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MULTI-OBJECTIVE OPTIMAL DESIGN OF SATMD INCLUDING SOIL-STRUCTURE INTERACTION USING NSGA-II

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ABSTRACT

In this paper, a procedure has been introduced to the multi-objective optimal design of semi-active tuned mass dampers (SATMDs) with variable stiffness for nonlinear structures considering soil-structure interaction under multiple earthquakes. Three bi-objective optimization problems have been defined by considering the mean of maximum inter-story drift as safety criterion of structural components, absolute acceleration as the criterion of occupants' convenience, and safety of non-structural acceleration sensitive components, as well as SATMD relative displacement as the cost criterion of the control device. The parameters of the weighting matrices of the instantaneous optimal control algorithm and the maximum and minimum level of variable stiffness of the semi-active device have been considered as design variables. An improved version of the non-dominated sorting genetic algorithm (NSGA-II), has been employed to solve the optimization problems and figure out the set of Pareto optimal solutions. SATMDs with different mass ratios have been designed for an eight-story shear type building with bilinear elasto-plastic stiffness model where the soil-structure interaction has been incorporated by Cone model with three degrees of freedom for the soil. Results show the capability and simplicity of the proposed procedure to design SATMDs considering multiple performance criteria. It is observed that this procedure can offer a wide range of optimal solutions throughout the Pareto front which can be chosen by the designer based on desired performance and application of the structure.

Keywords: multi-objective optimization; non-dominated genetic algorithm version II (NSGA-II); semi-active tuned mass damper (SATMD); soil-structure interaction (SSI).

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1. INTRODUCTION

The Tuned Mass Damper (TMD) devices are extensively employed as passive structural control systems for vibration mitigation of buildings under dynamic loads such as earthquakes and winds. During past decades, many studies have been addressed the capabilities of this device in structural response reduction and even implementation in many actual buildings [1-4]. TMDs consist of mass, spring, and dashpot and are usually more effective when installed at the top floor of buildings. Since TMDs are tuned to a specific frequency, they may be ineffective under the earthquakes that induce the building to vibrate in other frequency bands. Furthermore, the structures undergo nonlinear behavior under severe earthquakes that can alter the structural properties and detune the TMD. Alongside, the interaction between the soil and structure may also have detuning effects on the TMD performance. Accordingly, to overcome the disadvantageous of conventional TMDs, active tuned mass damper [5] and semi-active tuned mass damper (SATMD) [6-8] have been proposed in many investigations. Semi-active control strategies provide adaptability during earthquakes due to variable characteristics with small power requirements and could enhance reliability because of not having the potential of destabilizing the structure.

Several studies available in the literature have been addressed designing optimum TMD parameters through tuning formulas [9] for linear structures. Further, the design methodologies of this device have also been presented incorporating the nonlinear behavior of the host structure [10]. A significant portion of the studies performed in this topic have concentrated the hypothesis of a fixed-base structural system and neglected the influence of interaction between soil and structure. However, the seismic behavior of structures and the efficiency of TMDs may heavily be affected by this phenomenon.

During the last two decades, several studies have considered the soil-structure interaction (SSI) within the TMD tuning problem. Wu et al. [11] have assessed the efficiency of TMD when SSI effects are included for frequency-independent structural modeling and found that a strong SSI coupling can cause a severe limitation in RMS response reduction. Ghosh and Basu [12] have analyzed the seismic behavior of TMD attached in a single degree of freedom primary structure on a flexible base within the frequency domain and illustrated the loss of reliability of TMD tuning as compared to the fixed-base case. Liu et al. [13] have investigated a wind-induced linear forty-story frame building equipped with a TMD on its top with the assumption of three different soil cases compared to the fixed-base case. The main outcome was that the presence of SSI would become crucial for the case of soft soil in terms of TMD tuning and efficiency, whereas its effect appeared to be smaller for the instance of a stiff soil. Farshidianfar and Soheili [14] have presented an optimum TMD tuning approach including SSI through an optimization technic and have utilized different effective optimization algorithms such as ant colony optimization, artificial bee colony, and shuffled complex evolution. The optimum TMD parameters have been derived with the objective of displacement minimization for six earthquakes separately. Khatibinia et al. [15] have proposed a multi-objective optimization framework to design TMDs considering the SSI effect. Optimal TMD parameters have been determined for a linear forty-story frame building using multi-objective particle swarm optimization method with objectives of displacement and acceleration under El-Centro earthquake. Kamgar et al. [16] have also addressed the criteria of stroke length in addition to displacement and acceleration in the design process of TMD including the SSI effects. As a result of all studies, it has been demonstrated that the effects of SSI could significantly affect the TMD tuning problem.

The main contribution of the present study with respect to previous works is the application and designing semi-active tuned mass dampers. On the other hand, all the aforementioned studies in the area of designing TMDs considering SSI effect have been performed for linear structures. However, nonlinearity in behavior of the structural systems under severe earthquakes may follow the variation of structural properties and therefore detuning effects of TMDs. Besides, since it has been demonstrated that the seismic performance of mass damper devices is heavily influenced by the characteristics of the input earthquakes it is more appropriate to consider the uncertainty of the applied excitation in the design process of these devices. To this end, the best probabilistic estimate of structural demand could be the mean responses under multiple probable earthquakes. Therefore, in this study, multi-objective optimal design of SATMDs for nonlinear structures considering SSI effect under multiple probable earthquakes has been addressed.

2. SATMD-STRUCTURE INCLUDING SOIL-STRUCTURE INTERACTION

In this section, the equation of motion of the nonlinear shear building frame equipped with a SATMD with variable stiffness on the top story and including the soil-structure interaction have been formulated. Also, the semi-active control algorithm based on instantaneous optimal control has been presented. It should be noted that the interaction between soil and structure has been incorporated in order to the seismic behavior of the structural system to be more realistic and the control system design to be more practical.

2.1 Equation of motion

Fig. 1 schematically shows the structural model comprising of an N-story nonlinear shear-type building equipped with a SATMD with variable stiffness on its top, lying on a rigid cylindrical foundation embedded in the soil with Cone model. The Cone model is based on assuming the soil as homogenous half-space [17,18] and can provide sufficient engineering accuracy with respect to the accurate finite element methods [19]. The foundation is modeled with mass m_f, mass moment of inertia I_f, and equivalent radius r. Also, M_0 denotes the trapped mass moment of inertia which is connected to the foundation and moves in the same phase with the foundation. Two sway and rocking degrees of freedom (DOFs) are considered for the foundation. The additional DOF, so-called the monkey tail [18], is modeled to account for the frequency-dependency of the soil dynamic stiffness and let other model coefficients to remain frequency-independent. The parameters of the soil including the spring and dashpot coefficients are defined in Table 1 [20]. Along with three DOFs of the soil model and one DOF of the added SATMD, the entire system have N+4 DOFs. In this paper, the equation of motion and structural property matrices have been derived for the considered multi-story structure based on extending the motion equation of a single DOF system including the SSI effect presented by Ghannad and Jahankhah [21]. The equation of motion of the considered structural system under ground acceleration of $\ddot{u}_g$ can be expressed as follows:

$$\boldsymbol{M}.\ddot{\boldsymbol{u}}(t)+\boldsymbol{C}.\dot{\boldsymbol{u}}(t)+\boldsymbol{K}.\boldsymbol{u}(t)=\boldsymbol{M}.\boldsymbol{e}.\ddot{u}_g(t) \tag{1}$$

in which **u**=[u_d, u_N, u_{N-1}, ... , u_1, u_f, θ, θ_1]^T denotes the displacement vector of the system and dot represents derivative with respect to time. u_d denotes the displacement of the SATMD with respect to the top story; u_i is the inter-story drift of the i-th story; u_f denotes the displacement of the foundation with respect to the ground; θ and θ_1 are respectively the rocking of the foundation and monkey tail. **e**=[-1,...,-1]^T denotes the ground acceleration-mass transformation vector.

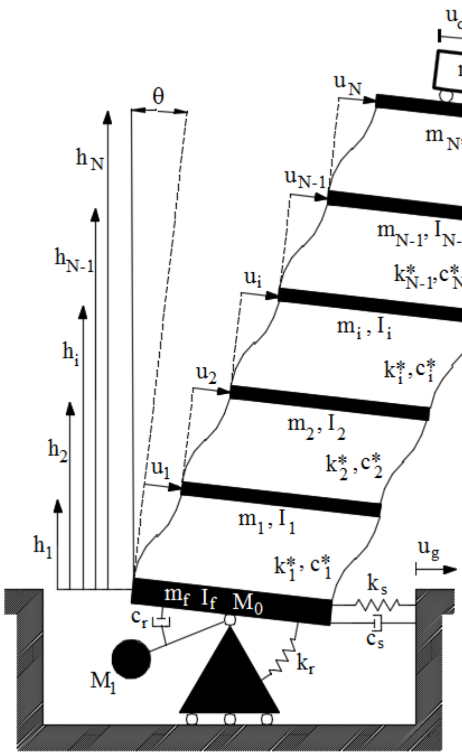

Figure 1. Schematic of SATMD-structure model including SSI

Table 1: Parameters of SSI model [20]

Rocking motion	Sway motion
$k_r=\dfrac{8\rho {V_S}^2 r^3}{3(1-\upsilon)}$	$k_s=\dfrac{8\rho {V_S}^2 r}{2-\upsilon}$
$c_r=\dfrac{\pi}{4}\rho V_a r^4$	$c_s=\pi\rho V_S r^2$
$M_1=\dfrac{9\pi^2}{128}\rho r^5(1-\upsilon)\left(\dfrac{V_a}{V_S}\right)^2$	
$M_0=0.3\pi\beta(\upsilon-0.33)\rho r^5$	
If $\upsilon\le 0.33$ then $\beta=0$ *and* $V_a=V_S$	
If $0.33\le\upsilon\le 0.5$ then $\beta=1$ *and* $V_a=2V_S$	

ρ: mass density of soil; υ:Poisson's ratio of soil; V_S:shear wave velocity; V_a:axial wave velocity

The (N+4)×(N+4) global matrices **M**, **C**, and **K** are respectively mass, nonlinear damping, and nonlinear stiffness matrices which can be represented as follows:

$$\mathbf{M}=\begin{bmatrix} m_d & m_d & m_d & \cdots & m_d & m_d & m_d h_d & 0 \\ & m_d+m_N & m_d+m_N & \cdots & m_d+m_N & m_d+m_N & m_d h_d+m_N h_N & 0 \\ & & m_d+m_N+m_{N-1} & \cdots & m_d+m_N+m_{N-1} & m_d+m_N+m_{N-1} & m_d h_d+m_N h_N+m_{N-1}h_{N-1} & 0 \\ & & & \ddots & \vdots & \vdots & \vdots & \vdots \\ & & & & m_d+\sum_{i=1}^{N} m_i & m_d+\sum_{i=1}^{N} m_i & m_d h_d+\sum_{i=1}^{N} m_i h_i & 0 \\ & & & & & m_d+\sum_{i=1}^{N} m_i+m_f & m_d h_d+\sum_{i=1}^{N} m_i h_i & 0 \\ & & & & & & m_d h_d^2+\sum_{i=1}^{N}\left(m_i h_i^2+I_i\right)+I_f+M_0 & 0 \\ Symmetry & & & & & & & M_1 \end{bmatrix} \quad (2)$$

$$\boldsymbol{C}=\begin{bmatrix} c_d & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ & c_N^* & \cdots & 0 & 0 & 0 & 0 & 0 \\ & & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & c_i^* & 0 & 0 & 0 & 0 \\ & & & & c_1^* & 0 & 0 & 0 \\ & & & & & c_h & 0 & 0 \\ & & & & & & c_\theta & -c_\theta \\ Symmetry & & & & & & & c_\theta \end{bmatrix} \quad (3)$$

$$\boldsymbol{K}=\begin{bmatrix} k_d(t) & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ & k_N^* & \cdots & 0 & 0 & 0 & 0 & 0 \\ & & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & k_i^* & 0 & 0 & 0 & 0 \\ & & & & k_1^* & 0 & 0 & 0 \\ & & & & & k_h & 0 & 0 \\ & & & & & & k_\theta & 0 \\ Symmetry & & & & & & & 0 \end{bmatrix} \quad (4)$$

where m_i, c_i^*, and k_i^* are respectively mass, nonlinear damping coefficient, and nonlinear stiffness of the i-th story. The parameters of SATMD with variable stiffness include mass, m_d, damping coefficient, c_d, and variable stiffness coefficient, $k_d(t)$. The proper variable stiffness coefficient of the semi-active device is determined in each time step by instantaneous optimal control algorithm based on the previous study performed by the authors [8] and has been addressed in the next subsection. Note that the passive TMD can be simulated by replacing the constant stiffness coefficient of the TMD, k_d, with the variable stiffness of SATMD.

2.2 Instantaneous optimal control algorithm and clipped control concept

The semi-active control algorithm is comprised of two stages. The first stage is calculating active control force, f_{active}, using the instantaneous optimal control algorithm as follows [8]:

$$f_{active}(t)=-\boldsymbol{R}^{-1}\boldsymbol{D}^T\boldsymbol{K}_n^{*-T}(t)\left(\boldsymbol{Q}_1\boldsymbol{x}(\boldsymbol{t})+a_4\boldsymbol{Q}_2\dot{\boldsymbol{x}}(t)+a_1\boldsymbol{Q}_3\ddot{\boldsymbol{x}}(t)\right) \quad (5)$$

in which, $\boldsymbol{x}$, $\dot{\boldsymbol{x}}$, and $\ddot{\boldsymbol{x}}$ are respectively the displacement, velocity and acceleration vector with the dimension of (N+4)×1. The elements of these vectors for the primary structure and SATMD have used the displacements with respect to the foundation. Also, $\mathbf{Q}_1$, $\mathbf{Q}_2$, and $\mathbf{Q}_3$

are (N+4)×(N+4) positive semi-definite weighting matrices related to the penalty of structural responses and have been assumed as follows:

$$\boldsymbol{Q}_1 = \begin{bmatrix} q_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

$$\boldsymbol{Q}_2 = \begin{bmatrix} q_2 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

$$\boldsymbol{Q}_3 = \begin{bmatrix} q_3 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

where q_1, q_2, and q_3 are the parameters of the weighting matrices. **R** is the penalty matrix of the control force which is a scalar in the case of SATMD. **D**=[1,-1,0,...,0]^T is the location matrix of the SATMD device with dimension of (N+4)×1. $\boldsymbol{K}^*_{n_k}$ is generalized stiffness matrix which in time step k is as follows:

$$\boldsymbol{K}^*_{n_k} = a_1\boldsymbol{M} + a_4\boldsymbol{C}_{k-1} + \boldsymbol{K}_{k-1} \tag{9}$$

$\boldsymbol{C}_{k-1}$ and $\boldsymbol{K}_{k-1}$ are the nonlinear damping and stiffness matrix in time step k-1. a_1 and a_4 are the coefficients of Newmark's numerical integration method [22]. Then, in the second stage, the characteristics of the semi-active device adjust such that the most similar control force to the calculated active control force be produced based on clipped control concept. Therefore, the variable stiffness of the SATMD device, $k_d(t)$, in each time step is determined as [8]:

$$k_d(t) = \begin{cases} k_{min} & f_{active}/u_d \le k_{min} \\ f_{active}/u_d & k_{min} < f_{active}/u_d < k_{max} \\ k_{max} & f_{active}/u_d \ge k_{max} \end{cases} \tag{10}$$

In which, u_d denotes the relative displacement of the SATMD with respect to the top story. Also, k_{min} and k_{max} are respectively the minimum and maximum sfiffness coefficients of the SATMD device which are determined by the designer. These parameters are the limits that the variable stiffness of the SATMD is expected to be altered between them during operation time.

3. MULTI-OBJECTIVE OPTIMAL DESIGN OF SATMD

In this study, a procedure has been introduced to design SATMDs with variable stiffness for the nonlinear structure using a multi-objective optimization framework. In this method, parameters of the weighting matrices q_1 and q_2 of the control force applied to the structure as well as minimum and maximum level of variable stiffness of the semi-active device, k_{min} and k_{max}, have been considered as design variables. The reduction in the maximum inter-story drift of the entire structure as a safety criterion has been widely intended in most of the previous researches on designing structural control systems as this demand is highly correlated to the damage of structural system and drift sensitive non-structural components. Moreover, other structural responses such as absolute floor accelerations as the criterion of occupants' convenience and/or safety of non-structural acceleration sensitive components can also be of interest especially for more important structures such as hospitals, fire stations, etc. Furthermore, the stroke length of mass damper devices is another important aspect that relates to the indirect cost of this device. TMDs and SATMDs occupy valuable floor space due to their required stroke. The stroke also influences the cost of the spring and the damper of these devices. Hence, limiting the relative displacement of mass dampers and consequently, the stroke length of these devices is desired in the design process. It should be noted that such multi-objective criteria that conflict with each other have been used previously for designing passive TMDs [23]. On the other hand, it is evident that the characteristics of the applied excitation could remarkably influence the performance of mass damper mechanisms. Hence, it is more appropriate to involve the uncertainty of the applied seismic excitation in the design process of SATMDs. To this end, the best probabilistic estimate of responses of the structure could be the mean response under multiple probable earthquakes. Therefore, in this paper, to design SATMD systems the following three objective functions have been considered: (1) minimization of the mean building inter-story drift; (2) minimization of the mean building absolute acceleration; (3) minimization of the mean SATMD relative displacement.

3.1 Multi-objective optimization problem

Indeed, many realistic engineering problems need to satisfy different objectives simultaneously which may usually conflict with each other and improvement of one can cause worsening of at least one other objective. To solve these problems, multi-objective optimization can be utilized which presents a set of optimal solutions known as Pareto optimal solutions or Pareto front, instead of a single optimal solution. In this set of optimal solutions, none of the answers dominates others. Multi-objective optimization methods could be attractive particularly for decision makers since several optimal solutions with different characteristics could be offered. In general, a multi-objective optimization problem is defined as follows:

$$\begin{array}{lll} \textit{Find}: & \mathbf{X}^* = [X_1^*, X_2^*, \ldots, X_n^*]^T & \\ \textit{Optimize}: & \mathbf{f}(\mathbf{X}) = [f_1(\mathbf{X}), f_2(\mathbf{X}), \ldots, f_m(\mathbf{X})]^T & \\ \textit{Subject to}: & g_i(\mathbf{X}) \geq 0, & i = 1,2,\ldots,l \\ & h_j(\mathbf{X}) = 0, & j = 1,2,\ldots,p \end{array} \tag{11}$$

where **f** denotes the vector of *m* objective functions; g and h are respectively *l* inequality and *p* equality constraints. **X** is the vector of design variables that may have several solutions, $\mathbf{X}^*$, to optimize the objective functions satisfying all inequality and equality constraints.

3.2 Bi-objective optimization problems to design SATMDs

As already mentioned, three objectives including maximum interstory drift, absolute acceleration, and SATMD relative displacement have been considered. In this paper, three sets of bi-objective optimization problems have been formulated and solved. In one, minimization of mean of maximum inter-story drift and SATMD relative displacement under multiple earthquakes have been considered as objectives as follows:

$$\min_{q_1,q_2,k_{\min},k_{\max}} \quad f_1 = \frac{1}{n}\sum_{j=1}^{n} \max\left(\left|drift_i(k)\right|\right),\ f_2 = \frac{1}{n}\sum_{j=1}^{n} \max\left(\left|RD(k)\right|\right) \tag{12}$$

In the second bi-objective optimization problem, the mean of maximum absolute acceleration and maximum relative displacement of SATMD under multiple earthquakes have been considered as:

$$\min_{q_1,q_2,k_{\min},k_{\max}} \quad f_1 = \frac{1}{n}\sum_{j=1}^{n} \max\left(\left|acc_i(k)\right|\right),\ f_2 = \frac{1}{n}\sum_{j=1}^{n} \max\left(\left|RD(k)\right|\right) \tag{13}$$

Finally, the mean of maximum inter-story drift ratio and absolute acceleration under multiple earthquakes have been considered in the third optimization problem where the constraint on the mean of maximum SATMD relative displacement have also been intended as following:

$$\begin{aligned} \min_{q_1,q_2,k_{\min},k_{\max}} \quad & f_1 = \frac{1}{n}\sum_{j=1}^{n} \max\left(\left|drift_i(k)\right|\right) + \alpha.\max[0,g] \\ & f_2 = \frac{1}{n}\sum_{j=1}^{n} \max\left(\left|acc_i(k)\right|\right) + \alpha.\max[0,g] \\ & g = \frac{\frac{1}{n}\sum_{j=1}^{n}\max\left(RD(k)\right)}{U_L} - 1 \end{aligned} \tag{14}$$

In these equations, the responses should be calculated over story *i* and time step *k*. Also, *n* denotes the number of earthquakes.

4. OPTIMIZATION USING NSGA-II

Among several optimization methods, genetic algorithm, GA, is one of the most capable methods which search for optimal answer using technics inspired by the evolution process in nature and has been developed first by Holland [24]. GA has been widely utilized in civil engineering [25-27] as well as structural control system design [28-30], since its simplicity and effectiveness for solving nonlinear optimization problems even with large number of design variables. This algorithm has three main operations such as selection, crossover, and mutation [31].

The multi-objective optimization problem could be converted to a single objective problem and solved by classical optimization algorithms which is a common technic in the literature [32]. However, the main disadvantage of these technics is the requirement of several simulation runs with different adjustments, whereas a significant portion of the Pareto front may stay undetected. Accordingly, different algorithms have been developed to directly solve the multi-objective optimization problem and figure out the Pareto optimal solutions even in a single simulation run.

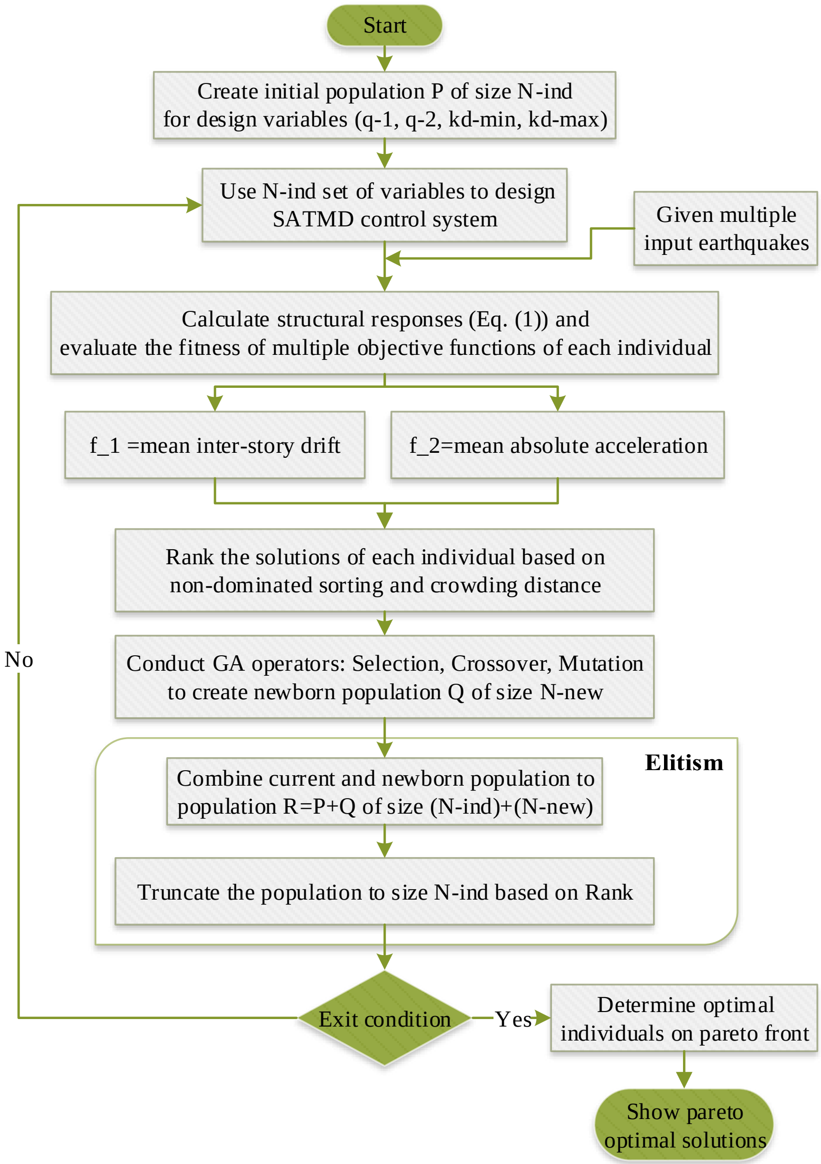

Figure 2. Flowchart for multi-objective optimal design of SATMD system based on NSGA-II algorithm

The non-dominated sorting genetic algorithm, NSGA, developed by Srinivas and Deb [33] was more efficient to solve such problems. Subsequently, this algorithm has been improved to version II [34] to improve the algorithm regarding some criticisms such as high computational burden, lack of elitism, and requirement of what called sharing parameter. Further, version III [35] has been developed for the problems with several design variables. Nevertheless, for the problems with a few design variables such as this study, the NSGA-II seems still more proper and has been used for solving the multi-objective optimization problem. Fig. 2 illustrates the flowchart of the proposed procedure for multi-objective optimal design of SATMD based on NSGA-II algorithm.

There are nine operations incorporated in this algorithm such as: Initialization, fitness evaluation, non-dominated sorting, crowding distance calculation, selection, crossover, mutation, combination, truncate. A random individual population P of size N_{ind} is generated in the initialization process. Thereafter, the fitness of each individual is calculated by evaluating multiple objective functions. These solutions are sorted based on non-domination concept and take place into successive Pareto fronts. Then, the crowding distance, which calculates how close a solution is to its neighbors in the same front, is evaluated for each solution. The higher crowding distance is better since it improves the diversity of the population. Afterward, the traditional GA operators including selection, crossover and mutation are performed to generate newborn population Q of size N_{new}. The current and newborn populations are incorporated into population R=P+Q of size $N_{ind}+N_{new}$ which ensures the elitism of the best individuals. Finally, based on the rank values, the population is truncated to the size of N_{ind}. The main process of the optimization is repeated while the exit condition is not satisfied which is reaching the maximum number of generations.

5. NUMERICAL ANALYSIS AND DISCUSSION

In this section, the methodology of multi-objective design of SATMDs for nonlinear structures considering soil-structure interaction has been explained through numerical analysis. The objectives of minimization of inter-story drift as the structural safety criterion, absolute acceleration as the criterion of occupants' convenience and safety of non-structural acceleration sensitive components, as well as SATMD relative displacement as the cost criterion of the control system have been considered in three bi-objective optimization problems.

By using the capable NSGA-II method, the optimization problems have been solved and different SATMDs with different mass ratios have been designed. SATMD has been installed on the top of an eight-story nonlinear shear building frame with bilinear hysteretic model shown in Fig. 3. K_E and K_{PE} are respectively elastic stiffness and post-elastic stiffness. The yielding inter-story drift is u_y=2.4cm. The parameters of this building case study have been reported in Table 2. All parameters are identical for all stories. The first vibration mode of the structure has 0.5% damping ratio. The fundamental period of the structure based on its initial stiffness and fixed base condition is $T_1=1.087s$. It has been assumed that the building is located on soft soil with mass density of soil ρ=1800 kg/m^3, Poisson's ratio of soil υ =0.49, and shear wave velocity V_S=100 m/s [36].

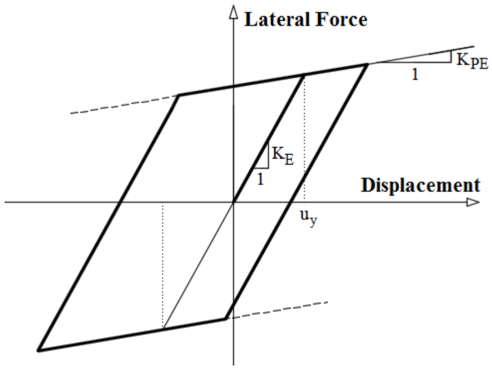

Figure 3. Bilinear elasto-plastic stiffness model

Table 2: Parameters of the nonlinear eight-story shear building

Story mass (m)	3.456e5 (kg)
Story linear viscous damping (c)	7.343e5 (N.s/m)
Story elastic atiffness (K_E)	3.404e8 (N/m)
Story post-elastic atiffness (K_{PE})	3.404e7 (N/m)
Story moment of inertia (I)	1.152e7 (kg.m^2)
Story height (h)	3.5 (m)
Faoundation radius (r)	10 (m)
Faoundation mass (m_f)	5.184e5 (kg)
Faoundation moment of inertia (I_f)	1.296e7 (kg.m^2)

5.1 Earthquake ground motions set used in this study

The most significant uncertainty within this problem is the inherent random nature of earthquakes. To involve this uncertainty into the design process of the SATMD, the mean responses under multiple probable earthquakes have been considered to be minimized. An ensemble of 20 real earthquake ground motions has been employed proportional to the specific site where the structure is located in. It has been assumed that the structure is located in downtown Los Angeles. Table 3 reports the characteristics of the adopted earthquakes which have been recorded on soft soil condition and proposed for FEMA/SAC project [37] for this site.

Fig. 4 shows the acceleration response spectrum of the selected earthquakes and the design acceleration spectrum of ASCE/SEI-7-05 code [38] for soft soil (i.e. class E) and building site with coordinates of (34.038˚N, 118.247˚ W). Based on this code, the earthquake records are scaled such that no value of the mean acceleration response spectrum to be less than the design spectrum between periods of $0.2T_1$ and $1.5T_1$.

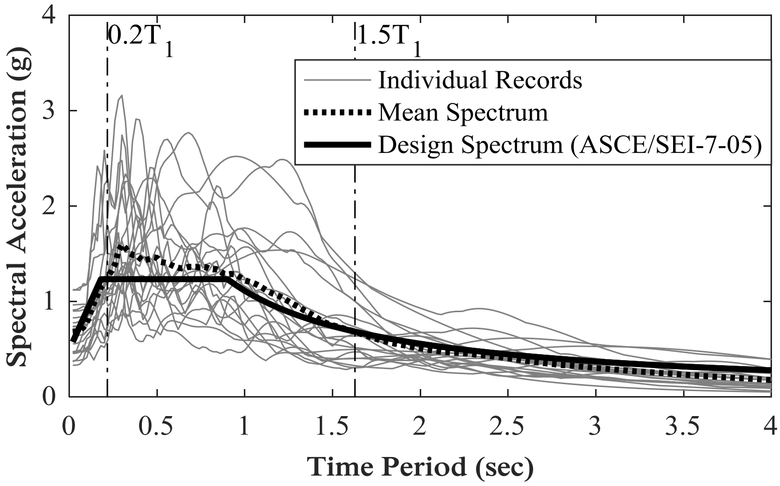

Figure 4. Acceleration response spectrum of the selected earthquake records and the design response spectrum at downtown Los Angeles for site class E

Table 3: Selected earthquake records used in this study

Earthquake code			Description	PGA (g)
ls01E	IVIR	N	Los Angeles 10% in 50 Years, 40-IVIR, Soil Type 2, Depth 1	0.40
ls02E	IVIR	P	Los Angeles 10% in 50 Years, 40-IVIR, Soil Type 2, Depth 1	0.71
ls03E	AR05	N	Los Angeles 10% in 50 Years, IV-AR05, Soil Type 2, Depth 1	0.39
ls04E	AR05	P	Los Angeles 10% in 50 Years, IV-AR05, Soil Type 2, Depth 1	0.40
ls05E	AR06	N	Los Angeles 10% in 50 Years, IV-AR06, Soil Type 2, Depth 1	0.40
ls06E	AR06	P	Los Angeles 10% in 50 Years, IV-AR06, Soil Type 2, Depth 1	0.32
ls07E	BARS	N	Los Angeles 10% in 50 Years, LN-BARS, Soil Type 2, Depth 1	0.55
ls08E	BARS	P	Los Angeles 10% in 50 Years, LN-BARS, Soil Type 2, Depth 1	0.28
ls09E	YERM	N	Los Angeles 10% in 50 Years, LN-YERM, Soil Type 2, Depth 1	0.61
ls10E	YERM	P	Los Angeles 10% in 50 Years, LN-YERM, Soil Type 2, Depth 1	0.48
ls11E	GIL3	N	Los Angeles 10% in 50 Years, LP-GIL3, Soil Type 2, Depth 1	0.96
ls12E	GIL3	P	Los Angeles 10% in 50 Years, LP-GIL3, Soil Type 2, Depth 1	0.61
ls13E	NEWH	N	Los Angeles 10% in 50 Years, NR-NEWH, Soil Type 2, Depth 1	0.77
ls14E	NEWH	P	Los Angeles 10% in 50 Years, NR-NEWH, Soil Type 2, Depth 1	0.67
ls15E	RRS.	N	Los Angeles 10% in 50 Years, NR-RRS., Soil Type 2, Depth 1	0.82
ls16E	RRS.	P	Los Angeles 10% in 50 Years, NR-RRS., Soil Type 2, Depth 1	0.31
ls17E	SYLM	N	Los Angeles 10% in 50 Years, NR-SYLM, Soil Type 2, Depth 1	0.78
ls18E	SYLM	P	Los Angeles 10% in 50 Years, NR-SYLM, Soil Type 2, Depth 1	0.63
ls19E	DHSP	N	Los Angeles 10% in 50 Years, PS-DHSP, Soil Type 2, Depth 1	0.96
ls20E	DHSP	P	Los Angeles 10% in 50 Years, PS-DHSP, Soil Type 2, Depth 1	0.63

5.2 Multi-objective optimal design of SATMD using NSGA-II

In this section, the variable stiffness SATMDs with mass ratios of μ=5%,7%,10% installed on the top floor have been designed for the nonlinear structure including soil-structure interaction through solving multi-objective optimization problems under multiple earthquakes using NSGA-II method. For designing SATMD mechanisms, first, the properties of TMDs should be determined for the structure. In this regard, optimal TMDs for different mass ratios and assuming 0.5% damping ratio for the first vibration mode have been designed based on Sadek et al. [9] procedure where their properties have been presented in Table 4. Three different bi-objective optimization problems have been solved including inter-story drift, absolute acceleration and SATMD relative displacement in pair forms. The SATMD system has four design variables including parameters of weighting matrices q_1 and q_2 related to the instantaneous optimal control algorithm as well as the minimum and maximum level of semi-active stiffness, k_{min} and k_{max}. The upper and lower bounds of the search domains for the design variables have been presented in Table 5.

Table 4: Optimum parameters of TMDs

Mass ratio μ(%)	m_d (ton)	k_d (kN/m)	c_d (kN.s/m)
5	118.379	3503.623	363.084
7	165.731	4677.910	579.820
10	236.759	6237.180	940.360

Table 5: The boundaries of design variables

	q_1	q_2	k_{min}	k_{max}
Lower bound	1e8	1e5	$0.1k_d$	$0.1k_d$
Upper bound	1e12	1e8	k_d	$10k_d$

Table 6 reports the mean responses of the uncontrolled structure and the structure equipped with passive TMDs under multiple earthquakes. For different values of SATMD mass ratios, the optimization problems defined in Equation (12) to (14) with constraint on SATMD relative displacement equal to U_L=1.0m has been solved frequently by NSGA-II. The parameters of the NSGA-II have been chosen as presented in Table 7. To ensure the accuracy of the optimization method, at least four different simulation runs of NSGA-II with different initial random population have been performed for the considered optimization problems.

Table 6: Optimum parameters of TMDs

Mechanism	Drift (cm)	Acceleration (cm/s^2)
Uncontrolled	9.52	1026.1
TMD (μ=5%)	8.76	995.8
TMD (μ=7%)	8.46	978.3
TMD (μ=10%)	8.05	949.0

Table 7: Parameters of genetic algorithm

N_{ind}	Number of individuals in each generation	25
N_{New}	Number of newborns	18
m_r	Mutation rate	0.02
N_{max}	Maximum number of generation	50

Fig. 5 shows the Pareto-optimal solution sets in which the objectives of inter-story drift as safety criterion of structural components and SATMD relative displacement as the cost of the control system have been considered as defined in bi-objective optimization problem in Equation (12). These solutions have been derived for SATMDs with different mass ratios and all 25 individuals have stood on Pareto fronts. Hence, various optimal answers are provided and selecting suitable solution depends on the decision of the designer. It is observed that SATMDs with different mass ratios have relative displacement demand lower than 1.0m which could be sufficient for this device. Also, the capability of SATMDs in reducing the inter-story drift has been improved by increasing the mass ratio. If reducing the inter-story drift as the safety criterion is in priority, the solutions at the up-left of the Pareto fronts are more suitable. As an example, these SATMDs with mass ratios of μ=5%, 7%, and 10% have reduced the mean inter-story drift respectively about 13%, 16%, and 20% with respect to the uncontrolled structure.

Along with SATMD relative displacement, the absolute acceleration as the criterion of occupants' convenience and safety of non-structural acceleration sensitive components have been considered in bi-objective optimization problem according to Equation (13) where the corresponding Pareto fronts have been illustrated in Fig. 6. Similarly, the Pareto fronts offer several optimal solutions which are available for decision makers. Results show that the SATMDs relative displacements are still in an acceptable range and for reducing the acceleration of the structure the solutions placed in the up-left corner of the Pareto fronts can be selected. As an instance, SATMDs with mass ratio of μ=10% placed in the up-left corner of the Pareto fronts can reduce the absolute acceleration up to 12% with respect to the uncontrolled structure.

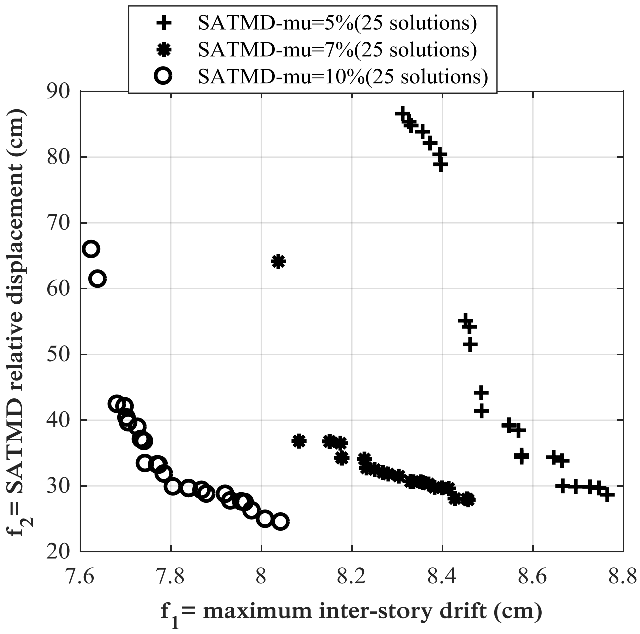

Figure 5. Pareto-optimal solution sets of bi-objective optimization problem considering drift and SATMD displacement

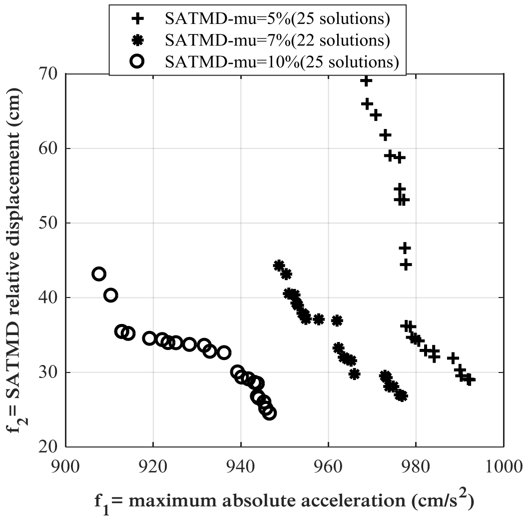

Figure 6. Pareto-optimal solution sets of bi-objective optimization problem considering acceleration and SATMD displacement

Finally, by considering the objectives of drift and acceleration and constraint on SATMD relative displacement, the bi-objective optimization problem defined in Equation (14) have been solved for different mass ratios and the related Pareto fronts have been shown in Fig. 7. It is evident that still several solutions are at hand and the appropriate SATMD could be selected based on the required controller proportional to the application of structure. In this regard, if the safety of structural system is more important, the solution in up-left of the Pareto front could be selected. On the contrary, when the criterion of convenience or the safety of non-structural elements is in priority, the solution in the down-right of the Pareto front could be chosen. Eventually, a wide range of optimal solutions within the Pareto front are available when both safety and convenience criteria are crucial especially for more complex and important structures such as hospitals, fire stations, etc.

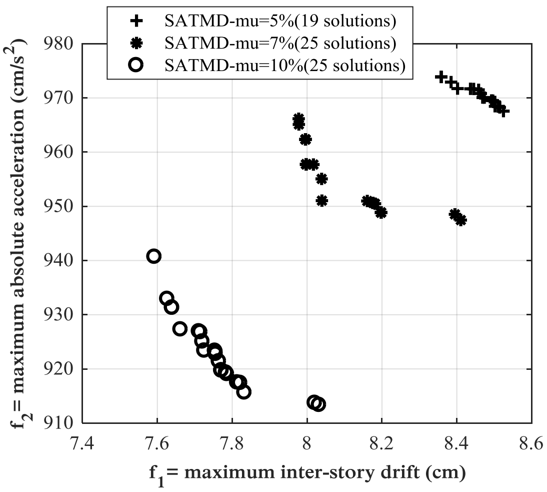

Figure 7. Pareto-optimal solution sets of bi-objective optimization problem considering drift and acceleration

As a sample, the optimum design variables of the SATMDs with different mass ratios considering the objectives of drift and acceleration have been reported in Table 8. For each Pareto front which relates to a specific mass ratio, two individuals including the ones placed within up-left and down-right individuals of the Pareto front have been presented. In this table, the upper row corresponds to up-left individual and the lower row relates to the down-right individual. Also, as an instance, time histories of inter-story drift of the first three floors as well as absolute acceleration of the last floor correspond to the uncontrolled structure and the structure equipped with SATMD with mass ratio of μ=10% and designed based on the drift and acceleration objectives and up-left individual of the Pareto front under earthquake number one have been compared in Fig. 8 and 9, respectively. As can be seen, the SATMD system has been optimally designed using the proposed method and can properly mitigate the responses of the nonlinear frame and thus eliminate severe damage potential to the structural and non-structural components.

Table 8: Optimum design variables of SATMDs considering drift and acceleration objectives

	q_1	q_2	k_{min} (N/m)	k_{max} (N/m)
μ=5%	512923243904	51042025	1388242	13544906
	536243978176	51833305	1464840	13158543
μ=7%	545024883875	32085379	2081781	30505818
	939391797299	42861790	3037187	17401767
μ=10%	707612222901	23715618	3074596	44786330
	626289212609	55902456	4995173	29240744

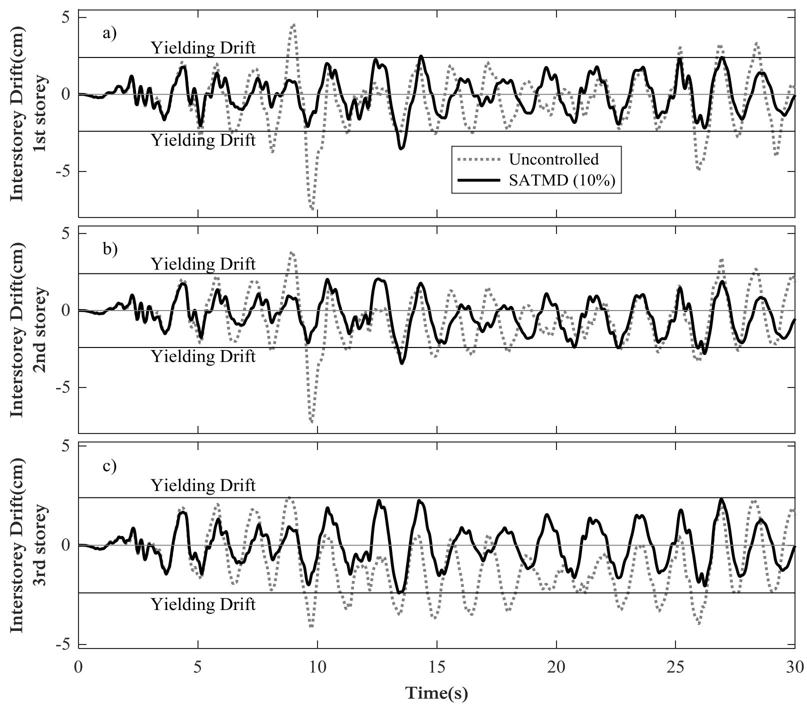

Figure 8. Time histories of a) first floor, b) second floor, and c) third floor inter-story drift of the uncontrolled structure and equipped with SATMD with mass ratio μ=10%

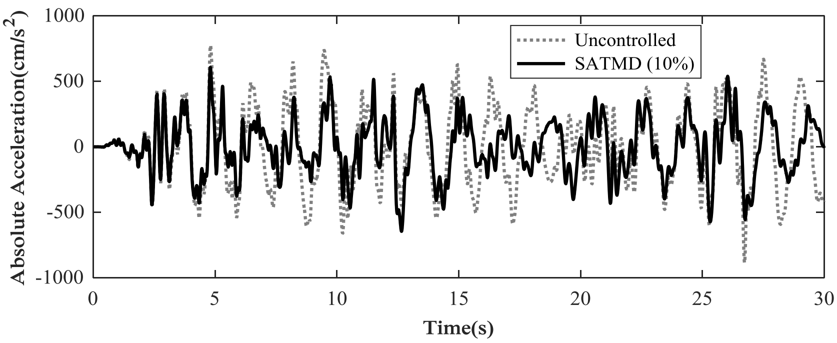

Figure 9. Time history of the last floor absolute acceleration of the uncontrolled structure and equipped with SATMD with mass ratio μ=10%

6. CONCLUSIONS

In this study, it has been aimed to present a methodology to multi-objective optimal design of variable stiffness SATMDs for nonlinear structures considering soil-structure interaction under multiple earthquakes. Different objectives including mean of maximum inter-story drift as safety criterion of structural components, absolute acceleration as the criterion of occupants' convenience and safety of non-structural acceleration sensitive components, as well as SATMD stroke length as the cost criterion of the control device have been considered and three bi-objective optimization problems have been defined incorporating these criteria in paired form. The parameters of the weighting matrices of the instantaneous optimal control algorithm and the maximum and minimum level of variable stiffness of the semi-active device have been considered as design variables. An improved version of non-dominated sorting genetic algorithm, NSGA-II, has been used to solve the optimization problems and represent the set of Pareto optimal solutions. For numerical analysis, an eight-story shear type building with bilinear elasto-plastic stiffness model has been used where the soil-structure interaction has been incorporated by Cone model with three degrees of freedom for the soil. Variable stiffness SATMDs with mass ratios of μ=5%, 7%, and 10% have been designed for the site assumed to be located in soft soil. The results show the capability and simplicity of the introduced procedure to design the SATMDs with a wide range of optimal solutions provided for decision makers. It is observed that the efficiency of the SATMDs improves by increasing the mass ratio of the device. As an instance, SATMD with mass ratio of μ=10% can reduce the mean inter-story drift up to 20% and also can mitigate the mean absolute acceleration about 12%. Consequently, if the safety of structural components is more important the SATMDs with the minimum inter-story drift could be selected. On the other hand, SATMDs with minimum absolute acceleration can be used when the the criterion of convenience and safety of non-structural acceleration sensitive components is in priority, especially for more important and complex structures such as hospitals, fire stations, etc, that contain sensitive equipment. Accordingly, when providing both safety and convenience criterion is intended, a wide range of optimal solutions are at hand that can be chosen by the designer considering the application of the structure.

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