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OPTIMUM DESIGN OF ARCH DAMS FOR FREQUENCY LIMITATIONS

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ABSTRACT

An efficient methodology is proposed to find optimal shape of arch dams on the basis of constrained natural frequencies. The optimization is carried out by virtual sub population (VSP) evolutionary algorithm employing real values of design variables. In order to reduce the computational cost of the optimization process, the arch dam natural frequencies are predicted by properly trained back propagation (BP) and wavelet back propagation (WBP) neural networks. The WBP network provides better generalization compared with the standard BP network. The numerical results demonstrate the computational merits of the proposed methodology for optimum design of arch dams.

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KEY WORDS: arch dam, natural frequency, optimum design, continuous evolutionary algorithm, wavelet transform, neural networks.

1. INTRODUCTION

It is well known that the natural frequencies are fundamental parameters which affect dynamic behavior of arch dams. Therefore, some constraints should be considered on natural frequency range so that the dynamic behavior of arch dam is modified. Furthermore, the eventual resonance phenomenon is eliminated. Traditionally, to achieve this purpose, the dam must be frequently analyzed and designed, that is an initial scheme is given and analyzed. If it satisfies the demands of design specifications, the scheme is adopted. Otherwise, the shape of the dam is modified and reanalyzed [1]. The shape of dam obtained

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in this way is feasible but not necessarily optimal. Moreover, this procedure is very tedious. The process can be easily and reliably implemented by employing optimization techniques. In the last years, some progress has been made in optimum design of arch dams considering stress constraints. Almost all of them have used conventional methods for analysis approximation and optimization [2-4]. These methods usually employ derivative calculations and may be trapped into local optima.

In this study, an efficient method is presented to find optimal shape of arch dams with constrained natural frequencies. The arch dam cost including concrete volume and the casting areas is considered as objective function. The design variables are geometric parameters of arch dam. To implement a practical design optimization, many constraints such as stress, displacement, stability requirement, and frequency constraints should be considered. In the present study, in order to simplify the optimization operation, only frequency and some geometrical constraints are taken.

In the field of structural optimization, one of the most popular evolutionary algorithms is genetic algorithm (GA) [5-10]. The standard GA is not efficient to find the solution in the problems with a great number of design variables. In order to eliminate this shortcoming of GA, virtual sub population (VSP) method is employed [11]. In this method all the necessary mathematical models of the natural evolution operations are implemented on the small initial population to access optimal solution on iterative basis. Nevertheless, the stochastic nature of evolutionary search techniques makes the convergence of the process slow. To accelerate the optimization process and reduce the computational effort, two strategies are adopted. The first strategy is to employ the real values of the design variables instead of their binary cods. The second one is to predict the natural frequencies of arch dams using properly trained neural networks instead of direct evaluation. Back propagation (BP) and wavelet back propagation (WBP) neural networks are employed for this purpose. a number of neural networks such as radial basis function (RBF), generalized regression (GR), counter propagation (CP), back propagation (BP) and wavelet back propagation (WBP) neural networks have been used in civil engineering applications [11-18]. However, a few researchers have been worked on designing and applying of the wavelet neural networks [12-14]. In the present study, a daughter wavelet function with the fixed position and dilation is considered as the hidden layer neurons activation function. The results of BP and WBP testing indicate that the WBP possesses the best performance generality.

The numerical results reveal the robustness and high performance of the suggested methods for optimum design of arch dams. Also, it is demonstrated that, the optimum design obtained by VSP using the WBP network is the best compared with the other results.

2. GEOMETRICAL MODEL OF ARCH DAM

2.1. Shape of central vertical section

For the central vertical section of double-curvature arch dam, as shown in Figure 1, one polynomial of *n*th order is used to determine the curve of upstream boundary and another polynomial is used to determine the thickness. In this study, a parabolic function is considered for the curve of upstream face as [2, 3]:

$$y(z) = b(\mathrm{z}) = -s\,z + s\,z^2/(2\beta\,h) \tag{1}$$

where h and s are the height of the dam and the slope at crest, respectively and the point where the slope of the upstream face equals to zero is $z=\beta\,h$ in which β is constant.

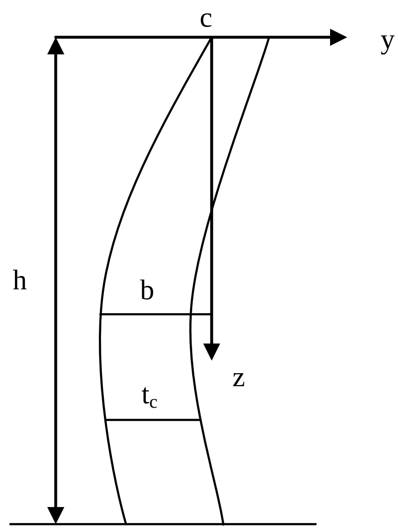

Figure 1. Central vertical section of arch dam

A quadratic function for the thickness of central vertical section is also chosen as:

$$t_c(z) = n_1(z)\,.t_{c1} + n_2(z)\,.t_{c2} + n_3(z)\,.t_{c3} \tag{2}$$

where

$$n_1(z) = \frac{(z/h-\lambda)(z/h-1)}{\lambda}\;,\; n_2(z) = \frac{(z/h)(z/h-1)}{\lambda(\lambda-1)}\;,\; n_3(z) = \frac{(z/h)(z/h-\lambda)}{1-\lambda} \tag{3}$$

in which t_{c1}, t_{c2} and t_{c3} are the thicknesses of the central vertical section at z=0, $z=\lambda\,h$ and $z=h$, respectively and λ is a factor in the range of (0,1) and in this study is considered as λ=0.55.

2.2. Shape of horizontal section

As shown in Figure 2, for the purpose of symmetrical canyon and arch thickening from crown to abutment, the shape of the horizontal section of a parabolic arch dam is determined by the following two parabolas [3]:

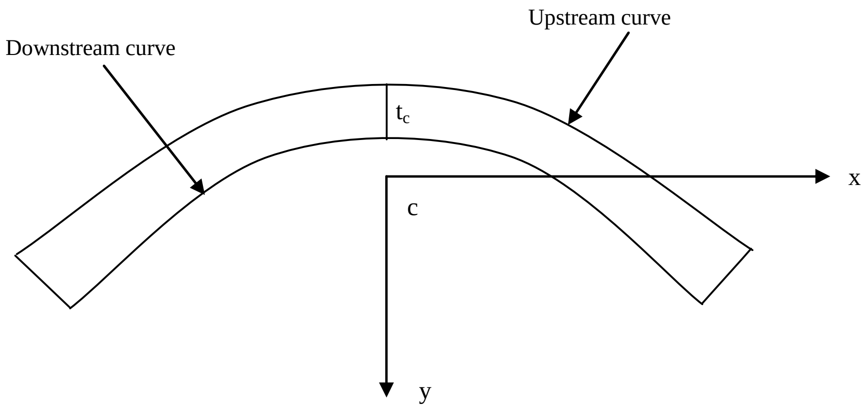

Figure 2. the shape of horizontal section of a parabolic

At the upstream face of dam:

$$y_{\mathrm{u}}(x,z) = \frac{1}{2\,r_u(z)}x^2 + b(z) \tag{4}$$

At the downstream face of dam:

$$y_{\mathrm{d}}(x,z) = \frac{1}{2\,r_d(z)}x^2 + b(z) + tc(z) \tag{5}$$

where r_u and r_d are radii of curvatures correspond to upstream and downstream curves, respectively and functions of *n*th order with respect to z can be used for those radii.

In this study, assuming n=2, r_u and r_d are considered as quadratic functions:

$$r_u = n_1(z)r_{u1} + n_2(z)r_{u2} + n_3(z)r_{u3} \tag{6}$$

$$r_d = n_1(z)r_{d1} + n_2(z)r_{d2} + n_3(z)r_{d3} \tag{7}$$

where r_{u1}, r_{u2}, r_{u3} and r_{d1}, r_{d2}, r_{d3} are values of r_u and r_d at z=0, $z=\lambda h$ and $z=h$, respectively.

3. FINITE ELEMENT MODEL OF ARCH DAM

A finite element model based on modal analysis for double-curvature arch dam is presented. The arch dam is treated as a three dimensional linear structure. To mesh of the arch dam body twenty-node isoperimetric solid element is used. It is assumed that the reservoir is empty and dam foundation is rigid to avoid the extra complexities that would otherwise arise. The physical and mechanical properties involved here are the concrete density (2.4kN.s^2/m^4), the concrete poison’s ratio (0.2) and the concrete elasticity (2.1×10^7 kN/m^2). The finite element model of a parabolic arch dam is depicted in Figure 3.

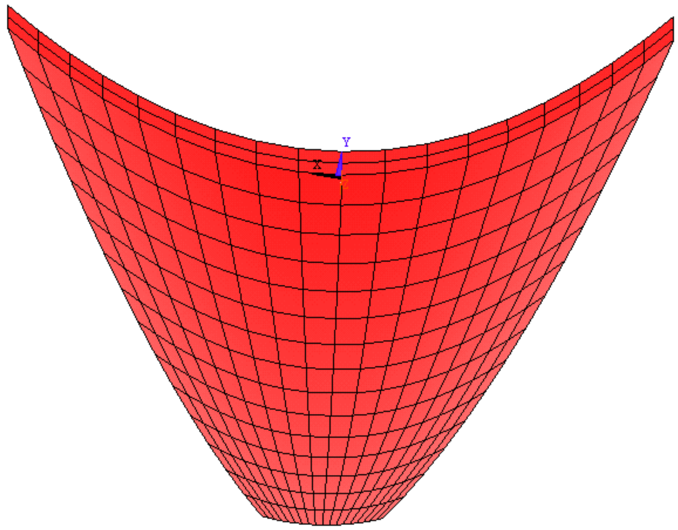

Figure 3. The finite element model of the parabolic arch dam

The Lanczos method is used for eigenvalue extraction. This method is useful for large-scale structures and typically, is applicable to the type of problems solved using the subspace eigenvalue method, however, at a faster convergence rate [19].

4. NEURAL NETWORKS

In the recent years, neural networks are considered as more appropriate techniques for simplification of complex and time consuming problems. The interest shown to neural networks is mainly due to their ability to process and map external data and information based on past experiences.

Neural networks are not programmed to solve specific problems. Indeed, neural networks never use rules or physic equations related to the specific problem in which they are employed. Neural networks use the knowledge gained from past experiences to adapt themselves to solve the new problems. As a matter of fact, learning is never selective and it is not limited only to the explicit and desired knowledge but it also involves implicit information that sometimes are not well-known a priori even to the designer. Indeed, one of the most important limitations of neural networks is that they are not able to provide explanations and justifications for their results and answers. This fact is the direct consequence of the intrinsic nature of a neural network in which knowledge and experiences are not well localized but are redistributed to each neuron. Then a shared out representation of concepts, quantities and data is obtained.

4.1. BP neural networks

Back propagation was created by generalizing the Widrow-Hoff learning rule [20] to multilayer networks and nonlinear differentiable transfer functions. Input vectors and the corresponding target vectors are used to train a network until it can approximate a function, associate input vectors with specific output vectors. Networks with a sigmoid layer and a linear output layer are capable of approximating any function with a finite number of discontinuities. Standard back propagation is a gradient descent algorithm, in which the network weights are moved along the negative of the gradient of the performance function. In this study Levenberg-Marquardt (LM) [21] methods is employed.

4.2. Wavelet neural networks

Wavelets must have at least a minimum oscillation and a fast decay to zero of its amplitude. This property is analogous to an admissibility condition of a function that is required for the wavelet transform [22]. Sets of wavelets are employed to approximate a signal and the goal is to find a set of daughter wavelets constructed by a dilated and translated original wavelets or mother wavelets that best represent the signal. The daughter wavelets are generated from a single mother wavelet $h(t)$ by dilation and translation:

$$h_{j,k}(t) = \frac{1}{\sqrt{j}} h\left(\frac{t-k}{j}\right) \tag{8}$$

where $j > 0$ and k are the dilation and the translation factors, respectively [23].

Wavelet neural networks employing wavelets as the activation functions recently have been researched as an alternative approach to the neural networks with sigmoidal activation functions. The combination of wavelet theory and neural network concepts has led to the development of wavelet networks. In wavelet networks, both the position and the dilation of the wavelets may be optimized besides the weights. In the present study, wavelet neural network is referred to neural network using wavelets as activation function of hidden layer neurons with the fixed position and the dilation.

4.3. WBP neural networks

BP network is now the most popular mapping neural network. Substituting of BP neurons activation function with some wavelet functions may improve its performance generality. Activation function of hidden layer neurons in BP network is sigmoidal function. To design wavelet back propagation (WBP) network the hidden layer sigmoidal activation function of BP network is substituted with POLYWOG1 wavelet [23]:

$$h_{\text{POLYWOG1}}(t) = \sqrt{e}\,(t)\, e^{(-(t)^2)/2} \tag{9}$$

The daughter POLYWOG1 wavelet is obtained by substituting Eq. (9) into Eq. (8):

$$h_{j,k}(t) = \sqrt{\frac{e}{j}}\left(\frac{t-k}{j}\right) \mathrm{e}^{(-(\frac{t-k}{j})^2)/2} \tag{10}$$

In this study, to design WBP network, the position and dilation of the POLYWOG1 wavelets are fixed and only the network weights are optimized by LM algorithm. The best results are obtained by considering j = 2 and k = 0 in Eq. (10).

One of the problems that occur during the neural network training is called overfitting. One method for preventing of overfitting and improving network generalization is called regularization [24]. This procedure is employed in the present work to enhance the generalization of the networks.

5. ARCH DAM OPTIMIZATION

5.1. Mathematical model and optimization variables

The optimization problem is formally stated as follows:

$$\begin{aligned} &\text{Minimize}: w(X) \\ &\text{Subject to}: g(X) \leq 0 \,, i = 1,..., m \\ &X^{l} \leq X \leq X^{u} \end{aligned} \tag{11}$$

where X is the vector of design variables with n unknowns, g_j is jth constraint from m inequality constraints and $w(X)$ represents the objective function that should be minimized. Also, X^l and X^u are stood for the lower and upper bounds of design variable vector.

5.1.1. Design variables

The most effective parameters for creating the arch dam geometry were mentioned in section 2. The parameters can be adopted as design variables:

$$X^{\mathrm{T}} = \{s \;\; \beta \;\; t_{c\,1} \;\; t_{c\,2} \;\; t_{c\,3} \;\; r_{u\,1} \;\; r_{u\,2} \;\; r_{u\,3} \;\; r_{d\,1} \;\; r_{d\,2} \;\; r_{d\,3}\} \tag{12}$$

where the vector of design variables cantinas 11 shape parameters of arch dam.

5.1.2. Design constraints

Design constraints are divided into some groups including the structural, geometrical and stability constraints. The structural constraints are the restricted natural frequencies that are defined as follows:

$$frl_n \leq fr_n \leq fru_n \;\Rightarrow \begin{cases} 1 - \dfrac{fr_n}{frl_n} \leq 0 \\ \dfrac{fr_n}{fru_n} - 1 \leq 0 \end{cases} \quad , \quad n = 1,2,\ldots, n_{\mathrm{fr}} \tag{13}$$

where fr_n , frl_n and fru_n are the nth natural frequency, lower bound and upper bound of the nth natural frequency, respectively. Also, n_{fr} is the number of natural frequencies.

The most important geometric constrains are those that prevent from intersection of upstream face and downstream face as:

$$rd_n \le ru_n \Rightarrow \frac{rd_n}{ru_n} - 1 \le 0 \;, n = 1,2,3 \tag{14}$$

where rd_n and ru_n are radii of curvatures at the down and upstream faces of the dam in nth position in z direction.

The geometric constrain that is applied for facile construction, is defined as:

$$s \le s_{all} \Rightarrow \frac{s}{s_{all}} - 1 \le 0 \tag{15}$$

where s is the slope of overhang at the downstream and upstream faces of dam and s_{all} is its allowable value. Usually s_{all} is taken as 0.3.

The constraints ensuring the sliding stability of the dam may be expressed as:

$$\varphi^{\text{l}} \le \varphi \le \varphi^{\text{u}} \tag{16}$$

where φ is the central angle of arch dam and usually: $90 \le \varphi \le 110$ [1].

5.1.3. Objective function

The objective function is the cost of the dam, which may be expressed as:

$$w(X) = p_v v(X) + p_a a(X) \tag{17}$$

where v(X) and $a(X)$ are the concrete volume and the casting area of dam body. The unit price of concrete and casting are chosen as p_v=\$33.34 and p_a=\$6.67, respectively.

To evaluate $v(X)$ and $a(X)$ a computer program is coded using MATLAB [24].

6. EVOLUTIONARY ALGORITHMS

In structural optimization problems, where the objective function and the constraints are highly non-linear functions of the design variables, the computational effort spent in gradient calculations required by the mathematical programming algorithms is usually large. In the recent years, it was found that probabilistic search algorithms are computationally efficient even if greater number of optimization cycles is needed to reach the optimum. These cycles are computationally less expensive than in the case of mathematical programming algorithms since they do not need gradient evaluation. Furthermore, probabilistic methodologies were found to be more robust in finding the global optima, due to their random search, whereas mathematical programming algorithms may be trapped into local optima [25].

6.1. Continuous VSP methods

In the present study, to obtain the optimum shape of arch dams under frequency constraints the VSP method is employed based on the real values [26] of design variables instead of their binary codes. By using this strategy the discrete nature of VSP optimization turns into continuous one and design variables are presented to the crossover and mutation operators with their real values. Continuous optimization methods require less computer effort comparing with the discrete methods. Numerical results show that, employing continuous VSP method in shape optimization can lead to appropriate solution.

7. MAIN STEPS OF ARCH DAM OPTIMIZATION

The main steps for the optimization of arch dams by the proposed methodology are summarized as follows:

(a) Generating a number of arch dams considering their geometric parameters.
(b) Evaluating natural frequencies of the generated dams by ANSYS [19].
(c) Using the provided data, BP and WBP networks are trained to predict the natural frequencies.
(d) Selecting some parent vectors from the design variables space.
(e) Evaluating natural frequencies of the dams using trained BP and WBP networks.
(f) Checking the constraints for feasibility of parent vectors.
(g) Generating offspring vectors using continuous crossover and continuous mutation operators.
(h) Employing the trained BP and WBP networks for predicting the natural frequencies of the offspring.
(i) Checking the constraints.
(j) Checking convergence.
(k) Selecting a number of parent vectors from the previous solution and some random variables as a VSP.
(l) Repeating steps (h) to (k) until the proper solution is met.

8. TEST EXAMPLE

In order to assess the effectiveness of proposed methodology, the shape optimization of an arch dam with a height of 180 m is considered. The width of the valley in its bottom and top are 40 m and 220 m, respectively. The lower and upper bounds of design variables using empirical design methods are considered as:

$$\begin{array}{llll} 0 \le s \le 0.3 & 4\text{m} \le t_{c1} \le 12\text{m} & 50\text{m} \le r_{u1} \le 180\text{m} & 50\,\text{m} \le r_{d1} \le 180\,\text{m} \\ 0 < \beta \le 1.0 & 8\,\text{m} \le t_{c2} \le 30\,\text{m} & 40\,\text{m} \le r_{u2} \le 120\,\text{m} & 40\,\text{m} \le r_{d2} \le 120\,\text{m} \\ & 12\,\text{m} \le t_{c3} \le 40\,\text{m} & 10\,\text{m} \le r_{u3} \le 50\,\text{m} & 10\,\text{m} \le r_{d3} \le 50\,\text{m} \end{array} \qquad (18)$$

In current study, natural frequency constraints are imposed as:

$$fr_1 \geq 3\text{Hz} \quad fr_2 \geq 6\text{Hz} \quad fr_3 \geq 7\text{Hz} \quad fr_4 \geq 8Hz \tag{19}$$

The errors between exact and approximate frequencies are also calculated using the following equation:

$$error = \left| \frac{fr_{ap} - fr_{ex}}{fr_{ex}} \right| \times 100 \tag{20}$$

where fr_{ap} and fr_{ex} represent the approximate and exact frequencies, respectively.

With the mentioned conditions, the optimization is carried out by the following methods:

(a) GA using exact analysis.
(b) GA using approximate analysis by BP network.
(c) GA using approximate analysis by WBP network.
(d) VSP using exact analysis.
(e) VSP using approximate analysis by BP network.
(f) VSP using approximate analysis by WBP network.

The parameters of GA and VSP methods are given in Table 1. The time of all computations is evaluated in clock time by a personal Pentium IV 3000MHz.

Table 1. The parameters of GA and VSP method

Parameters	**GA**	**VSP**
Population size Crossover method	50 three points	30 three points
Crossover rate	0.9	0.9
Mutation rate	0.001	0.001
Maximum generation	100	30

8.1. Data selection for training the networks

In this study, the input space consists of design variables of the arch dams. The corresponding natural frequencies of the dams are considered as the target space components. A total number of 343 arch dam samples are randomly generated based on design variables and their natural frequencies are evaluated using the finite element analysis. From which, 260 and 83 samples are used for training and testing the networks, respectively.

8.2. Training and testing the networks

To train and test the neural networks, MATLAB is utilized. The times of BP and WBP training are about 1.5 min and 1.1 min, respectively. Information of the performance generality of the networks in testing mode is given in Table 2.

Table 2. Maximum and mean errors of BP and WBP networks in testing mode

Network	**Maximum errors (%)**				**Mean errors (%)**			
	fr_1	fr_2	fr_3	fr_4	fr_1	fr_2	fr_3	fr_4
BP	22.81	18.82	24.17	12.43	3.51	3.16	2.39	2.49
WBP	10.58	10.16	7.25	9.39	2.86	2.11	1.94	2.17

8.3. Results of optimization

Optimum solutions obtained by the various methods are given in Table 3. As observed in this table the solutions found by VSP are more economical than that of the GA and the best solution is attained by VSP using WBP.

Table 3. Optimum designs of the arch dam obtained by the various methods

Variable No.	**Optimum design (m)**					
	GA			**VSP**		
	Exact	**BP**	**WBP**	**Exact**	**BP**	**WBP**
1	0.2254	0.2254	0.2254	0.2703	0.2673	0.2508
2	0.8172	0.8172	0.9752	0.6747	0.6703	0.7376
3	4.4306	4.4306	4.4306	4.2636	4.0027	4.0027
4	23.2672	23.2672	23.2672	23.0512	23.0512	18.0125
5	14.1411	13.6326	13.6326	12.0258	12.258	12.0258
6	129.3949	129.3949	129.3949	139.1293	139.1293	149.9189
7	95.0685	95.0685	92.0741	100.2709	100.2709	94.6904
8	39.4704	39.4704	39.4704	49.9067	49.9067	49.2262
9	129.0289	129.0289	129.0289	131.5843	138.6503	148.2564
10	40.8120	40.8120	40.8120	72.7854	72.7854	53.0239
11	31.0890	31.0890	31.0890	42.8869	39.314	48.2745
Cost ($\$10^6$)	8.809	8.806	8.740	7.540	7.407	6.576
Generations	90	92	87	78	95	90
Time (min)	375	2.5	2.0	195	2.4	2.3

The errors of approximate frequencies of optimum dams predicted by the BP and WBP networks are compared with their corresponding actual ones, computed by via the finite element analysis, in Table 4. It can be observed that, although the accuracy of approximate frequencies obtained by all the methods is good, the accuracy of results obtained by VSP method using WBP network is the best.

Table 4. Error percentage of approximate frequencies of optimum dams

Frequency No.	**GA**		**VSP**	
	BP	**WBP**	**BP**	**WBP**
1	1.317	1.165	2.177	0.677
2	3.664	2.130	3.724	1.656
3	2.853	0.492	3.149	0.117
4	3.330	0.929	1.858	0.559
Ave.	2.791	1.179	2.727	0.752

The present study demonstrates that the combination of VSP method with WBP neural networks creates a reliable and powerful tool for optimization of arch dams with multiple natural frequency constraints.

9. CONCLUSIONS

In the present study, an efficient optimization procedure is developed to find the optimal shape of arch dams with frequency constraints employing real values of the design variables. To achieve this task, a combination of the evolutionary algorithm and neural networks is utilized. The evolutionary algorithm used in this investigation is continuous VSP method. It has been observed that continuous VSP method results in a better solution and a greater efficiency compared with the standard GA. Performing arch dam optimization using the accurate modal analysis through finite element analysis is a time consuming procedure. To reduce the computational time of the optimization process, the natural frequencies of the arch dam are predicted by properly trained BP and WBP neural networks. The results of the network testing show the higher performance generality of the WBP compared with BP network. Numerical results indicated that the best optimization solution has been attained by VSP method using WBP network. Also, it is observed that using the proposed optimization strategy the time of optimization can be considerably reduced while the computational errors appeared due to approximation, are negligible.

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