

## DAMAGE IDENTIFICATION IN STRUCTURES USING TIME DOMAIN RESPONSES BASED ON DIFFERENTIAL EVOLUTION ALGORITHM

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### ABSTRACT

An effective method utilizing the differential evolution algorithm (DEA) as an optimisation solver is suggested here to detect the location and extent of single and multiple damages in structural systems using time domain response method. Changes in acceleration response of structure are considered as a criterion for damage occurrence. The acceleration of structures is obtained using Newmark method. Damage is simulated by reducing the elasticity modulus of structural members. Three illustrative examples are numerically investigated, considering also measurement noise effect. All the numerical results indicate the high accuracy of the proposed method for determining the location and severity of damage.

**Keywords:** damage identification; differential evolution algorithm; time domain response; acceleration response; analytical model.

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### 1. INTRODUCTION

The process of damage identification and condition assessment of building structures has attracted increasing interest in the research community during the last several decades as many building structures are now or will soon be, approaching the end of their design lives because of long term deterioration and after facing extreme events such as earthquakes. If appropriate retrofitting is not carried out, buildings can suffer partial or complete collapse without prior warning resulting in loss of human lives and a large economic impact. Therefore the need of damage identification and condition assessment of building structures is essential, during their life, especially when the building is old or is suspected to have been subjected to overloads. Damage in building structures is defined as intentional or unintentional changes to the material or geometric properties of these systems, including

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changes to the boundary conditions and system connectivity, which adversely affect the current or future performance of that structure [1,2].

Damage detection methods are divided into two groups, static and dynamic methods. Dynamic methods compared with static methods are more exact and favorite. Dynamic approaches use from dynamical response of a structure, such as natural frequencies, mode shapes, damping, etc. During the last few years, a great number of nondestructive techniques have been introduced to determine the site and extent of eventual damage in structural systems [3-12]. One type of the methods employs the optimisation algorithms for solving the damage detection problem. An application of genetic algorithms (GA) for determining the location and quantity of structural damage maximizing a correlation coefficient, named the multiple damage location assurance criterion (MDLAC) has been proposed by Koh and Dyke [13]. Doebeling et al. [14,15] have presented comprehensive review of literature mainly focusing on frequency-domain methods for damage detection in linear structures. The relationship between model updating methods and damage detection problem has been explored by He et al. [16]. Alampalli et al. [17] conducted laboratory and field studies on bridge structures to investigate the feasibility of measuring bridge vibration for inspection and evaluation. These studies focused on sensitivity of measured modal parameters to damage. Cross diagnosis using multiple signatures involving natural frequencies, mode shapes, modal assurance criteria and co-ordinate modal assurance criteria was shown to be necessary to detect the damages. Liu [18] examined the influence of input errors on identification process in the context of identification and damage detection in truss structures. Combined experimental and finite element modelling studies has been carried out by Chen et al. [19] on steel channel beams, to detect reduction in load carrying capacity using dynamic response. Zimin and Zimmerman [20] compared frequency domain analysis with time domain analysis and also developed an experimental test based on structural health monitoring. These results have shown to be a reliable indicator of the existence of structural damage. This is indicated using simulated and actual experimental data. Studies by FU et al. [21] have been used to detect damage in a cantilevered steel plate. Two explanatory test examples were considered in order to show the efficiency of the proposed method for determining single or multiple damage cases. Studies by Kaveh et al. [22,23] have been used to identify damage in different truss structures. Enhanced Vibrating Particles system (EVPS) is presented by Kaveh et al. [24] and then Vibrating Particles system (VPS) and EVPS algorithm are employed for damage detection of truss structures. Results indicate that the EVPS algorithm has reached better answer compared to the VPS algorithm for damage identification problems. The investigation of different studies demonstrated that fewer researches are done about damage detection by time domain responses.

This paper is organized as follows. The theoretical background is provided in Section 2. A brief description of the differential evolution algorithm (DEA) is presented in section 3. In section 4, damage identification steps using time domain responses and the differential evolution algorithm (DEA) method are represented. Section 5 contains three numerical examples. Finally, the conclusions are provided in section 6.

## 2. THEORETICAL BACKGROUND

### 2.1 Finite element modeling of the structures

The equations of motion of a structure with  $n$  degrees of freedom and viscous damping coefficients can be expressed as [25]:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{P(t)\} \quad (1)$$

where  $[M]$ ,  $[C]$  and  $[K]$  represent the mass, damping and stiffness matrices, respectively.  $\{u\}$ ,  $\{\dot{u}\}$  and  $\{\ddot{u}\}$  are nodal displacement, nodal velocity and nodal acceleration vectors, respectively, and  $\{P(t)\}$  is load vector.

Rayleigh damping is used, in which the damping matrix is considered as proportional to the combination of the mass and stiffness matrices, as follows:

$$[C] = a_0[M] + a_1[K] \quad (2)$$

where  $a_0$  and  $a_1$  are Rayleigh coefficients parameters to be determined from two modal damping ratios.

The dynamic responses of the structures can be achieved by direct integration using Newmark time integration method [25]. Based on this method, we have to obtain the coefficients of Newmark and then displacement, velocity and acceleration responses. In this study, the damage of structure is evaluated via the variations of acceleration responses.

### 2.2 Damage identification strategy

The optimisation problem to identify damage can be defined as follows:

$$\begin{aligned} \text{Find: } & X^T = \{x_1, x_2, \dots, x_n\} \\ \text{Minimize: } & W(X) \\ \text{Subject to: } & X^l \leq X_i \leq X^u \end{aligned} \quad (3)$$

where  $X^T = \{x_1, x_2, \dots, x_n\}$  is a damage variable vector including the sites and sizes of  $n$  unknown damages,  $X^u$  and  $X^l$  are the upper and lower bounds of the damage vector. Also,  $W$  is an objective function that should be minimized.

Due to occurrence of damage in each structural element, the element stiffness reduces. Hence, in most studies, damage has been simulated by decreasing one of the stiffness parameters of the element such as the elasticity module (E), moment of inertia (I), cross sectional area (A) and etc. In this research, the damage variables are defined via a relative reduction of elasticity modulus of an element.

### 2.2.1 Objective function

Various correlation indices have been selected as the objective function in the literature. In this research, the multiple damage location assurance criteria (MDLAC) introduced in [26] is employed as the objective function for the optimisation given by:

$$W(X) = -\frac{|a_d^T \cdot a(X)|^2}{(a_d^T a_d)(a(X)^T \cdot a(X))} \quad (4)$$

where  $a_d$  and  $a(X)$  are the acceleration vector of the damaged structure and an analytical model due to the acceleration vector of undamaged structure, respectively.  $W$  changes from a minimum value of  $-1$  to a maximum value of  $0$ . It is minimum when the vector of analytical model is equal to the acceleration vector of the damaged structure, that is,  $a(X) = a_d$ .

The optimisation process stops when the objective function is smaller than  $-0.999$  or does not change significantly after a number of successive iterations. In this study, DEA [27] is used to properly solve the problem.

## 3. THE DIFFERENTIAL EVOLUTION ALGORITHM (DEA)

The selection of an appropriate algorithm for solving the optimisation based damage identification problem is an important issue, because the damage detection problem has many local solutions. Thus, the optimisation algorithm should be capable of obtaining the global optimum needing fewer structural analyses without trapping into local optima. In this study, the differential evolution algorithm (DEA) is utilized to correctly solve the damage identification problem. The framework of DEA is alike to a genetic algorithm (GA), however, the classical crossover and mutation operators in GA have been substituted by other operators and consequently came up to a suitable differential operator. The DEA can be fulfilled very easily and needs a small number of parameter tuning. The step by step summary of the DEA, shown in Fig. 1, can be explained as [27]:

a) Initialization. The original parameters, constants and initial population are recognized.

Like other evolutionary algorithms, DEA starts to search from an initial population. The initial population is generated arbitrarily in the search space as:

$$X^l \leq X_i \leq X^u, i = 1, 2, \dots, np \quad (5)$$

where  $X^u$  and  $X^l$  are the upper and lower vectors of a design variable vector, respectively. Also,  $np$  is the number of initial population that must be at least 4.

b) Mutation. For a given vector  $X_i$  ( $i = 1, 2, \dots, np$ ), a mutant vector is defined by a particular combination of three different current solutions as:

$$V_i = X_{r_1} + F(X_{r_2} - X_{r_3}), r_1 \neq r_2 \neq r_3 \neq i \quad (6)$$

where the three different indices  $r_1, r_2$  and  $r_3 \in \{1, 2, 3, \dots, np\}$  are randomly selected to be different for each index  $i$ . Also,  $F \in [0, 2]$  is a real and constant factor which controls the amplification of the differential variation ( $X_{r2} - X_{r3}$ ).

c) Crossover. In order to increase the variety of the perturbed parameter vector, crossover is introduced by producing the trial vectors  $U_i$  ( $i = 1, 2, \dots, np$ ) as:

$$u_{ji} = \begin{cases} v_{ji} & \text{if } (rand_{ji} \leq cr \quad \text{or} \quad j = rand_i) \\ x_{ji} & \text{if } (rand_{ji} > cr \quad \text{and} \quad j \neq rand_i) \end{cases} \quad (7)$$

where  $rand_{ji}$  is a uniformly random number  $\in [0, 1]$ ,  $cr$  is the crossover constant  $\in [0, 1]$  and  $rand_i$  is a random integer  $\in \{1, 2, \dots, n\}$  which ensures that  $U_i$  gets at least one parameter from  $V_i$ .

d) Selection. For final selection, the trial vector  $U_i$  and target vector  $X_i$  are compared. If the vector  $U_i$  yields a smaller objective function value than  $X_i$ , then  $X_i$  is set to  $U_i$ ; otherwise, the old value  $X_i$  is retained.

e) Convergence. In this step, solution convergence is controlled. If the solution is converged, then the optimisation is stopped otherwise return to step b.

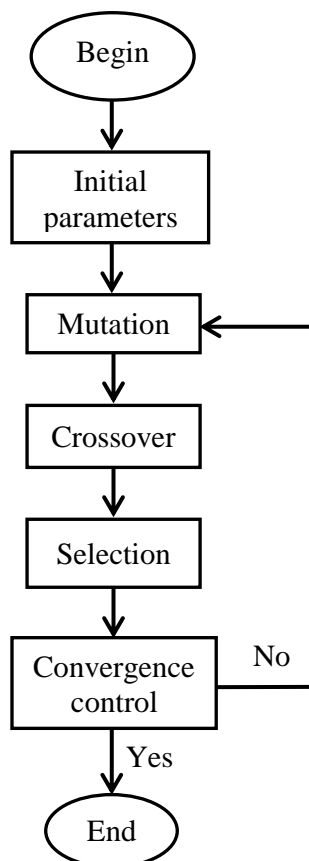


Figure 1. General process of DEA

#### 4. THE EXECUTIVE STEPS OF THE PROPOSED DAMAGE IDENTIFICATION METHOD

Damage identification steps using time domain responses and the differential evolution algorithm (DEA) method are given as follows:

Step 1) An impact loading as shown in Fig. 2 is applied to structures.

Step 2) Analysis of damaged and undamaged structure is done using Newmark method numerically and structure acceleration is extracted at two different points.

Step 3) According to equation 4, the objective function is defined of analytical model acceleration response and damaged structure acceleration.

Step 4) The objective function obtained in the previous step is minimized using DEA and then location and extent of damage is specified.

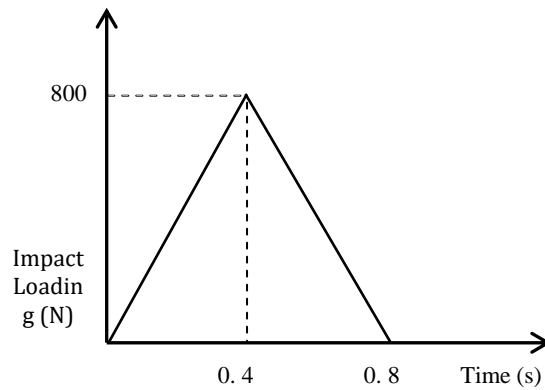


Figure 2. The applied impact loading to structures

## 5. NUMERICAL RESULTS

In this section, the capabilities of the proposed method for detecting structural damage is assessed through three numerically simulated damage detection tests; a 26-bar planar truss, a 31-bar planar truss and a 15-element planar frame are considered with various damage cases. In order to investigate the noise effects on the performance of the proposed method, measurement noise is considered here by a standard error of  $\pm 1\%$  to  $\pm 3\%$ . The effect of measurement noise on the damaged structure acceleration is considered from:

$$N\text{Acceleration } D = \text{Acceleration } D \cdot [1 + (2[\text{random}] - 1) \cdot \text{noise}] \quad (8)$$

where *noise* is the quantity of measurement noise in %, *random* is a positive random function which is smaller than 1, *Acceleration D* is damaged structure acceleration vector and *NAcceleration D* is the damaged structure acceleration vector by considering effects of measurement noise.

### 5.1. Twenty six -bar planar truss

The 26-bar planar truss shown in Fig. 3 is considered to display the robustness of the proposed method. The structure has twenty six members and twelve nodes. All members are made of steel. The elasticity modulus and material density are 210 GPa and 7900 kg/m<sup>3</sup>, respectively. A damage variable in the structure is defined here via a relative reduction in the elasticity modulus of individual bars. Thus, the problem eventually has 26 damage variables. Three different damage scenarios given in Table 1 are induced in the structure and the proposed method is considered including noise effect. In this table damage ratio is defined as the relative reduction of elasticity modulus which is induced at different elements of structure. The DEA is now used to solve the damage identification problem to specify the damage severity. The initial parameters of DEA, counting the number of initial population (*np*), the crossover constant (*cr*) and the constant factor (*F*) are set to 25, 0.4 and 0.7 respectively. The maximum number of generations (*ng*) and a number of successive iterations (*si*) for optimisation are also set to 1000 and 150 respectively. Also an impact

load, as shown in Fig. 2, is applied at node 6. In order to obtain the acceleration response, two sensors are considered to have been installed, one at node 5 and the other at node 9.

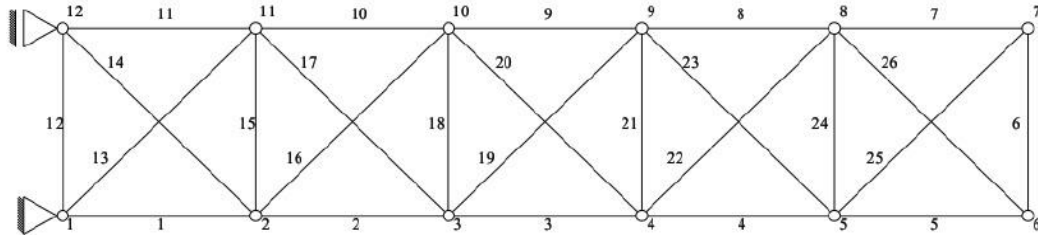
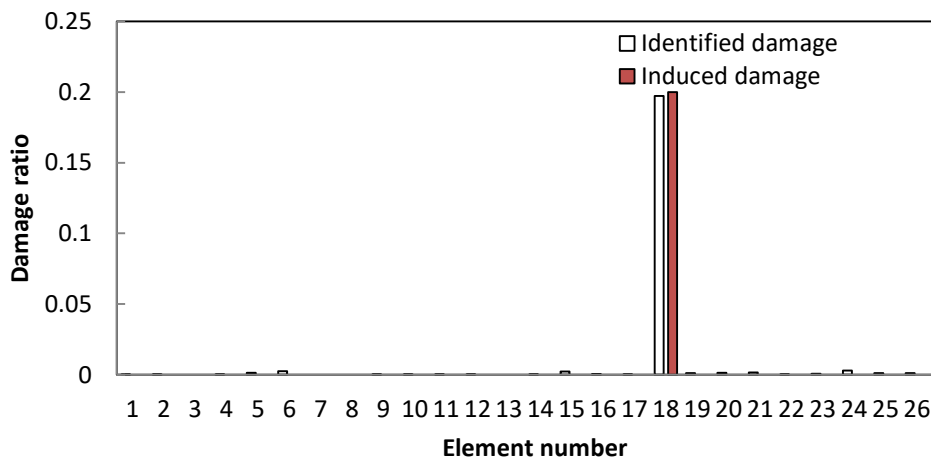


Figure 3. The planar truss having 26 elements

The damage detection results for various damage scenarios achieved by DEA for ten sample runs are shown in Figs. 4-6. It is observed that the optimisation obtains the location and severity of actual damage truthfully. It should be noted that the optimisation process for scenarios 1 to 3 converges to the actual damage after about 30 iterations (4575, 5815 and 6990 finite element analyses, respectively). The final results of various damage scenarios reveal the efficiency of DEA for determining the damage location and severity.

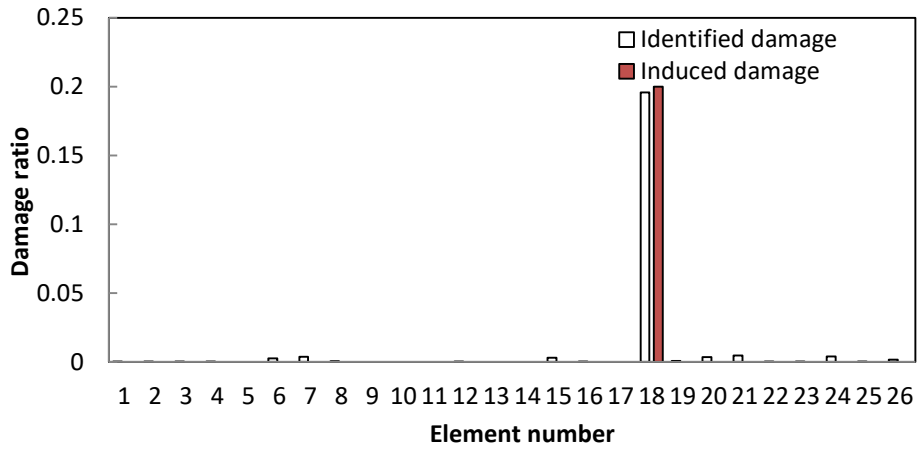
Table 1: Three different damage cases induced in the 26-bar planar truss

Case 1		Case 2		Case 3	
Element Number	Damage Ratio	Element Number	Damage Ratio	Element Number	Damage Ratio
18	0.2	3	0.2	2	0.3
		12	0.2	21	0.3

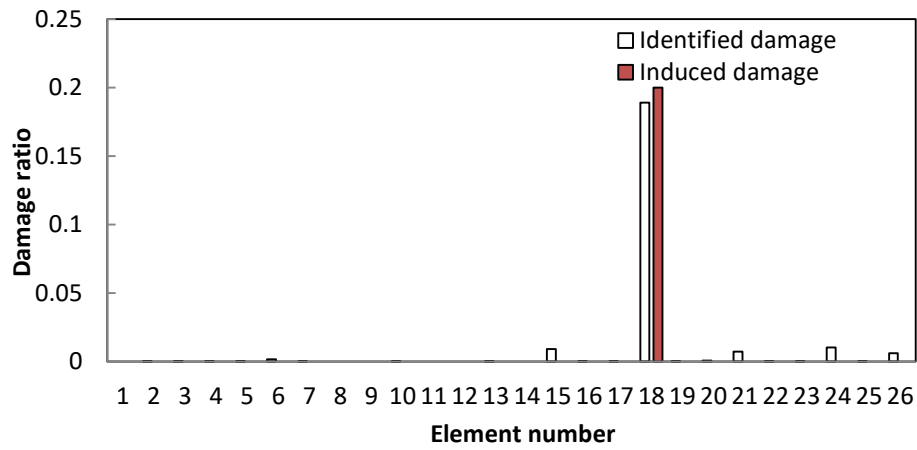


(a)



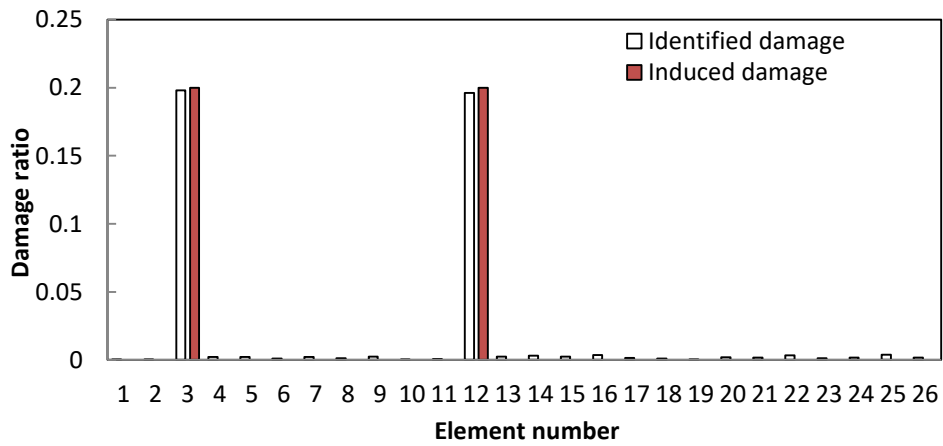


(b)

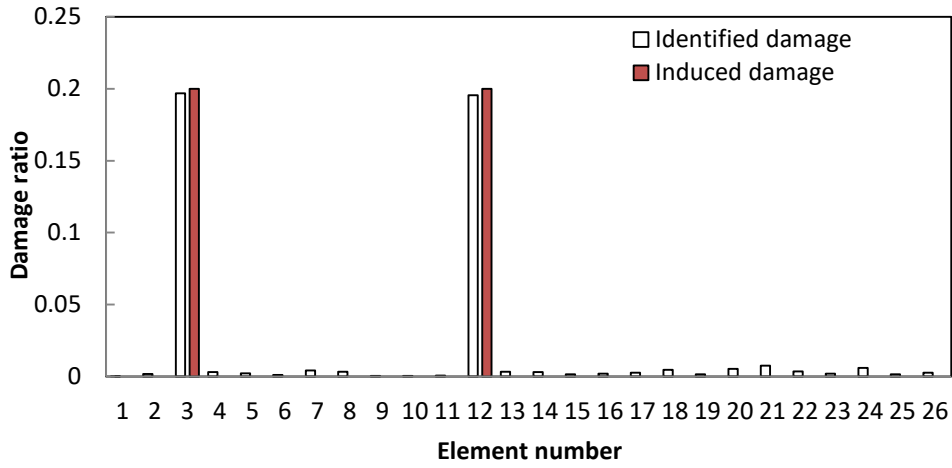


(c)

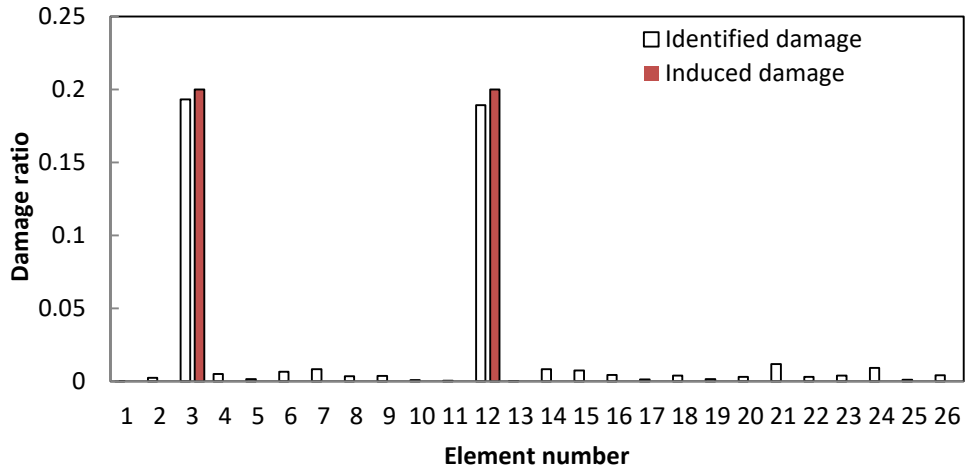
Figure 4. Final identified damage variables of the 26 element truss for case 1 considering noise (a) 1%, (b) 2% and (c) 3%



(a)

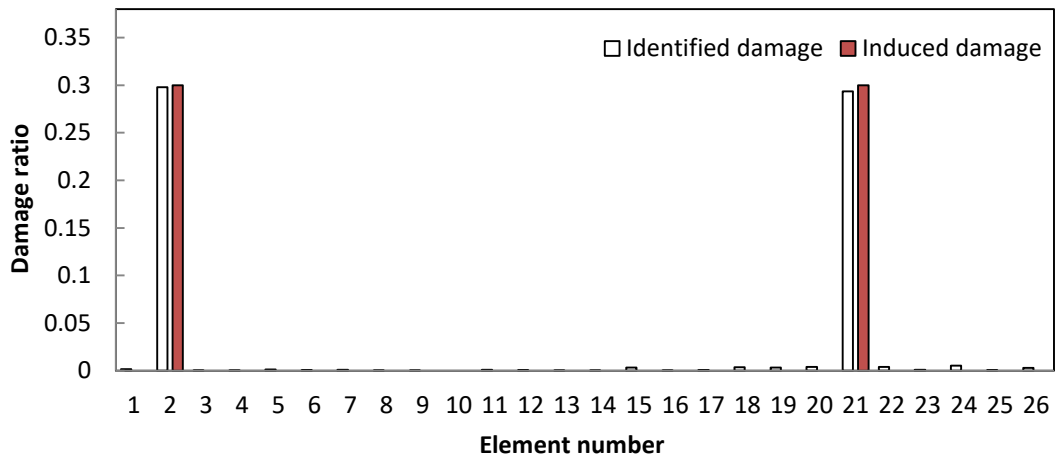


(b)



(c)

Figure 5. Final identified damage variables of the 26 element truss for case 2 considering noise (a) 1%, (b) 2% and (c) 3%



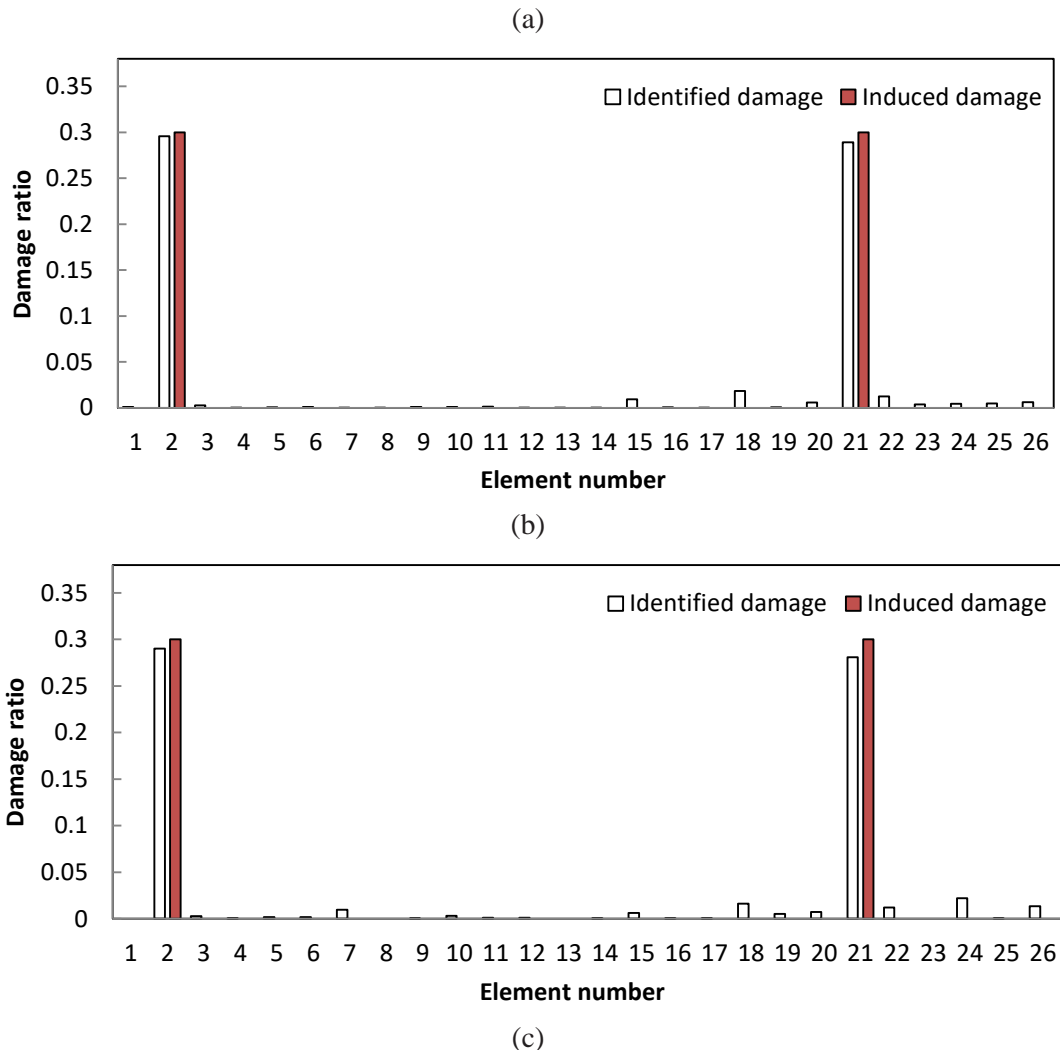


Figure 6. Final identified damage variables of the 26 element truss for case 3 considering noise (a) 1%, (b) 2% and (c) 3%

5.2. Thirty-one-bar planar truss

The 31-bar planar truss [28] shown in Fig. 7 is modelled using the conventional finite element method without internal nodes, leading to 25 degrees of freedom. The elasticity modulus and material density of aluminum truss are 70 GPa and 2770 kg/m<sup>3</sup>, respectively. Damage in the structure is also simulated as a relative reduction in the elasticity modulus of individual bars. Five various damage cases, given in Table 2, are induced in the structure and the DEA is tested for each scenario. In this Table, damage ratio is defined as the relative reduction of elasticity modulus which is induced at different elements of structure. All the optimisation parameters have been the same as in the first example. An impact load, as shown in Fig. 2, is applied at node 13. In order to determine the acceleration response, two sensors are considered to have been installed, one at node 3 and the other at node 11.

In order to consider the stochastic nature of the optimisation based damage identification problem, ten different optimisation runs are made for the damage cases. The damage

detection results of different damage scenarios with considering effects of measurement noise are shown in Figs. 8–12. The numerical results indicate the efficiency of the method for determining the damage location and severity. It is observed that the optimisation process has been able to determine the site and severity of actual damage truthfully. For detection of damage cases 1 to 5 by DEA as shown in figures, 10686, 9667, 11924, 14400 and 16200 finite element analyses (FEA), respectively, were averagely required.

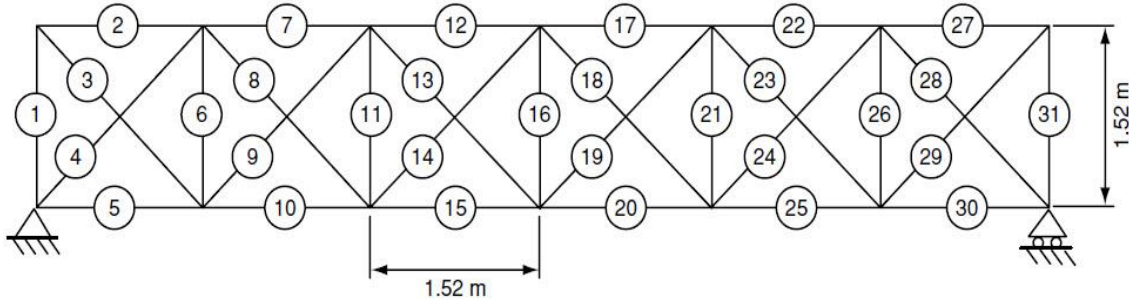
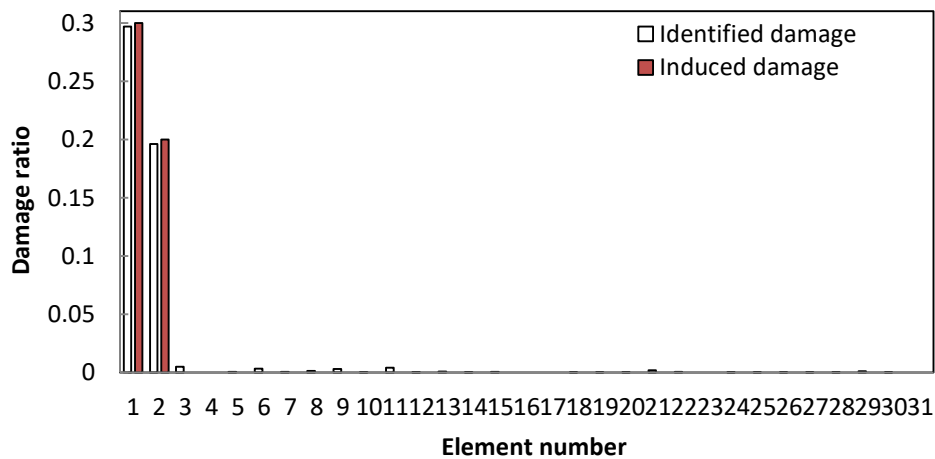


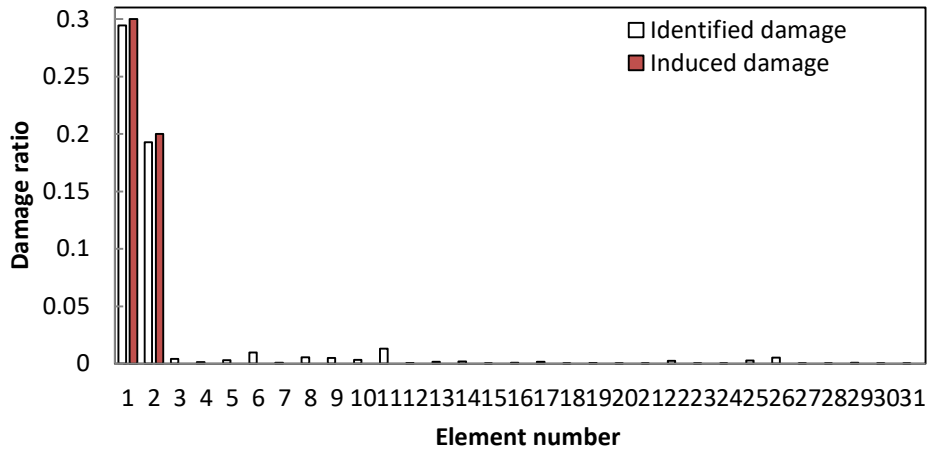
Figure 7. The planar truss having 31 elements

Table 2: Five different damage cases induced in the 31-bar planar truss

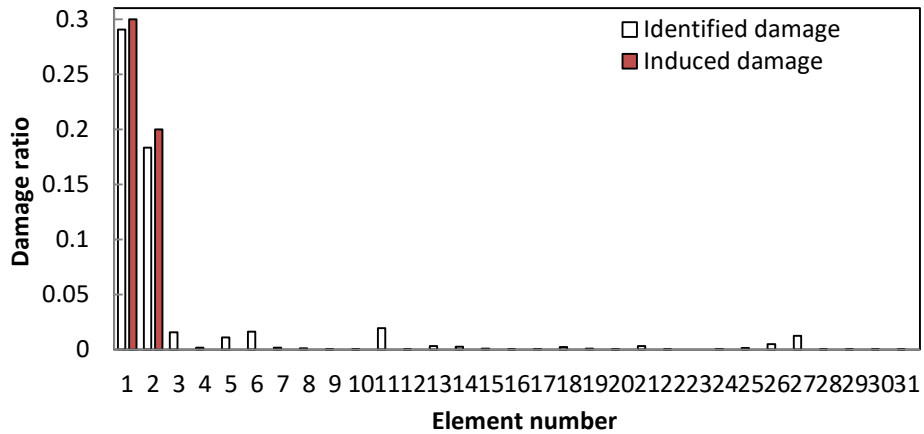
Case 1		Case 2		Case 3		Case 4		Case 5	
Element Number	Damage Ratio	Element Number	Damage Ratio	Element Number	Damage Ratio	Element Number	Damage Ratio	Element Number	Damage Ratio
1	0.3	16	0.3	11	0.25	3	0.1	2	0.05
2	0.2			25	0.15	11	0.2	6	0.1
						13	0.3	15	0.2
								27	0.3



(a)

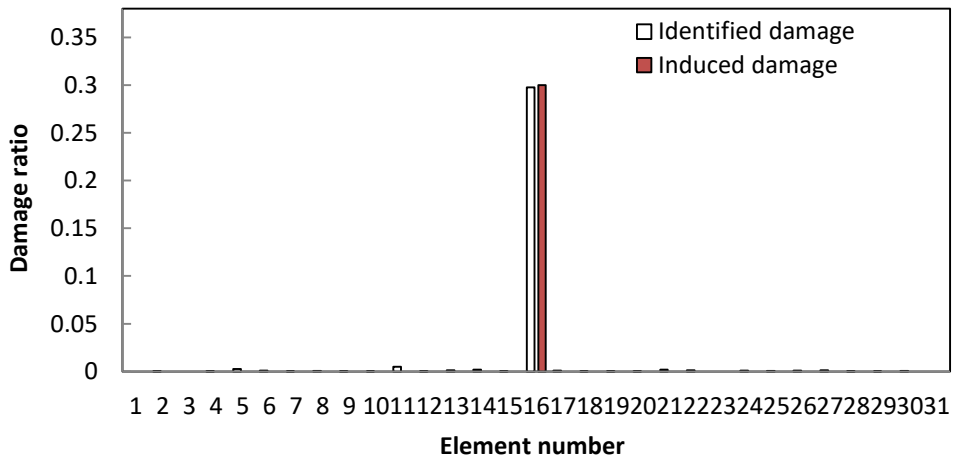


(b)

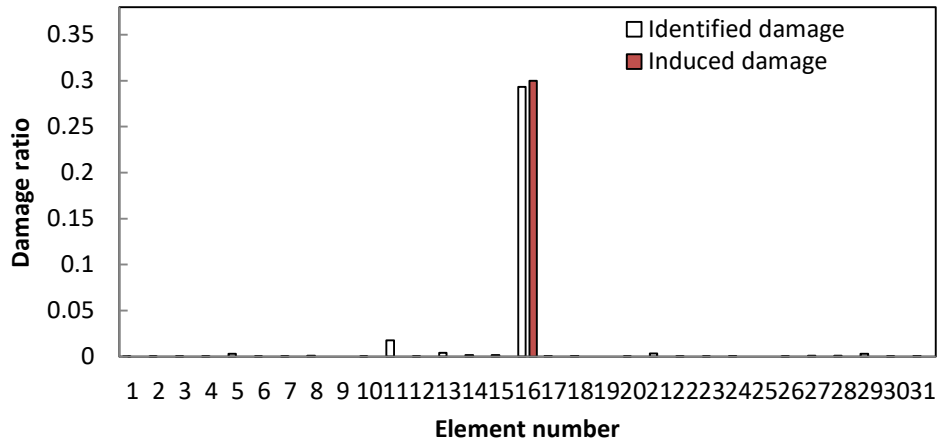


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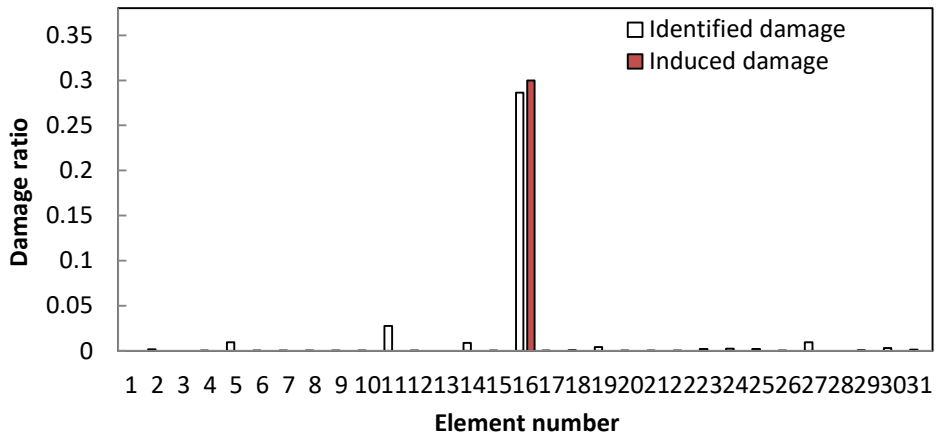
Figure 8. Final identified damage variables of the 31 element truss for case 1 considering noise (a) 1%, (b) 2% and (c) 3%



(a)

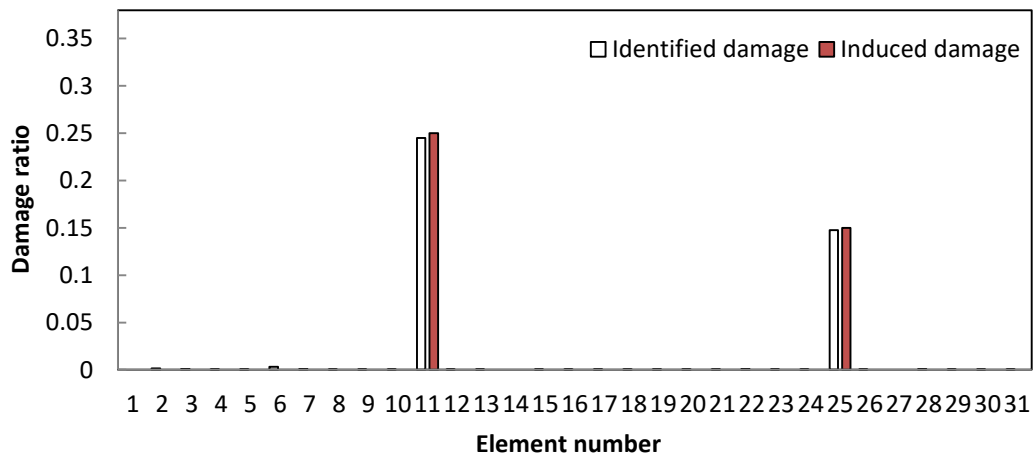


(b)

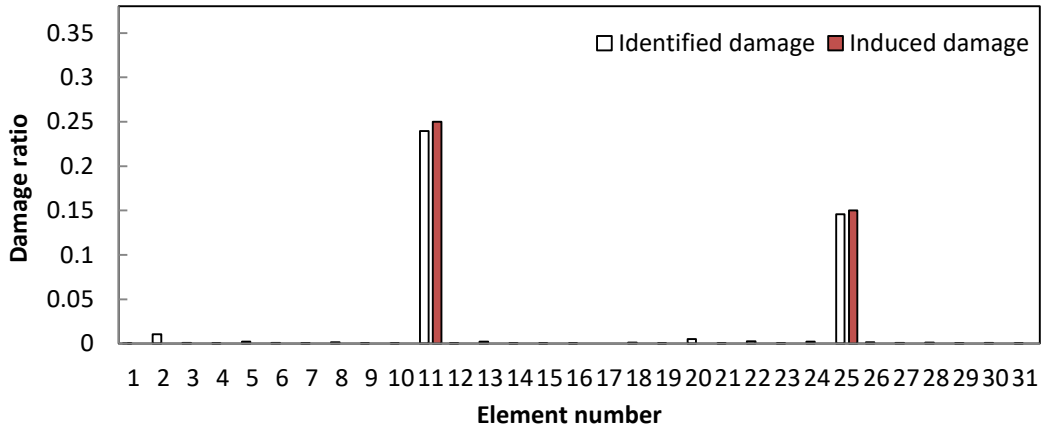


(c)

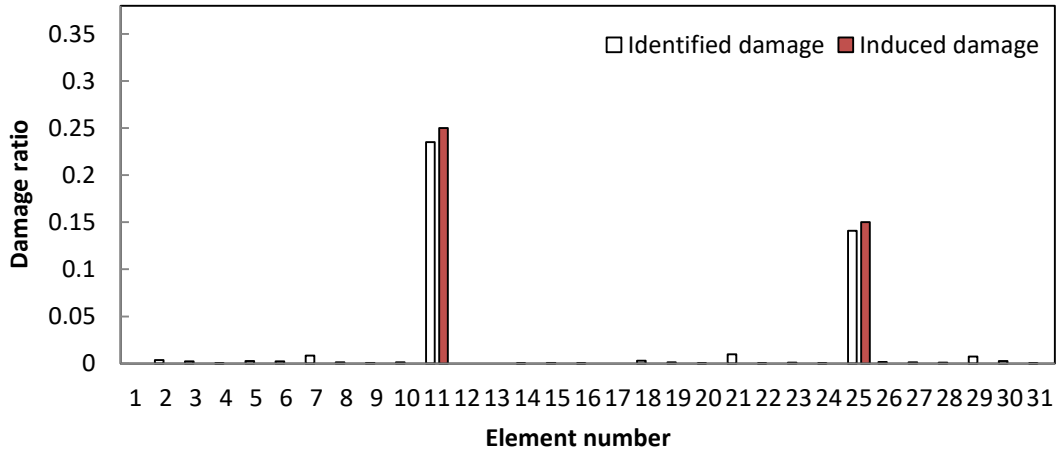
Figure 9. Final identified damage variables of the 31 element truss for case 2 considering noise (a) 1%, (b) 2% and (c) 3%



(a)

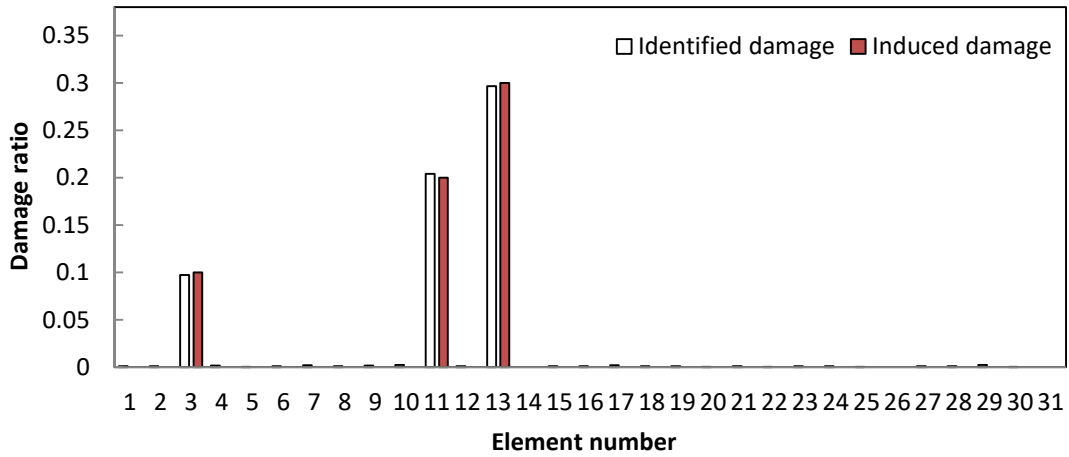


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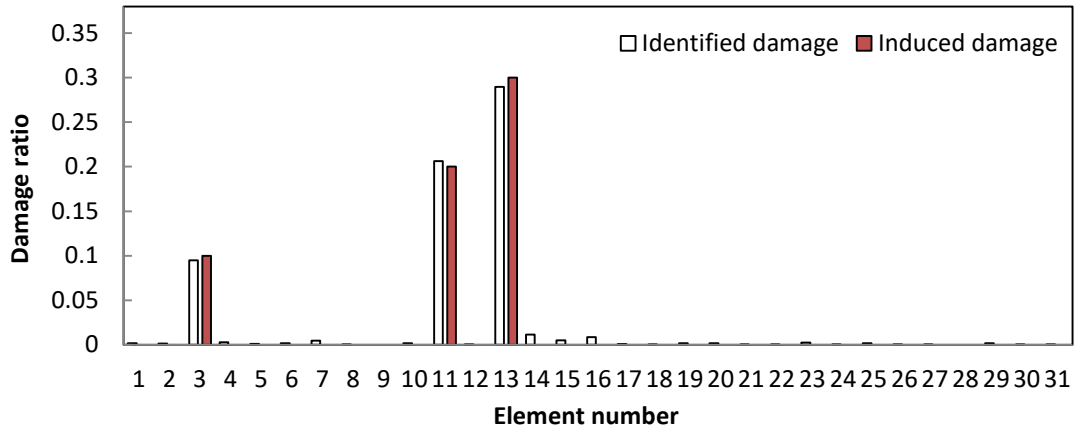


(c)

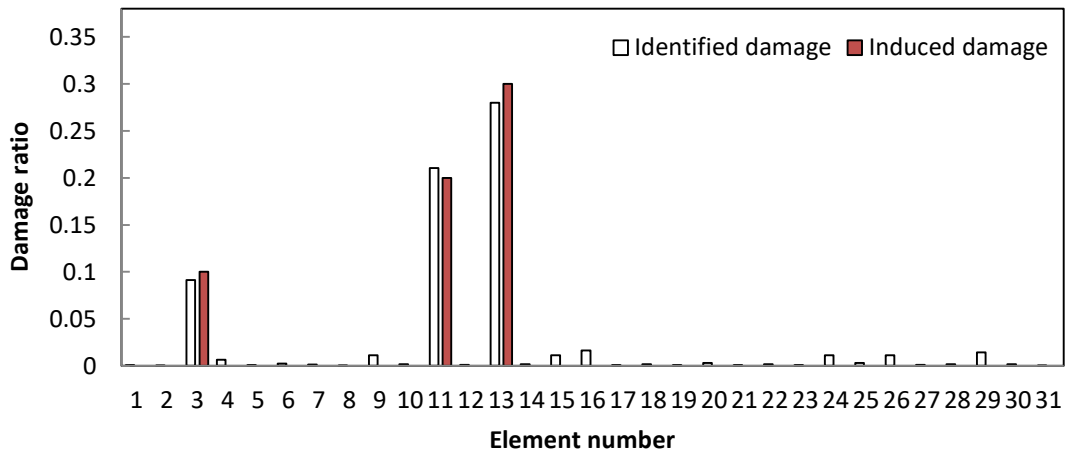
Figure 10. Final identified damage variables of the 31 element truss for case 3 considering noise (a) 1%, (b) 2% and (c) 3%



(a)

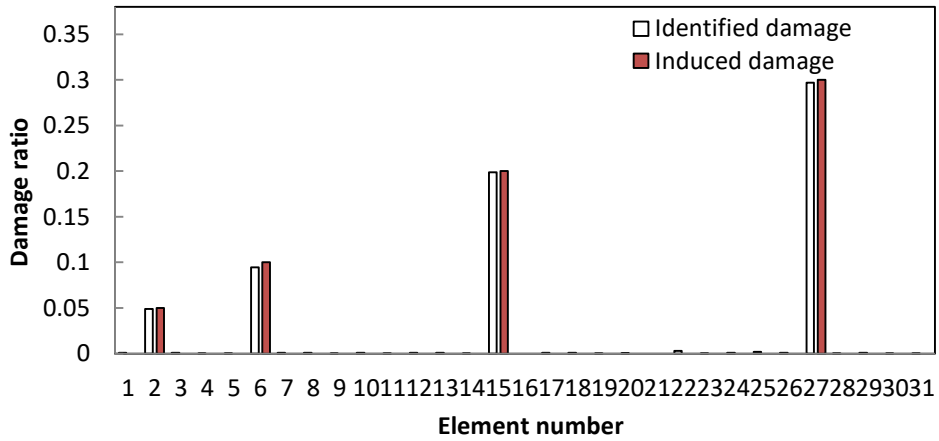


(b)



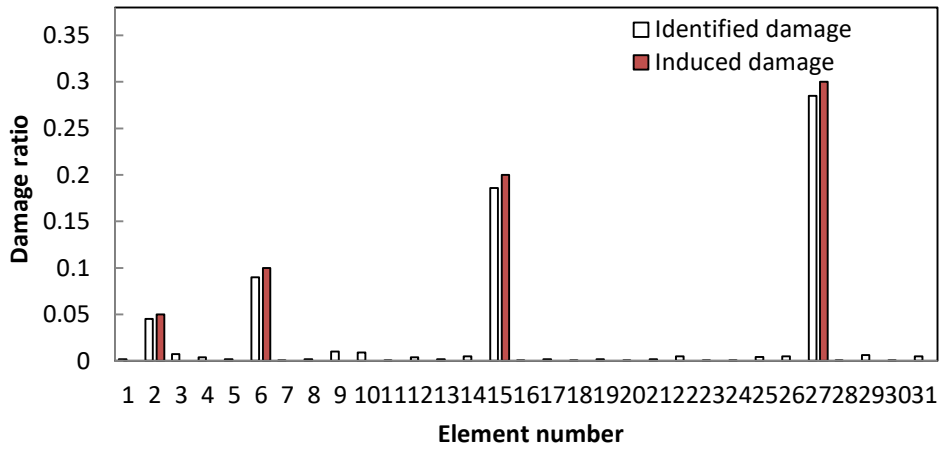
(c)

Figure 11. Final identified damage variables of the 31 element truss for case 4 considering noise (a) 1%, (b) 2% and (c) 3%

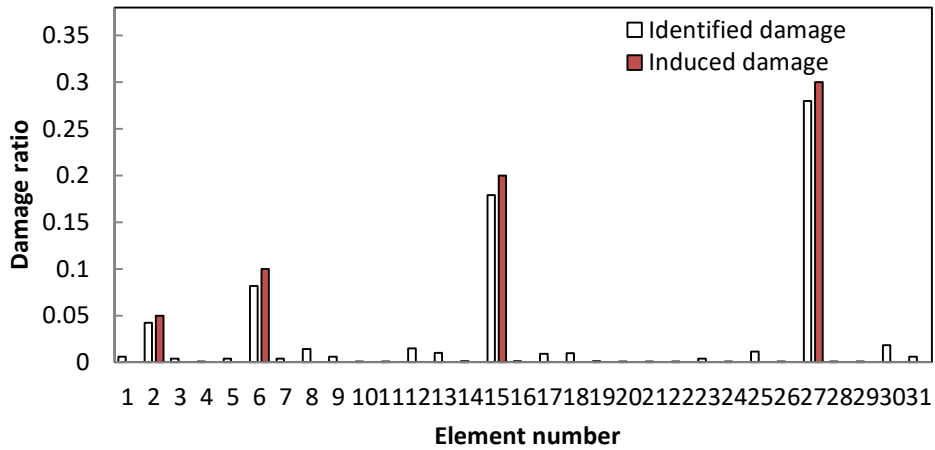


(a)





(b)



(c)

Figure 12. Final identified damage variables of the 31 element truss for case 5 considering noise (a) 1%, (b) 2% and (c) 3%

5.3. Fifteen-element planar frame

The third example considered in this study is a fifteen element planar frame used by other researchers too [29]. The frame has a rectangular cross sectional area of  $A= 0.0336 \text{ m}^2$ . The material has a mass density of  $\rho = 7860 \text{ kg/m}^3$  and elasticity modulus of  $E = 25 \text{ GPa}$ . Fig. 13 shows a sketch of the structural dimensions and the numbering of the discretized elements used in the finite element analysis. The 2D beam element with three degrees of freedom per node (one rotational and two translational) is employed for finite element discretization of the structure. In order to investigate the method, five damage scenarios created in Table 3 are numerically simulated here by reducing the elasticity modulus of some elements and the method is tested. The final set up parameters used in this research, have been the same as in the first example. Also the impact load used for excitation is as shown in Fig. 2 which is applied at node 5. In order to obtain the acceleration response, two sensors are considered: one at node 3 and the other one at node 12.

The damage detection results corresponding to the different damage cases for ten sample

runs while considering noise, are indicated in Figs. 14–18. It is observed that the optimisation has been able to determine the location and quantity of actual damage truthfully. It should be noted that the optimisation process for cases 1 to 5 has converged to the actual damage after about 45 iterations (3760, 3845, 4182, 4829 and 5225 finite element analyses, respectively). The final results of various damage cases reveal the efficiency of DEA for obtaining the site and quantity of damage.

Table 3: Five different damage cases induced in the 15- element planar frame

Case 1		Case 2		Case 3		Case 4		Case 5	
Element Number	Damage Ratio	Element Number	Damage Ratio	Element Number	Damage Ratio	Element Number	Damage Ratio	Element Number	Damage Ratio
5	0.1	13	0.1	5	0.1	4	0.1	3	0.05
				13	0.1	7	0.2	8	0.1
						8	0.3	12	0.2
								13	0.3

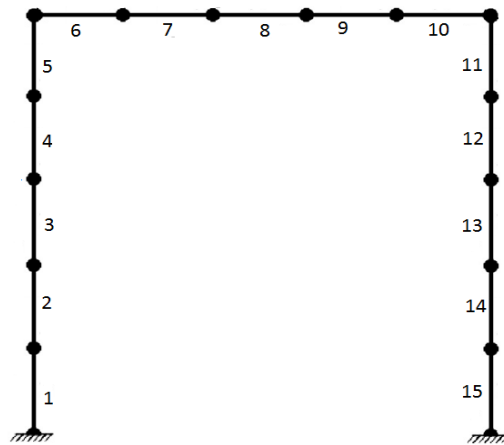
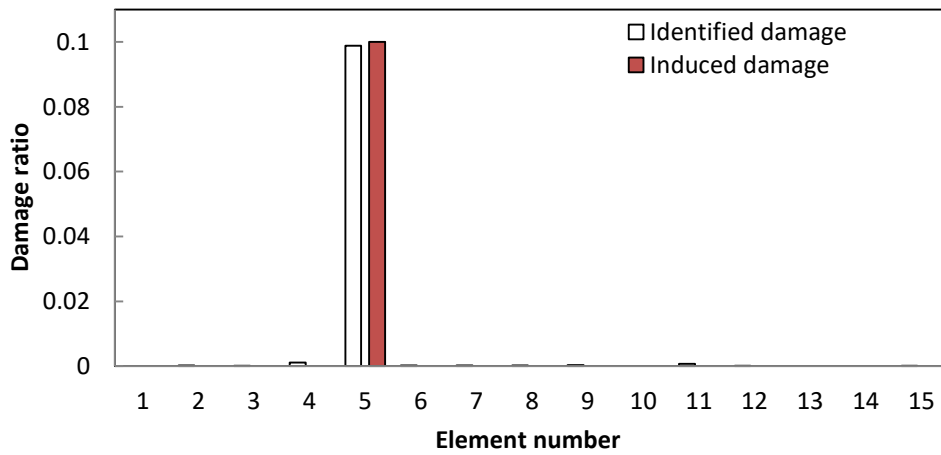
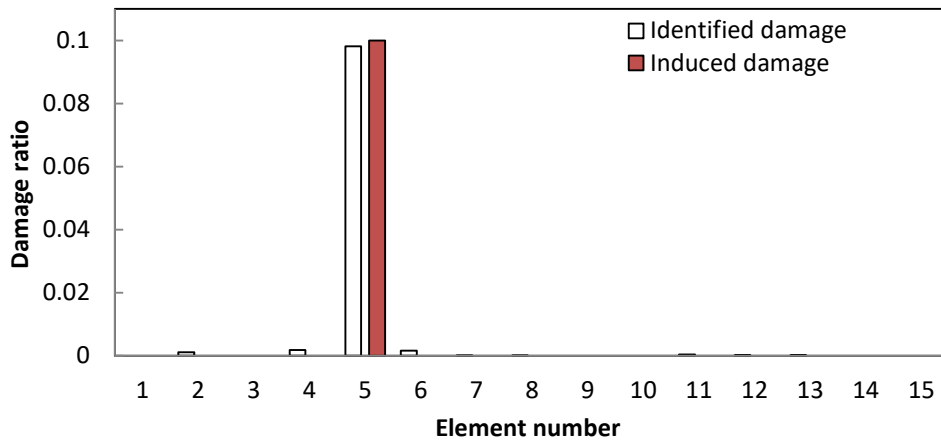


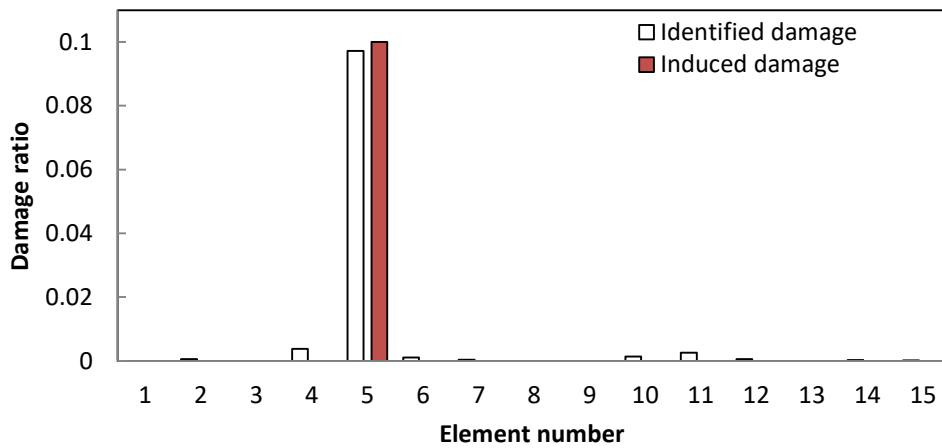
Figure 13. The planar frame having 15 elements



(a)

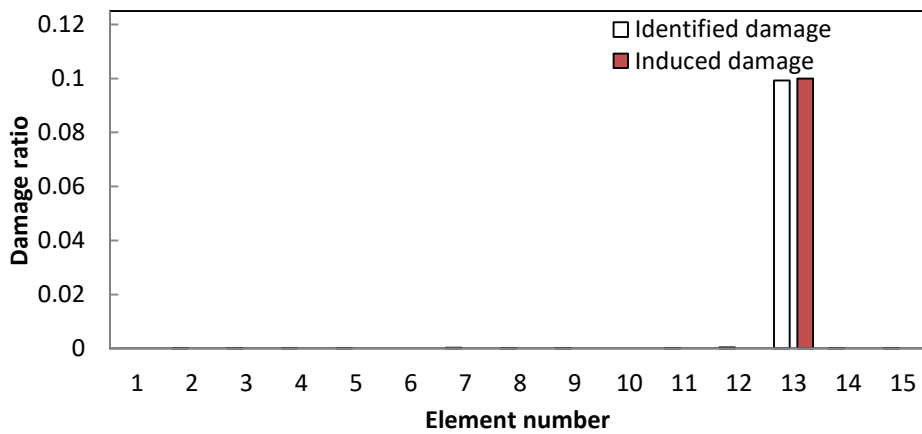


(b)

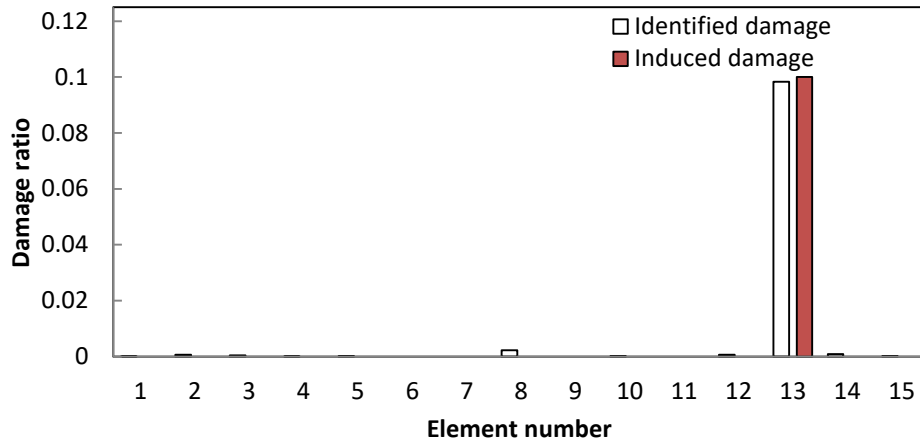


(c)

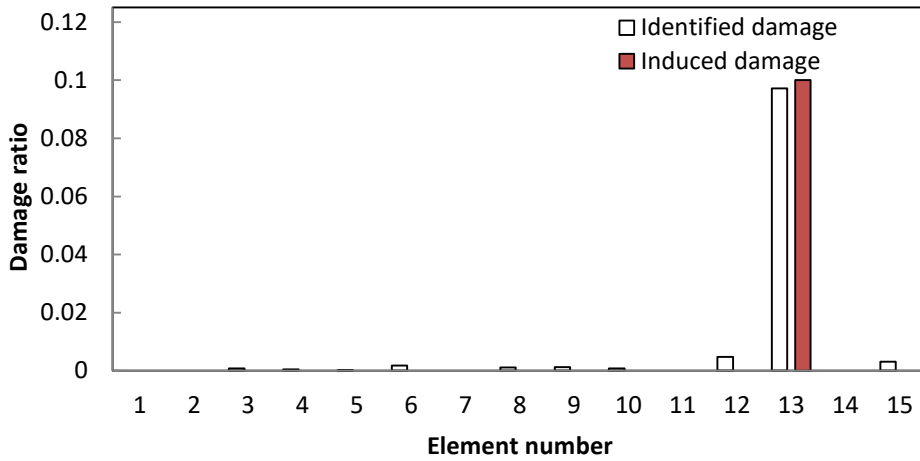
Figure 14. Final identified damage variables of the 15 element frame for case 1 considering noise (a) 1%, (b) 2% and (c) 3%



(a)

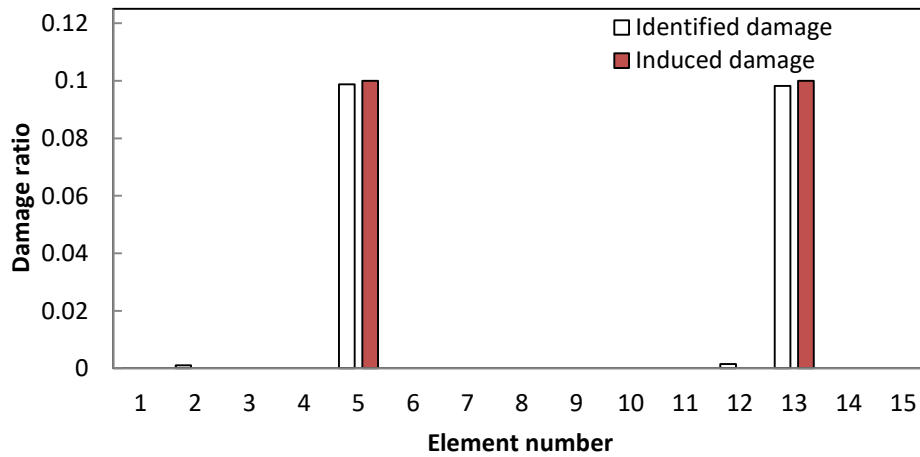


(b)



(c)

Figure 15. Final identified damage variables of the 15 element frame for case 2 considering noise (a) 1%, (b) 2% and (c) 3%



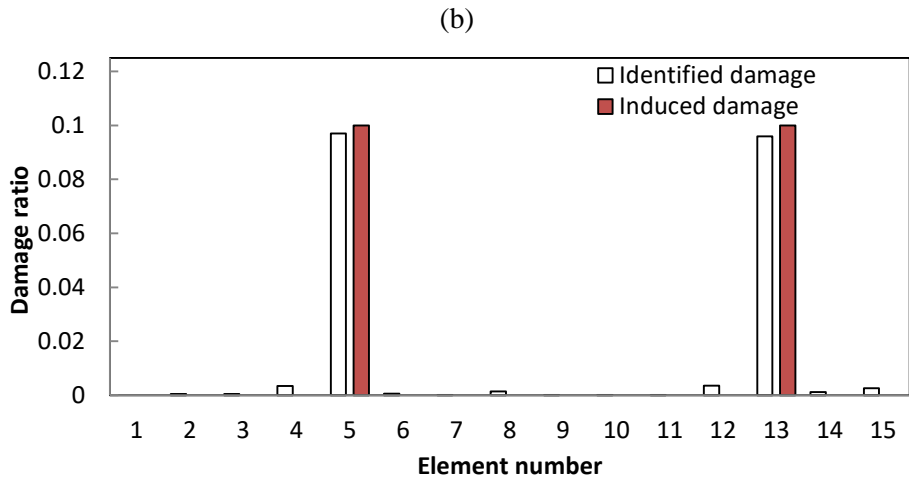
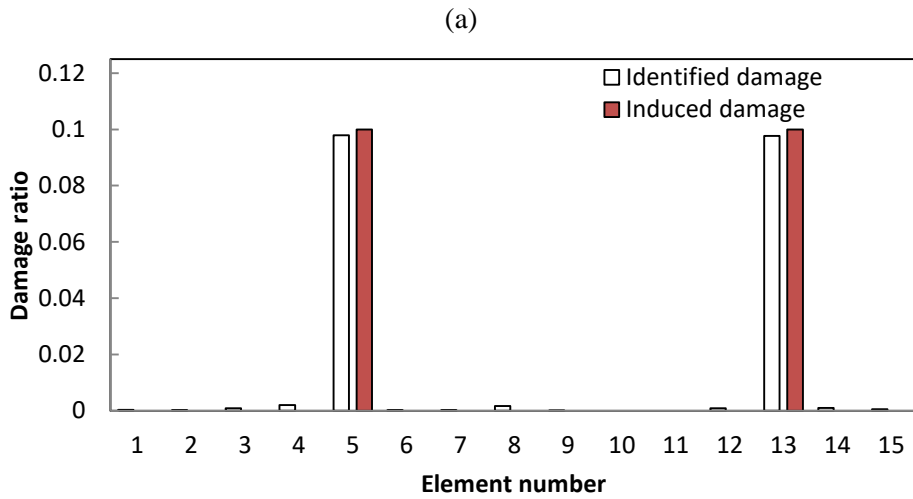
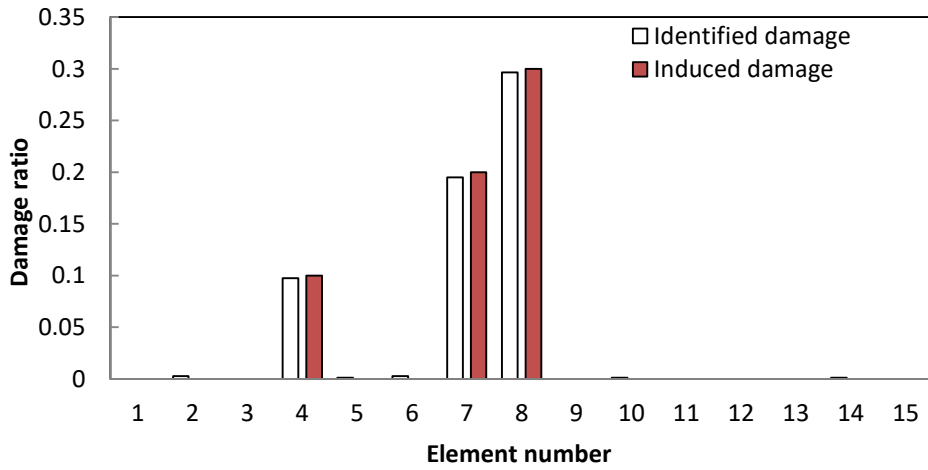
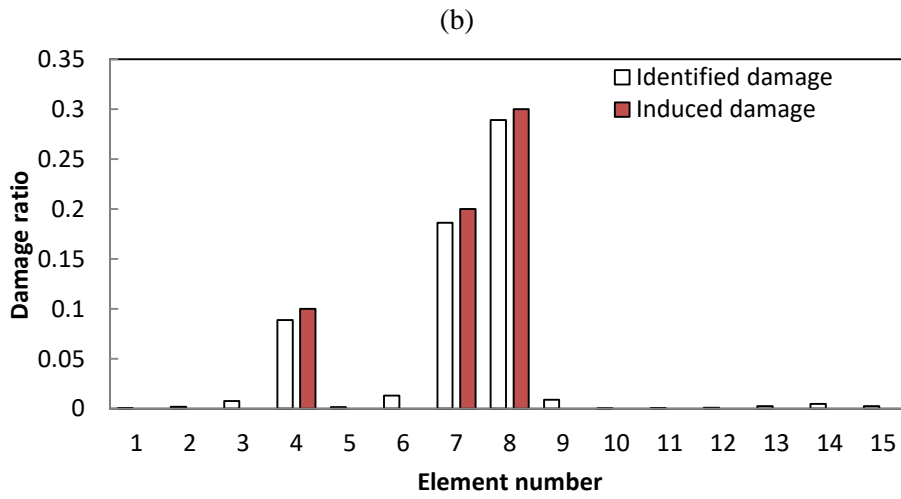
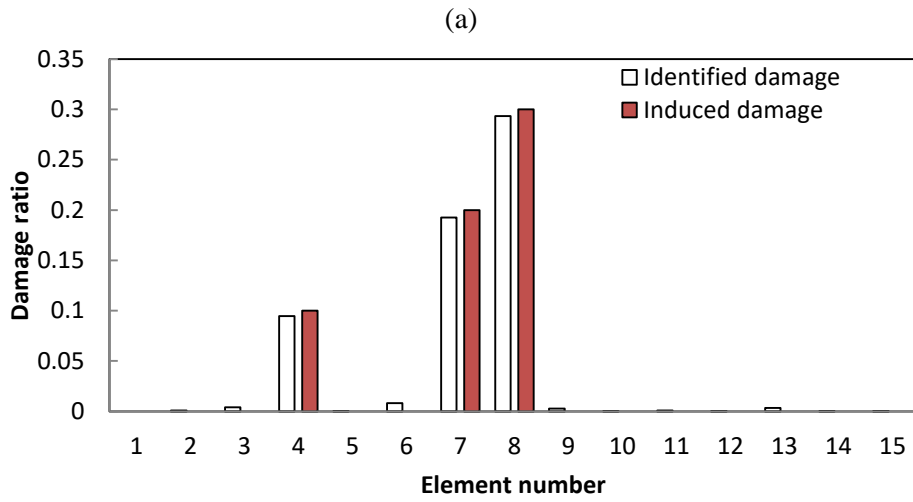


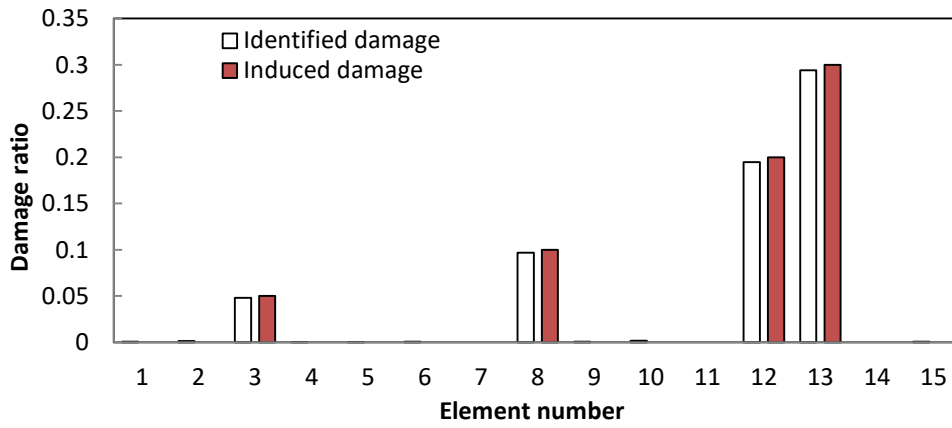
Figure 16. Final identified damage variables of the 15 element frame for case 3 considering noise (a) 1%, (b) 2% and (c) 3%



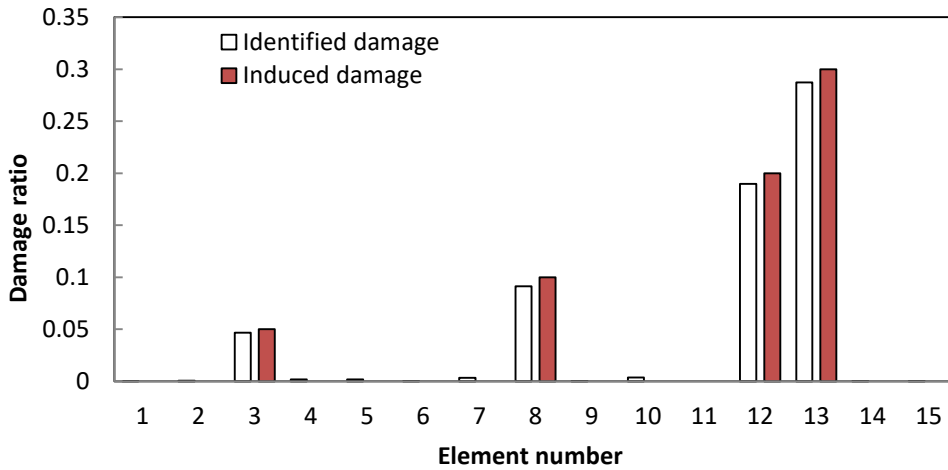


(c)

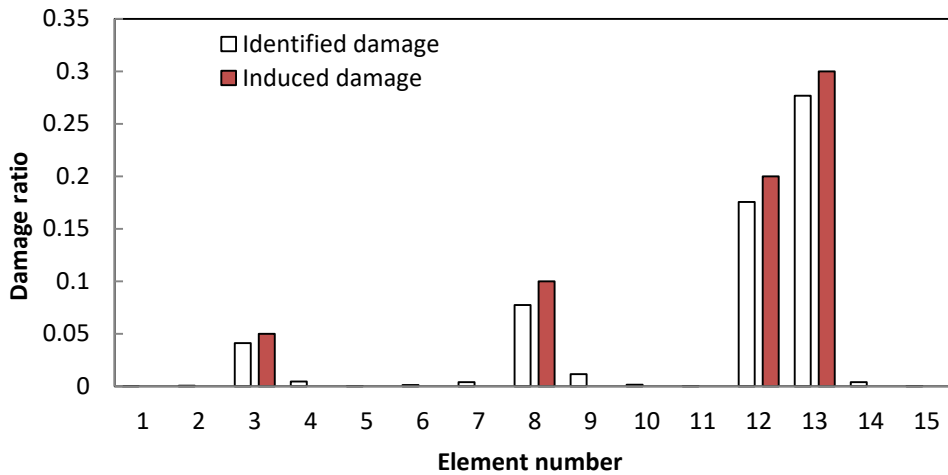
Figure 17. Final identified damage variables of the 15 element frame for case 4 considering noise (a) 1%, (b) 2% and (c) 3%



(a)



(b)



(c)

Figure 18. Final identified damage variables of the 15 element frame for case 5 considering noise (a) 1%, (b) 2% and (c) 3%

**6. CONCLUSIONS**

In this paper, an effective optimisation technique has been introduced to solve the problem of structural damage identification which is a highly nonlinear problem. The proposed approach includes, measuring acceleration responses of the time-domain and also creating a finite element model of civil structures (i.e., truss and frame). Thereafter, an objective function for solving the inverse problem of damage identification is defined and by the use of DEA, the problem is solved. Damage occurring on one location or more than one was successfully found. In order to assess the competence of the proposed approach for structural damage detection, three explanatory examples are numerically investigated by considering effects of measurement noise. Numerical results indicate that the DEA creates a robust tool to accurately identify the location and extent of single and multiple damage cases. The results of the proposed method have shown a very high performance for the method when

compared with actual damage induced.

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