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SIZE AND SHAPE RELIABILITY-BASED OPTIMIZATION OF DOME TRUSSES

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ABSTRACT

Nowadays, the optimal design of structures based on reliability has been converted to an active topic in structural engineering. The Reliability-Based Design Optimization (RBDO) methods provide the structural design with lower cost and more safety, simultaneously. In this study, the optimal design based on reliability of dome truss structures with probability constraint of the frequency limitation is discussed. To solve the RBDO problem, nested double-loop method is considered; one of the loops performs the optimization process and the other one assesses the reliability of the structure. The optimization process is implemented using ECBO and EVPS algorithms and the reliability index is calculated using the Monte Carlo simulation method. Finally, the size and shape reliability-based optimization of 52-bar and 120-bar dome trusses has been investigated.

Keywords: Monte Carlo simulation method; reliability index; truss structures; metaheuristic algorithms.

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1. INTRODUCTION

Nowadays, due to the fact that optimization problems are very important, its application in engineering sciences and optimal design of structures has been considered more than before.

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The main purpose of optimal design of structures is to use the most efficient optimization methods to achieve the most economical design while providing all the constraints. Eventually, this design in addition to providing the necessary constraints for each problem significantly reduces construction costs and avoids wasting materials. In recent years, many researchers have used meta-heuristic algorithms to perform optimization problems and the usage of these algorithms in solving optimization problems has been developed. Meta-heuristic algorithms can solve many different problems and the ability of these algorithms to cover the search space and avoid local optima has led to finding appropriate answers; therefore, the use of meta-heuristic algorithms is an appropriate method to perform optimal designs. Various types of these algorithms have been developed in recent decades [1-8].

Safety has always been one of the main goals in the design of structures. Structures confront with many uncertainties, so it is difficult to achieve a completely safe design; reliability theory is used to consider the effect of these uncertainties. Reliability is a theory that evaluates the probability of structural failures due to the uncertainty of design parameters and in order to determine the level of system safety, it uses an index called the reliability index. In the reliability evaluation, if a structure is designed with sufficient safety, the considered probabilistic variables satisfy all the constraints of the problem. By considering the uncertainties in the design variables, the design can be created based on a reliability level (Reliability-Based Design Optimization). Optimal design of structures based on reliability is an issue that has recently been developed by researchers to optimize structural systems. To solve RBDO optimization problems, various methods are proposed which are divided into three categories: double-loop, single-loop and decoupled.

The single-loop method used by Fan Li et al. to perform the optimal design based on reliability, converts the probabilistic constraints into approximate deterministic constraints, and the RBDO problem becomes a deterministic design optimization (DDO) problem in single loop [9]. In the double-loop method, using the outer optimization loop, the inner reliability analysis loop is performed and replicated [10, 11]. The reliability analysis loop is a separate problem that can be evaluated using direct methods such as the reliability index approach [11] or inverse methods such as the inverse reliability strategy [12, 13]. In the double-loop method, it is very important to select an optimization algorithm in the optimization loop to solve a specific RBDO problem [10]. Kaveh and Ilchi proposed a model for reliability based design optimization problem using several meta-heuristic algorithms. The meta-heuristic algorithms used to calculate the reliability index were IRO, DPSO, CBO and ECBO. The results show that the proposed algorithms have a desirable performance [14]. Keshtegar evaluated the nonlinear probabilistic constraints of RBDO problems to improve the performance of the inverse reliability method. In the paper, a Modified Mean Value (MMV) method based on the double-loop method for evaluating the reliability of RBDO is proposed [15]. Gholizadeh and Aligholizadeh performed an optimal seismic design based on reliability. They used CECBO and ECBO algorithms and Monte Carlo simulation method to solve the RBDO problem and investigated the efficiency of the proposed method for reinforced concrete moment frames [16]. The sequential optimization and reliability assessment (SORA) is one of the RBDO problem solving methods. In this approach, a decoupled strategy including optimization and reliability assessment is used [17]; which the problem is divided into a sequential optimization loop and a reliability assessment loop. Vinh Ho et al. proposed a new hybrid method (SORA-ICDE) to solve

RBDO problems of truss structures. This method is a combination of SORA method and improved constrained differential evolution algorithm (ICDE) which helps to improving the efficiency of SORA method and convergence of optimal solution. [18]. In another study, they also investigated the optimal design of truss structures based on the reliability. In their study, the optimization problem is defined by considering the frequency constraints under uncertainty of loading and material properties. They also proposed a double-loop method with a new combination of improved differential evolution algorithm, which uses the inverse reliability method to solve this problem [19]. Bataleblu and Ebrahimi presented an enhanced version of the SORA method to improve computational efficiency and expand the scope of SORA application. In the mentioned approach, a criterion is used to identify the probabilistic constraints and separate the satisfied constraints from the reliability assessment loop to reduce the computational costs [20]. Today, the wide range of applications of truss structures in structural engineering has made their optimal design valuable. The use of RBDO methods also makes it possible to design the best structure with the lowest cost and maximum reliability for truss structures. In solving the optimization problems of truss structures with frequency constraints, minimizing the total weight of the structure while satisfying the frequency constraints has been studied and is important. In this type of optimization problems, the frequency constraints applied to prevent the phenomenon of structural resonance as much as possible [21].

In this study, the optimal design of dome truss structures based on reliability is performed by considering the probability constraint of the frequency limitation. The method considered to solve this RBDO problem is the nested double-loop method. The Monte Carlo simulation method is used to calculate the reliability index, which is a suitable method for analyzing the reliability of structures. The optimization process is performed using Enhanced Colliding Bodies Optimization (ECBO) algorithm and Enhanced Vibrating Particles System (EVPS) algorithm. To evaluate the optimal design based on reliability, two numerical examples of 52-bar and 120-bar dome trusses have been considered.

2. RELIABILITY-BASED DESIGN OPTIMIZATION PROBLEM

2.1 Reliability assessment

The reliability assessment of the structure is one of the applications of reliability theory for evaluating the structural safety. The RBDO is the optimal design of a structure considering the probability variables; this topic of structural engineering has recently attracted the attention of many researchers. Probabilistic uncertainties of structural parameters such as material properties, external loads, geometric dimensions, etc. affect the final design and the safety of structural systems; therefore, to evaluating structural reliability, parameters with uncertainty are considered as random variables which each of them include a statistical distribution. Due to the random behavior of these variables, it is not possible to determine the safety or failure of the structure certainly; in this way the possibility of structural failure is evaluated. In general, in the optimization method based on reliability theory design safety is evaluated based on the probability of failure and uncertainties are modeled with probabilistic distribution of random variables. Satisfying of each problem constraints by the structure considering probabilistic uncertainties indicates the value of safety of structure. To investigate the probability of satisfying one of the considered constraints, a function of random variables (**X**) called limit state function (g (**X**)) is defined as Eq. (1):

$$g(\mathbf{X}) = R - Q \tag{1}$$

where, R is the value of system's ability to satisfy the considered constraint and Q is the constraint limit. If the value of g is positive ($g > 0$), the system is in the safe region and if the value of g is negative or zero ($g \leq 0$), the system is in the unsafe (failure) region. Also, R and Q can represent the values of strength and loading results on the structure, respectively. In fact, structural reliability assessment can determine appropriate estimation of the safety level of the structure by changing the applied loads and strength. One of the methods for evaluating the reliability of structures is the Monte Carlo simulation method which is a computational algorithm uses random sampling to calculate the results. In this method, first N samples are generated based on random variables and then the limit state function is calculated for each sample. Finally, the probability of structural failure is obtained by dividing the number of failure region samples (N_f) by the total number of selected samples (N) according to Eq. (2).

$$P_f = P\left(g \leq 0\right) = \frac{1}{N}\sum_{i=1}^{n} I\left(g(\mathbf{X}_i)\right) = \frac{N_f}{N} \tag{2}$$

In Eq. (2), I is an index function. If the limit state function for the ith sample of random variables ($g(\mathbf{X}_i)$) is positive, the value of I is equal to zero, otherwise it is 1. Finally, the reliability index is calculated by Eq. (3).

$$\beta = \Phi^{-1}(1 - P_f) \tag{3}$$

where, Φ^{-1} is the inverse of the normal cumulative distribution function.

2.2 Formulation of RBDO problem

The RBDO problem is defined as follows:

Find : $\{\mu_x\}$

To minimize : $f\ (\mu_x)$

Subject to : $P_f\ \{g_i(K,X) \leq 0\} \leq \Phi\ (-\beta_i),\ i = 1,2,...,n$

where, $f\ (\mu_x)$ is the objective function; K is the vector of random parameters; X is the vector of variables; g_i is the ith constraint function; n is the number of probabilistic constraints; Φ is the standard cumulative function of the normal distribution; β_j^i is the target reliability index for the ith probabilistic constraint; μ is the mean of these variables. In this study, f is considered as the weight of the structure.

3. META-HEURISTIC OPTIMIZATION ALGORITHM

3.1 ECBO algorithm

In this study, an efficient meta-heuristic optimization algorithm called ECBO has been used. Kaveh and Ilchi Ghazan [2] proposed the ECBO algorithm to improve convergence and performance of CBO (Colliding Bodies Optimization) by adding a memory to store some of the best solutions during the optimization process. The ECBO steps are as follows:

1. The initial positions of all colliding bodies (CBs) are randomly determined in an m-dimensional search space, using Eq. (4).
2. The value of the mass for each CB is calculated by Eq. (5).
3. Colliding memory is obtained to store some of the best CB vectors. Solution vectors stored in colliding memory are added to the population and the same number of the current worst CBs is removed. Eventually, CBs are sorted in descending order of mass.
4. CBs are divided into two groups: (a) Stationary group, (b) Moving group.
5. The velocities of stationary and moving bodies before collision and after this are calculated using Eqs. (6) and (7).
6. The new position of each CB is calculated by Eq. (9).

$$\mathbf{X}_i^0 = \mathbf{X}_{\min} + rand \circ (\mathbf{X}_{\max} - \mathbf{X}_{\min}),\ i = 1, 2, ..., n \tag{4}$$

$$m_i = \frac{1}{F(\mathbf{X}_i)} \tag{5}$$

$$V_{i_S} = 0\ ,\quad V_{i_M} = V_{i_S} - V_{i_M} \tag{6}$$

$$V'_{i_S} = \left(\frac{(1+\varepsilon)m_{i_M}}{m_{i_S} + m_{i_M}}\right)V_{i_M}\ ,\ V'_{i_M} = \left(\frac{(m_{i_M} - \varepsilon m_{i_S})}{m_{i_S} + m_{i_M}}\right)V_{i_M} \tag{7}$$

$$\varepsilon = 1 - \frac{iter}{iter_{\max}} \tag{8}$$

$$\mathbf{X}_{i_S}^{new} = \mathbf{X}_{i_S} + rand \circ V'_{i_S}\ ,\ \ \mathbf{X}_{i_M}^{new} = \mathbf{X}_{i_M} + rand \circ V'_{i_M} \tag{9}$$

$$\mathbf{X}_j^i = \mathbf{X}_{\min} + rand\ .(\mathbf{X}_{\max} - \mathbf{X}_{\min}) \tag{10}$$

where, $\mathbf{X}_i^0$ is the initial solution vector of the *i*th CB; *rand* is a random vector in the [0, 1] range; $\mathbf{X}_{max}$ and $\mathbf{X}_{min}$ are the upper and lower bounds of design variables in the search space, respectively; *n* is the number of CBs; $F(\mathbf{X}_i)$ is the objective function value of the *i*th CB; ε is a coefficient that decreases linearly from one to zero as shown in Eq. (8); *iter* is the current number of iterations; $iter_{max}$ is the total number of iterations; In ECBO, the *Pro* parameter is introduced in the [0, 1] range, and specifies whether each variable should change. In this study, the value of *Pro* is set to 0.25. For each CB, *Pro* is compared to *rni*; *rni* is a random number that is uniformly distributed in the [0, 1] range. If *rni* <*Pro*, one dimension of the *i*th CB is randomly selected and its value is recalculated by Eq. (10); $\mathbf{X}_j^i$ is the *j*th variable of the *i*th CB.

3.2 EVPS algorithm

Another algorithm used for the optimization problem is the EVPS algorithm, which has replaced the VPS (Vibrating Particles System) algorithm due to the increased convergence speed and efficiency of the VPS algorithm [7]. The implementation process of this meta-heuristic algorithm is as follows:

1. First, the initial population in the permissible range is generated by Eq. (11).
2. In this algorithm, another parameter called memory parameter is defined which stores the number of memory sizes from the best obtained positions for the population.
3. The parameter defined according to Eq. (12) determines the effect of the damping level in the vibration.
4. Eventually, the new positions of population are updated by Eq. (13).

$$\mathbf{X}_j^i = \mathbf{X}_{\min} + random.(\mathbf{X}_{\max} - \mathbf{X}_{\min}) \tag{11}$$

$$D = \left(\frac{iter}{iter_{\max}}\right) - \alpha \tag{12}$$

$$\mathbf{X}_i^j = \begin{cases} \left[D.A.rand1 + OHB^j\right] & , A = (\pm 1)\left(OHB^j - \mathbf{X}_i^j\right) \quad (a) \\ \left[D.A.rand2 + GP^j\right] & , A = (\pm 1)\left(GP^j - \mathbf{X}_i^j\right) \quad (b) \\ \left[D.A.rand3 + BP^j\right] & , A = (\pm 1)\left(BP^j - \mathbf{X}_i^j\right) \quad (c) \end{cases} \\ \omega_1 + \omega_2 + \omega_3 = 1 \tag{13}$$

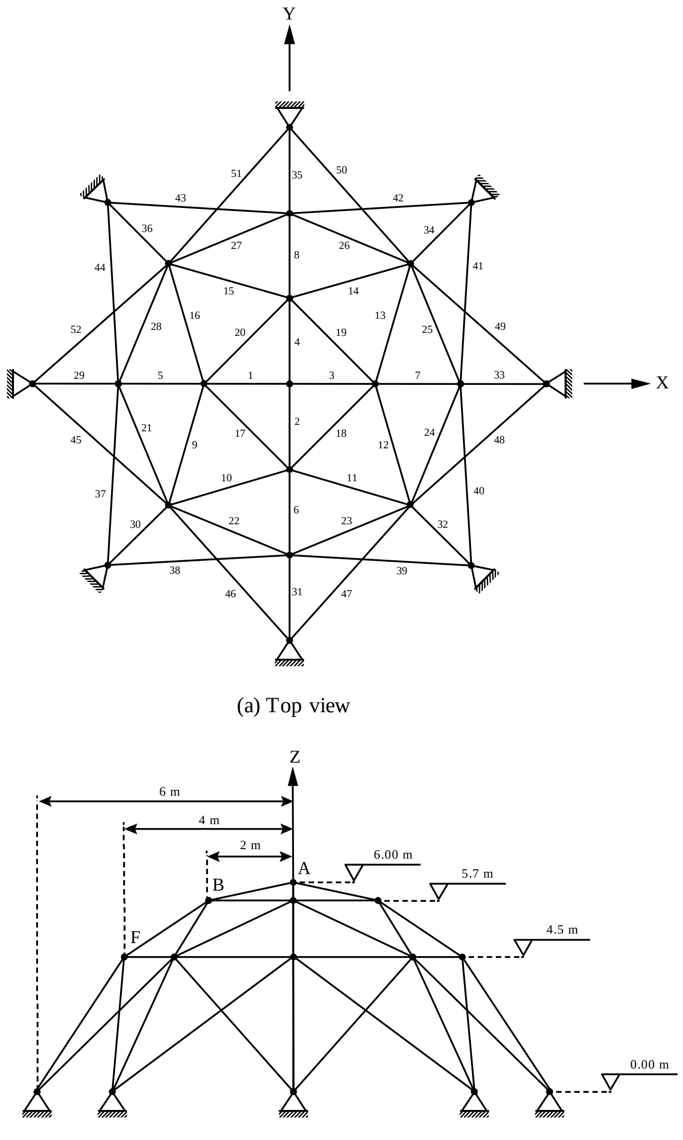
where, $\mathbf{X}_j^i$ is the *j*th variable of the *i*th particle; $\mathbf{X}_{\max}$ and $\mathbf{X}_{\min}$ are the upper and lower bounds of design variables in the search space, respectively; *iter* is the current number of iterations; $iter_{\max}$ is the total number of iterations and α is a parameter with a constant value; ±1 used randomly; *OHB*, *GP* and *BP* are determined independently for each of the variables; The coefficients ω_1, ω_2 and ω_3 are the relative importance for *OHB*, *GP* and *BP*, respectively; *rand*1, *rand*2 and *rand*3 are random numbers uniformly distributed in the [0, 1] range.

4. NUMERICAL EXAMPLES

In this section, two numerical examples are considered for reliability based design optimization problem of dome truss structures using ECBO and EVPS algorithms. The optimization process of both examples is performed in some independent runs. In this study for each of these optimization algorithms a population of 30 and a maximum number of iterations of 300 are selected. To perform the reliability analysis by Monte-Carlo simulation, the number of Monte-Carlo samples is assumed to be 10^4. The cross-sectional area of elements, modulus of elasticity, material density and the added masses are considered to be the random parameters which have a uniform distribution with a coefficient of variation of 5%. The modulus of elasticity is assumed 2.1×10^{11} (N/m^2).

4.1 A 52-bar dome-like truss

In this example size and shape optimization (simultaneously) of the 52-bar dome truss structure is considered which the initial schematic of this truss is shown in Fig. 1. All truss elements are arranged into eight groups as reported in Table 1. The Material density (ρ) and added mass are assumed 7800 (kg/m^3) and 50 (kg), respectively. The value of design variable range, frequency constraints and Allowed value of all free nodes are given in Table 2. The geometry of the structure changes that its symmetry is established. This example was previously studied [22-24]. Table 3. shows the optimization results obtained for the 52-bar dome-like truss.



(a) Top view

(b) Side view

Figure 1. Schematic of the 52-bar dome-like truss

Table 1: Element grouping adopted in the 52-bar dome-like truss problem

Group number	Elements	Group number	Elements
1	1-4	5	21-28
2	5-8	6	29-36
3	9-16	7	37-44
4	17-20	8	45-52

Table 2: Data for the 52-bar dome truss structure

Parameters (unit)	Value
Constraints on first two frequencies (Hz)	$\omega_1 \leq 15.961$, $\omega_2 \geq 28.648$
Allowable range of cross sections (m^2)	$0.0001 \leq A \leq 0.001$
Allowed value of all free nodes (m)	±2

Table 3: Optimization results obtained for the 52-bar dome-like truss

Design variables		RBDO		DDO	
		ECBO	EVPS	CSS [25]	DPSO [26]
Z_A (m)		4.1712	4.0828	5.2716	6.164
X_B (m)		3.2656	3.0906	1.5909	2.261
Z_B (m)		3.7000	3.7392	3.7093	3.832
X_F (m)		4.3743	4.3951	3.5595	4.046
Z_F (m)		2.5072	2.5633	2.5757	2.509
A_1 (cm^2)		1.0108	1.1157	1.0464	1.002
A_2 (cm^2)		1.0002	1.0291	1.7295	1.117
A_3 (cm^2)		1.7272	2.0154	1.6507	1.221
A_4 (cm^2)		2.4524	2.0387	1.5059	1.464
A_5 (cm^2)		1.3864	1.6380	1.721	1.513
A_6 (cm^2)		1.3557	1.1728	1.002	1.001
A_7 (cm^2)		2.6430	2.1287	1.7415	1.526
A_8 (cm^2)		1.7488	2.0398	1.2555	1.384
Best weight (kg)		275.946	271.564	205.237	195.624
Mean weight (kg)		341.262	294.791	213.101	-
Worst weight (kg)		470.116	335.751	-	-
Standard deviation (kg)		69.378	18.005	7.391	-
Natural frequencies (Hz)	f_1	14.033	11.386	9.246	11.236
	f_2	31.710	31.742	28.648	28.648
Reliability index β (Probability of safety %)	$\beta(f_1)$	3.239 (99.94%)	Inf (100%)	Inf (100%)	Inf (100%)
	$\beta(f_2)$	3.036 (99.88%)	3.011(99.87)	0.012 (50.46%)	0.0 (44.79%)

The convergence curves of each algorithm for the best and average solution of the 52-bar dome-like truss are shown in Fig. 2. and Fig. 3, respectively. Figures indicate the comparison of the convergence rate of these algorithms. The results show the appropriate performance of the proposed algorithms.

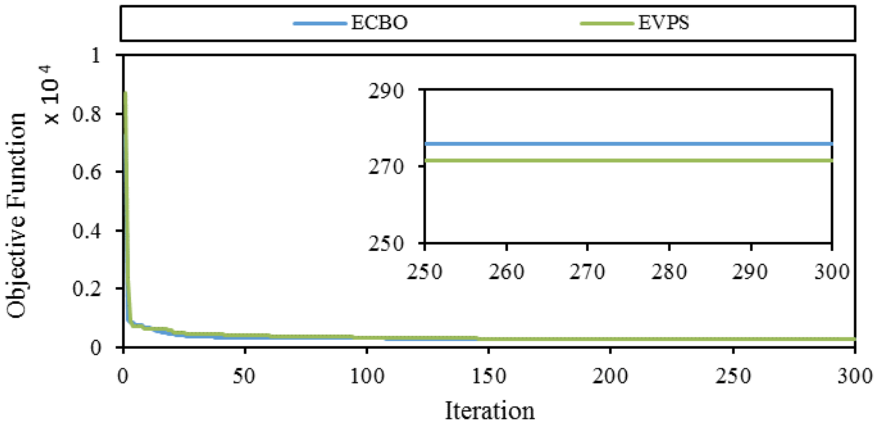


Figure 2. Comparison of the convergence curves for the best run obtained by the algorithms for the 52-bar dome-like truss

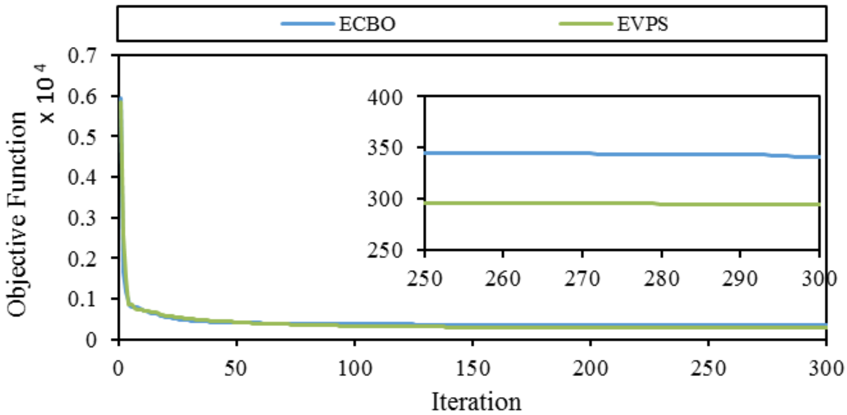
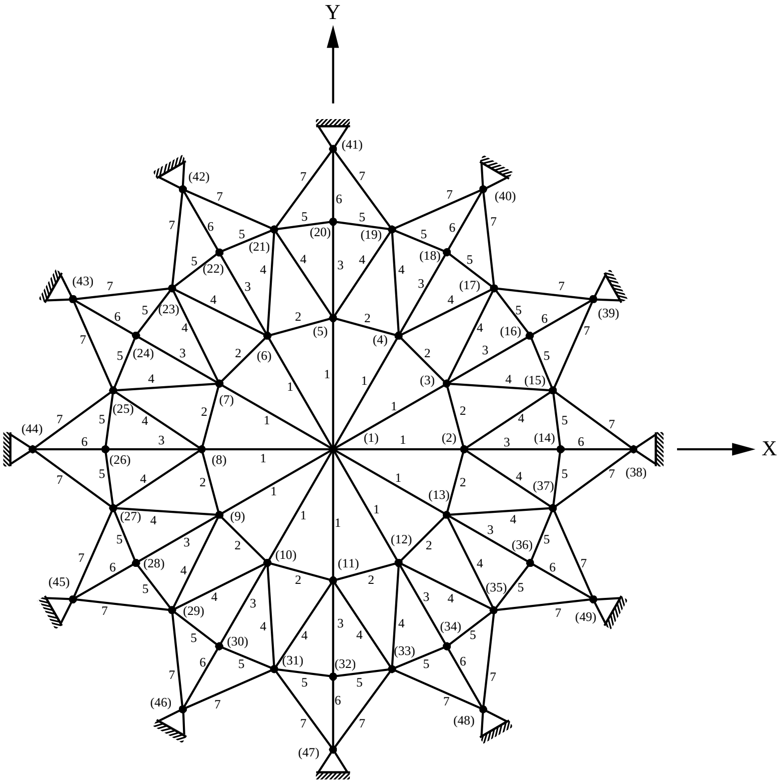


Figure 3. Comparison of the convergence curves for the average of runs obtained by the algorithms for the 52-bar dome-like truss

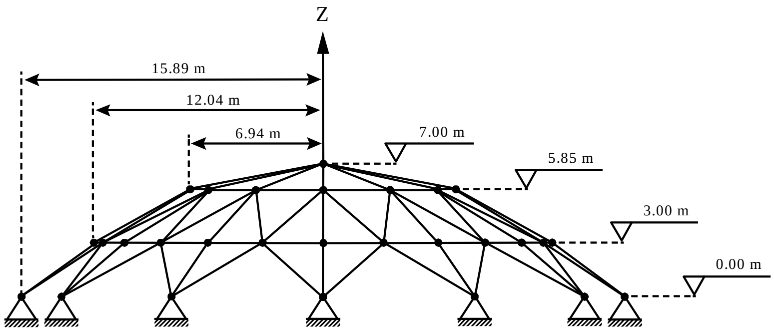
4.2 A 120-bar dome-like truss

In this example, a 120-bar dome truss structure is considered as shown in Fig. 4. This problem has already been evaluated in some researches as optimization problem with frequency constraints [25-29]. Non-structural masses are attached to all free nodes as follows: 3000 kg at node 1, 500 kg at nodes 2 through 13 and 100 kg in the remaining nodes and the material density (ρ) is assumed 7971.81 (kg/m^3) for this example. The variable range and frequency constraints are summarized in Table 4. The selected grouping of elements is shown in Fig. 4.

Table 5. shows the optimization results obtained for the 120-bar dome truss structure.



(a) Top view



(b) Side view

Figure 4. Schematic of the 120-bar dome truss

Table 4: Data for the 120-bar dome truss structure

Property (unit)	Value
Constraints on first two frequencies (Hz)	$\omega_1 \geq 9$, $\omega_2 \geq 11$
Allowable range of cross sections (m^2)	$0.0001 \leq A \leq 0.01293$

Table 5: Optimization results obtained for the 120-bar dome truss

Design variables		RBDO		DDO	
		ECBO	EVPS	CSS [26]	DPSO [26]
A_1 (cm^2)		25.870	23.531	19.454	19.571
A_2 (cm^2)		54.625	51.465	44.174	41.148
A_3 (cm^2)		10.294	21.335	10.860	11.439
A_4 (cm^2)		36.945	31.510	21.003	21.315
A_5 (cm^2)		20.848	16.160	9.060	10.094
A_6 (cm^2)		12.736	17.176	13.144	12.514
A_7 (cm^2)		18.954	19.729	15.447	15.080
Best weight (kg)		12692.36	12414.45	8922.85	8886.92
Mean weight (kg)		13035.42	12874.92	-	-
Worst weight (kg)		13351.26	13001.02	-	-
Standard deviation (kg)		315.711	269.263	-	-
Natural frequencies (Hz)	f_1	10.113	10.062	9.001	9.000
	f_2	12.124	12.161	11.001	11.000
Reliability index β (Probability of safety %)	$\beta(f_1)$	3.156 (99.92%)	3.291 (99.95%)	0.190 (57.52%)	0.186 (57.36%)
	$\beta(f_2)$	3.090 (99.90%)	3.011 (99.87%)	0.0 (47.74%)	0.0 (47.85%)

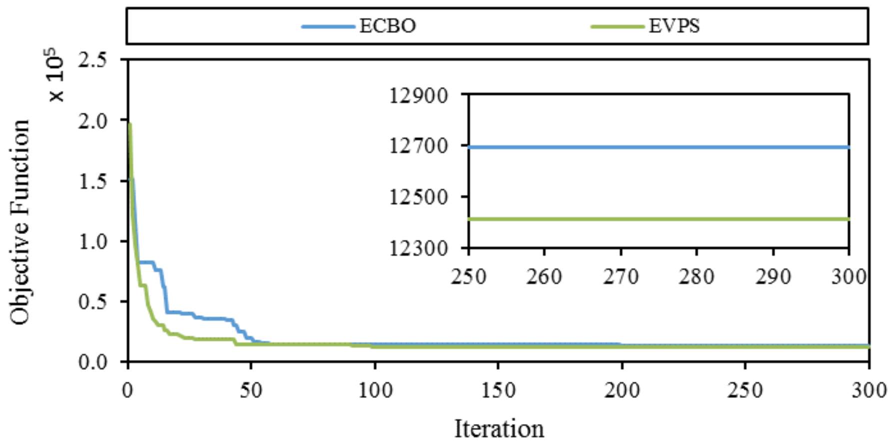


Figure 5. Comparison of the convergence curves for the best run obtained by the algorithms for the 120-bar dome truss

The convergence curves of each algorithm for the best and average solution of the 120-bar dome-like truss are shown in Fig. 5. and Fig. 6, respectively. Figures indicate the comparison of the convergence rate of these algorithms. The results show the appropriate performance of the proposed algorithms.

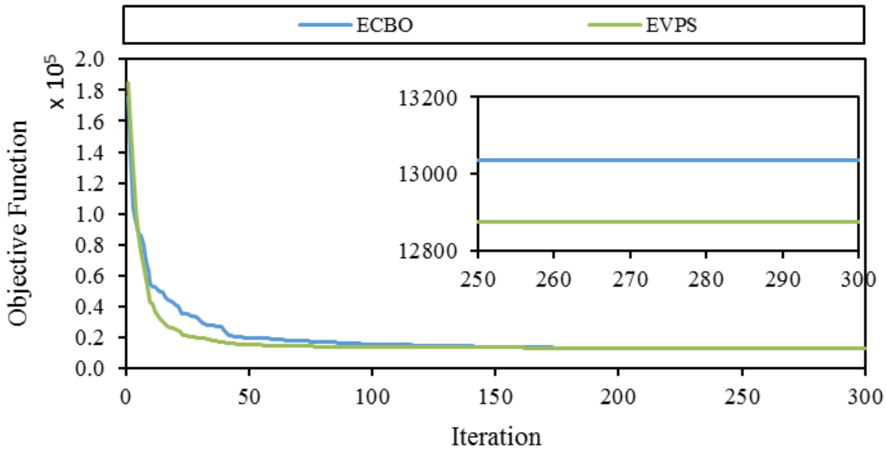


Figure 6. Comparison of the convergence curves for the average of runs obtained by the algorithms for the 120-bar dome truss

5. CONCLUSIONS

There are always many uncertainties in the design and execution of structures, so it is important to consider this effect. Reliability-based design optimization considers the effect of the mentioned effect and leads to the design of the structure with the desired reliability. These uncertainties are effective in various cases such as material, external loads, structural properties and etc. In this study, the RBDO of two dome structures with frequency limitation is considered. The EVPS and ECBO algorithms are used to optimize the problems. The results show that the EVPS algorithm is slightly more successful in finding the best optimal answer in compare with the ECBO algorithm. The Monte-Carlo simulation method is used to evaluate the reliability of each response of metaheuristic algorithm in the optimization process.

Table 3 and Table 5 show the results of the RBDO design and optimal design of some other researchers (without considering the reliability based design). According to the results, the weight of the structure, regardless of reliability, is lower and, of course, the reliability index is lower.

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