

EVALUATING EFFICIENCY OF BIG-BANG BIG-CRUNCH ALGORITHM IN BENCHMARK ENGINEERING OPTIMIZATION PROBLEMS

S. Kazemzadeh Azad^{a,*}, O. Hasançebi^a and O. K. Erol^b

^a*Middle East Technical University, Department of Civil Engineering, Ankara, Turkey*

^b*Istanbul Technical University, Electrical & Electronics Engineering Faculty, Istanbul,
Turkey*

ABSTRACT

Engineering optimization needs easy-to-use and efficient optimization tools that can be employed for practical purposes. In this context, stochastic search techniques have good reputation and wide acceptability as being powerful tools for solving complex engineering optimization problems. However, increased complexity of some metaheuristic algorithms sometimes makes it difficult for engineers to utilize such techniques in their applications. Big-Bang Big-Crunch (BB-BC) algorithm is a simple metaheuristic optimization method emerged from the Big Bang and Big Crunch theories of the universe evolution. The present study is an attempt to evaluate the efficiency of this algorithm in solving engineering optimization problems. The performance of the algorithm is investigated through various benchmark examples that have different features. The obtained results reveal the efficiency and robustness of the BB-BC algorithm in finding promising solutions for engineering optimization problems.

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1. INTRODUCTION

Daily life is full of instances which involve decision making about the best possible solution.

* Corresponding author: S. Kazemzadeh Azad, Middle East Technical University, Department of Civil Engineering, Ankara, Turkey

† E-mail address: saeid.azad@metu.edu.tr

By using the shortest path to reach the destination, shopping with a certain budget, or ordering daily tasks, implicitly one tries to find an optimum solution. Generally, time and cost limitations are the two most common restrictions faced in real life optimization instances. Similar to frequent daily problems, the field of engineering design includes a wide range of optimization problems as well. It can even be stated that engineering design without optimization is not meaningful.

Generally, in engineering design optimization the aim is to find the best set of design variables which leads to a final cost efficient design regarding a predefined set of constraints. Typically, by increasing the number of design variables and constraints involved in an engineering optimization problem, finding the optimal solution becomes a cumbersome task which needs an efficient optimization algorithm. The most recent techniques capable of dealing with different types of engineering optimization problems are the so called metaheuristic search algorithms. In the past decades, genetic algorithms [1], simulated annealing [2], particle swarm optimization [3], ant colony optimization [4, 5], harmony search method [6] etc. have proved to be quite robust and versatile in solving practical optimization instances. The stochastic characteristics of metaheuristics provide efficient search mechanisms for finding the optimal results from the broad solution spaces of complex engineering optimization problems. Further, metaheuristics do not need gradient information of objective functions and can handle both discrete and continuous design variables.

Big Bang–Big Crunch (BB-BC) algorithm [7] is a newly proposed metaheuristic optimization method inspired from the theories of the universe evolution. Due to the simple algorithmic outline of the algorithm and its efficiency in solving optimization problems, it has become one of the popular metaheuristics of the recent years [8-12]. The present study involves a performance evaluation of the BB-BC algorithm in engineering optimization problems. The remaining sections of the paper are arranged as follows. The second section includes a description of the BB-BC algorithm and the related formulation. In the third section, the efficiency of the BB-BC algorithm is investigated using three well known benchmark problems and the numerical results are presented. A brief conclusion of the study is given in the last section.

2. BIG-BANG BIG-CRUNCH ALGORITHM

Big-Bang Big-Crunch optimization method has been first appeared in Erol and Eksin's study [7]. It is emerged from the Big Bang and Big Crunch theories of the universe evolution. As its name implies, the method is based on the continuous application of two successive stages, namely Big Bang and Big Crunch phases. During Big Bang phase, new solution candidates, which are the parameters that affect the fitness function, are randomly generated around a "center of mass", that is later calculated in the Big Crunch phase with respect to their fitness values.

The algorithm is quite simple and is comprised of a few steps:

- i. Form the initial population by spreading randomly solution candidates over all search space (first Big Bang) in a uniform manner. This step has to be applied once.
- ii. Calculate the fitness value of every individual point and assign this value as its mass (if a minimization is to be carried out, form the "mass value" either by inverting the

fitness/cost value or by subtracting it from a constant number chosen bigger than the maximum possible value).

- iii. Calculate the “center of mass” by taking the weighted average using the coordinates and the mass values of every single individual (Big Crunch phase) or choose the fittest individual among all as their center of mass.
- iv. Generate new solution candidates by using Normal Distribution (Big Bang phase).
- v. Keep the fittest individual found so far in a separate place or as a member of the population (elitism) and go to step ii until a stopping criterion is accomplished.

In the present study, in each iteration of the BB-BC algorithm equation (1) is used to generate the new candidates around the center of mass which is taken as the fittest individual of the population.

$$x_i^{new} = x_i^c + \alpha r_i \frac{(x_i^{max} - x_i^{min})}{k} \quad (1)$$

where x_i^c is the value of i-th design variable in the fittest individual, x_i^{min} and x_i^{max} are the lower and upper bounds of the i-th variable, respectively, r_i is a randomly generated number according to a standard normal distribution, k is the iteration number, and α is a constant.

3. NUMERICAL EXAMPLES

This section covers three well known benchmark engineering optimization examples, which are used for performance evaluation of the BB-BC algorithm. Here, for BB-BC algorithm a population of 50 individuals is employed and the value of parameter α in equation (1) is set to 0.5. The optimum solution located using the BB-BC algorithm in each benchmark example is compared to the previously reported results in the literature.

3.1. Example 1: welded beam design

In order to evaluate the performance of the BB-BC algorithm, the optimum design of the welded beam (A), shown in Figure 1, is considered as the first benchmark example. Many researchers considered this benchmark problem so far [13-21]. Here, the objective is to find the best set of design variables to minimize the total fabrication cost of the structure subject to shear stress (τ), bending stress (σ), buckling load (P_c), and end deflection (δ) constraints. Considering $x_1 = h$, $x_2 = l$, $x_3 = t$, and $x_4 = b$ as the design variables, the mathematical formulation of the problem can be stated as follows [21]:

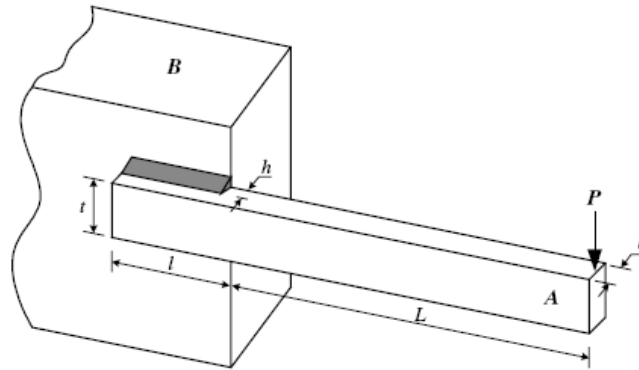


Figure 1. Welded beam structure [13]

$$\text{Find } x = \{x_1, x_2, x_3, x_4\} \quad (2)$$

$$\text{to minimize } \text{Cost}(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2) \quad (3)$$

subject to

$$\begin{aligned} g_1(x) &= t(x) - t_{\max} \leq 0 \\ g_2(x) &= s(x) - s_{\max} \leq 0 \\ g_3(x) &= x_1 - x_4 \leq 0 \\ g_4(x) &= 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5 \leq 0 \\ g_5(x) &= 0.125 - x_1 \leq 0 \\ g_6(x) &= d(x) - d_{\max} \leq 0 \\ g_7(x) &= P - P_c(x) \leq 0 \end{aligned} \quad (4)$$

where

$$\begin{aligned} t(x) &= \sqrt{(t')^2 + 2t't''\frac{x_2}{2R} + (t'')^2} \\ t' &= \frac{P}{\sqrt{2}x_1x_2} \quad t'' = \frac{MR}{J} \quad M = P(L + \frac{x_2}{2}) \quad R = \sqrt{\frac{x_2^2}{4} + (\frac{x_1 + x_3}{2})^2} \\ J &= 2 \left\{ \sqrt{2}x_1x_2 \left[\frac{x_2^2}{12} + (\frac{x_1 + x_3}{2})^2 \right] \right\} \quad s(x) = \frac{6PL}{x_4x_3^2} \quad d(x) = \frac{4PL^3}{Ex_3^3x_4} \\ P_c(x) &= \frac{4.013\sqrt{E(x_3^2x_4^6/36)}}{L^2} \left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \end{aligned} \quad (5)$$

$P = 6000$ lb, $L = 14$ in., $E = 30 \times 10^6$ psi, $G = 12 \times 10^6$ psi, $\tau_{\max} = 13600$ psi, $\sigma_{\max} = 30000$ psi, and $\delta_{\max} = 0.25$ in.

Here, the bounds on the design variables are:

$$0.1 \leq x_1 \leq 2, \quad 0.1 \leq x_2 \leq 10, \quad 0.1 \leq x_3 \leq 10, \quad 0.1 \leq x_4 \leq 2 \tag{6}$$

For this example, the BB-BC algorithm is executed 100 times, and the best design is found as $x^* = \{x_1, x_2, x_3, x_4\} = \{0.205440287437146, 3.47829687981258, 9.03860804181735, 0.205723869899450\}$. A comparison of this design with those of other studies in the literature is carried out in Table 1. As can be seen from this table, the BB-BC algorithm finds a near optimum solution using only 20000 objective function evaluations which is considerably lesser than those of other approaches. On the other hand, a statistical evaluation of 100 independent runs of BB-BC is presented in Table 2 in terms of the best, worst, average, and standard deviation (S. D.) of the designs attained in these runs.

Table 1. The best solutions of welded beam design problem obtained with various methods

Design variables	Gandomi et al.[18]	Fesanghary et al. [19]	Mahdavi et al. [20]	Lee and Geem [13]	This work
x_1	0.2015	0.20572	0.20573	0.2442	0.2054
x_2	3.562	3.47060	3.47049	6.2231	3.4783
x_3	9.0414	9.03682	9.03662	8.2915	9.0386
x_4	0.2057	0.20572	0.20573	0.2443	0.2057
Cost(x)	1.73121	1.7248	1.7248	2.38	1.72576
No. evaluations	50000	90000	200000	110000	20000

Table 2. The statistical performance of BB-BC algorithm in the welded beam design problem

Performance	This work
Best	1.72576
Average	1.773
Worst	2.1376
S. D.	0.0824

3.2. Example 2: design of a pressure vessel

The optimum design of the cylindrical pressure vessel capped at both ends by hemispherical heads (Figure 2) is considered as the second benchmark example [22]. The aim is to minimize the total manufacturing cost of the vessel regarding the combination of welding, material and forming costs. The vessel will be designed for a working pressure of 3000 psi and a minimum volume of 750 ft³ according to the provisions given in ASME boiler and pressure vessel code. The shell and head thicknesses should be multiples of 0.0625 in. The thickness of the shell and head is limited to 2 in. The shell and head thicknesses are not to be less than 1.1 in. and 0.6

in., respectively. The design variables of the problem are x_1 as the shell thickness (T_s), x_2 as the spherical head thickness (T_h), x_3 as the radius of cylindrical shell (R), and x_4 as the shell length (L). The formulation of the problem is as follows:

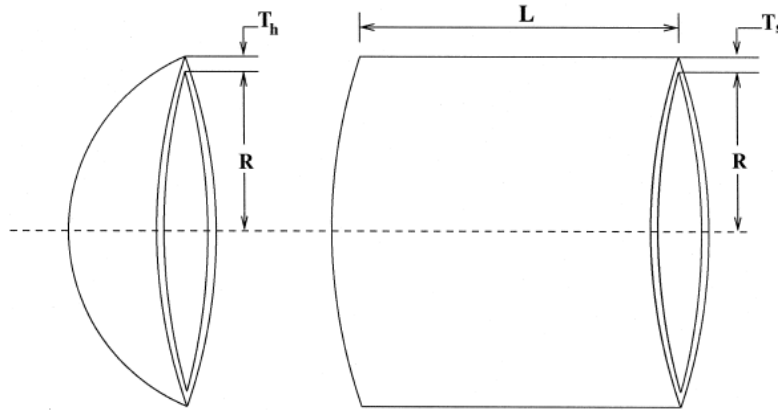


Figure 2. Spherical head and cylindrical shell of the pressure vessel [17]

$$\text{Find } x = \{x_1, x_2, x_3, x_4\} \quad (7)$$

$$\text{to minimize } Cost(x) = 0.6224x_3x_1x_4 + 1.7781x_3^2x_2 + 3.1611x_1^2x_4 + 19.8621x_3x_1^2 \quad (8)$$

subject to

$$\begin{aligned} g_1(x) &= 0.0193x_3 - x_1 \leq 0 \\ g_2(x) &= 0.00954x_3 - x_2 \leq 0 \\ g_3(x) &= 750 \times 1728 - px_3^2x_4 - \frac{4}{3}px_3^3 \leq 0 \\ g_4(x) &= x_4 - 240 \leq 0 \end{aligned} \quad (9)$$

where the bounds on the discrete design variables are:

$$1.125 \leq x_1 \leq 2, \quad 0.625 \leq x_2 \leq 2 \quad (10)$$

In addition, in the present study the bounds on the continuous design variables, x_3 and x_4 , are taken as follows:

$$10 \leq x_3 \leq 240, \quad 10 \leq x_4 \leq 240 \quad (11)$$

The best solution obtained from 100 independent runs of the BB-BC algorithm is given in Table 3, along with optimum solutions reported for this problem with other techniques in the literature. In this example BB-BC algorithm finds a near optimum solution vector of $x^* = \{x_1, x_2, x_3, x_4\} = \{1.125, 0.625, 58.2895313658313, 43.6964127951347\}$ through 25000 objective function evaluations. A statistical evaluation of the designs obtained through 100 independent runs of BB-BC is presented in Table 4.

Table 3. The best solutions of pressure vessel design problem obtained with various methods

Design variables	Mahdavi et al. [20]	Lee and Geem [13]	Sandgren [22]	This work
x ₁	1.125	1.125	1.125	1.125
x ₂	0.625	0.625	0.625	0.625
x ₃	58.29015	58.2789	48.97	58.2895
x ₄	43.69268	43.7549	106.72	43.6964
Cost(x)	7197.730	7198.433	7982.5	7199.412

Table 4. The statistical performance of BB-BC algorithm in the pressure vessel design problem

Performance	This work
Best	7199.412
Average	7347.105
Worst	9770.499
S. D.	420.07

3.3. Example 3: one hundred twenty-bar truss

As the third benchmark example, the weight minimization problem of the 120-bar truss structure shown in Figure 3 is considered. Both sizing and geometry optimization of the structure is carried out in [23]. In the present study, only the sizing optimization of the truss is performed. The structure is subjected to vertical loading at all unsupported nodes. The acting loads are -13.49 kips (-60 kN) at node 1, -6.744 kips (-30 kN) at nodes 2 to 14, and -2.248 kips (-10 kN) in the rest of the nodes. The minimum allowable cross-sectional area of each member is limited to 0.775 in.² (5 cm²). The allowable tensile stress is 0.6F_y and the compressive stress constraint σ_i^b of member i is computed as follows [24]:

$$\sigma_i^b = \begin{cases} \left[\left(1 - \frac{\lambda_i^2}{2C_c} \right) F_y \right] / \left(\frac{5}{3} + \frac{3\lambda_i}{8C_c} - \frac{\lambda_i^3}{8C_c^3} \right) & \text{for } \lambda_i < C_c \\ \frac{12\pi^2 E}{23\lambda_i^2} & \text{for } \lambda_i \geq C_c \end{cases} \quad (12)$$

In equation (12), F_y is the yield stress of steel, E is the modulus of elasticity, λ_i is the slenderness ratio (λ_i = kL_i/r_i) of i-th member, k is the effective length factor, L_i is the length of i-the member, r_i is the radius of gyration, and C = √(2π²E/F_y). Here, the material density is 0.288 lb/in.³ (7971.81 kg/m³), F_y = 58 ksi (400 MPa), and E = 30,450 ksi (210,000 MPa). In this example, two cases of displacement constraints are considered.

Case-1: no displacement constraints is imposed;

Case-2: the displacement of all nodes in directions x, y and z is limited to ± 0.1969 in.

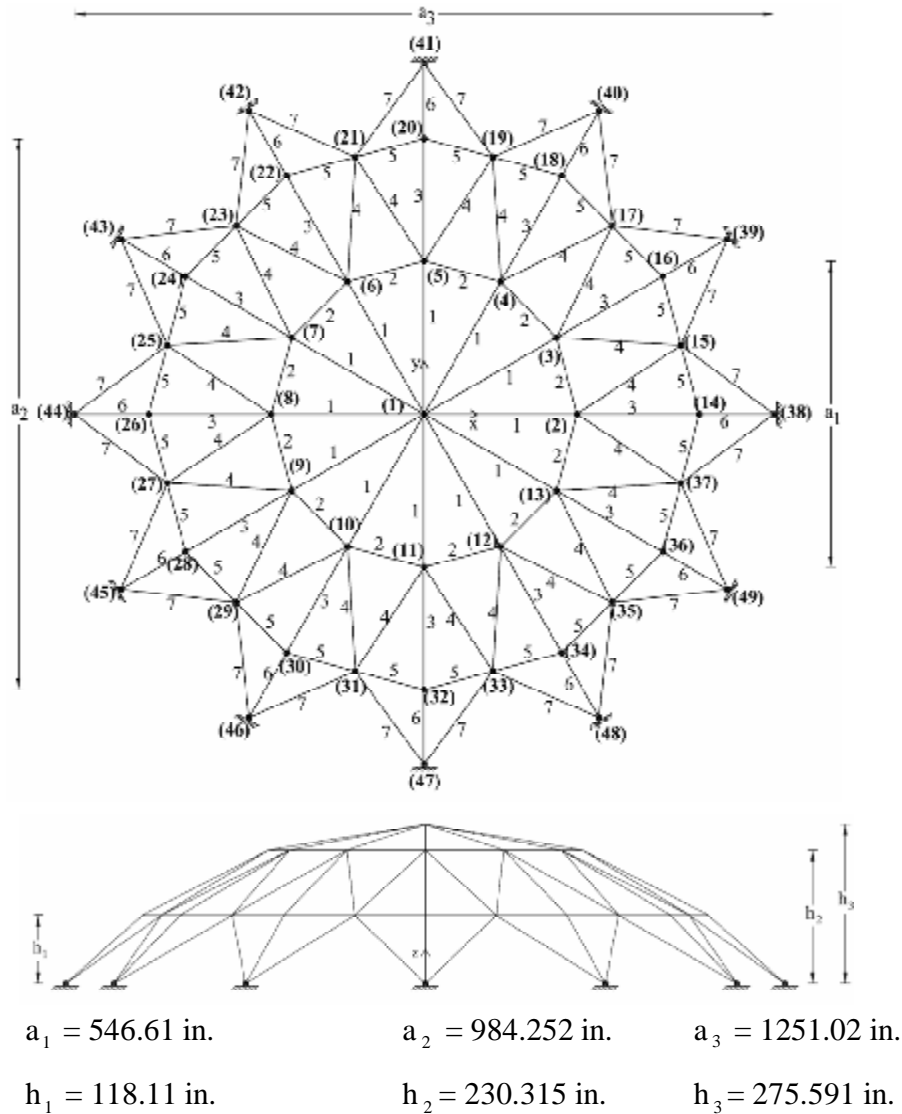


Figure 3. 120-bar truss structure

As shown in Figure 3, the members of the truss are grouped into seven sizing design variables (i.e. A_1 to A_7). This problem is solved by Lee and Geem [6] using a harmony search algorithm based approach. Recently, the same problem is studied with an improved firefly algorithm by Kazemzadeh Azad and Kazemzadeh Azad [25]. Tables 4 and 5 tabulate the optimum solutions of the problem in both cases obtained in this study in comparison to those of the other aforementioned approaches. These solutions are obtained by running the BB-BC algorithm independently a total of five times over 15,000 function evaluations each time. A statistical performance of the BB-BC algorithm in five independent runs is presented in Table 6.

Table 4. The best solutions of 120-bar truss structure (Case-1) obtained with various methods

Design variables (in. ²)	Lee and Geem [6]	Ref. [25]	This work
A ₁	3.295	3.3293	3.3180
A ₂	2.396	2.4384	2.4584
A ₃	3.874	4.0168	3.8925
A ₄	2.571	2.5918	2.5715
A ₅	1.15	1.1823	1.1535
A ₆	3.331	3.4513	3.3382
A ₇	2.784	2.7854	2.7879
Weight (<i>lb</i>)	19707.77	20016.67	19784.015
No. evaluations	35000	15000	15000

Table 5. The best solutions of 120-bar truss structure (Case-2) obtained with various methods

Design variables (in. ²)	Lee and Geem [6]	Ref. [25]	This work
A ₁	3.296	3.3005	3.3028
A ₂	2.789	2.7481	2.7853
A ₃	3.872	3.9036	3.8972
A ₄	2.57	2.5713	2.5787
A ₅	1.149	1.2889	1.1587
A ₆	3.331	3.4089	3.3415
A ₇	2.781	2.8150	2.7883
Weight (<i>lb</i>)	19893.34	20125.35	19953.016
No. evaluations	35000	15000	15000

Table 6. The statistical performance of BB-BC algorithm in the 120-bar truss structure problem

Performance	This work (Case-1)	This work (Case-2)
Best	19784.015	19953.016
Average	19967.247	20198.946
Worst	20515.951	20871.959
S. D.	310.34	385.65

4. CONCLUSION

In this study, the efficiency of the BB-BC algorithm in solving engineering optimization problems is evaluated through typical benchmark problems. Optimum design of a welded beam, design optimization of a pressure vessel as well as weight minimization of a 120-bar truss structure are carried out based on the BB-BC algorithm and the numerical results are compared to the previously reported results in the literature. Beside the best solutions found using the BB-BC algorithm, the worst, average, and standard deviation of results, obtained through independent runs of the algorithm, are also reported to provide a general outline of the performance. The numerical results indicate that complex engineering optimization problems can be effectively tackled using the BB-BC algorithm based approaches.

REFERENCES

1. Goldberg DE, Samtani MP. Engineering optimization via genetic algorithm. *Proceeding of the Ninth Conference on Electronic Computation, ASCE* 1986; pp. 471–482.
2. Kirkpatrick S, Gelatt CD, Vecchi MP. Optimization by simulated annealing. *Science* 1983; **220**: 671–80.
3. Kennedy J, Eberhart R. Particle swarm optimization. In: *IEEE international Conference on Neural Networks*, IEEE Press, 1995, Vol. 4, pp. 1942–1948.
4. Colomi A, Dorigo M, Maniezzo V. Distributed optimization by ant colony. In: *Proceedings of the First European Conference on Artificial Life*, USA, 1991, pp. 134–142.
5. Dorigo M. *Optimization, Learning and Natural Algorithms*, PhD thesis. Dipartimento Elettronica e Informazione, Politecnico di Milano, Italy, 1992.
6. Lee KS, Geem ZW. A new structural optimization method based on the harmony search algorithm. *Comput Struct* 2004; **82**: 781–98.
7. Erol OK, Eksin, I. A new optimization method: Big Bang-Big Crunch, *Adv Eng Software* 2006; **37**(2): 106–11.
8. Camp CV. Design of space trusses using Big Bang-Big Crunch optimization. *J Struct Eng ASCE* 2007; **133**: 999–1008.
9. Kaveh A, Talatahari S. Size optimization of space trusses using Big Bang-Big Crunch algorithm. *Comput Struct* 2009; **87**(17-18): 1129–40.
10. Kaveh A, Talatahari S. Optimal design of Schwedler and ribbed domes via hybrid Big Bang-Big Crunch algorithm. *J Construct Steel Res* 2009; **66** (3): 412–9.
11. Afshar MH, Motaie I. Constrained Big Bang-Big Crunch algorithm for optimal solution of large scale reservoir operation problem. *Int J Optim Civil Eng* 2011; **1**(2): 357–75.
12. Tang H, Zhou J, Xue S, Xie L. Big Bang-Big Crunch optimization for parameter estimation in structural systems, *Mech Syst Signal Proc* 2010; **24**(8): 2888–97.
13. Lee KS, Geem ZW. A new meta-heuristic algorithm for continuous engineering optimization: harmony search theory and practice. *Comput Methods Appl Mech Eng* 2005; **194**(36–38): 3902–33.
14. Siddall JN. *Analytical Design-making in Engineering Design*. Prentice Hall, 1972.

15. Ragsdell KM., Phillips DT. Optimal design of a class of welded structures using geometric programming. *ASME J Eng Indust* 1976; **98**(3):1021–5.
16. Deb K. Optimal design of a welded beam via genetic algorithms. *AIAA J* 1991; **29**(11): 2013–5.
17. Coello Coello CA. Use of a self-adaptive penalty approach for engineering optimization problems. *Comput Indus* 2000; **41**(2):113–27.
18. Gandomi AH, Yang XS, Alavi AH. Mixed variable structural optimization using firefly algorithm, *Comput Struct* 2011; **89**(23–24): 2325–36.
19. Fesanghary M, Mahdavi M, Minary-Jolandan M, Alizadeh Y. Hybridizing harmony search algorithm with sequential quadratic programming for engineering optimization problems. *Comput Methods Appl Mech Eng* 2008; **197**: 3080–91.
20. Mahdavi M, Fesanghary M, Damangir E. An improved harmony search algorithm for solving optimization problems. *Appl Math Comput* 2007; **188**: 1567–79.
21. Rao SS. *Engineering Optimization: Theory and Practice*. John Wiley and Sons, 3rd edition, 1996.
22. Sandgren E. Nonlinear integer and discrete programming in mechanical design optimization, *J Mech Des ASME*, 1990; **112**: 223–9.
23. Soh CK, Yang J. Fuzzy controlled genetic algorithm search for shape optimization. *J Comput Civil Eng* 1996;**10**: 143–50.
24. American Institute of Steel Construction (AISC). *Manual of steel construction-allowable stress design, 9th ed.*, Chicago, 1989.
25. Kazemzadeh Azad S, Kazemzadeh Azad S. Optimum design of structures using an improved firefly algorithm. *Int J Optim Civil Eng* 2011; **1**(2): 327–40.