LEAK DETECTION IN WATER DISTRIBUTION SYSTEM USING NON-LINEAR KALMAN FILTER

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ABSTRACT

Leakage detection in water distribution systems play an important role in storage and management of water resources. Therefore, to reduce water loss in these systems, a method should be introduced that reacts rapidly to such events and determines their occurrence time and location with the least possible error. In this study, in order to determine position and amount of leakage in distribution system, a detection method based on hydraulic model was evaluated using Extended Kalman Filter (EKF), which is a non-linear Kalman Filter. The results indicated that the method was well able to predict leakage position and its amount. Using a numerical model, a leakage was placed in 25.4 m distance of its upstream, amounting to 1.33 lit/sec which was equal to 10 percent of overall flow. The calculated mean position and leakage value by EKF were 27.17 m and 1.11 lit/sec, respectively.

1. INTRODUCTION

Water is the most valuable natural resource on earth. This resource plays an important role in environment, economy and human health. Distribution systems which transfer water from its resource to consumers on other parts, have an essential impact on economy and survival of human life. The pipeline networks are complex systems that require a high level of investment for their construction and maintenance. Failures of some components and communication lines in the municipal water supply system, which often result the catastrophic consequences, are inevitable [1]. Loss control in water distribution systems was addressed in the present study. Since water distribution systems are a set of buried equipment, finding damaged part is very difficult. Leakage issue in water distribution

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systems associates with impacts such as Water loss, energy consumption increase, contamination of drinking water due to infiltration of contaminants into pipe, water pressure loss and, consequently, decrease of delivery to consumers, as well as environmental damages. Water loss is high in worn-out systems. Large leakages or fractures may be detected by penetrating water to passage surface and insufficient water supply due to system pressure loss as well. Determination of smaller leakages, which does not lead to water accumulation or high pressure loss in system, is much more difficult and may be detected only by continuous system monitoring. It is clear that to determine leakage at a time close to real time, better and more accurate system monitoring is required. Various methods were used to determine leakage in water distribution systems, such as artificial neural networks [2], state estimation [3, 4, 5], statistical process control [6] and time series modeling [7]. All of these methods use measurements of system, such as pipe flow rate and pressure head.

Pudar and Liggett [2] solved a reverse problem using flow or pressure measurement. First, pressure values were obtained from a real system. Then a leakage, which came out uniformly from a number of holes along a pipe, was modeled and pressure value was calculated by Orifice Equation in each hole. Difference between measured and calculated values was minimized using least squares method and through this way true value of leakage was calculated. Liou [9] suggested leakage determination method based on mass equilibrium, i.e. a mass entering a system must come out with the same amount. A steady state model was used in this study. Liou and Tian [10] developed a model for a pipeline using transient flow and compared its results to field data. They employed Cauchy Algorithms and time marching to define pattern differences between modeled flow and pressure values on a pipe ends. They mentioned that existing noise often made leakage determination problematic. Brunone [11] calculated position of a leakage on pipes ends through return time of pressure wave. He compared pressure effect, which was measured on upstream end of pipe, to those on pipes suspected of leakage. Position of leakage was detected by occurrence of temporary fluctuations, occurrence of leakage, and duration of fluctuations. Vitkovsky et al. [12] modeled an inverse problem using genetic algorithm (GA). GA is not also guaranteed to converge to the desired position, but because of considering random factors in solution, an extensive search space was covered. Convergence was an extremely time-consuming process in complex problem. Haghighi et al. [13] used Inverse Frequency Response Analysis (IFRA) method. To implement an IFRA for leak detection, a transient state is initiated in the pipe by fast closure of the downstream end valve. Then, the pressure time history at the valve location is measured. Using the Fast Fourier Transform (FFT) the measured signal is transferred into the frequency domain. Besides, using the transfer matrix method, a frequency response analysis model for the pipeline is developed as a function of the leak parameters including the number, location and size of leaks. Then, a nonlinear inverse problem is defined to minimize the discrepancies between the observed and predicted responses at the valve location. Tang et al. [14] used genetic algorithm to calibrate model of a system and to solve unknown leakage of pipe. In this research, convergence was also slowly occurred, therefore, in order to determine error, state estimation techniques were applied on liquid systems.

Kang and Lansey [15] compared linear Kalman Filter to detection state estimator in order to estimate demand of a set of nodes. By measuring root of meansquares error, they found that detection state estimator method had better performance in detection. Kang et al. [16\

found that urban water distribution systems, even loop systems, were not generally high order non-linear systems, hence first order covariance of non-linear Kalman Filter gave acceptable results in those systems. Ye and Fenner [3], introduced a new method for determining the time of leakage in a specified area using Kalman Filter. They determined the amount of leakage by calculating the residuals of the filter that is the difference between the predicted flow and the measured flow. In order to study and detect fracturing, Kaplan et al. [17] introduced indexes of residual flow/pressure (difference between measured and calculated values), normal residual flow/pressure, moving average of residual flow/pressure, normalized moving average of residual flow/pressure. They found that flow-based indexes showed better performance in leakage detection.

Since dynamic equations of studied system were non-linear, therefore extended Kalman filter, which is a non-linear filter, should be used. Hence, in this research, to fully express non-linear relations between system parameters, for its rapid convergence and its high capability of noise control, a technique based on hydraulic model using EKF was used and evaluated in order to determine leakage through water distribution system of Fath Abad Ferdows Village, located in Southern Khorasn Province.

2. MATERIALS AND METHODS

2.1 Studied area

Fath Abad Village, is included in Baghestan County, which is a subsidiary of Central District of Ferdows Township, and is located 25 km southeast of Ferdows Town. Water distribution system of the village consisted of 86 polyethylene pipes, 87 consumption nodes, a pressure reducing valve and a 300 m3 reservoir. The highest and the lowest network elevation were 1845 m and 1815 m, respectively, and inlet discharge was 16 lit/sec. Fig. 1 indicates geographical position of water distribution network.

2.2 Watergems software

WaterGems is an efficient software for analysis of transferring pipelines and water distribution systems which may analyze network schematically or on scale. When plotting distribution system, the software denoted pipes, nodes and other elements. System analysis was done in both steady state and transient state using continuity and energy equations by gradient method. This software is capable of energy loss calculation by equations of Chezy, Colebrook-White, Hazen-Williams, Darcey-Weisbach, Swamee- Jain and Manning. In this study, WaterGems Software was employed to model distribution network in steady state conditions without any leakage. Discharge Values of pipes and nodal pressure head, obtained from the program analysis, were involved as initial conditions for solving transient system equations.

Figure 1. Geographical position of water distribution network of Fath abad Village

3. KALMAN FILTER

Kalman Filter was initially introduced by Rudolf Emil Kalman. He presented an article titled as "Filtering problem of linear discrete data" describing a recursive method [18]. Kalman Filter is a technique of state estimation in linear dynamical systems which are perturbed by Gaussian white noise and uses measurements that are functions of state and include another Gaussian white noise [19]. Covariance error is minimized by using this filter in a linear stochastic system. There are two types of noises in stochastic systems, process noise and measurement noise. Process noise is defined as the difference between a real system and a model. Measurement noise is related to noise of system sensors and equipment. Kalman filter may tolerate conditions including frequent noises or data with high uncertainties, and, consequently, it may be accepted as an appropriate method for leakage detection in pipes. This filter requires a discrete model of state space. State space models are easy to use in estimation and control problems, and change complicated problems to simple ones. Equations (1) to (6) indicate recursive algorithm of state estimation by Kalman Filter.

Prediction equations:

$$X(k + 1|k) = A(k)X(k|k) + B(k)u(k) + w_k, \tag{1}$$

$$Y(k + 1|k) = H(k)X(k + 1|k) + v_k, \tag{2}$$

$$P(k + 1|k) = A(k)P(k|k)A(k)' + Q_k \tag{3}$$

Correction equations:

$$K(k+1) = P(k|k)H(k)'(H(k)P(k|k)H(k)' + R_k)^{-1}k) \tag{4}$$
$$X(k+1|k+1) = X(k+1|k) + K(k+1)[Z(k+1) - Y(k+1|k)] \tag{5}$$
$$P(k+1|k+1) = [I - K(k+1)H(k)]P(k+1|k) \tag{6}$$

where, x_k is the (n × 1) state vector at time t_k, A_k is the (n × n) system or transition matrix of constants for time t_k, B_k is the (n × r) input matrix, u_k is the (r ×1) input vector, w_k (n ×1) vector of random system disturbances characterized by zero mean white noise, Y_{k+1} is the (p ×1) vector of defined measurements, H_k is the (p × n) output matrix that linearly connects outputs and states, and v_{k+1} is the (p ×1) vector of white measurement noise, P is the (n × n) error covariance matrix , K is the kalman gain, Q(k) and R(k) are the covariances of w_k and v_k.

4. EXTENDED KALMAN FILTER

When dynamical equations of studied system are non-linear, extended Kalman filter should be used [19]. This filter acts similar to Kalman filter, but, in EKF, Equations must be linearized at the last state estimation for each time step. Time update equations are linearized at a posteriori state ($\hat{x}_k$) and measurement equations are linearized at a priori state ($\hat{x}_k^-$). Linearization process is done by using Taylor series approximation. It is assumed that a non-linear model is defined by state and measurement equations as the following:

$$x_{k+1} = f[x_k] + u_k + w_k, \tag{7}$$
$$z_k = h[x_k] + v_k, \tag{8}$$

In general, linearization of a non-linear function such as h[x_k], by Taylor series is as follows:

$$h[x_k] \approx h[\hat{x}_{k+1}^-] + J_h[x_k - \hat{x}_{k+1}^-]. \tag{9}$$

Where J_h is Jacobian of matrix h[x_k] at $\hat{x}_{k+1}^-$. Jacobian matrix is given as follows:

$$J_h = \frac{\partial h[x_k]}{\partial x}\bigg|\hat{x}_{k+1}^- = \begin{bmatrix} \frac{\partial h[x_{1k}]}{\partial x_1} & \cdots & \frac{\partial h[x_{1k}]}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial h[x_{nk}]}{\partial x_1} & \cdots & \frac{\partial h[x_{nk}]}{\partial x_n} \end{bmatrix} \tag{10}$$

A non-linear function f[x_k] is linearized at $\hat{x}_k$ using Taylor series:

$$f[x_k] \approx f[\hat{x}_k] + J_x[x_k - \hat{x}_k], \tag{11}$$

Jacobian matrix J_x is defined as follows:

$$J_x = \frac{\partial f[x_k]}{\partial x}\bigg|\hat{x}_k = \begin{bmatrix} \dfrac{\partial f[x_{1k}]}{\partial x_1} & \cdots & \dfrac{\partial f[x_{1k}]}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f[x_{nk}]}{\partial x_1} & \cdots & \dfrac{\partial f[x_{nk}]}{\partial x_n} \end{bmatrix} \tag{12}$$

Using above-mentioned issues, two groups of equations are obtained in EKF which include predict and update equations. The equations are summarized below:

Prediction equations:

$$\hat{x}_{k+1}^- = f[\hat{x}_k] + u_k, \tag{13}$$
$$P_{k+1}^- = J_x P_k J_x^T + Q_k, \tag{14}$$

Correction equations:

$$K_k = P_k^- J_h^T (J_h P_k^- J_h^T + R_k)^{-1}, \tag{15}$$
$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - J_h \hat{x}_k^-). \tag{16}$$
$$P_k = (I - K_k J_h)P_k^-. \tag{17}$$

After achieving state variables by EKF algorithm, amount and position of real leakage may be estimated.

5. DISCUSSION AND RESULTS

The studied model included a pipeline of 51 m length, 125 mm outer diameter and 106.6 mm inner diameter. Discharge of the pipe was 13.3 lit/sec in steady state conditions without any leakage.

To estimate leakage, the pipe was divided in 3 equal segments of 16.93 m length and 4 nodes, as well, as can be seen in Fig. 2. In order to enhance calculation accuracy, the nodes 1 and 4 were located on the pipe ends, each by 10 cm distance from the end, in this model. Pressure heads of the nodes located on the beginning and the end of the pipe were assumed to be constant during steady flow and were introduced to the model as input variables in u vector. State vector, x, included 12 variables in this model. Four of its variables represented pressure heads of 4 mentioned nodes, 6 variables represented inlet and outlet discharges to those 4 nodes and 2 of its variables represented leakage discharges in two middle nodes. EKF was used to estimate two assumed leakages in two middle nodes. The assumed leakages were employed in determining any inter-nodal leakage, so that their sum was equal to real leakage. Values of these two leakages fluctuated around zero and their sum would be equal to zero, provided there were no leakage in the pipe. Position of the real leakage was determined by linear interpolation based on magnitude and position of two assumed leakages [20]. In Fig. 3, pipe (a) consists of a leakage QL in a distance XL upstream of the pipe. Pipe (b) indicates EKF predict model and two assumed leakages Q_{L1} and Q_{L2} on given

positions X_{L1} and X_{L2}. Considering steady state conditions as well as similar border conditions, these two pipes are the same.
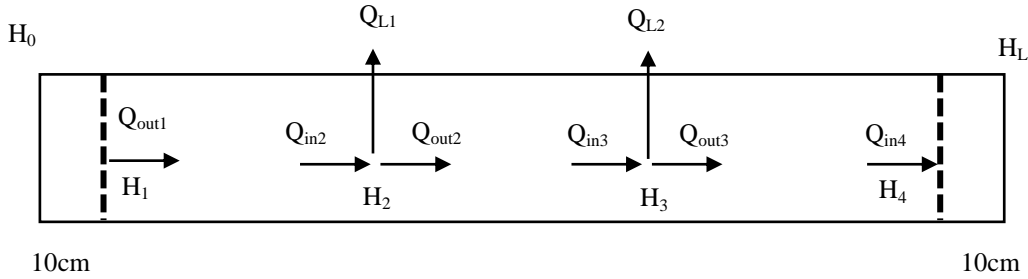

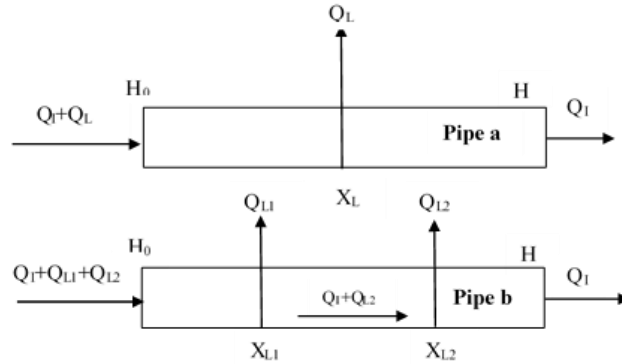Figure 2. State variables of EKF

Figure 3. Flow state in two corresponding pipes

Regarding Equation (18), amount of real leakage was equal to sum of two assumed leakages. Besides, the position of real leakage was achieved by interpolation of these two leakages denoted in equation (19).

$$Q_L = Q_{L1} + Q_{L2}, \tag{18}$$

$$X_L \approx \frac{Q_{L1}X_{L1} + Q_{L2}X_{L2}}{Q_L}. \tag{19}$$

In transient flow model, a leakage was taken into account amounting to 10% of the total flow in a distance of 25.4 m upstream of the pipe. Mean flow in the pipe was 13.3 lit/sec and, consequently, leakage value was 1.33 lit/sec. Regard to this leakage, pressure head values calculated on 4 nodes were combined with white noise to simulate real measured values by sensors. Fig. 4 shows pressure heads of 4 nodes which were used as measurement variables in predict process. As can be seen in the figure, pressure heads of middle nodes greatly fluctuated at the beginning of the time period due to the leakage and then reached a steady state.
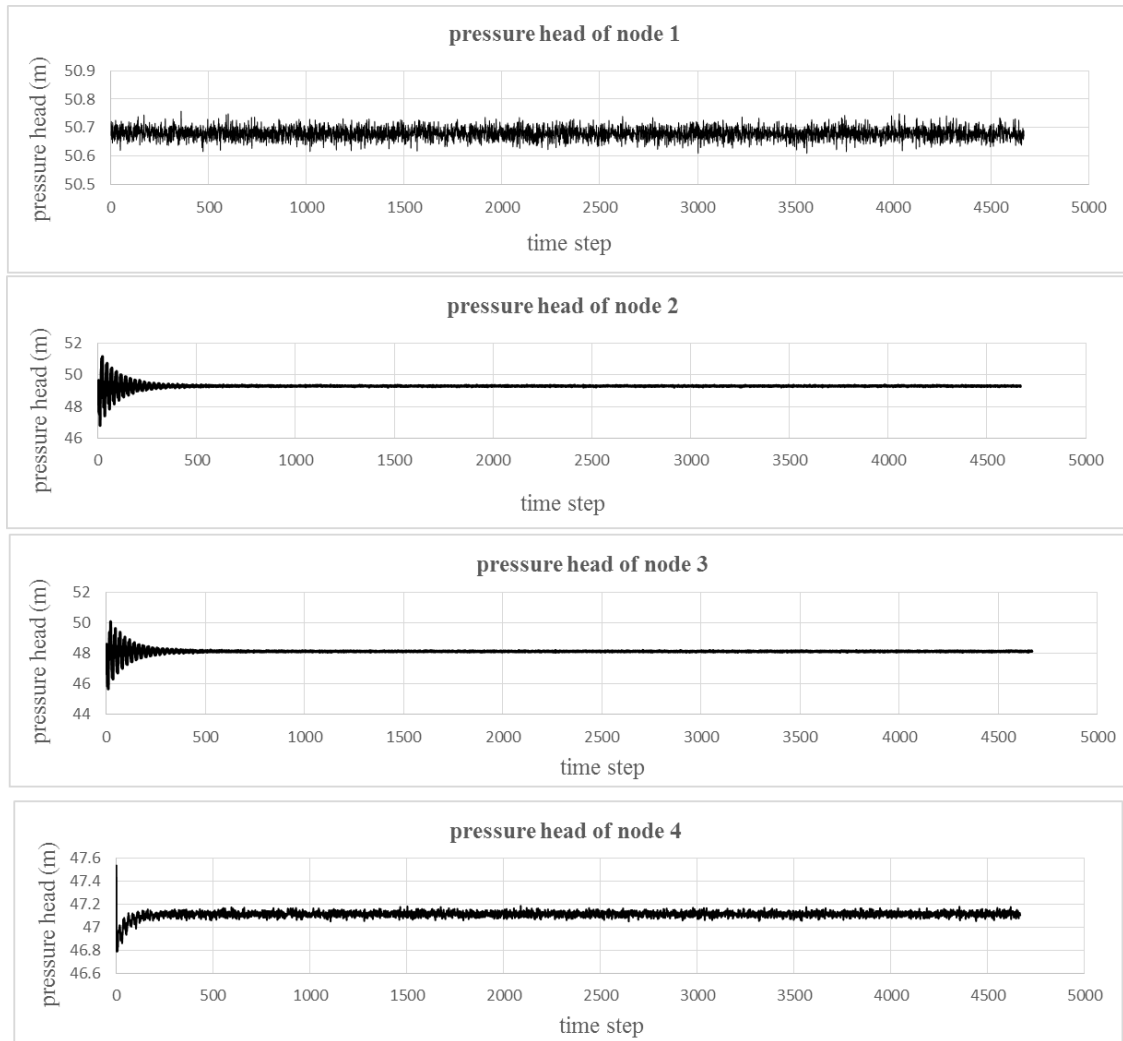
Figure 4. Measurement pressure heads as input for predict model

Using EKF in modeling, assumed leakages positioned at 16.93 m and 33.86 m upstream of the pipe. Fig. 5 indicates leakage values on middle nodes, estimated by EKF. Mean values of QL1 and QL2 were 0.45×10^{-3} and 0.60×10^{-3} m3/sec, respectively. These values were used in equations (18) and (19) to calculate amount and position of real leakage. Since QL2 was greater than QL1, estimated leakage should locate at a farther distance from middle of the pipe.
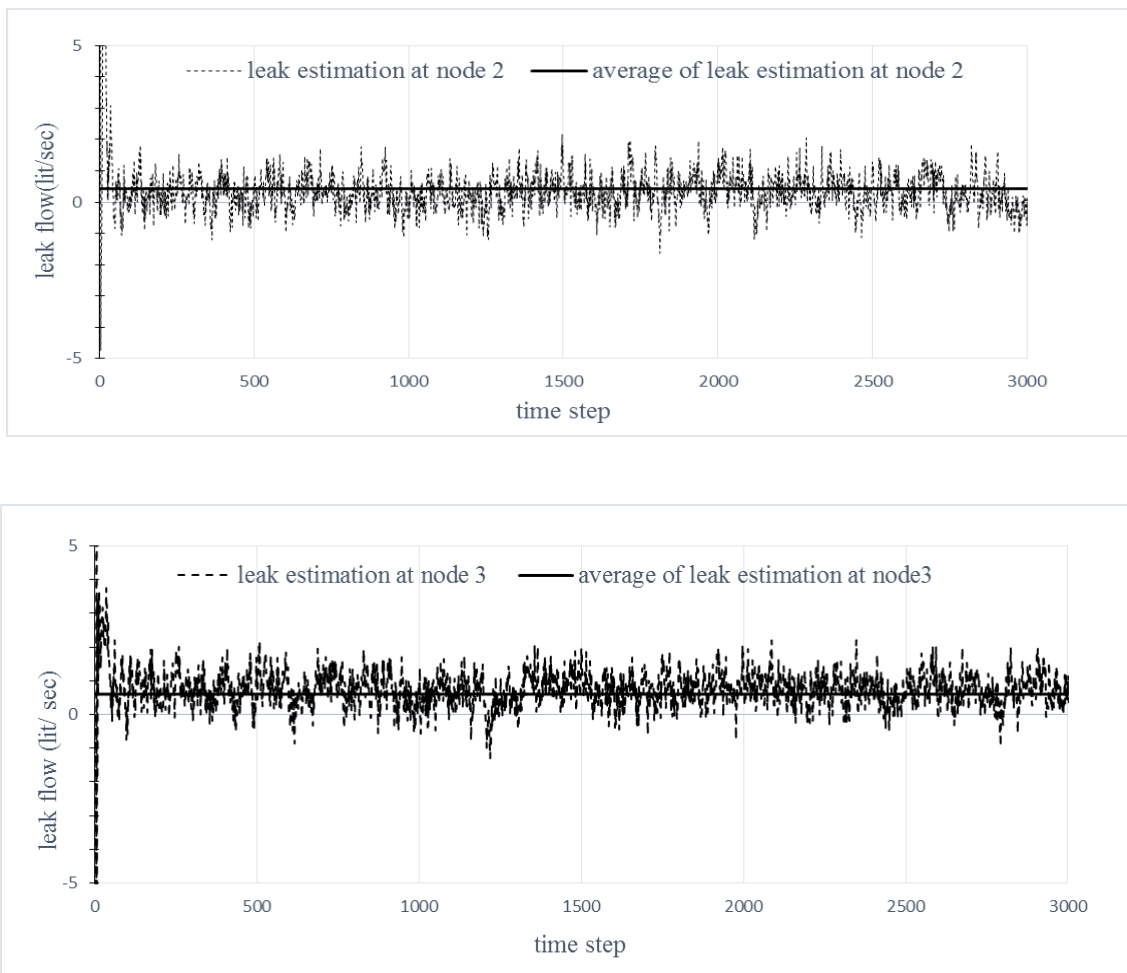
Figure 5. Detection of two assumed leakages by EKF

Fig. 6 shows predicted leakage for all of time steps. Regarding that real leakage was positioned on 25.4 m distance from upstream of the pipe, estimated values fluctuated at this value. Mean position predicted by Kalman filter was 27.17 m. Position fluctuations and their mean values are shown in Fig. 7 for limited number of time steps.

In Fig. 8 predicted leakage value and its fluctuations due to noise occurrence is shown. These fluctuations occurred at real value of leakage, 1.33 lit/sec. Mean value of predicted leakage over time is 1.11 lit/sec. Fig. 9 indicates these fluctuations and their mean within a short period of time.
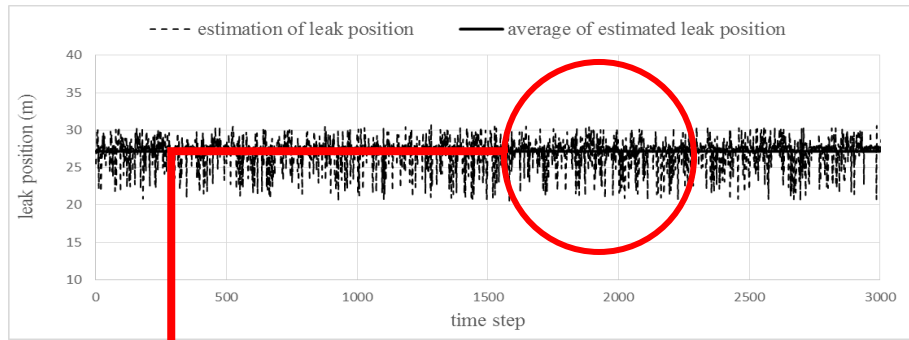
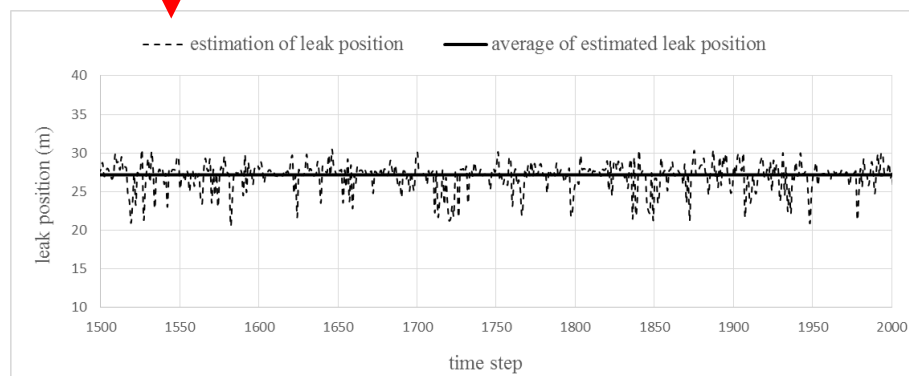Figure 6. Position of leakage predicted by EKF

Figure 7. Details of leakage position predicted by EKF and its mean value
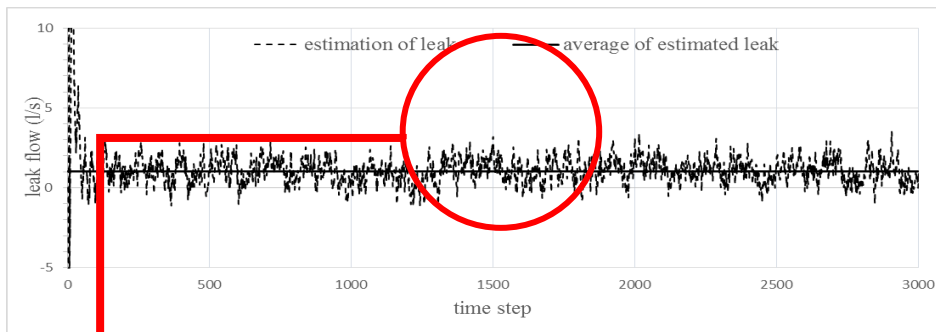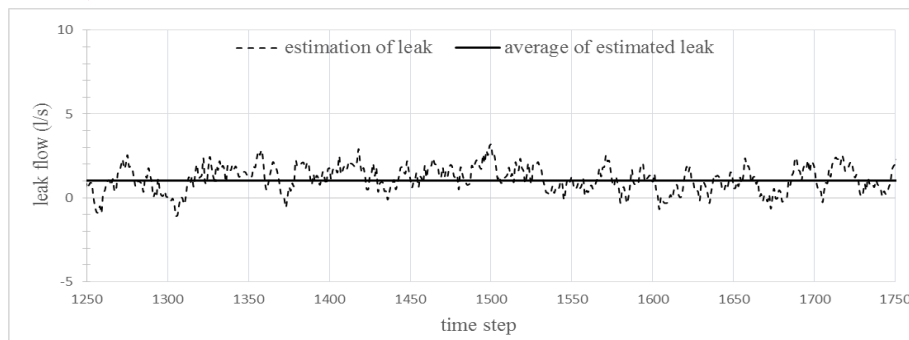

Figure 8. Amount of leakage predicted by EKF

Figure 9. Details of leakage value predicted by EKF and its mean value

6. COMPARISON OF NON-LINEAR KALMAN FILTER AND EKF

In order to use non-linear Kalman filter, momentum and continuity equations, which were employed in leakage modeling, changed to linear equations by using finite difference method, then required coefficients for Kalman filter relations were calculated. The results obtained from this method, for a pipe having properties mentioned in Fig. 2, compared to the results of EKF for 10% leakage positioned at 21.25 m distance from upstream of pipe and is shown in following table:

Table 1: Comparison of non-linear Kalman filter and EKF

Position of real leakage(m)	Real leakage value (m^3/sec)	Predicted position by non-linear KF	Predicted value by non-linear KF	Predicted position by non-linear EKF	Predicted value by non-linear EKF
21.25	0.00133	30.42	0.004	23.40	0.0012
Relative error value		0.4315	2.01	0.1011	0.0769
Calculation time (sec)		302		154	

As can be seen in above table, percent of calculation error for predicting leakage value as well as calculation time were much greater in nonlinear Kalman filter than nonlinear EKF. Higher error percent in non-linear Kalman filter might be due to error occurred during linearization process.

7. CONCLUSIONS

Leakage issue in water distribution system is important considering several aspects such as economic, social, environmental impacts etc. There are different methods to detect leakage in water distribution systems to prevent water loss. One of these method is modeling a system in transient conditions to determine leakage and its position. In this study, in order to determine leakage in distribution system, a technique based on hydraulic model was studied using Extended Kalman Filter (EKF). A leakage was placed in 25.4 m distance of its upstream, amounting to 1.33 lit/sec which was equal to 10 percent of overall flow. The calculated mean position and leakage value by EKF were 27.17 m and 1.11 lit/sec, respectively. Further, both non-linear KF and EKF were compared for the same amount of leakage on another position. The results came out of these two methods demonstrated that EKF could determine amount and position of leakage on pipeline with less error compared to non-linear KF. So that, for a leakage equal to 1.33 lit/sec at a distance of 21.25 m from upstream, mean value of predicted amount and position of leakage were obtained respectively 1.2 lit/sec and 23.4 m by EKF and 4 lit/sec and 30.42 m by non-linear KF.

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