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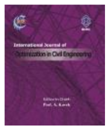

A NOVEL FUZZY MULTI-OBJECTIVE ENHANCED TIME EVOLUTIONARY OPTIMIZATION FOR SPACE STRUCTURES

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ABSTRACT

This research presents a novel design approach to achieve an optimal structure established upon multiple objective functions by simultaneous utilization of the Enhanced Time Evolutionary Optimization method and Fuzzy Logic (FLETEO). For this purpose, at first, modeling of the structure design problem in this space is performed using fuzzy logic concepts. Thus, a new problem creates with functions and constraints regarding the design in fuzzy space as well as membership functions corresponded to every single of them. Then, the problem is solved by means of the Enhanced Time Evolutionary Optimization method (ETEO), eventually, based on the acquired results, the values of optimal design variables are obtained in the main problem. In the current paper, to validate the proposed approach and evaluate its performance, the optimal design of several standard structures has been carried out. Comparing the acquired results and previous ones is an indication of the high power of the proposed method in finding the best possible design with high convergence speed and deprived of contravening the constraints governing the problems.

Keywords: Multi-Objective Optimization, Truss, Enhanced Time Evolutionary Optimization, Fuzzy Logic.

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1. INTRODUCTION

Engineering design problems consist of more than one purpose in the real world; hence, in most of them, the desired purposes show an antithetical behavior so that improving one of them leads to making the other worse [1]. Therefore, to achieve the best desired design, using optimization methods, which can be made a balance between all important goals in the design of the problems and attained the best answer to the problem with high efficiency, is

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necessary. In this regard, Pareto, for the first time, announced the concept of multi-objective optimization [2].

Since designing most of the engineering problems is not based on only a single criterion, this makes solving the problems more complicated. On the other hand, the restrictions and parameters, which should be considered to analyze these problems, increase, so, the designer encounters a difficult task for mathematically accurately modeling them. One of the methods that researchers can use to model the complications of these issues in a better way is the utilization of the fuzzy logic. The fuzzy logic was presented by Zadeh for the first time [3], and since then it is at the center of interest for the researchers as one of the active and fascinating fields.

Metaheuristic algorithms are one of the most robust procedures to solve engineering problems. By simultaneous producing several designs having mechanisms motivated by nature, physical rules, animal social life etc., these methods proceed toward getting better [4].

Some of the methods that have been proposed in recent years are NCO [5], DEACO [6], PFA [7], ACRO [8], AIG [9], ICLBO [10] and etc. Recently, in 2019 a robust metaheuristic optimization method named Enhanced Time Evolutionary Optimization (ETEO) has been presented by Sheikhi *et al* [11]. According to the unique characteristics of the aforementioned method in comparison with the other metaheuristic ones in solving engineering problems, in this research, the optimal analyzing and designing of the structures under multiple objectives have been investigated using the ETEO and fuzzy logic.

Many quantitative studies have been conducted in the field of structural optimization of multi-objective engineering problems via fuzzy logic. For example, Rao *et al.* [12] carried out the multi-objective optimization of engineering problems using the fuzzy logic method and mathematical scheduling. Chen and Shieh [13] also performed the multi-objective optimization of structures by the fuzzy logic. They wanted to minimize and maximize the flexibility and natural frequencies related to the structure in that order. Moreover, Coello and Christiansen [14] used the genetic algorithm as well as the Min-Max method to optimize multi-objective structural problems. The main goal in this research was to reach the minimum weight, maximum displacement and stress in the structure through fining the cross-sections optimal area of the components. Furthermore, Papadrakakis *et al.* [15] studied the multi-objective optimization of skeletal and truss structures under static and seismic load. It should be mentioned that Luh and Chueh [16] used the metaheuristic algorithm inspired by the body defensive system in order to find Pareto's optimal answers for truss structures. In this research work, the volume and displacement of the structure nodes were minimized by means of finding cross-section optimal area of its components. Besides, Kelesoglo and Ulker [17] obtained the minimum weight and node displacement for space truss structures with the use of the fuzzy logic and ANSYS software as well as by considering the multi-objective optimization problem. They also examined the multi-objective optimization of space trusses using fuzzy logic and Ms Excel software [18]. Additionally, Kelesoglo [19] utilized the genetic algorithm and fuzzy logic for the multi-objective optimization of truss structures. Kaveh and Laknejadi [20] also accomplished the multi-objective optimization for truss structures through combining particle swarm optimization algorithm and DM. In addition, Pholdee and Bureerat [21] scrutinized the multi-objective optimization of trusses using a combination of a gradual method and a gradient approximation-based searching strategy.

The current study, using a combination of fuzzy logic manner and ETEO presented a novel method as called (FLETEO), and optimally designs structures by considering several objective functions under different constraints. In the results presented in previous articles, the problem constraints in the optimal state were contravened somewhat with the use of the fuzzy logic. Here, according to that the problem constraints are directly applied based on the ETEO method, the constraint violation is completely obstructed. Besides non-violation of the constraint, comparing the obtained results to earlier research works points out the high power of the proposed approach in finding optimal Pareto points so that it has been occasionally able to dominate the answer presented in them.

2. ENHANCED TIME EVOLUTIONARY OPTIMIZATION

ETEO is a novel metaheuristic method which was presented by Sheikhi *et al.* in 2019 [11]. This method has been inspired by the evolution of living creatures on the Earth during the lifetime. Based on the evolution law in nature, the creatures that have less flexibility in special environmental conditions and are unable to familiarize themselves with environmental conditions, will vanish after some time. In these conditions, the species will remain in nature which are more adaptable to their surroundings. It should be mentioned that evolution, which occurs under the influence of time pass, was existed all the time and will endure. This evolution creates based on births and consists of the reproduction of elite species. Therefore, time and living environment are two significant factors. The ETEO method flowchart is presented in Fig. 1 [11]. The respected readers can refer to the reference [11] to know more details of how to mathematically model and search process in design space and convergence toward the optimal design of the aforementioned method if necessary.

3. FUZZY MULTI-OBJECTIVE OPTIMIZATION PROBLEMS

In this section, how to mathematically model the multi-objective optimization problem with the use of the fuzzy logic is presented. A multi-objective optimization problem is generally defined as follows [22]:

$$\begin{aligned} &Min \;\; F(X) = \{f_1(X),\ldots,f_k(X)\}^T \\ &S.to \begin{cases} g_i(X) \le 0, & i = 1,2,\ldots,m \\ h_j(X) = 0, & j = 1,2,\ldots,l \end{cases} \end{aligned} \tag{1}$$

where $F(X)$ is the vector of the objective functions including k elements (k is the number of the objective functions), m and l are the number of unequal and equal constraints of $g(X)$ and $h(X)$, respectively. To solve this problem by fuzzy logic, at first, the problem modeling in fuzzy space should be performed [23]. For this purpose, the following steps should be carried out.

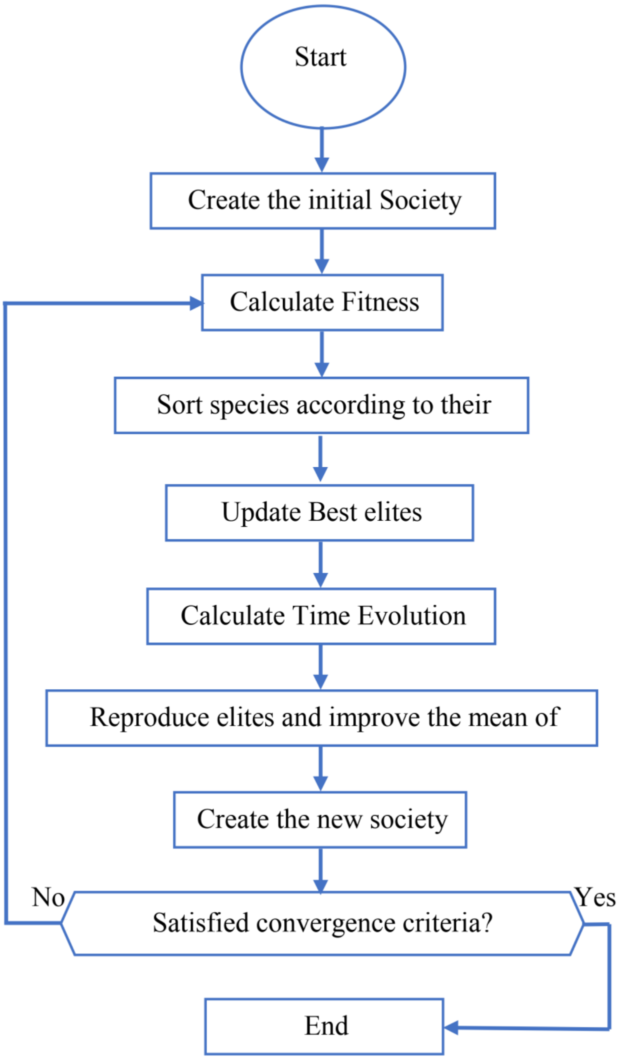


Figure 1. The flowchart of Enhanced Time Evolutionary Optimization [11].

The first step: single-objective-based finding the optimal point for every one of the objective functions under all governing constraints on the problem (Eq. 2.) [12]; also, finding the optimal point (X_r^*) and then the formation of matrix p in the form of relation (3).

$$\begin{aligned} &Minimize \;\; f_r(X), \;\; r = 1,2,\dots,k \\ &S.to \begin{cases} g_i(X) \le 0, \;\; i = 1,2,\dots,m \\ h_j(X) = 0, \;\; j = 1,2,\dots,l \end{cases} \end{aligned} \tag{2}$$

$$p=\begin{bmatrix} f_1(X_1^*) & f_2(X_1^*) & \dots & f_k(X_1^*) \\ f_1(X_2^*) & f_2(X_2^*) & \dots & f_k(X_2^*) \\ \vdots & \vdots & \ddots & \vdots \\ f_1(X_k^*) & f_2(X_k^*) & \dots & f_k(X_k^*) \end{bmatrix} \tag{3}$$

The second step: finding the best and worst possible design for each of the objective functions after the formation of matrix *p*. In this matrix, the elements on the main diameter have the lowest value in their own column. Thus, the best possible design of the problem is in the form of single-objective as Eq. 4.

$$\begin{cases} f_i^{\min} = \min_j f_i(X_j^*) = f_i(X_i^*) \\ f_i^{\max} = \max_j f_i(X_j^*) \end{cases} i, j = 1, 2, \dots, k \tag{4}$$

The third step: using the found values of the best and worst objective functions of the problem to make it fuzzy and defining the membership functions concerning the objective ones in the form of Eq. (5).

$$\mu_{f_i}(X) = \begin{cases} 0, & f_i(X) > f_i^{\max} \\ \dfrac{-f_i(X) + f_i^{\max}}{f_i^{\max} - f_i^{\min}}, & f_i^{\min} \le f_i(X) \le f_i^{\max} \\ 1, & f_i(X) \le f_i^{\min} \end{cases} \tag{5}$$

The fourth step: defining the membership functions of the constraints in the form of equations (6) and (7) by considering a permissible range (Eq. 8).

$$\mu_{g_j^{(l)}}(X) = \begin{cases} 1, & g_j(X) \ge g_j^{(l)} \\ \dfrac{g_j(X) - g_j^{(l)} + \Delta g_j^{(l)}}{\Delta g_j^{(l)}}, & g_j^{(l)} - \Delta g_j^{(l)} < g_j(X) < g_j^{(l)} \\ 0, & g_j(X) \le g_j^{(l)} - \Delta g_j^{(l)} \end{cases} \tag{6}$$

$$\mu_{g_j^{(u)}}(X) = \begin{cases} 1, & g_j(X) \le g_j^{(u)} \\ \dfrac{-g_j(X) + g_j^{(u)} + \Delta g_j^{(u)}}{\Delta g_j^{(u)}}, & g_j^{(u)} < g_j(X) < g_j^{(u)} + \Delta g_j^{(u)} \\ 0, & g_j(X) \le g_j^{(u)} + \Delta g_j^{(u)} \end{cases} \tag{7}$$

$$g_j^{(l)} - \Delta g_j^{(l)} \le g_j(X) \le g_j^{(u)} + \Delta g_j^{(u)} \tag{8}$$

where $g_j^{(l)}$ and $g_j^{(u)}$ are the lower and upper limit for constraint jth of the problem in that order and $\Delta g_j^{(l)}$ and $\Delta g_j^{(u)}$ are negligible deviation value which are considered for the problem limits.

The fifth step: changing the optimization problem to the fuzzy state. In this phase, using the defined membership functions for objective functions and problem constraints and by considering them together, first, the problem converts from multi-objective state to single-objective one, then the single-objective optimization problem is solved. The λ formulation method is an efficient approach to solve the multi-objective optimization problems with fuzzy logic. In this method, the multi-objective optimization problem in the fuzzy state converts in the form of Eq. 9.

$$\left| \begin{array}{l} Max \;\; \lambda \\ S.to \begin{cases} \lambda \le \mu_{f_i}(X), \; i = 1,2,\ldots,k \\ \lambda \le \mu_{g_j^{(l)}}(X), \; j = 1,2,\ldots,m \\ \lambda \le \mu_{g_j^{(u)}}(X), \; j = 1,2,\ldots,m \end{cases} \end{array} \right. \tag{9}$$

The sixth step: solving the problem in the fuzzy state by using an efficient optimization method. One can analyze the problem using the ETEO method after converting the optimization problem from multi-objective to single-objective with the use of the fuzzy logic.

According to that the ETEO method has a high ability in solving the conditional optimization problems, the constraints are directly applied to the optimization algorithm to solve the problems in this research.

4. NUMERICAL RESULT

In this section, to validate the power and efficiency of the proposed method, the optimization design of several structures is studied that investigated by the other researchers. For this purpose, these problems were solved by simultaneous utilization of the ETEO method and fuzzy logic and compared the acquired results to the other ones.

4.1 Spatial four-bar truss structure

As the first structure, the optimal design of a spatial four-bar truss structure shown in Fig. 2 is considered in the form of Eq. 10 to achieve two goals of the minimum total volume of the structure (*V*) and total displacement at node "1" (δ_1).

$$Min \begin{cases} V = \sum_{i=1}^{4} A_i l_i \\ \delta_1 = \sqrt{\delta_{1x}^2 + \delta_{1y}^2 + \delta_{1z}^2} \end{cases} \tag{10}$$

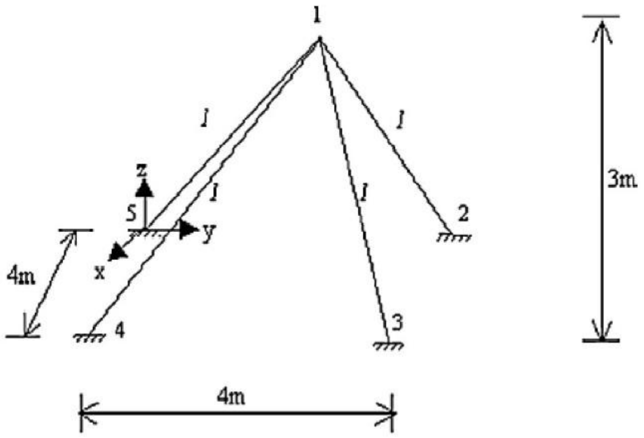


Figure 2. Schematic of the spatial four-bar truss structure [18].

In the design of this structure, the displacement of node "1" in each of the three coordinate directions should be less than $1mm$ alone. The highest point of the structure is under the influence of a $150kN$ force in y-direction. In addition, the elasticity modulus of all member of the structure is equal to $210kN/mm^2$. The minimum and maximum considered area for the cross-section of the structure members are equal to $200mm^2$ and $600mm^2$ respectively. The λ formulation method in fuzzy logic was utilized to solve this problem. The results obtained from single-objective optimization under all constraints of the problem (Eq. 11.) are presented in Table 1 using ETEO. The convergence curve to optimal volume in the single-objective function with the aforementioned method is shown in Fig. 3.

$$\left| \begin{array}{l} \begin{cases} MinV \\ S.to \ \ \delta_j(x) \le 1, j = 1, \ldots, 5 \end{cases} \\ \begin{cases} Min \ \delta \\ S.to \ \ \delta_j(x) \le 1, \ j = 1, \ldots, 5 \end{cases} \end{array} \right. \tag{11}$$

Three items are presented in Fig. 4: 1) the convergence curve to the optimal design in the fuzzy logic to reach the structure under the multi-objective functions, 2) how to change the maximum value of all constraints (Max C) governing the structure, and 3) the final values of the design variables in the optimal design. In the first column of the bar diagram related to this figure, the λ variable value and cross-section area of the structure member are presented. There is no violation of the problem constraints during designing this structure. Eventually, the problem constraint is activated in the optimal point (the maximum violation difference value in the all constraints is close to zero and equal to -8.9253×10^{-8}).

Figure 5 illustrates the objective functions variation diagram including the total volume of the structure and upper node displacement of the structure during the optimization process. As can be seen, the displacement amount of the structure increases in an iteration of the algorithm by diminishing the total volume value of the structure. This is because of the antithetical behavior of the objective functions relative to each other based on the change in the structure design variables.

Table 1. Single objective results for the spatial four-bar truss structure

Sectional areas (mm^2)	Objective functions	
	Volume (mm^3)	Displacement (mm)
312.9143	$V^* = 5.1607\times10^6$	$\delta = 1.0000$
600.0000	$V = 9.8954\times10^6$	$\delta^* = 0.5215$

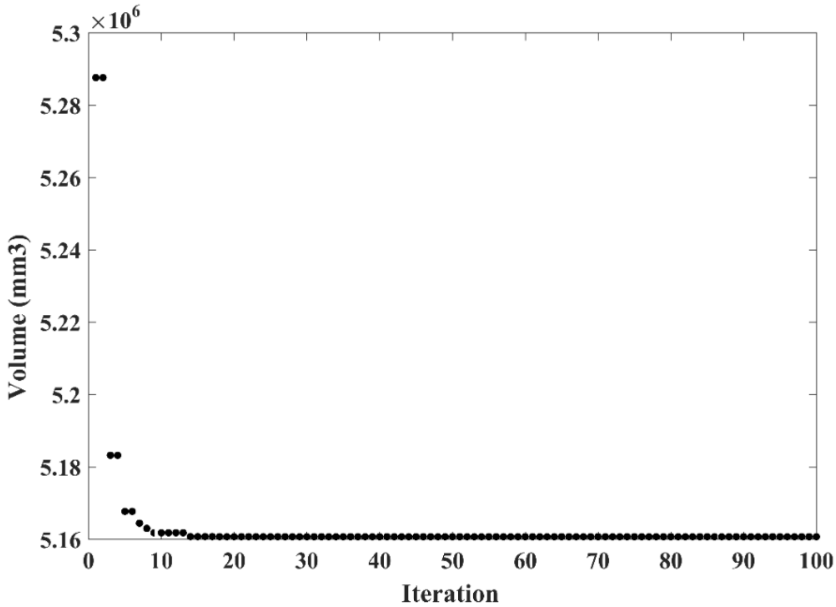


Figure 3. Convergence curve for the spatial four-bar truss structure

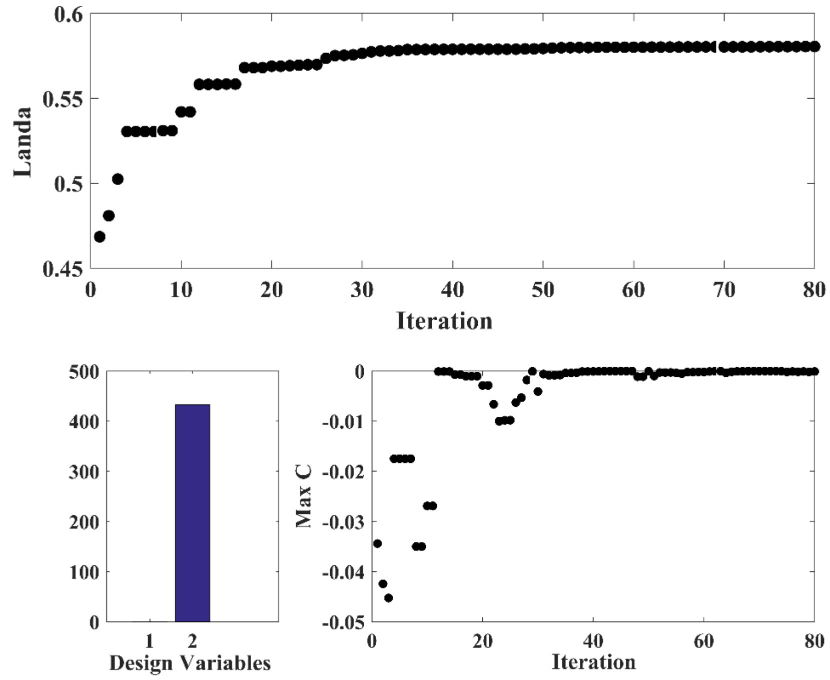


Figure 4. Convergence curve to optimal design in the fuzzy logic, optimal design variables and maximum value of all constraints for the spatial four-bar truss structure.

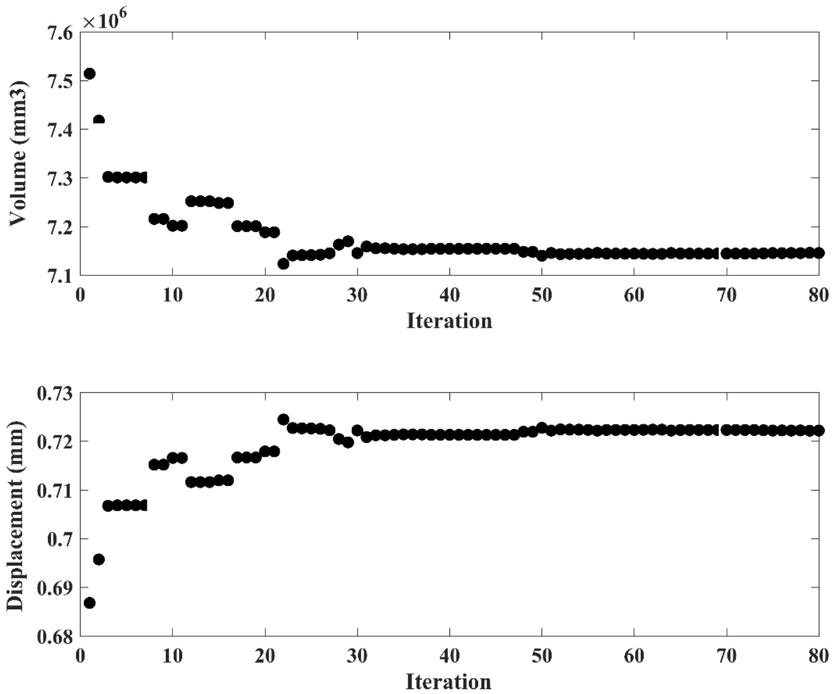


Figure 5. The variation of objective functions for the spatial four-bar truss structure.

For instance, in iteration 10 of the algorithm, the volume value of the structure is increased, but its displacement value is decreased. This exhibits that the algorithm should pursue a compromise amongst the problem objective functions.

The comparison of results between proposed method and reference [18] are presented in Table 2. As can be seen, none of the obtained answers have any preference to the other and do not dominate each other. It is worthy to note that the decline amount of the node 1 displacement and increase structure volume in the proposed method relative to the reference [18] are about 10% and 9%, respectively.

Table 2. Fuzzy multi objective results for the spatial four -bar truss structure

Method	Sectional areas (mm^2)	Objective function		λ^*
		Volume ($\times 10^3 mm^3$)	Displacement (mm)	
[18]	394.91	6513	0.79	0.6031
FLETEO	433.271	7145.683	0.722	0.5805

4.2 Transmission tower truss structure

The optimal design of transmission tower truss structure as well as minimum displacement totality of the upper joints in the structure and volume of it (Eq. 12), which is schematically shown in Fig. 6, are two goals of this section. All members in this structure were categorized into 8 groups. The minimum and maximum values of the cross-section area and the modulus of the structure members are equal to $45.6mm^2$, $2000mm^2$ and $207kN/mm^2$, correspondingly.

$$Min\begin{cases} V(x) = \sum_{i=1}^{25} A_i l_i \\ \delta(x) = \sum_{i=1}^{2} \sqrt{\delta_{ix}^2 + \delta_{iy}^2 + \delta_{iz}^2} \end{cases} \tag{12}$$

Table 3 shows the loading conditions on the structure. The maximum upper joints displacement of the structure should be less than $8.89mm$ in every single of transverse directions.

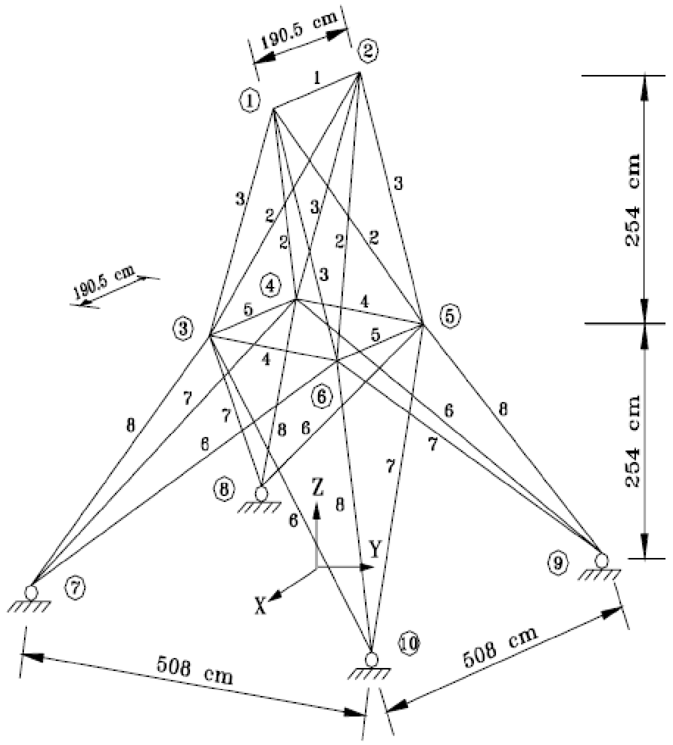


Figure 6. Schematic of the transmission tower truss structure

Table. 3. The loading for the transmission tower truss structure

No. of Joint	Loading (*kN*)		
	x	*y*	*z*
1	4.54	45.40	-22.70
2	0.00	45.40	-22.70
3	2.27	0.00	0.00
6	2.27	0.00	0.00

The results acquired from the single-objective optimization of the problem including two separate objective functions under all constraints of the problem are presented in Table 4 using ETEO. The convergence curve to optimal volume in the single-objective state with the proposed method is shown in Fig. 7.

Table 4. Single objective results for the transmission tower truss structure by using ETEO.

Elements group	Sectional areas (mm^2)	
	For Volume	For Displacement
1	12.5031	2000.0000
2	6.4500	2000.0000
3	766.4321	2000.0000
4	6.4500	2000.0000
5	450.1320	2000.0000
6	165.1574	2000.0000
7	6.4500	2000.0000
8	894.8326	2000.0000
Objective functions	$V^* = 2.545478 \times 10^7\ mm^3$	$\delta^* = 4.347540\ mm$
	$\delta = 24.139294\ mm$	$V = 1.68006 \times 10^8\ mm^3$

The convergence curve of the multi-objective functions, how to change the maximum violation value of all constraints governing on the structure, and the final values of the design variables in the optimal designs are accessible in Fig. 8. In the optimal design, the problem constraint is activated and the maximum violation difference value between all constraints is close to zero and equal to -1.6445×10^{-7}. The variation diagram of the objective functions consisting of the structural volume and displacement totality of two upper nodes of the structure during the optimization process is illustrated in Fig. 9. The results obtained using the developed algorithm are presented in Table 5.

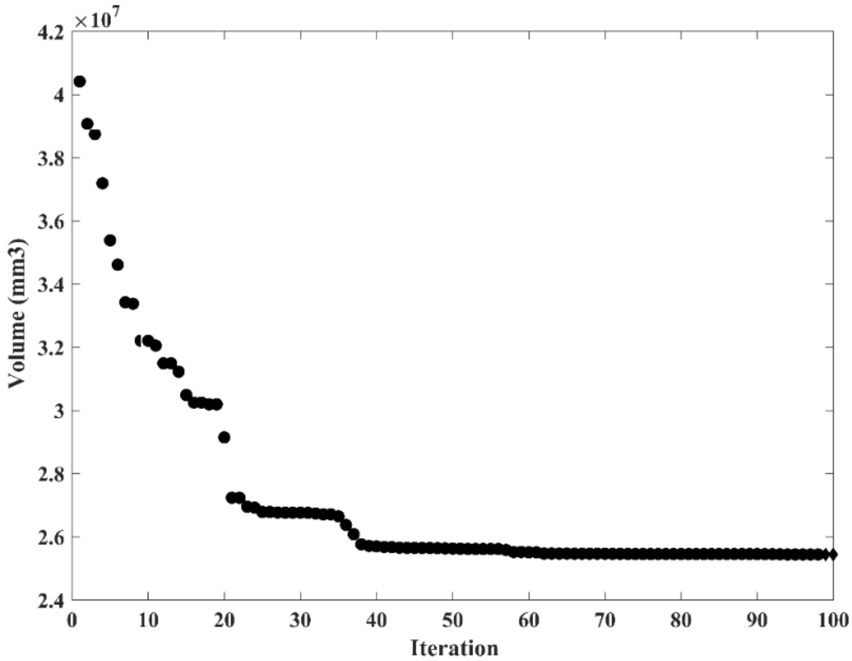


Figure 7. Convergence curve for the transmission tower truss structure.

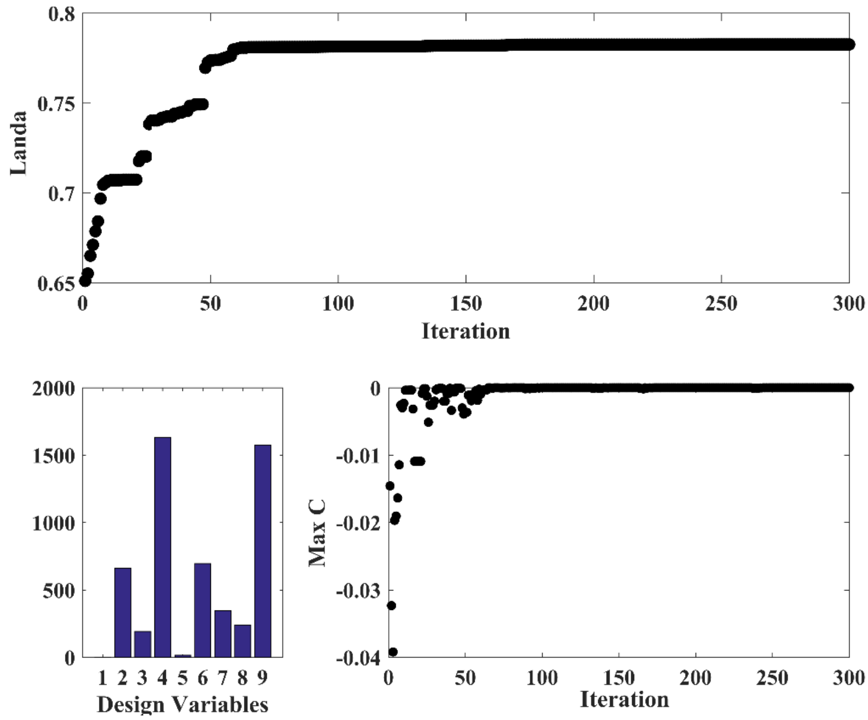


Figure 8. Convergence curve to optimal design in the fuzzy logic, optimal design variables and maximum value of all constraints for the transmission tower truss structure.

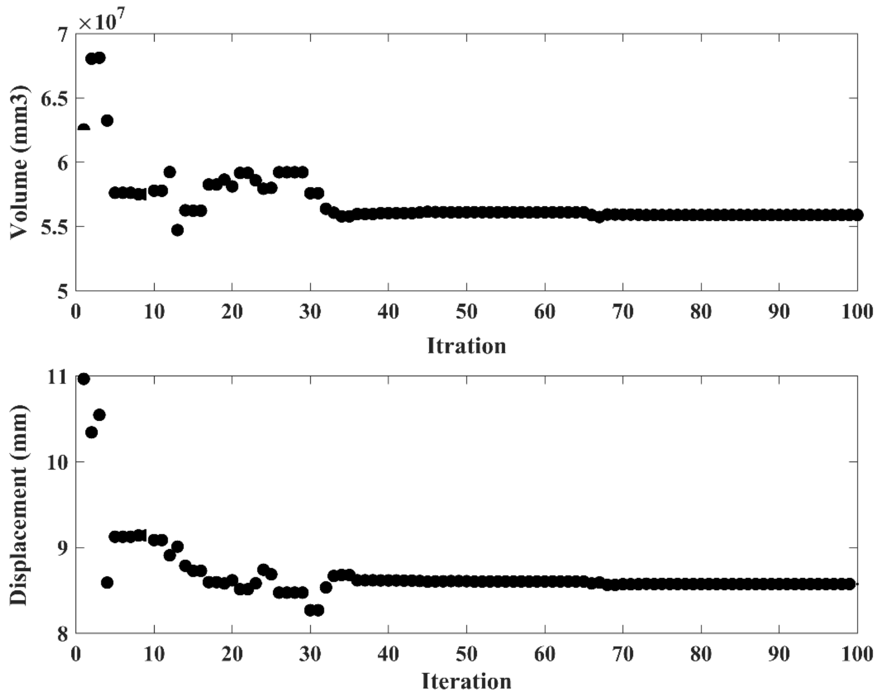


Figure 9. The variation of objective functions for the transmission tower truss structure.

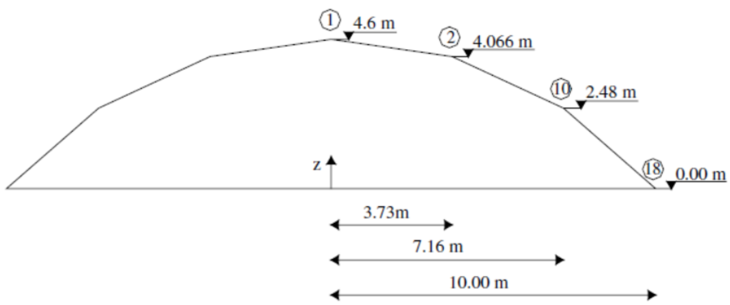
Table 5. Fuzzy multi objective results for transmission tower truss structure.

Element group	Sectional areas (mm^2)
1	104.974
2	74.286
3	1849.194
4	94.744
5	743.933
6	356.700
7	126.042
8	1663.616
λ^*	0.787
Objective functions	$V = 5.589 \times 10^7 \, mm^3$ $\delta = 8.572 mm$

4.3 Dome spatial truss structure

In this section, the structure of the dome spatial truss structure was considered in the form of Fig. 10. Several forces having various magnitudes are applied to different parts of the structure, e.g. a $4kN$ force in y-direction and $30kN$ force in z-direction to the highest members connection point in the structure, and a $4kN$ and $10kN$ force in y- and z-direction to the other joints of the structure in that order. The vertical displacement of some nodes (4, 5, 6, 12, 13 and 14) and node 8 displacement in y-direction should be less than $4mm$ and $2mm$, respectively. The cross-section area of all structure members has three types. Moreover, the elasticity modulus, minimum and maximum cross-section area of the members are equal to $210 \, kN/mm^2$, $200 \, mm^2$ and $2000 \, mm^2$, respectively. The objective functions in this structure are the minimum structure volume and the highest joint displacement (see Eq. 13).

$$Min \begin{cases} V = \sum_{i=1}^{56} A_i \, l_i \\ \delta_1 = \sqrt{\delta_{1x}^2 + \delta_{1y}^2 + \delta_{1z}^2} \end{cases} \quad (13)$$



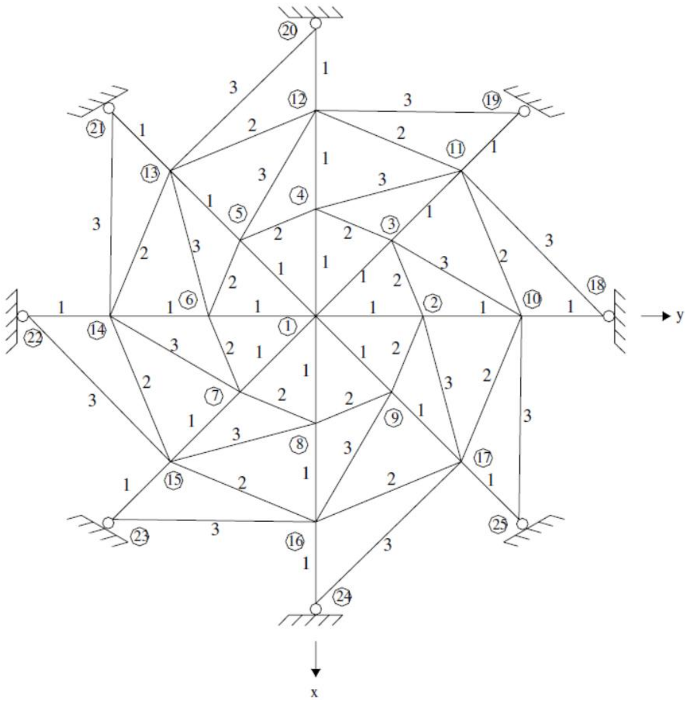


Figure 10. Schematic of the dome spatial truss structure [19].

Table 6. Single objective results for dome spatial truss structure.

Method	Sectional areas (mm^2) Elements group 1	2	3	Objective functions
GA [19]	773	477	832	$V^* = 1.88 \times 10^8 \, mm^3$
GA [19]	2000	2000	2000	$\delta_1^* = 2.7694 \, mm$
ETEO	729.334	294.240	329.108	$V^* = 1.1979 \times 10^8 \, mm^3$ $\delta = 7.3978 \, mm$
ETEO	2000.000	208.639	2000.000	$\delta_1^* = 2.2138 \, mm$ $V = 4.0242 \times 10^8 \, mm^3$

The reported optimal values regarding to the single-objective optimization problem are shown in Table 6 using genetic and ETEO. Besides, the second objective function values in each of the aforementioned states are expressed in Table 6 as well.

The convergence curve of the single objective functions consisting of the structural volume and maximum violation difference value between all constraints (Max C) during the optimization process is illustrated in Fig. 11.

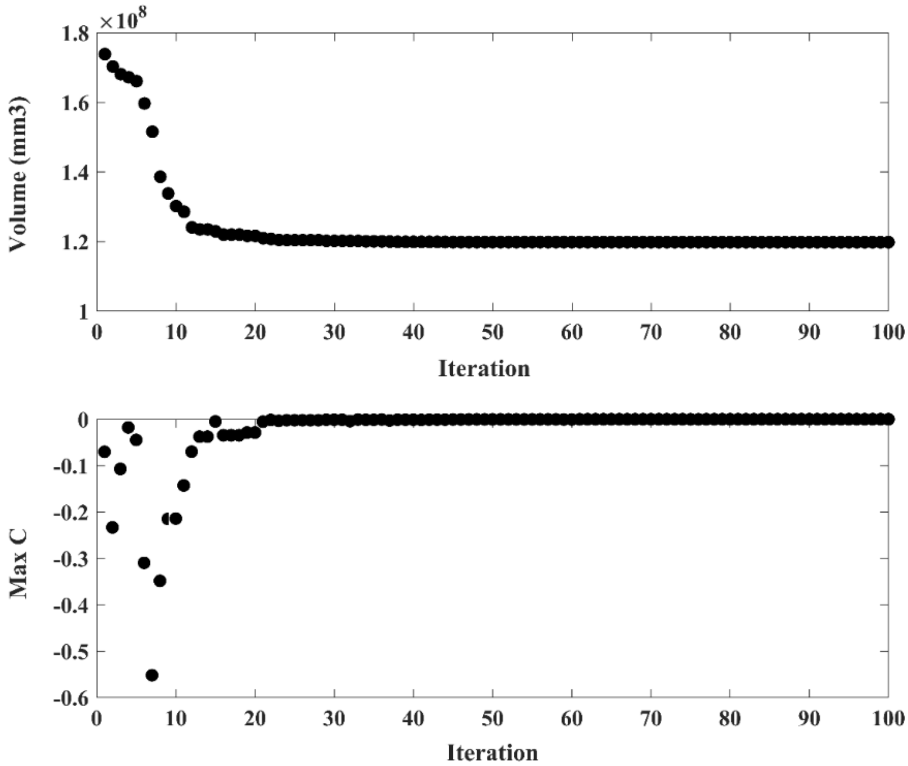


Figure 11. Convergence curve and maximum constraint violation for the dome spatial truss structure.

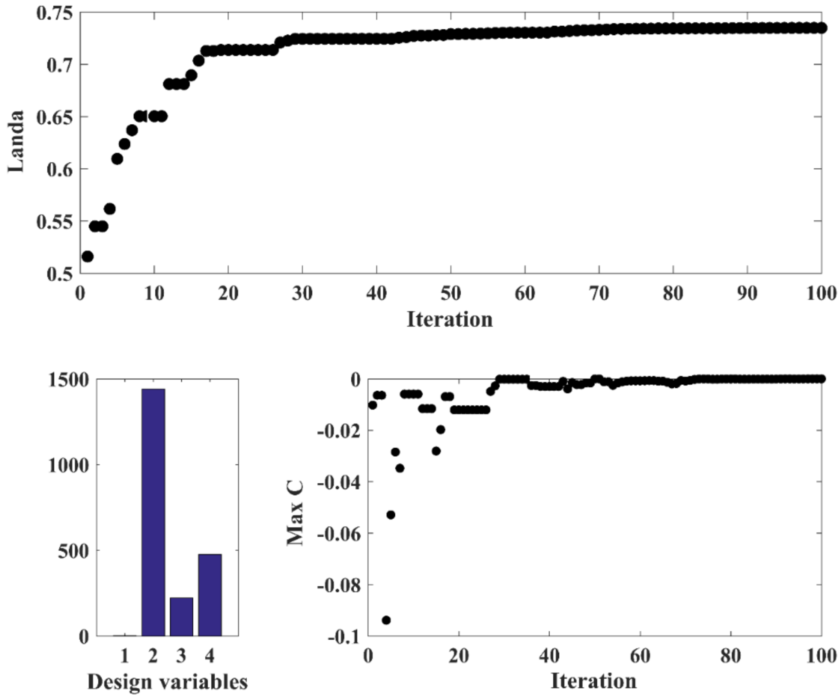


Figure 12. Convergence curve of the multi-objective functions, how to change the maximum violation value of all constraints, and the final optimal design variables.

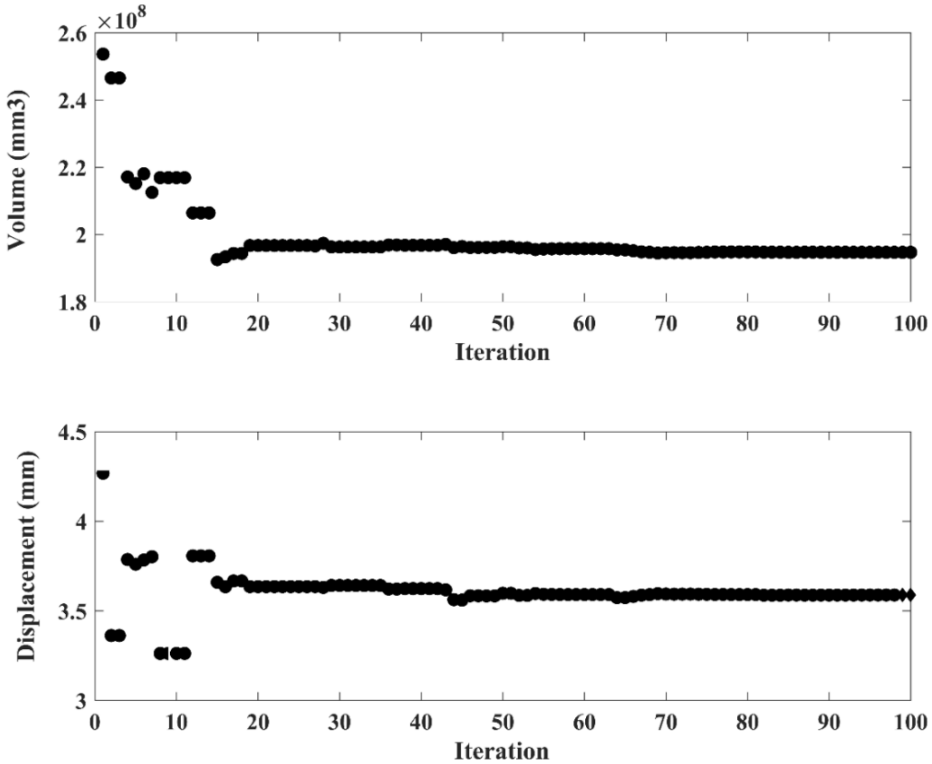


Figure 13. Convergence curve of objective functions for the dome spatial truss structure.

The convergence curve of the multi-objective functions, how to change the maximum violation value of all constraints, and the final optimal design variables are proposed in Fig. 12. As mentioned earlier, the first design variable in this figure is related to the λ parameter in fuzzy logic and the rest concerns the type corresponding to the cross-section area of the structure members. Figure 13 shows how to change the objective functions during the optimization process for the minimum volume and upper node displacement of the structure.

Table 7 presents the obtained results for the multi-objective optimization of the space dome structure using genetic [19] as well as utilizing λ formulation with fuzzy logic and its combination with the ETEO method. As can be seen, the proposed algorithm reaches much better result than genetic [19] and dominates it. Hence, the acquired optimal design in the reference [19] is not a part of Pareto's answers. In the optimal design obtained by the FLETEO, the total volume value and node 1 displacement concerning the structure have a decline of about 40.35% and 18.83% than the genetic algorithm [19] in that order. However, none of the problem constraints aren't exceeded and their maximum value relative to its admissible value is equal to -6.0500×10^{-7}. It should be noted that the constraints governing the problem are activated in optimal design. The problem constraints are contravened in the reference [19] and the maximum constraints violation value is equal to 0.3211. Therefore, the presented answer is in the infeasible region.

Table 7. Fuzzy multi objective results for transmission tower truss structure

Method	Sectional areas (mm^2) Elements group 1	2	3	Objective functions	λ^*	Max C
GA [19]	1232.17	1188.22	1308.63	$V = 3.2621\times10^8 mm^3$, $\delta = 4.4208 mm$	0.5897	0.3211
ETEO	1440.165	222.519	477.224	$V = 1.9472\times10^8 mm^3$, $\delta = 3.5883 mm$	0.7351	-6.05×10^{-7}

5. CONCLUSION

Lots of criteria should be considered to optimally design of a structure. In this paper, by combining fuzzy logic and ETEO, a novel metaheuristic algorithm (FLETEO) is proposed. For this purpose, the conversion process of the multi-objective optimization problem to the single-objective one and vice versa was carried out. Furthermore, designing several structures under different constraints was implemented. The proposed method is capable to use in each engineering structure including frames.

Based on the previous research works, there is a minor violation in the optimal design of the problem constraints which was initiated from fuzzy logic. In this paper, the problem constraints were directly applied by the ETEO method so that it led to completely preventing the constraint violation. In addition to non-violation of the constraint, comparing the acquired results in this research to the previous ones indicates the power of the presented method in finding Pareto's optimal points. This method could also dominate some of the earlier answers.

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