3$ - in^2 (cm^2)	0.2 (1.29)	0.1 (0.645)	0.1 (0.645)	0.8 (5.16)	0.1 (0.645)	0.1 (0.645)
$A4$ - in^2 (cm^2)	0.2 (1.29)	0.1 (0.645)	0.1 (0.645)	0.8 (5.16)	0.1 (0.645)	0.1 (0.645)
$A5$ - in^2 (cm^2)	0.2 (1.29)	0.1 (0.645)	0.1 (0.645)	0.8 (5.16)	0.1 (0.645)	0.1 (0.645)
$A6$ - in^2 (cm^2)	1.1 (7.095)	0.9 (5.805)	0.9 (5.805)	0.6 (3.87)	0.9 (5.805)	1.0 (6.45)
$A7$ - in^2 (cm^2)	1.1 (7.095)	0.9 (5.805)	0.9 (5.805)	0.6 (3.87)	0.9 (5.805)	1.0 (6.45)
$A8$ - in^2 (cm^2)	1.1 (7.095)	0.9 (5.805)	0.9 (5.805)	0.6 (3.87)	0.9 (5.805)	1.0 (6.45)
$A9$ - in^2 (cm^2)	1.1 (7.095)	0.9 (5.805)	0.9 (5.805)	0.6 (3.87)	0.9 (5.805)	1.0 (6.45)
$A10$ - in^2 (cm^2)	0.2 (1.29)	--	--	--	--	--
$A11$ - in^2 (cm^2)	0.2 (1.29)	--	--	0.1 (0.645)	--	--
$A12$ - in^2 (cm^2)	0.3 (1.935)	--	--	0.1 (0.645)	--	--
$A13$ - in^2 (cm^2)	0.3 (1.935)	--	--	--	--	--

A14 - in^2 (cm^2)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)
A15 - in^2 (cm^2)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)
A16 - in^2 (cm^2)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)
A17 - in^2 (cm^2)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)	0.1 (0.645)
A18 - in^2 (cm^2)	0.2 (1.29)	0.1 (0.645)	0.1 (0.645)	--	0.1 (0.645)	0.1 (0.645)
A19 - in^2 (cm^2)	0.2 (1.29)	0.1 (0.645)	0.1 (0.645)	0.2 (1.29)	0.1 (0.645)	0.1 (0.645)
A20 - in^2 (cm^2)	0.2 (1.29)	0.1 (0.645)	0.1 (0.645)	0.2 (1.29)	0.1 (0.645)	0.1 (0.645)
A21 - in^2 (cm^2)	0.2 (1.29)	0.1 (0.645)	0.1 (0.645)	0.2 (1.29)	0.1 (0.645)	0.1 (0.645)
A22 - in^2 (cm^2)	0.9 (5.805)	1.0 (6.45)	1.0 (6.45)	0.9 (5.805)	1.0 (6.45)	0.9 (5.805)
A23 - in^2 (cm^2)	0.9 (5.805)	1.0 (6.45)	1.0 (6.45)	--	1.0 (6.45)	0.9 (5.805)
A24 - in^2 (cm^2)	0.9 (5.805)	1.0 (6.45)	1.0 (6.45)	0.9 (5.805)	1.0 (6.45)	0.9 (5.805)
A25 - in^2 (cm^2)	0.9 (5.805)	1.0 (6.45)	1.0 (6.45)	0.9 (5.805)	1.0 (6.45)	0.9 (5.805)
X4 - *in* (*cm*)	41.07 (104.318)	39.91 (101.371)	38.7913 (98.5299)	20.0 (50.8)	38.9206 (98.8583)	38.8645 (98.7158)
Y4 - *in* (*cm*)	53.47 (135.814)	61.99 (157.455)	66.1110 (167.922)	60.0048 (152.412)	60.4932 (153.6527)	62.9959 (160.0096)
Z4 - *in* (*cm*)	124.6 (316.484)	118.23 (300.304)	112.9787 (286.966)	122.4981 (311.145)	120.3199 (305.6125)	118.0183 (299.7665)
X8 - *in* (*cm*)	50.8 (129.032)	53.13 (134.95)	48.7924 (123.933)	41.4456 (105.2718)	50.0024 (127.0061)	51.1868 (130.0144)
Y8 - *in* (*cm*)	131.48 (333.96)	138.49 (351.764)	138.8910 (352.7831)	137.9194 (350.315)	137.6654 (349.6701)	139.9532 (355.4811)
Weight *lb* (*N*)	**136.2 (605.847)**	**114.74 (510.388)**	**114.3701 (508.743)**	**111.9508 (497.981)**	**114.4200 (508.965)**	**113.5081 (504.909)**
Max σ *ksi* (N/cm^2)	15.589 (10748.237)	17.353 (11964.472)	17.753 (12240.262)	35.074 (24182.67)	17.239 (11885.87)	19.896 (13717.81)
Max Δ *in* (*cm*)	0.347 (0.88138)	0.350 (0.889)	0.34999896 (0.888997)	0.350 (0.889)	0.350 (0.889)	0.350 (0.889)

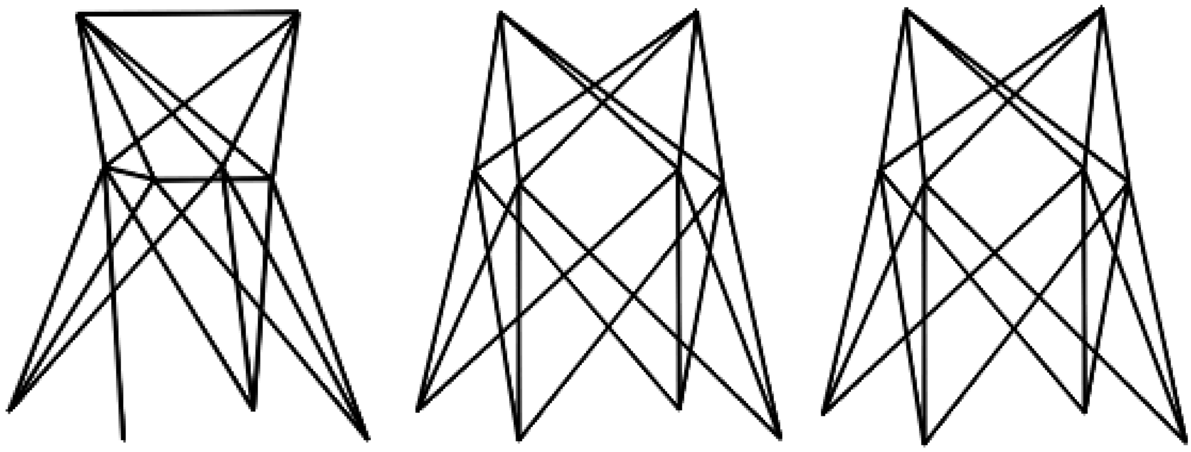


Figure 9. State 1 Figure 10. State 2 Figure 11. State 3

Optimum design for 25-bar space truss based on MHSM

4.4. Design of a 131-bar truss

In order to study the industrial applicably of the MHSM algorithm, a 131-bar truss, which is proposed and studied for the first time in this article, is designed. Figure 12 shows the geometry and initial topology of the considered structure.

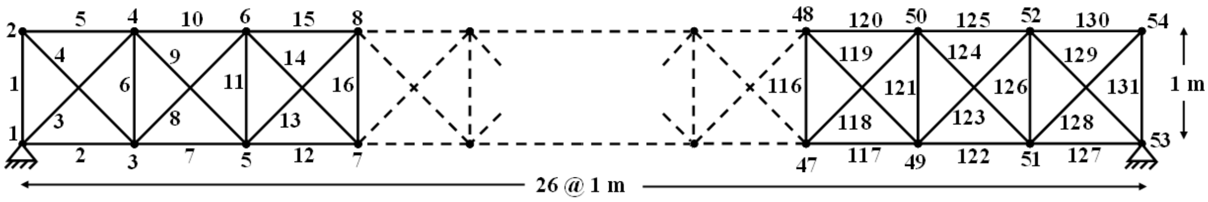


Figure 12. Ground structure for the 131-bar truss

In the case, the permissible compressive and tensile stresses and also the permissible nodal displacements are selected according to the AISC-ASD [56] as follows:
when $\lambda<C_c$:

$$\sigma_{all(com)}=\left(F_y\left[1-\frac{\lambda^2}{2C_c^2}\right]\right)/\left(\frac{5}{3}+\frac{3\lambda}{8C_c}-\frac{\lambda^3}{8C_c}\right) \tag{34}$$

and when $\lambda\geq C_c$:

$$\sigma_{all(com)}=\frac{12\pi^2E}{23\lambda^2} \tag{35}$$

In the above equations, E is the module of elasticity and F_y is yield stress of steel that is taken as 2400 kg/cm^2. λ is the slenderness ratio which is calculated for compressive member around x and y axis as:

$$\lambda=l/r_i \qquad i=x\ ,y \tag{36}$$

where l is the length of the member and r_i is the governing radius of gyration. Maximum slenderness ratio is limited to 300 for the members under tension, and to 200 for the members under compression loads.

In Eqs. (34), C_c is the slender ratio dividing the elastic and inelastic buckling regions, which is calculated as follow:

$$C_c=\sqrt{\frac{2\pi^2E}{F_y}} \tag{37}$$

On the other hand, according to AISC-ASD, the allowed tensile stress is calculated by the following equation:

$$\sigma_{all(Ten)}=0.6F_y \tag{38}$$

The permissible nodal displacement is also considered as 10.83 *cm*, according to AISC-ASD code [56].

In this example, in order to make the final executable design, the nodal coordinates of the upper members in vertical direction, are determined based on the positive angle of each member with respect to horizon. This issue leads to enlargement of the design space and increases the design variables (angle design variable), however, in final configuration, none of the upper member's slope should be negative, and an executable design will be resulted.

Designing and optimization information of the structural configuration can be seen in Table 8.

Table 8. Data for design of the 131-bar planar truss

Design variables
Size variables: A_i ; i = 1, 2, …, 66
Geometry variables:
$X3 = X4 = -X51 = -X52$, $X5 = X6 = -X49 = -X50$
$X7 = X8 = -X47 = -X48$, $X9 = X10 = -X45 = -X46$
$X11 = X12 = -X43 = -X44$, $X13 = X14 = -X41 = -X42$
$X15 = X16 = -X39 = -X40$, $X17 = X18 = -X37 = -X38$
$X19 = X20 = -X35 = -X36$, $X21 = X22 = -X33 = -X34$
$X23 = X24 = -X31 = -X32$, $X25 = X26 = -X29 = -X30$; $X2 = X54$, $Y2 = Y54$
Angle variables:
$\alpha5 = \alpha130$, $\alpha10 = \alpha125$, $\alpha15 = \alpha120$, $\alpha20 = \alpha115$, $\alpha25 = \alpha110$
$\alpha30 = \alpha105$, $\alpha35 = \alpha100$, $\alpha40 = \alpha95$, $\alpha45 = \alpha90$, $\alpha50 = \alpha85$
$\alpha55 = \alpha80$, $\alpha60 = \alpha75$, $\alpha65 = \alpha70$
Constraint data
$\sigma_{all\ (Com)}$ = *Eq.* (35) , *Eq.* (36) ; $\sigma_{all\ (Ten)}$ = *Eq.* (40) ; Δ_{all} = 10.83 *cm*
$0° < \alpha_i < 30°$ i=5, 10, 15, …, 125, 130 ; ε for angel variable = 1°
$-1340\ cm < X2 < -1260\ cm$, $-1240\ cm < X4 < -1160\ cm$
$-1140\ cm < X6 < -1060\ cm$, $-1040\ cm < X8 < -960\ cm$
$-940\ cm < X10 < -860\ cm$, $-840\ cm < X12 < -760\ cm$
$-740\ cm < X14 < -660\ cm$, $-640\ cm < X16 < -560\ cm$
$-540\ cm < X18 < -460\ cm$, $-440\ cm < X20 < -360\ cm$
$-340\ cm < X22 < -260\ cm$, $-240\ cm < X24 < -160\ cm$
$-140\ cm < X26 < -60\ cm$, $50\ cm < Y2 < 150\ cm$; ε for geometry variable = 1 *cm*
List of available profiles
S_1 = {2UNP80, 2UNP100, 2UNP120, 2UNP140, 2UNP160, 2UNP180, 2UNP200, 2UNP220, 2UNP240}
S_2 = {UNP80, UNP100, UNP120, UNP140, 2UNP80, 2UNP100, 2UNP120, 2UNP140, 2UNP160}
Loading data
Distributed load on upper members ; $w = -17.5$ *kN/m*
Material properties
$E = 6895000\ N/cm^2 = 10^4\ ksi$; $\rho = 0.0272\ N/Cm^3 = 0.1\ lb/in^3$

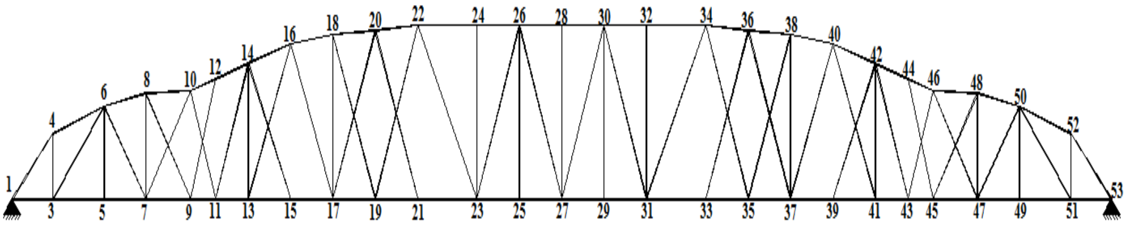


Figure 13. Optimal geometry and topology for the 131-bar truss based on the MHSM

Table 9. Results of optimum design for the 131-bar truss

Sizing and topology design variables								**Geometry design variables**			
A1	--	*A23*	UNP 80	*A45*	2UNP 160	*X2*	--	*Y2*	--
A2	2UNP 80	*A24*	UNP 80	*A46*	--	*X4*	–1201	*Y4*	88.76
A3	2UNP 100	*A25*	2UNP 140	*A47*	2UNP 80	*X6*	–1080	*Y6*	125.666
A4	--	*A26*	--	*A48*	UNP 80	*X8*	–981	*Y8*	143.122
A5	--	*A27*	2UNP 80	*A49*	UNP 80	*X10*	–876	*Y10*	146.788
A6	2UNP 80	*A28*	UNP 80	*A50*	2UNP 160	*X12*	–817	*Y12*	162.597
A7	2UNP 100	*A29*	--	*A51*	--	*X14*	–740	*Y14*	183.228
A8	UNP 140	*A30*	2UNP 160	*A52*	2UNP 80	*X16*	–640	*Y16*	210.022
A9	--	*A31*	UNP 80	*A53*	--	*X18*	–540	*Y18*	222.3
A10	2UNP 80	*A32*	2UNP 80	*A54*	UNP 80	*X20*	–440	*Y20*	227.541
A11	UNP 80	*A33*	UNP 80	*A55*	2UNP 160	*X22*	–340	*Y22*	234.534
A12	2UNP 80	*A34*	UNP 80	*A56*	UNP 80	*X24*	–200	*Y24*	234.534
A13	--	*A35*	2UNP 160	*A57*	2UNP 80	*X26*	–100	*Y26*	234.534
A14	UNP 80	*A36*	--	*A58*	UNP 80				
A15	2UNP 120	*A37*	2UNP 80	*A59*	--				
A16	UNP 80	*A38*	--	*A60*	2UNP 160				
A17	2UNP 80	*A39*	UNP 80	*A61*	UNP 80				
A18	2UNP 80	*A40*	2UNP 160	*A62*	2UNP 80				
A19	UNP 80	*A41*	UNP 80	*A63*	--				
A20	2UNP 120	*A42*	2UNP 80	*A64*	UNP 80				
A21	--	*A43*	UNP 80	*A65*	2UNP 160				
A22	2UNP 80	*A44*	UNP 80	*A66*	UNP 80				

In order to attain the executive goals and beauty of the structure, design variables of sizing, topology, geometry and slope are somehow classified to make the final design symmetric. Therefore, the base of structure is considered to be the lower chord and middle node (node 27) of the truss to make the structure crosswise symmetric. Thus, sizing design variable are classified into 66 groups so that all of them have two members except the last one which has one corresponding to the vertical member connecting nodes 27 and 28. In this regard, a topologic variable is considered for each group. On the other hand, the variables of the geometry and slope are respectively classified into 14 and 13 categories as shown in Table 8. The base of the variation for slope design variable is the horizontal line in the first node of each member.

Figure 13 is resulted utilizing the MHSM. As it is illustrated in Figure 13, the structure tends to be curved which is referred to logical and correct determination of the upper members by MHSM. Zero slopes of the upper members at the middle of structure is due to the type and number of sections in S_2 collection, because the length of vertical and oblique members at the middle of structure, has decreased by zeroing the slope. Therefore, the middle members are prevented from becoming slender.

Table 9 shows the results of the optimum design for design variables on which basis the weight of truss is obtained as 2238.137 *kg*.

6. CONCLUSION

- Using the method of island distribution in the proposed algorithm (MHSM), answers have remarkable diversity and design space is more vastly searched. This is because of allocating different meta-heuristic methods to each island. Hence, design space is wisely searched by various algorithms and the probability of local optimum is nearly diminished.
- Using MHSM causes the optimization issue to be studied by several meta-heuristic methods, simultaneously, and thus all advantageous of meta-heuristic algorithms are incorporated.
- In meta-heuristic algorithms, due to the effect of parameters and governing relations on the results, subsequent executions are used in which the amount of parameters are changed to obtain better answers. Although, due to relative parameters independence and governing relations of the MHSM, this algorithm is free of subsequent executions for not being trapped in local optima. Therefore, it moves to global optimum with a constant and reliable rate, and the probability of getting trapped in local optimum is declined.
- Since in the first variant of the proposed algorithm (MHSM.1), best members of each island are transferred to other islands, during migration process, or in the second variant (MHSM.2), best members are transferred to the selected island and substituted with the members of lower fitness, it is anticipated the convergence speed and average growth rate of the population fitness to be enhanced.
- Employing different value for the parameters of the meta-heuristic methods based on the type of design variables causes each variable to be searched proportional to the related space. This process positively effects on the optimization process of the configuration according to meta-heuristic methods.

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