INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING
Int. J. Optim. Civil Eng., 2016; 6(1):1-12

INVERSE FREQUENCY RESPONSE ANALYSIS FOR PIPELINES LEAK DETECTION USING THE PARTICLE SWARM OPTIMIZATION

M. H. Ranginkaman, A. Haghighi[*, †] and H.M. Vali Samani
Civil Engineering Department, Faculty of Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran

ABSTRACT

Inverse Transient Analysis (ITA) is a powerful approach for leak detection of pipelines. When the pipe transient flow is analyzed in frequency domain the ITA is called Inverse Frequency Response Analysis (IFRA). To implement an IFRA for leak detection, a transient state is initiated in the pipe by fast closure of the downstream end valve. Then, the pressure time history at the valve location is measured. Using the Fast Fourier Transform (FFT) the measured signal is transferred into the frequency domain. Besides, using the transfer matrix method, a frequency response analysis model for the pipeline is developed as a function of the leak parameters including the number, location and size of leaks. This model predicts the frequency responses of the pipe in return for any random set of leak parameters. Then, a nonlinear inverse problem is defined to minimize the discrepancies between the observed and predicted responses at the valve location. To find the pipeline leaks the method of Particle Swarm Optimization (PSO) is coupled to the transient analysis model while, the leak parameters are the optimization decision variables. The model is successfully applied against an example pipeline and in both terms of efficiency and reliability the results are satisfactory.

Kewords: leak detection; inverse transient analysis; frequency response; particle swarm optimization.

Received: 20 May 2015; Accepted: 20 September 2015

---

[*]Corresponding author: A. Haghighi, Department of Civil Engineering, Faculty of Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran
[†]E-mail address: a.haghighi@scu.ac.ir (A. Haghighi)

1. INTRODUCTION

Leaks occur in all pipelines because of poor quality of materials and fittings, error in operation and maintenance, corrosion, high pressures, soil moving and traffic loads. Leaks would result in serious operational and environmental costs. Therefore, development of leak detection methods and devices is highly required. In recent years, several model-based methods for leak detection of pressurized pipes have been introduced. In view point of hydraulic modeling, these methods can be categorized into time and frequency domain analysis approaches. Inverse Transient Analysis (ITA) is a well-known model-based method for leak detection and calibration of pressurized pipelines [1-8]. The hydraulic simulation model is quite important to the ITA success. In most of the previous studies the transient flow is simulated in time domain and analyzed numerically by the Method of Characteristics (MOC). There are several standard references [9-10] that comprehensively describe the MOC and its applications for the transient analysis of pipelines from simple to complex systems. The MOC is well-developed, capable of handling various boundary conditions as well as the nonlinearity of the governing equations. In spite of all that, it needs to discretize the pipes and boundary conditions in both time and space. To increase the model accuracy and capture all the propagating waves the system must be discretized in very small reaches resulting in very small computational time steps. This issue have made the MOC computationally expensive rather than analytical solutions. However, the time-domain analytical solutions are limited to very simple systems and boundary conditions.

An alternative approach to analyze a pipeline under transient flow state is the frequency domain analysis. For this purpose, the transient governing equations are linearized and transferred into the frequency domain and solved analytically. The frequency response method provides a different insight into the system and would manifest the system features and faults e.g., leak more clearly. Accordingly, many researches have been recently concentrated on the frequency characteristics of the transient flow for leak detection of pipelines [11-19]. The frequency response method needs no discretization and is computationally very fast. The leak parameters are more clearly evident in the frequency response diagram of the system and therefore, the predicted leak parameters would be more reliable. Nevertheless, the method has some limitations associated with linearization of the governing equation as well as detection of multiple leaks directly from the system frequency response diagram.

This study introduces an ITA scheme for leak detection of pipelines. The conventional hydraulic simulation model in time domain is replaced by the frequency response model and, the Particle Swarm Optimization (SOP) is used to solve the raised inverse problem. In what follows, the governing equations of pressurized pipes in both time and frequency domains are introduced. Then, the method of transfer matrix [10] is applied to the frequency domain equations. The inverse problem of pipe leak detection in the frequency domain is developed and coupled to the SOP. The model is applied against an example pipeline and, the results are presented and discussed.

2. TRANSIENT HYDRAULICS AND GOVERNING EQUATIONS

The continuity and momentum equations governing the transient pipe flow for most engineering applications are as the following [10]:

$$\frac{\partial Q}{\partial x} + \frac{gA}{a^2}\frac{\partial H}{\partial t} = 0 \tag{1}$$

$$\frac{\partial H}{\partial x} + \frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{fQ|Q|}{2gDA^2} = 0 \tag{2}$$

where, $x =$ distance along pipe, $t =$ time, $a =$ wave speed, $g =$gravitational acceleration, $A =$ cross-sectional area of pipe, $D =$ pipe diameter, $Q =$ instantaneous discharge, $H =$ instantaneous piezometric head and $f =$ Darcy–Weisbach friction factor.

For frequency response analysis, the above equations must be linearized and transferred into the frequency domain. The main concept behind the frequency response method is that; when a pipe flow is excited by a sinusoidal valve maneuver with a constant amplitude and frequency, after a transient period, the pressure and flow in the system will have sinusoidal oscillations as well. The generated periodic flow is called the steady oscillatory flow [10]. Based on the superposition principle in the linear control theory, the response of each arbitrary excitation is the summation of responses of frequency contents of the excitation bandwidth.

For a steady oscillatory flow with frequency ω, the instantaneous flow and head in the governing Eqs. (1 and 2) are considered $Q = (q + Q_0)$ and $H = (h + H_0)$. The subscript "0" is standing for the initial steady conditions and, q and h are respectively, the flow and head oscillations (deviations) with respect to the initial conditions. Substituting Q and H in Eqs. (1 and 2) and linearizing the friction loss term in the momentum equation, the following equations are derived for the steady oscillatory flows [10].

$$\frac{\partial q}{\partial x} + \frac{gA}{a^2}\frac{\partial h}{\partial t} = 0 \tag{3}$$

$$\frac{\partial h}{\partial x} + \frac{1}{gA}\frac{\partial q}{\partial t} + \frac{f|Q_0|}{gDA^2}q = 0 \tag{4}$$

Then, using the Fourier transform, the above equations are transferred in frequency domain as the following.

$$i\omega\hat{h} + \frac{a^2}{gA}\frac{\partial\hat{q}}{\partial x} = 0 \tag{5}$$

$$\frac{\partial\hat{h}}{\partial x} + \left(\frac{i\omega}{gA} + \frac{f|Q_0|}{gDA^2}\right)\hat{q} = 0 \tag{6}$$

in which, $i = \sqrt{-1}$, $\omega =$ angular frequency and hat sign " $\widehat{}$ "is to indicate the flow and head variables as a function of angular frequency. Solving the above equations for a pipeline (Fig. 1) with length l (from the upstream end U to the downstream end D) under a steady oscillatory flow with frequency ω results in the following relations.

$$\hat{q}_D = (\cosh \mu l)\hat{q}_U - \frac{1}{Z}(\sinh \mu l)\hat{h}_U \tag{7}$$

$$\hat{h}_D = -Z(\sinh \mu l)\hat{q}_U + (\cosh \mu l)\hat{h}_U \tag{8}$$

where, $\mu = \frac{i\omega}{a}\sqrt{(1 + \frac{f|Q_0|}{i\omega DA})}$ and $Z = \frac{a^2\mu}{igA\omega}$ are respectively termed the propagation constant and the characteristic impedance.

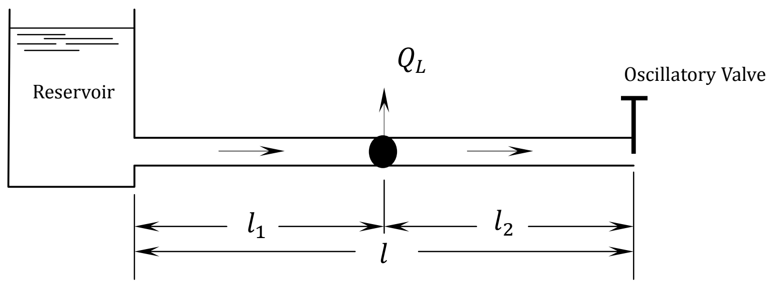


Figure 1. A pipe system with oscillatory valve

For frequency analysis of a pipe system based on the above linear equations, both impedance [9] and transfer matric [10] methods can be used. Nevertheless, the transfer matrix method is more popular since, it is easier to be understood and implemented. The application of the transfer matrix method to analyze the pipe steady-oscillatory flow was first introduced by Chaudhry [20-21] and then, was developed by other researchers specially for leak detection of pipelines [11,14-18]. However, the previous applications of this method are limited to the single leak detection and mostly to find patterns a leak affects the frequency response diagram.

Through the transfer matrix method, the pressure head and flow discharge at the upstream and downstream ends of the pipe are related to each other by a transfer matrix. In this condition, the flow and head at the pipe ends are the system state variables. For a pipeline analysis, there are three types of transfer matrix including; 1- the field matrix “F” to relate the state variables at the upstream and downstream ends of the pipe, 2- the point matrix “P” to relate the state variables at the left and right hand sides of a discontinuity e.g., an orifice or a valve in the pipe and 3- the overall matrix “U” which is the multiplication of the field and point matrixes to relate the states at the upstream and downstream ends of the system. The following equations respectively represent the aforementioned matrices for a simple pipe with a leak in its midpoint (Fig. 1). According to Eqs. (7) and (8) the field matrices for reaches 1 and 2 can be written as follows.

$$F_1 = \begin{bmatrix} \cosh \mu_1 l_1 & -\frac{1}{Z_{c1}} \sinh \mu_1 l_1 \\ -Z_{c1} \sinh \mu_1 l_1 & \cosh \mu_1 l_1 \end{bmatrix} \tag{3}$$

$$F_2 = \begin{bmatrix} \cosh \mu_2 l_2 & -\frac{1}{Z_{c2}} \sinh \mu_2 l_2 \\ -Z_{c2} \sinh \mu_2 l_2 & \cosh \mu_2 l_2 \end{bmatrix}$$

For determining the leak point matrix (P_L), the following orifice equation is used.

$$Q_L = A_e \sqrt{2gH_L} \tag{10}$$

where Q_L= leak discharge, A_e= effective leak area and H_L = is the pressure head at leak location. Substituting $Q_L = (q_L + Q_{L0})$ and $H_L = (h_L + H_{L0})$ into the above equation and linearizing it results in the leak point matrix.

$$P_L = \begin{bmatrix} 1 & -\frac{Q_{L0}}{2H_{L0}} \\ 0 & 1 \end{bmatrix} \tag{11}$$

The overall matrix of a reservoir-pipe-valve system including a single leak (Fig. 1) is $\boldsymbol{U} = \boldsymbol{F_2 P_L F_1}$ as follows.

$$\boldsymbol{U} = \begin{bmatrix} \cosh \mu_2 l_2 & -\frac{1}{Z_{c2}} \sinh \mu_2 l_2 \\ -Z_{c2} \sinh \mu_2 l_2 & \cosh \mu_2 l_2 \end{bmatrix} \begin{bmatrix} 1 & -\frac{Q_{L0}}{2H_{L0}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cosh \mu_1 l_1 & -\frac{1}{Z_{c1}} \sinh \mu_1 l_1 \\ -Z_{c1} \sinh \mu_1 l_1 & \cosh \mu_1 l_1 \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} \hat{q}_3 \\ \hat{h}_3 \end{bmatrix} = \boldsymbol{U} \begin{bmatrix} \hat{q}_1 \\ \hat{h}_1 \end{bmatrix} \tag{13}$$

It is worth noting that, if the system includes multiple leaks, for each individual leak the corresponding point matrix is written (P_{L1}, P_{L2} ,…) and applied to the overall matrix.

In Eq. (13), the water level at the reservoir is constant therefore, $\hat{h}_1 = 0$ and $\hat{q}_3$ is equal to the variation of discharge at the valve during the excitation. Solving the above equation results in the transient pressure heads at upstream of the valve in frequency domain ($\hat{h}_3$). On this basis, a simulation model has been developed to predict $\hat{h}_3$ as a function of leak parameters in the point matrices P_{L1}, P_{L2} ,…

3. LEAK DETECTION METHODOLOGY

Based on the inverse transient analysis method, the Inverse Frequency Response Analysis (IFRA) method is developed here for leak detection of pipelines. The IFRA algorithm includes the following main steps.

1. A transient-state flow is generated in the pipe by fast closure and opening of the downstream end valve. The valve excitation pattern in time is transferred into frequency domain by the FFT and considered as the system's downstream boundary conditions. It is worth noting that, the maximum valve perturbation size must be small enough so that, the valve maneuver can

be assumed linear. In general, the valve perturbation size is considered less than 25% of the initial flow across the valve to decrease the linearization errors in the frequency response analysis [22].

2. The transient pressures at the valve location are measured and transferred into the frequency domain using the FFT. The observed frequency response diagram of pressure heads at the valve location is called $\hat{h}_o$.
3. The pipe transient analysis model based on the transfer matrix method is set up as explained in the previous section. This model is used to analyze the system in frequency domain as a function of leak parameters. For each given leak parameters, the predicted frequency response diagram of pressure heads at the valve location is called $\hat{h}_c$.
4. To find the leak parameters, the observed and predicted frequency responses, $\hat{h}_o$ and $\hat{h}_c$, must be identical. Accordingly an inverse problem with the following least-squares criterion objective function is defined. The decision variables of this objective function are the leak parameters consisting of the number, size and location of leaks.

$$C = \sqrt{\sum_{i=1}^{K} \left(\left|\hat{h}_{oi}\right| - \left|\hat{h}_{ci}\right|\right)^2} \tag{14}$$

where, C is the objective function and K is the number of frequency contents in the observed head response. Note that, $\hat{h}_{oi}$ and $\hat{h}_{ci}$ are complex numbers. To evaluate the objective function the absolute values ($\left|\hat{h}_{oi}\right|$ and $\left|\hat{h}_{ci}\right|$) of them are used. To detect the pipeline leaks, the objective function should be minimized subject to the following constraints on the problem decision variables.

$$\begin{cases} 0 \le nl \le nl_{max} \\ 0 \le A_{e,j} \le A_{e,max} \\ 0 \le l_j \le L \end{cases} \tag{15}$$

in which, nl= number of leaks in the system, nl_{max}= maximum number of leaks, $A_{e,j}$ = effective area size of leak j, $A_{e,max}$ = maximum leak area size, l_j = location of leak j from the upstream end and L = pipe length.

The number of leaks is itself a decision variable hence, the dimension of the optimization problem (total number of decision variables) is prior-unknown. Therefore, in the beginning, the number of leaks nl_{max} is assumed large enough. If so, the problem become a conventional $2nl$- dimensional optimization since, there are nl unknown leak area sizes as well as nl unknown leak locations. After minimization of objective function C, those leaks that have non-zero area size and are not located at the upstream and downstream ends of the pipe are counted as the number of leaks nl in the system so that $nl < nl_{max}$. If finally it is found that $nl = nl_{max}$, the initial assumption for the number of leaks is not sufficient and a larger number is required to be sure that the model has captured all leaks.

To minimize the objective function, a nonlinear optimization model is required. For this purpose, an optimization solver based on the Particle Swarm Optimization (PSO) method is coupled to the proposed IFRA model. The following section describes the applied PSO.

4. PARTICLE SWARM OPTIMIZATION (PSO)

The PSO is a stochastic metaheuristic optimization method that was first introduced by Kennedy and Eberhart [23]. It very soon became a popular metaheuristic optimization model for the sake of its simplicity, low cost of computation and good performance [24]. The main idea behind the PSO is inspired by the behavior of birds flock or fish school when seeking out food in their environment. The PSO is a population-based optimization method so that, its population is called "swarm" and each individual design alternative into the swarm is called "particle". Each particle is assumed to have two characteristics: a position and a velocity. Each particle wanders around in the design space and remembers the best position (in terms of the food source or objective function value) it has discovered. The particles communicate information or good positions to each other and adjust their individual positions and velocities based on the information received on the good positions [25]. The PSO updates the velocity vector of each particle j according to the best local position (solution) $\vec{P}_{best,j}$ associated with the lowest objective value ever visited by itself and the best global position $\vec{G}_{best}$ associated with the lowest objective value encountered in all the previous iterations by any of the swarm particles. Also, two parameters of cognitive and social learning rates, c_1 and c_2 respectively, would balance the relative importance of each particle best experience $\vec{P}_{best,j}$ to the entire swarm best experience $\vec{G}_{best}$. Then, using the updated velocities, the particle positions are updated too and, this procedure is continued until all particles converge to the same position.

In this study, each particle position $\vec{P}_j$ is consisting of the leak parameters for the initially assumed nl_{max} leaks; $\vec{P}_j = \{A_{e,1}, A_{e,2} \cdots, A_{e,nl_{max}} | l_1, l_1 \cdots, l_{nl_{max}}\}$. To find the global optimum particle that minimizes the IFRA objective function (Eq. 14) a simple scheme of PSO is used here as introduced below;

The swarm population size *N* is decided. Usually a size of 20 to 30 particles is assumed for the swarm as a compromise [25]. However, this issue is case-dependent and needs to be calibrated according to the problem size and complexity.

The initial population with *N* particles is randomly generated into the problem's feasible decision space (Eq. 15). Also, initially, all particle velocities are assumed to be zero.

The hydraulic simulation model is run against each particle to evaluate the problem objective function.

The historical best position of each particle $\vec{P}_{best,j}$ and the swarm historical best particle position $\vec{G}_{best}$ are identified.

The velocity vector of each particle j is updated using the following formula:

$$\vec{V}_j^{new} = \vec{V}_j^{old} + c_1 r_1 [\vec{P}_{best,j} - \vec{P}_j] + c_2 r_2 [\vec{G}_{best} - \vec{P}_j] \tag{16}$$

where, r_1 and r_2 are random numbers in the range 0 and 1. Also, the values of c_1 and c_1 are usually assumed to be 2 so that $c_1 r_1$ and $c_2 r_2$ ensure that the particles would overfly the target about half the time [25]. The particle positions are updated as the following.

$$\vec{P}_j^{new} = \vec{V}_j^{new} + \vec{P}_j^{old} \tag{17}$$

After updating the particles, some particle positions may be obtained outside of the feasible search space. For the reposition of the errant particles the following method has been proposed by Formato [26].

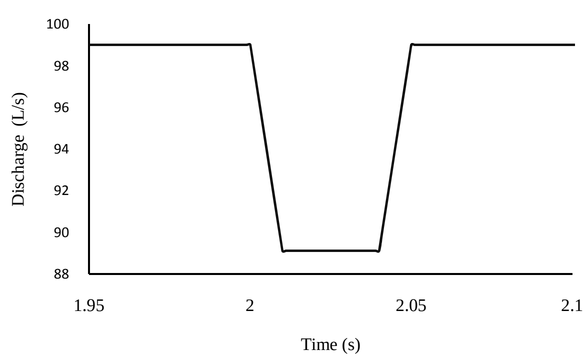
$$\begin{cases} if\ P_{i,j}^{new} > P_i^{max}, & P_{i,j}^{new} = P_i^{max} - F_{rep}\left(P_i^{max} - P_{i,j}^{old}\right) \\ if\ P_{i,j}^{new} < P_i^{min}, & P_{i,j}^{new} = P_i^{min} + F_{rep}\left(P_{i,j}^{old} - P_i^{min}\right) \end{cases} \tag{18}$$

where, P_i^{min} and P_i^{max} are the lower and upper bounds of each decision variable i (from Eq. (15)), and F_{rep}= the user-defined reposition factor in the range 0 and 1 however, it is generally assumed to be 0.5.

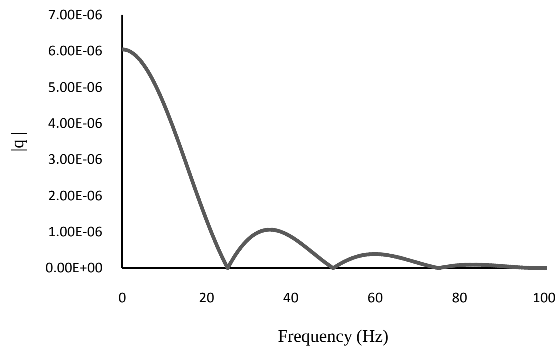
The PSO convergence is checked. If no further improvement was seen in the particle positions the convergence criterion is satisfied otherwise, the algorithm is repeated from step 3 until all particles converge to the same optimum solution.

5. ILLUSTRATIVE EXAMPLE

To investigate the proposed method, a hypothetic reservoir-pipe-valve system according to the schematic of Fig. 1 is analyzed here. The system includes a reservoir with the constant head $H = 20$ m and a pipeline with length $L = 1200$ m; diameter size $D = 250$ mm; Darcy–Weisbach friction factor $f = 0.02$ and wave speed $a = 1200$ m/s. There exists a valve at the downstream end of the pipe. Initially, the valve is partially open so that, the steady-state flow across the valve to atmosphere is 99.02 l/s. To investigate the proposed IFRA method, two leak detection scenarios including the single and multiple leaks in the pipeline are introduced to the pipeline. The transient flow in the system is initiated at $t = 2s$ by closing the valve in 0.01 s for 10% reduction in the initial flow and then after 0.03 s, opening the valve in the same duration to increase the flow to the initial steady state as shown in Fig. 2(a). The flow pattern across the valve is transferred into the frequency domain using the FFT as shown in Fig. 2(b). In each leak scenario, the generated transient flow is analyzed using the MOC in time domain and, the pressure time history at the valve location is considered as the observed data for the IFRA.



(a)



(b)

Figure 2. Pattern of the flow across the valve; (a) in time domain (b) in frequency domain

Two leak scenarios as the following are introduced to the pipeline and then, the pipeline is inversely analyzed using the PSO.

Single-leak detection: in this scenario a single tiny leak with area size of $A_e = 1.3$ cm^2 (about 0.25% of the pipe cross sectional area) is located at $l = 300$ m from the upstream end. A transient condition is initiated in the pipe by the described valve maneuver (Fig. 2a) and the pressure variations at the valve location are measured. The pressure signal is transferred into the frequency domain using the FFT as presented in Fig. 3 compared to the system response diagram for the intact conditions (without leak). The transferred signal is considered as the observed response into the IFRA objective function (Eq. 14). To minimize the objective function, the limits of decision variables, the leak area size and location, are assumed to be $0 \leq A_e \leq 10$ cm^2 and $0 \leq l \leq 1200$. Also, it is assumed the pipe has two leaks with unknown parameters. Accordingly, the problem has four decision variables including two leak area sizes and two leak locations. The PSO was run with 20 particles and very successfully found the leak parameters after about 350 iterations. Table 1 presents the model predications compared to the actual leak parameters. It is worth noting that, the PSO determined a leak area size null which means that the pipeline has only one single leak.

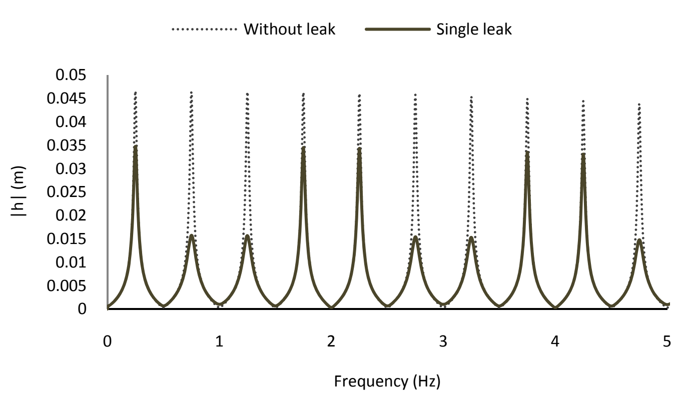


Figure 3. The pressure frequency response diagram at the valve location in the single leak scenario

Table 1: The model results for the single leak scenario

Leak parameters		Error (%)
Actual	Predicted	
$x_l = 300$	$x_l = 300.1$	0.033
$A_e = 1.3$	$A_e = 1.2999$	0.008

Multiple-leak detection: in this scenario the pipe has two leaks located at locations $l_1 = 300m$ and $l_2 = 900m$ respectively with area size $A_{e1} = 1.3\ \text{cm}^2$ and $A_{e2} = 2\ \text{cm}^2$. Similar to the previous scenario, the transient flow is initiated in the system. Then, the flow is analyzed by the MOC and the transient pressures at the valve location are measured and transferred into the frequency domain as depicted in Fig. 4. In this case, it is initially assumed that the pipeline has four leaks with unknown size and location. Therefore, the optimization problem will have eight decision variables. The limits of decision variables are similar to the previous example. The PSO was run with 20 particles and the optimum leak parameters were obtained after about 600 iterations. For two leaks the area size was obtained zero which means that the pipe has two leaks. The parameters of non-zero leaks have been presented in Table 2 in comparison with the actual leaks. The results clearly manifest that the IFRA model is also very good at detecting multiple leaks in the pipeline.

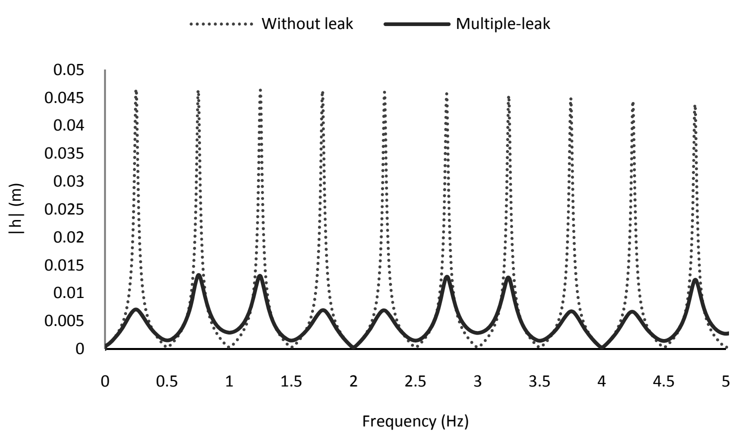


Figure 4. The pressure frequency response diagram at the valve location in the multiple-leak scenario

Table 2: The model results for the multiple-leak scenario

Leak parameters		Error (%)
Actual	Predicted	
$l_1 = 300\ m$	$l_1 = 301.8\ m$	0.600
$A_{e1} = 1.3\ cm^2$	$A_{e1} = 1.304\ cm^2$	0.310
$l_2 = 900\ m$	$l_2 = 899.4\ m$	0.067
$A_{e2} = 2\ cm^2$	$A_{e2} = 1.998\ cm^2$	0.100

6. CONCLUSION

This study introduced a model-based leak detection method for pipelines using the inverse transient analysis in frequency domain and the particle swarm optimization. The method was applied against a reservoir-pipe-valve system and evaluated for both single- and multiple-leak detection. Transient hydraulic simulation of pipelines in frequency domain is much faster than in time domain since, it needs no numerical computations. Also, the presence of leaks in the frequency diagram of the system is very evident. This issue causes the pipe frequency responses very sensitive to the leak parameters. Hence, application of the frequency response method to the inverse transient analysis makes the process of leak detection computationally more efficient and reliable. Also, by using the inverse frequency analysis the pipeline needs no discretization in time and space and therefore, unlike the time-domain methods, there is no limitation to simulate leaks only on the discretization nodes. In fact, through the proposed method leak locations are treated continuously. The previous studies that utilized the frequency response method directly (without optimization) for leak detection, mostly rely on the determined conditions of the system and present a relationship for the single leak detection. Formulating the problem as an inverse problem and using the particle swarm optimization makes it possible to analyze the system in underdetermined conditions and detect multiple leaks.

REFERENCES

1. Liggett JA, Chen LC. Inverse transient analysis in pipe networks, *J Hydraul Eng* 1994; **120**(8): 934-95.
2. Vitkovsky J, Lambert MF, Simpson A, Liggett JA. Experimental observation and analysis of inverse transients for pipeline leak detection, *J Water Resour Plann Manage* 2007; **133**(6): 519-30.
3. Al-Khomairi AM. Leak detection in long pipelines using the least squares method, *J Hydraul Eng* 2008; **46**(3): 392-401.
4. Shamloo H, Haghighi A. Leak detection in pipelines by inverse backward transient analysis, *J Hydraul Res, IAHR*, 2009; **47**(3): 311-18.
5. Shamloo H, Haghighi A. Optimum leak detection and calibration of pipe networks by inverse transient analysis, *J Hydraul Res* 2010; **48**(3): 371-76.
6. Haghighi A, Shamloo H. Transient generation in pipe networks for leak detection, *Proc Inst Civ Eng Water Manag* 2011; **164**(6): 311-18.
7. Haghighi A, Ramos H. Detection of leakage freshwater and friction factor calibration in drinking networks using central force optimization, *Water Resour Manage* 2012; **26**(8): 2347-63.
8. Shamloo H, Haghighi A. Leak detection in pipelines based on inverse transient modeling and mixed integer nonlinear programming, *J Hydraul* 2009; **3**(2): 27-42. (in Farsi).
9. Wylie EB, Streeter VL. *Fluid Transients in Systems*, Prentice Hall, Englewood Cliffs, New Jersey, USA, 1993.

10. Chaudhry MH. *Applied hydraulic transients*, Third Edition, Springer, New York Heidelberg Dordrecht, London, 2014.
11. Mpesha W, Chaudhry MH, Gassman SL. Leak detection in pipes by frequency response method using a step excitation, *J Hydraul Eng, ASCE* 2002; **40**(1): 55-62.
12. Mpesha W, Gassman SL, Chaudhry MH. Leak detection in pipes by frequency response method, *J Hydraul Eng, ASCE* 2001; **127**(2): 134-47.
13. Kim SH. Extensive development of leak detection algorithm by impulse response method, *J Hydraul Eng, ASCE* 2005; **131**(3): 201-8.
14. Lee PJ, Vítkovský JP, Lambert MF, Simpson AR, Liggett JA. Leak Location Using the Pattern of the Frequency Response Diagram in Pipelines: a Numerical Study, *J Sound Vibration* 2005; **284**(3): 1051-73.
15. Lee PJ, Vitkovsky´ JP, Lambert MF, Simpson AR, Liggett JA. Frequency domain analysis for detecting pipeline leaks, *J Hydraul Eng, ASCE* 2005; **131**(7): 596-604.
16. Lee PJ, Lambert MF, Simpson AR, Vitkovsky´ JP, Liggett J. Experimental verification of the frequency response method for pipeline leak detection, *J Hydraul Res* 2006; **44**(5): 693-707.
17. Sattar AM, Chaudhry MH. Leak detection in pipelines by frequency response method, *J Hydraul Res* 2008; **46**(EI1): 138-51.
18. Duan H, Lee PJ, Ghidaoui M, Tung Y. Leak detection in complex series pipelines by using system frequency response method, *J Hydraul Res* 2011; **49**(2), 213-23.
19. Duan H, Lee PJ, Ghidaoui M, Tung Y. System response function based leak detection in viscoelastic pipeline, *J Hydraul Eng* 2012; **138**(2): 143-53.
20. Chaudhry MH. *Resonance in Pressurized Piping Systems*, thesis presented to the University of British Columbia. Vancouver, British Columbia, Canada, in partial fulfillment of the requirements for the degree of doctor of philosophy, 1970.
21. Chaudhry MH. Resonance in Pressurized Piping Systems, *J Hydraul Div, ASCE* 1970; **96**: 1819-39.
22. Lee PJ, Vítkovský JP. Quantifying linearization errors when modelling of fluid transients using the frequency response method, *J Hydraul Eng* 2010; **136**(10): 831-36.
23. Kennedy J, Eberhart RC, Particle swarm optimization, *Proceedings of the 1995 IEEE International Conference on Neural Networks*, IEEE Service Center, Piscataway, NJ, 1995.
24. Kennedy J, Eberhart RC. *Swarm Intelligence*, Morgan Kaufmann Publishers, Inc, USA, 2001.
25. Rao SS. *Engineering Optimization: Theory and Practice*, John Wiley & Sons, Inc, New Jersey, 2009.
26. Formato RA. Central force optimization: a new metaheuristic with applications in applied electromagnetics, *J Progr Electromagn Res* 2007; **7**(1): 425-49.